STATE COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

TELANGANA, HYDERABAD

REMEDIAL SHEET

LEVEL - 1

Class: X   Medium: English   Subject: Mathematics

Unit: Polynomials   Topic/Concept: Variables and Algebraic expressions

Work sheet number: 10.EM.03.01

Learning Outcome: After completing this work sheet you are able to

1. Translate word expressions into algebraic expressions and vice-versa

2. Evaluate algebraic expressions

Conceptual Understanding/Model Problem/Model Example/Activity

1. Translating word expressions:

We know that numeric expressions are combinations of numbers and mathematical operations.
Example: $14 + 5,\ -28 - 3,\ 23 \times 5,\ 45 \div (-5)$
In an algebraic expression we use letters as symbols. A symbol representing only one number is called a **constant** i.e. each number is a constant. A symbol that can represent more than one number is called a **variable**.
In an expression$14 + x$, 14 is a constant and x is a variable.
Algebraic expressions are combinations of numbers, variables, and mathematical operations.

To write an algebraic expression

- ✓ Define the variable or variables
- ✓ Translate the words into numbers and symbols

Example: Translate the word expression “Thrice the square of a number exceeds 5 times the number” into an algebraic expression.

Let the number be x

$3x^2 - 5x$

Example: In a two digit number, ten’s place digit is 5 more than unit’s place digit

Let x = unit’s place digit and ten’s place digit = $x + 5$

$number = 10(x + 5) + x$

2. Translating algebraic expressions:

When we translate an algebraic expression into a word expression, more than one translation will be correct.

Example: Translate the algebraic expression $5x$ into a word expression.

Solution: There are several possibilities

Multiply 5 and a number

The product of 5 and a number

5 times a number

5 multiplied by a number.

Example: Translate the algebraic expression $3x + 7$ into a word expression.

Solution: There are several possibilities

The product of 3 and a number, increased by 7

7 more than 3 times a number

The sum of a number tripled and 7

3. Evaluating algebraic expressions:

To determine the value for the expression, we **substitute** the value of the variable into the expression.

To evaluate the algebraic expression

- ✓ Substitute the given values for the variables
- ✓ Simplify the resulting numerical expression using order of operations

Example: Evaluate $x + 12$ for $x = -2$

Solution: $x + 12 = -2 + 12$ (substitute -2 for x)

$= 10$ (Add)

Example: Evaluate $m^2 - \frac{n}{2}$ for $m = \frac{3}{4}$ and $n = 2$

Solution: $m^2 - \frac{n}{2} = \left(\frac{3}{4}\right)^2 - \frac{2}{2}$ Substitute $m = \frac{3}{4}$ and $n = 2$

$= \frac{9}{16} - \frac{2}{2} = \frac{9}{16} - \frac{16}{16}$ (Simplify)

$= \frac{-7}{16}$

WORK SHEET

1. Can you identify the following as numerical or algebraic expressions? Justify.

$(i) 3 + 7 - 4$ $(ii) 3x$ $(iii) -4 - 7$ $(iv)\ x - \frac{11}{4}$ $(v)\ 5 + (2 - 8)$

$(vi)\ 3z + 1$ $(vii)\ (13 - 6) + 2$ $(viii)\ 2x - y$ $(ix)\ 2xy + 7$ $(x)\ 3x^2$

Solution: ______________________________________

2. Translate the following word expressions into algebraic expressions.

(i) The product of two consecutive numbers

Solution: ______________________________________

(ii) A number which exceeds its reciprocal by 3

Solution: ___________________________________

(iii) Sum of squares of two consecutive even numbers

Solution: ____________________________________

(iv) 6 less than the product of a number and 8

Solution: ____________________________________

(v) Twice the square of a number exceeds 4 times the number

Solution: _____________________________________

(vi) A number less than its square

Solution: ____________________________________

(vii) The square of a number, increased by the product of 3 times the number

Solution: ____________________________________

(viii) Twice a number divided by the sum of the number and 5

Solution: ____________________________________

3. Translate the following algebraic expressions into word expressions.

(i) $2x - 25$

Solution: ____

(ii) $x + 20$

Solution: ____

(iii) $\frac{2}{3}\, m + \frac{1}{3}\, n$

Solution: ____

(iv) $14 - 5z$

Solution: ____

(v) $a^2 + b^2$

Solution: ____

(vi) $x^2 + 3x$

Solution: ____

4. Evaluate the following:

(i) $x^2 - 3x + 2$ at $x = 2, -3, \frac{1}{2}, \sqrt{2}$

Solution: ____

(ii) $x^3 - 3x^2 + 2x - 6$ at $x = 5, -2, \frac{2}{3}, \sqrt{3}$

Solution: ____

(iii) $x + 2$ at $x = -4, -2, 2$

Solution: ____

(iv) $x^2 - x - 2$ at $x = 1, 2, 3$

Solution: ____

(v) $x^2 - x - 6$ at $x = -3, -2, -1, 0, 1, 2, 3$

Solution: ____

(vi) $2x^2 + 3x - 5$ at $x = -3, -2, -1, 0, 1, 2, 3$

Solution: ____

Instructions:

For more practice of concepts, collect some more problems of the same type from your teacher or friends and solve.

What I have learnt?	Can do	can't do
1. I can identify the given expression as numerical or algebraic expressions.	☐	☐
2. I can translate word expressions into algebraic expressions.	☐	☐
3. I can translate algebraic expressions into word expressions.	☐	☐
4. I can evaluate the given algebraic expressions.	☐	☐

STATE COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

TELANGANA, HYDERABAD

REMEDIAL SHEET

LEVEL - 1

Class: X Medium: English Subject: Mathematics

Unit: Polynomials Topic/Concept: Terminology associated with algebraic expressions

Work sheet number: 10.EM.03.02

Learning Outcome: After completing this work sheet you are able to

1. Understand the terminology associated with algebraic expressions

2. Understand types of expressions based on number of terms

3. Understand degree of a term and degree of an expression

Conceptual Understanding/Model Problem/Model Example/Activity

1. Term of an algebraic expression:

- ✓ Only constants like $1,\ -2,\ 0,\ \frac{2}{3}$ ...
- ✓ Only variables like x, y, z ....
- ✓ Product of variables like $xy,\ x^2y,\ xyz,\ y^3$ ....
- ✓ Product of constant and variables like $5xy,\ 6x^2,\ \frac{2}{5}uv$ .....

To determine the number of terms of an algebraic expression, identify the parts of the expression separated by plus or minus sign

Example: How many terms are there in the algebraic expression $6x^2 + 7x - 5$? What are they?

There are three terms. They are $6x^2,\ 7x\ and\ -5$

The numeric **coefficient** is the numeric factor (sign and number) in a term.

Example: What are the coefficients in the algebraic expression $6x^2 + 7x - 5$?

The coefficients are $6,\ 7\ \ and\ \ -5$ respectively.

2. Like terms and Unlike terms:

Terms are called like terms if they contain the same variable with same exponent

Example: $6x^2\ and\ 7\ x^2$ because both terms have the same variable with same exponent.

The terms which are not like terms are called unlike terms.

Example: $3x\ and\ 4\ x^2$ because both have the same variable but exponents are different.

3. Types of Algebraic expressions based on number of terms:

Number of terms	**Name of the expression**	**Example**
One term	Monomial	$x,\quad 7xyz,\ \ 3x^2y$
Two unlike terms	Binomial	$7 + 4x,\ \ x^2 + 2y$
Three Unlike terms	Trinomial	$3x^2 + 4x - 7$
More than one unlike terms	Multinomial	$9p^2 - 11q + 19r + t$

4. Degree of a term:

The sum of all exponents of the variables present in a monomial is called the degree of the term

Example: Consider the term $9x^2y^3$

What are the exponents of x and y in the above term? They are 2 and 3 respectively.

What is the sum of these two exponents? 2+3 = 5

So degree of the term $9x^2y^3$ is 5.

What is the degree of the constant term 7? It is zero because 7 can be written as $7x^0$

5. Degree of an Algebraic expression:

The highest of the degrees of all the terms of an algebraic expression is called its degree.

Example: The expression $pq - 6p^2q^2 - p^2q + 9$ contains 4 terms.

Term	pq	$-6p^2q^2$	$-p^2q$	$+9$
Degree	2	4	3	0

The degree of the given expression is 4 because the highest degree is 4.

WORK SHEET

1. Complete the table

Algebraic Expression	Number of terms	Terms
1) $4x^2 - 3x + 7$		
2) $5x^2 + 3xy - 2y^2 + 11$		
3) $2x - 8 \ - 6x + 12$		

2. Complete the table

Algebraic expressions	Coefficients of terms
1) $12x^2 - 17x + 29$	
2) $-23y^2 + 16xy - 41x^2$	
3) $16x^3 - 8x^2 + 7x - 4$	

3. Complete the table

Algebraic Expressions	Like terms
1) $-15y + 12z - y + 9$	
2) $3x + 5z - 2 + x + 3z - 6$	
3) $2m + 3(n - 5) + 6(n - 5)$	

4. State whether the algebraic expressions given is monomial, binomial, trinomial or multinomial

(i) y^2 (ii) $1 + x + x^2$ (iii) 100 (iv) $p^2 - 3pq - r$

(v) $7x^2 - 2xy + 9y^2 - 11$

Solution: ____

5. Write the degree of each of the monomials

(i) $7y$ (ii) $-11yz^2$ (iii) $3mn$ (iv) xy^2z^2 (v) $4\,a^2b^2$

Solution: ___________________________________

6. Find degree of each algebraic expression

(i) $3x - 16$ (ii) $xy + yz$ (iii) $2y^2z + 9yz - 7z - 11x^2y^2$

(iv) $2y^2z + 10yz$ (v) $pq + p^2q - p^2q^2$ (vi) $ax^2 + bx + c$

Solution: ___________________________________

Instructions:

For more practice of concepts, ask your teacher or friends to give some more problems and solve them.

What I have learnt?	Can do	can't do
1. I understood the terminology associated with Algebraic expressions	☐	☐
2. I can tell types of expressions based on number of terms	☐	☐
3. I can find degree of a term and degree of an expression	☐	☐

STATE COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

TELANGANA, HYDERABAD

REMEDIAL SHEET

LEVEL - 1

Class: X | Medium: English | Subject: Mathematics

Unit: Polynomials | Topic/Concept: Division of polynomials

Work sheet number: 10.EM.03.03

Learning Outcome: After completing this work sheet you are able to

1. Factorize quadratic polynomials

2. Divide polynomials with monomials

3. Understand Remainder and factor theorems

Conceptual Understanding/Model Problem/Model Example/Activity

1. Factoring quadratic polynomial of the form $\boldsymbol{ax^2+bx+c\ (a=1)}$

Multiply $(i)\ (x+3)(x+2)\quad (ii)\ (x-3)(x-2)\quad (iii)\ (x+3)(x-2)\quad (iv)\ (x-3)(x+2)$

Let us understand this with a general example

$$(x+m)(x+n) = x^2 + mx + nx + mn$$

$$= x^2 + (m+n)x + mn$$

Compare with $\quad x^2 + \quad bx \quad + c$

We get $m+n = b\ and\ mn = c$.

In other words look for two numbers whose $product = c$ and whose $sum = b$

Example: factorize $x^2 + x - 6$

Here $a = 1,\ b = 1,\ c = -6$

First determine factors of $c, -6$. Next add pairs of factors to determine number $b, 1$.

Factor	Factor	Sum of factors	
1	-6	-5	
-1	6	5	
2	-3	-1	
$\mathbf{-2}$	$\mathbf{3}$	$\mathbf{1}$	$\leftarrow\ \boldsymbol{b}$

Choose the last pair $-2\ and\ 3$ because their sum is $1(b)$

So $x^2 + x - 6 = (x-2)(x+3)$ where $-2\ \ and\ \ 3$ are the **factors** obtained above.

Determining the signs for factors of c: Answer the following.

(i) $x^2 + 5x + 6$ = ____ _____

If c is positive and b is positive, then the factors of c are both ______________

(ii) $x^2 - 5x + 6$ = ____ _____

If c is positive and b is negative, then the factors of c are both ______________

(iii) $x^2 + x - 6$ = ____ _____

If c is negative and b is positive, then the factors of c have different signs. The factor with the larger absolute value is ______________

(iv) $x^2 - x - 6$ = ____ _____

If c is negative and b is negative, then the factors of c have different signs. The factor with the larger absolute value is ______________

Rule 1: If c is positive, then the factors both have the same sign as b

Rule 2: If c is negative, then the factors have different signs (the factor with the larger absolute value has the same sign as b)

2. Factoring quadratic polynomial of the form $\boldsymbol{ax^2 + bx + c}$ $(\boldsymbol{a \neq 1})$ [ac method]

Multiply $(i)\ (2x+3)(x+2)$ $(ii)\ (2x-3)(x-2)$ $(iii)\ (2x+3)(x-2)$ $(iv)\ (2x-3)(x+2)$

Let us understand this with an example

Factorize $2x^2 + 5x - 3$

Here $a = 2$, $b = 5$, $c = -3$

First determine the product of the coefficients a and c. So the product $ac = 2(-3) = -6$

Next, determine the factors of $ac = -6$ whose sum is b (5)

Factor	Factor	Sum of factors	
1	-6	-5	
-1	6	5	$\leftarrow \quad b$
2	-3	-1	
-2	3	1	

We use the factors -1 *and* 6 because their sum is same as b (5). Now we rewrite the middle term as a sum, using -1 *and* 6 as the new coefficients.

So $\quad 2x^2 + 5x - 3 = 2x^2 \overbrace{- x + 6x}^{+5x} - 3$

$$= (2x^2 - x) + (6x - 3) = x(2x - 1) + 3(2x - 1)$$

$$= (2x - 1)(x + 3)$$

3. Dividing a polynomial with a monomial:

Example 1: $\frac{2a^2+6a^2b-4b^2}{2a}$

$$= \frac{2a^2}{2a} + \frac{6a^2b}{2a} - \frac{4b^2}{2a}$$

$= \text{a + 3ab - } \frac{2b^2}{a}$

Example 2: : $\frac{4x^2y^3z-2xy^2z}{2xyz}$

$$= \frac{4x^2y^3z}{2xyz} - \frac{2xy^2z}{2xyz}$$

$$= 2xy^2 - y$$

4. Remainder Theorem:

If $p(x)$ is divided by $(x - a)$, then the remainder is $p(a)$ [degree of $p(x) \geq 1$]

Example: Find the remainder when $x^2 + 4x - 5$ is divided by $x + 1$

$x + 1 = 0 \Rightarrow x = -1$

Now substitute $x = -1$ in $p(x)$

$p(-1) = (-1)^2 + 4(-1) - 5 = 1 - 4 - 5 = -8$

So by remainder theorem, we can say that when $x^2 + 4x - 5$ is divided by $x + 1$, remainder is -8

5. Factor Theorem:

If $p(x)$ is a polynomial [degree of $p(x) \geq 1$] and $p(a) = 0$ then $(x - a)$ is a factor of $p(x)$

Example: Factorize $x^3 + 3x^2 - x - 3$

$p(x) = x^3 + 3x^2 - x - 3$

$p(1) = (1)^3 + 3(1)^2 - 1 - 3$

$= 1 + 3 - 1 - 3 = 0$

So $(x - 1)$ is a factor of $p(x)$

$x^3 + 3x^2 - x - 3 = (x - 1)(x^2 + 4x + 3)$

$= (x - 1)(x + 1)(x + 3)$

WORK SHEET

1. Factorize the following

(i) $x^2 + 11x + 24$ (ii) $x^2 + 4x - 7$ (iii) $2x^2 + 10x + 8$

(iv) $x^2 + 14x + 45$ (v) $p^2 + 14p + 48$ (vi) $y^2 - 15y + 56$

(vii) $u^2 + 11x - 26$ (viii) $p^2 - 9p - 36$ (ix) $z^2 + 6z + 12$

2. Factorize the following

(i) $6x^2 - 9x - 10$ (ii) $8x^2 - 26x + 15$ (iii) $12x^2 - 14x - 6$

(iv) $2x^2 - 9x - 5$ (v) $15x^2 + 7x + 2$ (vi) $30x^2 + 65x + 30$

(vii) $6x^2 + 19x + 8$ (viii) $3x^4 + 2x^2 - 8$ (ix) $6x^4 - 23x^2 + 20$

3. Solve the following

(i) $\frac{6x^3+12x^2-18x}{3x}$ (ii) $\frac{21a^4-14a^2+42}{7a^2}$ (iii) $\frac{4x^2-2x+12}{8x}$

4. Factorize the following

(i) $x^3 - 2x^2 - x + 2$ (ii) $x^3 - 3x^2 - 9x - 5$

(iii) $x^3 + 13x^2 + 32\,x + 20$ (iv) $y^3 + y^2 - y - 1$

5. Check whether $x + 1$ is a factor of $5x^3 + x^2 - 5x - 1$ using factor theorem

6. Check whether $x + 1$ is a factor of $x^3 + 3x^2 + 3x + 1$ using factor theorem

7. Check whether $x - 2$ is a factor of $x^3 - 4x^2 + x + 6$ using factor theorem

8. Check whether $3x - 2$ is a factor of $3x^3 + x^2 - 20x + 12$ using factor theorem

9. Check whether $2x + 3$ is a factor of $4x^3 + 20x^2 + 33x + 18$ using factor theorem

Instructions:

For more practice of concepts, ask your teacher or friends to give some more problems and solve them.

What I have learnt?	Can do	can't do
1. I understood how to factorize quadratic polynomials	☐	☐
2. I can divide polynomials with monomials	☐	☐
3. I can use Remainder and factor theorems for solving problems	☐	☐

STATE COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

TELANGANA, HYDERABAD

ACADEMIC YEAR 2020-21

LEVEL - 2

Class: X Medium: English Subject: Mathematics

Unit: 3. Polynomials Topic/Concept: Introduction to polynomials

Work sheet number: 10.EM.03.01

Learning Outcome: After completing this work sheet you are able to

1. Identify a polynomial
2. Find the degree of the given polynomial
3. Classify the polynomials based on degree and based on number of terms
4. Find value and zeroes of a polynomial

Conceptual Understanding/Model Problem/Model Example/Activity

1. Polynomial is an algebraic expression containing the sum of a finite number of terms of the form $a\,x^n$, where 'a' is a Real number called coefficient ($a \neq 0$) and $'n'$ is a **non-negative integer** (ie whole numbers $0, 1, 2, 3, 4, \ldots..$) called exponent.

   A Polynomial is of the form

   $a_0 + a_1 x + a_2 x^2 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots + a_{n-2} x^{n-2} + a_{n-1} x^{n-1} + a_n x^n$

   Where $a_0, a_1, a_2, \ldots\ldots. a_{n-2}, a_{n-1}, a_n$ are real numbers known as **coefficients** and $1, 2, 3, \ldots. n-2, n-1, n$ are **non-negative integers** known as exponents.

   Examples of polynomial are $x^2 + 5x + 6$, $x^8 + 8x^2 + 15x + 9$, $x^3 + 2x^2 + x + 4$

You want to know more about what is a polynomial?

Instructions to the students: Refer page 52 of chapter 3 of the text book

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2. Degree of the Polynomial:

Degree of a polynomial is the highest degree of the variable terms.

Example: Degree of the polynomial $3x^2y^2 + 4xy + 7$

It contains three terms. They are $3x^2y^2,\ 4xy\ and\ \ 7$

Terms of the polynomial	$3x^2y^2$	$4xy$	7
Degree of each term	4	2	0

So degree of the given polynomial is 4 because it is the highest degree of the terms.

3. Types of polynomials based on their degree

Degree of a polynomial	Name of the polynomial	Examples
Not defined	Zero polynomial	0
Zero	Constant Polynomial	$-12,\ 5,\ \frac{3}{4}$
1	Linear Polynomial	$ax + b$
2	Quadratic polynomial	$ax^2 + bx + c$
3	Cubic polynomial	$ax^3 + bx^2 + cx + d$
n	nth degree polynomial	$a_0\, x^n + a_1\, x^{n-1} + \cdots \ldots \ldots + a_{n-2}x^2 + a_{n-1}x + a_n$

You want to know more about degree of a polynomial?

Instructions to the students: Refer page 52, 53 of chapter 3 of the text book

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4. Value of a Polynomial:

$p(x)$ is a polynomial in x .If k is a real number, then the value obtained by replacing x by k in $p(x)$, is called the value of $p(x)\ at\ x = k$, and is denoted by $p(k)$.

Now consider the polynomial $p(x) = x^2 - 2x - 3$. what is the value of the polynomial at any value of x?

For example at $x = 1$, Substitute $x = 1$ in the given polynomial.

$\mathrm{p}(1) = (1)^2 - 2(1) - 3 = 1 - 2 - 3 = -4$ So the value of $\mathrm{p}(\mathrm{x})$ at $\mathrm{x} = 1$ is -4

We write the same as $\mathrm{p}(1) = -4$

You want to know more about value of a polynomial?

Instructions to the students: Refer page 53 of chapter 3 of the text book

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5. Zeroes of a polynomial:

A real number k is said to be a zero of a polynomial $p(x), if\ p(k) = 0$.

Example: Find zeroes of the polynomial $p(x) = x^2 - 3x + 2$

Substituting $x = 1$, we get $p(1) = (1)^2 - 3(1) + 2 = 1 - 3 + 2 = 0$

Since $p(1) = 0$ we say that 1 is a zero of $p(x)$

Substituting $x = 2$, we get $p(2) = (2)^2 - 3(2) + 2 = 4 - 6 + 2 = 0$

Since $p(2) = 0$ we say that 2 is a zero of $p(x)$

You want to know more about zeroes of a polynomial?

Instructions to the students: Refer pages 53 and 54 of chapter 3 of the text book

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WORK SHEET

1. Which of the following expressions are polynomials? Justify.

(i) $4x^2 + 6x - 2$ (ii) 7 (iii) $2x^2 + \frac{3}{x} - 5$ (iv) $\sqrt{3}\,x^2 + 5y$

(v) $y^2 - 9$ (vi) $2\sqrt{x - 4}$ (vii) $2x^{-3} + 3x - 7$ (viii) $3x^x + 2$

(ix) $4x^{\frac{1}{2}} - 3$ (x) $\frac{3}{2}\,x - \frac{1}{4}$ (xi) $0.07b^2 - 2.6b + 13.908$

Solution: ____

2. Find the degree of each of the polynomials given

(i) $6y^2 + y^3 - y$ (ii) $8x^4 + 5x^3y^3 - y^4$ (iii) $7x^5 + 2x^2y^2 - 12$

(iv) 5 (v) $5t - \sqrt{3}$ (vi) $3x^6 + 6y^3 + 7$

Solution: ____

3. Classify each expression as linear, quadratic or cubic polynomials.

(i) $5x^2 + x - 7$ (ii) $x - x^3$ (iii) $x - 1$

(iv) $5p$ (v) $\prod r^2$ (vi) $x^2 + x + 4$

Solution: ____

4. Classify each expression as a monomial, a binomial or a trinomial

(i) $3a + 4b - 5c$ (ii) $2z^2$ (iii) $2 - 4b + 7$ (iv) $\frac{1}{7}x + \frac{2}{7}y + \frac{3}{7}z$

(v) $x - y$ (vi) $-14x^3$ (vii) $b^2 - 4ac$ (viii) $b - 2b + 3b - 4b + 5b$

Solution: ____

5. Write the coefficient of x^3 in each of the following

(i) $x^3 + x + 1$ (ii) $2 - x^3 + x^2$ (iii) $\sqrt{2}x^3 + 5$ (iv) $\frac{2}{3}x^3 - 7$ (v) 7

Solution: ____

6. Evaluate $3x^2 - 4x + 1$ for $(i)\ x = 4\ (ii)\ x = -2\ (iii)\ x = 0$

Solution: ____

7. Evaluate $3x^3 + 2x^2 + x + 1$ for $(i)\ x = 2\ (ii)\ x = -1\ (iii)\ x = 0$

Solution: ____

8. Evaluate $a^3 + a^2b + ab^2 + b^3$ for $(i) a = 2, b = 1\ (ii) a = -1, b = -2\ (iii) a = 0, b = 3$

Solution: ____

9. Evaluate $\frac{2}{3}x^2 - x - 3$ for $(i) x = -\frac{3}{2}\quad (ii) x = \frac{1}{4}\quad (iii) x = 3$

Solution: ____

10. Check whether 1, 3 are zeroes of the polynomial $x^2 - 3x + 2$

Solution: ____

11. If 3 is a zero of the polynomial $x^2 + 2x - a$ then find the value of a

Solution: ____

12. If 0 and 1 are the zeroes of the polynomial $2x^3 - 3x^2 + ax + b$ then find the values of $a\ and\ b$

Solution: ____

Instructions:

For more practice of concepts, solve

1. “Do this” from Page no.52 of chapter 3 of the text book
2. “Do this” from Page no.53 of chapter 3 of the text book
3. “Do this” from Page no.54 of chapter 3 of the text book
4. “Exercise-3.1” from Page 54 of chapter 3 of the text book

What I have learnt?	Can do	can’t do
1. I can identify a polynomial	☐	☐
2. I can find the degree of the given polynomial	☐	☐
3. I can classify the polynomials based on degree and based on number of terms	☐	☐
4. I can find value and zeroes of a polynomial	☐	☐

STATE COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

TELANGANA, HYDERABAD

ACADEMIC YEAR 2020-21

LEVEL - 2

Class: X　　Medium: English　　Subject: Mathematics

Unit: 3. Polynomials　　Topic/Concept: Geometrical meaning of zeroes of a polynomial

Work sheet number: 10.EM.03.02

Learning Outcome: After completing this work sheet you are able to

1. Create tables of values for polynomials
2. Draw graphs of polynomials
3. Identify zeroes of the polynomials from the graph

Conceptual Understanding/Model Problem/Model Example/Activity

We know that a real number k is a zero of the polynomial $p(x)$ if $p(k) = 0$, but how will be the graphical representation of a polynomial? What is the geometrical meaning of its zeroes?

Graphs of Linear polynomials:

$p(x) = ax + b(a \neq 0)$ is a linear polynomial. Find $p(k)$ and equate it to zero. $p(k) = ak + b = 0$

So $k = -\frac{b}{a}$ is a zero of linear polynomial $ax + b$. A linear polynomial has only one zero.

Already you learnt how to plot the graph of a linear polynomial. You know that the graph of a linear polynomial is a straight line. But how do we get zero of a linear polynomial graphically?

The zero of polynomial $ax + b$ is the x-coordinate of the point where the graph of $y = ax + b$ intersects X-axis. Do you want to recollect what you have studied about graphs of linear polynomials and their zeroes?

Instructions to the students:

1. Refer pages 55 and 56 of the text book of chapter 3 (polynomials)

Graphs of Quadratic polynomials:

We know that $p(x) = ax^2 + bx + c(a \neq 0)$ is a quadratic polynomial.

How do we calculate points and plot graph for a quadratic polynomial?

What is the shape of the graph obtained?

How many zeroes will be there for a quadratic polynomial?

Do we always get the same number of zeroes for them?

The zeroes of a quadratic polynomial $ax^2 + bx + c\ (a \neq 0)$ are the x-coordinates of the points where the graph of $y = ax^2 + bx + c\ \ (a \neq 0)$ intersects X-axis. Some times graph cuts X-axis at 2 points, some times it touches at 1 point and some times it does not intersect. From that we understand they have 2 zeroes, 1 zero and no zeroes respectively. No zeroes means we have to understand that there is NO Real number k for which $p(k) = 0$.

Do you want to know how to plot graph of a quadratic polynomial and obtain zeroes from the graph?

Instructions to the students:

1. Refer pages 56, 57, 58 and 59 of chapter 3 (polynomials) of the text book to know about Graphs of Quadratic polynomials

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Geometrical meaning of zeroes of cubic polynomials:

We know that $ax^3 + bx^2 + cx + d\ (a \neq 0)$ is general form of a cubic polynomial. A cubic polynomial has at most 3 zeroes.

How do we calculate points and plot graph for a cubic polynomial?

How many zeroes will be there for a cubic polynomial?

How do we obtain zeroes of a cubic polynomial from the graph?

Do you want to study geometrical meaning of zeroes of cubic polynomials in a more detailed manner?

The zeroes of a cubic polynomial $ax^3 + bx^2 + cx + d\ (a \neq 0)$ are the x-coordinates of the points where the graph of $y = ax^3 + bx^2 + cx + d\ (a \neq 0)$ intersects X-axis.

Instructions to the students:

1. Refer pages 59, 60 and 61 of chapter 3 (polynomials) of the text book to know about Graphs of quadratic polynomials

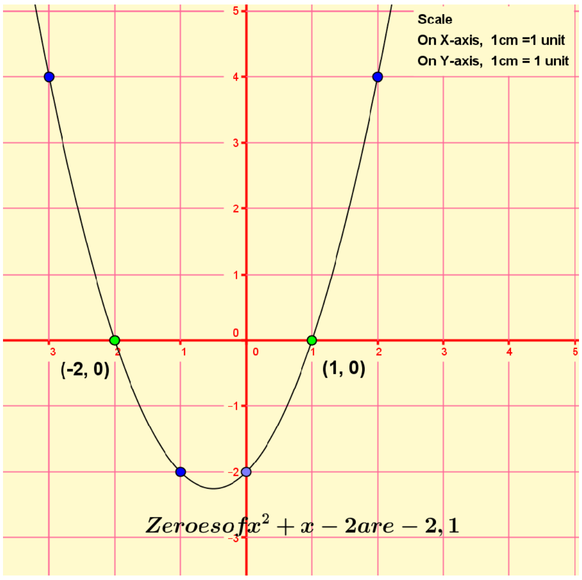
2. Also observe the solved examples in pages 62 and 63 of the text book of chapter 3 to understand the process.

Solved Examples:

1. Draw the graph of $x^2 + x - 2$ and find zeroes from the graph

Solution:

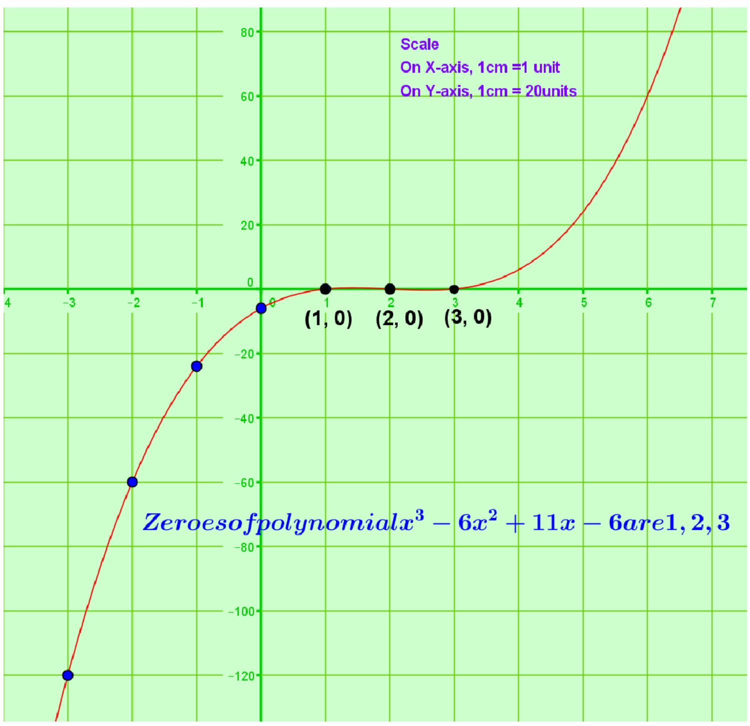
x	$y = x^2 + x - 2$	y	(x, y)
-3	$y = (-3)^2 + (-3) - 2$ $= 9 + (-3) - 2$ $= 4$	4	$(-3, 4)$
-2	$y = (-2)^2 + (-2) - 2$ $= 4 + (-2) - 2$ $= 0$	0	$(-2, 0)$
-1	$y = (-1)^2 + (-1) - 2$ $= 1 + (-1) - 2$ $= -2$	-2	$(-1, -2)$
0	$y = (0)^2 + (0) - 2$ $= 0 + 0 - 2$ $= -2$	-2	$(0, -2)$
1	$y = (1)^2 + (1) - 2$ $= 1 + 1 - 2$ $= 0$	0	$(1, 0)$
2	$y = (2)^2 + (2) - 2$ $= 4 + 2 - 2$ $= 4$	4	$(2, 4)$
3	$y = (3)^2 + (3) - 2$ $= 9 + 3 - 2$ $= 10$	10	$(3, 10)$



2. Draw the graph of $x^3 - 6x^2 + 11x - 6$ and find zeroes from the graph

Solution:

x	$y = x^3 - 6x^2 + 11x - 6$	y	(x, y)
-3	$y = (-3)^3 - 6(-3)^2 + 11(-3) - 6$ $= -27 - 6(9) - 33 - 6$ $= -120$	-120	$(-3, -120)$
-2	$y = (-2)^3 - 6(-2)^2 + 11(-2) - 6$ $= -8 - 6(4) - 22 - 6$ $= -60$	-60	$(-2, -60)$
-1	$y = (-1)^3 - 6(-1)^2 + 11(-1) - 6$ $= -1 - 6(1) - 11 - 6$ $= -24$	-24	$(-1, -24)$
0	$y = (0)^3 - 6(0)^2 + 11(0) - 6$ $= -0 - 6(0) - 0 - 6$ $= -6$	-6	$(0, -6)$
1	$y = (1)^3 - 6(1)^2 + 11(1) - 6$ $= 1 - 6(1) + 11 - 6$ $= 0$	0	$(1, 0)$
2	$y = (2)^3 - 6(2)^2 + 11(2) - 6$ $= 8 - 6(4) + 22 - 6$ $= 0$	0	$(2, 0)$
3	$y = (3)^3 - 6(3)^2 + 11(3) - 6$ $= 27 - 6(9) + 33 - 6$ $= 0$	0	$(3, 0)$



WORK SHEET

1. Prepare tables of values for the following polynomials

(i) $p(x) = 2x + 5$

(ii) $p(x) = x^2 + 2x + 7$

(iii) $p(x) = x^2 - 4x + 1$

(iv) $p(x) = x^3 + 2x^2 - 5x - 6$

2. Draw graphs for the following polynomials and find zeroes from their graphs

(i) $p(x) = 2x - 5$

(ii) $p(x) = -3x + 2$

(iii) $p(x) = -x^2 + 6x - 4$

(iv) $p(x) = x^2 + 5x + 4$

(v) $p(x) = x^2 - 3x + 2$

(vi) $p(x) = x^2 - 3x - 10$

(vii) $p(x) = 2x^2 + 4x + 1$

(viii) $p(x) = x^3$

(ix) $p(x) = x^3 + 2x$

(x) $p(x) = x^3 + 2$

3. Evaluate the polynomial $p(x) = -2x^3 + x^2 - 5x + 8$ at

(i) $p(-2)$ (ii) $p(-1)$ (iii) $p(2)$ (iv) $p(3)$

4. Complete the following table for the polynomial $x^2 - 2x - 8$

x	-3	-2	-1	0	1	2	3
x^2							
$-2x$							
-8							
y							
(x, y)							

Instructions:

For more practice of concepts, solve

1. "Do this" from Page no.56 of chapter 3

2. "Try this" exercises from Pages 57, 59 and 61 of chapter 3

3. Solve Exercise 3.2 from page 64 of chapter 3

What I have learnt?	Can do	can't do
1. I can create table of values for the given polynomial	☐	☐
2. I can draw graph of the given polynomial	☐	☐
3. I can identify zeroes of the given polynomial from the graph	☐	☐

STATE COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

TELANGANA, HYDERABAD

ACADEMIC YEAR 2020-21

LEVEL - 2

Class: X Medium: English Subject: Mathematics

Unit: 3. Polynomials Topic/Concept: Relationship between zeroes and coefficients of a polynomial

Work sheet number: 10.EM.03.03

Learning Outcome: After completing this work sheet you are able to

1. Understand the relation between zeroes and coefficients of a quadratic polynomial

2. Understand the relation between zeroes and coefficients of a cubic polynomial

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Conceptual Understanding/Model Problem/Model Example/Activity

We know that zero of a linear polynomial $ax + b(a \neq 0)$ is $\alpha = -\frac{b}{a}$. We know that $a\ and\ b$ are coefficients of a linear polynomial.

$$\alpha = -\frac{constant}{coefficient\ of\ x}$$

Can we give such relation for a quadratic polynomial?

Observe the following quadratic polynomials for which $\alpha\ and\ \beta$ are zeroes.

S.No.	Quadratic Polynomial	factors	Sum of zeroes	Product of zeroes
1	$x^2 - 3x - 4$	$(x-4)(x+1)$	3	-4
2	$x^2 + 5x + 6$	$(x+2)(x+3)$	-5	6
3	$x^2 - 4x + 3$	$(x-1)(x-3)$	4	3
4	$2x^2 - 3x + 1$	$(x-1)(2x-1)$	$\frac{3}{2}$	$\frac{1}{2}$

For polynomial $ax^2 + bx + c(a \neq 0)$, zeroes are $\alpha\ and\ \beta$

Sum of zeroes = $\alpha + \beta = -\frac{b}{a} = -\frac{x\ coefficient}{x^2 coefficient}$

Product of zeroes = $\alpha\beta = \frac{c}{a} = \frac{constant}{x^2 coefficient}$

Example: Find zeroes of the quadratic polynomial $x^2 + 2x - 8$ and check the relation between zeroes and coefficients.

Solution: First factorize the given quadratic polynomial $x^2 + 2x - 8$

$$= x^2 + 4x - 2x - 8 = x(x + 4) - 2(x + 4)$$

$$= (x + 4)(x - 2)$$

So zeroes of this polynomial are $-4\ and\ 2$

Sum of zeroes = $-4 + 2 = -2$

Product of zeroes = $(-4)\ 2 = -8$

Again for the polynomial $x^2 + 2x - 8$, $a = 1,\ b = +2,\ c = -8$

Sum of zeroes = $\alpha + \beta = -\frac{b}{a} = -\frac{2}{1} = -2$

Product of zeroes = $\alpha\beta = \frac{c}{a} = \frac{-8}{1} = -8$

From the above we can say that the relation between zeroes and coefficients is verified.

Writing a Quadratic polynomial when zeroes are given:

So when two zeroes $\alpha\ and\ \beta$ are given, quadratic polynomial is

$$x^2 - x(sum\ of\ zeroes) + product\ of\ zeroes = x^2 - (\alpha + \beta)x + \alpha\beta$$

Do you want to know more about them?

Instructions to the students:

Refer pages 65, 66, 67 and 68 of the text book of chapter 3 to know more about the relation between zeroes and coefficients of a quadratic polynomial. Also go through the examples given in those pages.

What about the relation between zeroes and coefficients of a cubic polynomial?

$ax^3 + bx^2 + cx + d\ (a \neq 0)$ is a cubic polynomial whose zeroes are $\alpha, \beta\ and\ \gamma$

Sum of zeroes = $\alpha + \beta + \gamma = -\frac{b}{a} = -\frac{x^2 coefficient}{x^3\ coefficient}$

Sum of product of zeroes (pair-wise) $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{x\ coefficient}{x^3 coefficient}$

Product of zeroes $\alpha\beta\gamma = -\frac{d}{a} = -\frac{constant}{x^3 coefficient}$

Let us consider one example.

Example: Find zeroes of the cubic polynomial $x^3 - x$ and check the relation between zeroes and coefficients.

Solution: First factorize the given cubic polynomial $x^3 - x$

$$= x(x^2 - 1) = x\,(x - 1)(x + 1)$$

So zeroes of the given cubic polynomial are $0, 1, -1$

$$\alpha + \beta + \gamma = 0 + 1 + (-1) = 0$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = 0(1) + 1(-1) + (-1)0 = 0 + (-1) + 0 = -1$$

$$\alpha\beta\gamma = 0(1)(-1) = 0$$

Now find the coefficients of the given cubic polynomial $x^3 - x,\ a = 1,\ b = 0,\ c = -1, d = 0$

Sum of zeroes = $\alpha + \beta + \gamma = -\frac{b}{a} = -\frac{0}{1} = 0$

Sum of product of zeroes (pair-wise) $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{-1}{1} = -1$

Product of zeroes $\alpha\beta\gamma = -\frac{d}{a} = -\frac{0}{1} = 0$

From the above we can say that the relation between zeroes and coefficients is verified.

Do you want to know more about them?

Instructions to the students:

Refer page 69 to know about the relation between zeroes and coefficients of a cubic polynomial. Also go through the example given in page 70.

WORK SHEET

1. Find zeroes of the polynomial $6x^3 + 5x^2 + 7x - 20$ and verify the relation between zeroes and coefficients.

Solution: ____

2. Find zeroes of the polynomial $x^3 + 4x^2 - 25x - 100$ and verify the relation between zeroes and

coefficients.

Solution: ____

3. Find zeroes of the polynomial $2x^2 - 8x + 6$ and verify the relation between zeroes and coefficients.

Solution: ____

4. Find zeroes of the polynomial $5x^2 + 6x + 1$ and verify the relation between zeroes and coefficients.

Solution: ____

5. Find zeroes of the polynomial $x^2 + 5x - 3$ and verify the relation between zeroes and coefficients.

Solution: ____

6. Find zeroes of the polynomial $5x^3 - 4x^2 + 7x - 8$ and verify the relation between zeroes and coefficients.

Solution: ____

7. Write quadratic polynomials with given zeroes

(i) $2 \; and - 3$ (ii) $4 \; and - 1$ (iii) $3 \; and \; 2$ (iv) $-2 \; and - 3$

Solution: ____

8. Check whether 2, 4 are zeroes of the quadratic polynomial $x^2 - 6x + 12$

Solution: ____

9. Check whether 1, 2, 3 are zeroes of the cubic polynomial $x^3 - 3x^2 + 12x$

Solution: ____

10. Check whether 1, 2, $\frac{1}{2}$ are zeroes of the cubic polynomial$x^3 + 3x^2 - x - 3$

Solution: ____

11. Check whether $\frac{1}{2}$, $\frac{3}{2}$ are zeroes of the quadratic polynomial $4x^2 - 8x + 3$

Solution: ____

12. If $1 \; and \; -1$ are two zeroes of the polynomial $x^3 + 2x^2 + ax + b$, then find the values of $a \; and \; b$

Solution: ____

13. If 1 and 2 are two zeroes of the polynomial $x^3 - 6x^2 + ax + b$, then find the values of $a \ and \ b$

Solution: ____

14. If 2 is a zero of the polynomial $x^3 - 3x^2 + 4x + k$, then find the value of k

Solution: ____

15. If 3 is a zero of the polynomial $x^4 - 2x^3 + 3x^2 - mx + 5$, then find the value of m

Solution: ____

Instructions:

For more practice of concepts, solve

1. "Do this" from Pages 66 and 70 of chapter 3

2. "Try this" from Page no. 68 of chapter 3

3. Exercise 3.3 from page 71 of chapter 3

What I have learnt?	Can do	can't do
1. I understood the relation between zeroes and coefficients of a quadratic polynomial	☐	☐
2. I understood the relation between zeroes and coefficients of a cubic polynomial	☐	☐

STATE COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

TELANGANA, HYDERABAD

ACADEMIC YEAR 2020-21

LEVEL - 2

Class: X Medium: English Subject: Mathematics

Unit: 3. Polynomials Topic/Concept: Division Algorithm for polynomials

Work sheet number: 10.EM.03.04

Learning Outcome: After completing this work sheet you are able to

1. Divide a polynomial with another polynomial

2. Solve problems using division algorithm of polynomials.

Conceptual Understanding/Model Problem/Model Example/Activity

1. **Long Division of Polynomials:**

Divide $4x^3 + 3x + 5$ by $2x - 3$

We write Dividend/divisor in the descending order of their powers. If any terms are missing, we write them with coefficient 0

So it becomes $4x^3 + 0\,x^2 + 3x + 5$ by $2x - 3$

We write the same as $2x - 3)\ 4x^3 + 0x^2 + 3x + 5$

Now divide first term of dividend with first term of divisor $\frac{4x^3}{2x} = 2x^2$ which will become the first term of the quotient.

$$2x - 3)\ 4x^3 + 0x^2 + 3x + 5(2x^2$$

Then multiply divisor with first term of the quotient obtained.

$2x - 3)\ 4x^3 + 0x^2 + 3x + 5(2x^2 + 3x + 6$

$4x^3 - 6x^2$

$(-) \quad (+)$ Subtract (change signs and add) and

bring down the next term.

$6x^2 + 3x$ Repeat the above procedure.

$6x^2 - 9x$

$(-) \quad (+)$

$12x + 5$

$12x - 18$

$(-) \quad (+)$

$+23$

2. Division Algorithm for polynomials:

We know division algorithm for real numbers.

$Dividend = (divisor \times quotient) + remainder$. Now we apply it for polynomials.

If $p(x)$ *and* $g(x)$ are any two polynomials with $g(x) \neq 0$, then we can find polynomials

$q(x)$ *and* $r(x)$ such that $p(x) = [g(x) \times q(x)] + r(x)$

where either $r(x) = 0$ *or degree of* $r(x) <$ *degree of* $g(x)$ *if* $r(x) \neq 0$

From the above we can obtain

(i) If $g(x)$ is a linear polynomial then $r(x) = r$ is a constant.

(ii) If *degree of* $g(x) = 1$, then *degree of* $p(x) = 1 +$ *degree of* $q(x)$.

(iii) If $p(x)$ is divided by $(x - a)$, then the remainder is $p(a)$ (Remainder theorem)

(iv) If $r = 0$, we say $q(x)$ *divides* $p(x)$ *exactly or* $q(x)$ *is a factor of* $p(x)$ (Factor Theorem)

Now let us verify division algorithm for the above example.

Dividend = $4x^3 + 0\,x^2 + 3x + 5$ Divisor = $2x - 3$

Quotient = $2x^2 + 3x + 6$ Remainder = + 23

$$Dividend = (divisor \times quotient) + remainder$$

$$= (2x - 3) \times (2x^2 + 3x + 6) + 23$$

$$= 4x^3 + 6x^2 + 12x - 6x^2 - 9x - 18 + 23$$

$$= 4x^3 + 3x + 5$$

In this way division algorithm is verified.

Do you want to know about it in a more detailed manner?

Instructions to the students:

Refer pages 71, 71, 73 and 74 of the text book of chapter 3 polynomials to know about division algorithm of polynomials and also for some solved examples to understand the process.

WORK SHEET

1. Find quotient and remainder when $2x^2 + 11x - 21$ is divided by $x + 7$. Also verify division algorithm.

Solution: ____

2. Find quotient and remainder when $6x^2 - x + 9$ is divided by $3x + 1$. Also verify division algorithm.

Solution: ____

3. Find quotient and remainder when $3b^2 + 22b - 16$ is divided by$b + 8$. Also verify division algorithm.

Solution: ____

4. Find quotient and remainder when$x^2 + 10x + 25$ is divided by $x + 3$. Also verify division algorithm.

Solution: ____

5. Find quotient and remainder when $x^3 - 1$ is divided by -1 . Also verify division algorithm.

Solution: ____

6. Find quotient and remainder when $x^4 + 4x^3 + 3x^2 - 4x - 4$ is divided by $x - 1$. Also verify Division algorithm.

Solution: ____

7. Find quotient and remainder when $x^4 - 4x^3 + 4x^2 - 2$ is divided by $x - 3$. Also verify Division algorithm.

Solution: ____

8. Find quotient and remainder when $4x^3 - 3x + 9$ is divided by $2x - 3$. Also verify Division algorithm.

Solution: ____

Instructions:

For more practice of concepts, solve

1. Exercise 3.4 from pages 74, 75 of chapter 3 (polynomials)

What I have learnt?	Can do	can't do
1. I understood how to divide a polynomial by another polynomial	☐	☐
2. I can solve problems using division algorithm of polynomials	☐	☐