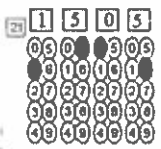
# JEE-MAIN

# MODEL GRAND TEST

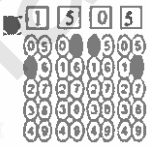
***IMPORTANT INSTRUCTIONS:***

1. Immediately fill in the Admission number on this page of the Test Booklet with **Blue/Black** Ball Point **Pen** only.
2. The candidates should not write their Admission Number anywhere (except in the specified space) on the Test Booklet/ Answer Sheet.
3. The test is of **3 hours** duration.
4. The Test Booklet consists of 90 questions. The maximum marks are **300.**
5. There are **three** parts in the question paper 1,2,3 consisting of **Physics, Chemistry** and **Mathematics** having **30 questions** in each subject and subject having **two sections.**
   **(I) Section –**I contains 20 **multiple choice** questions with only one correct option.
   **Marking scheme:** +4 for correct answer, 0 if not attempt and -1 in all other cases.
   **(II) Section-II** contains 10 **Numerical Value Type** questions. Attempt any 5 questions only, if more than 5 questions attempted, First 5 attempted questions will be considered.

- The Answer should be within **0 to 9999.** If the Answer is in **Decimal** then round off to the **nearest Integer** value (Example i,e. If answer is above **10** and less than **10.5** round off is **10** and If answer is from **10.5** and less than **11** round off is **11).**
  To cancel any attempted question bubble on the question number box.
  For example: To cancel attempted question 21. Bubble on 21 as shown below



**Question Answered for Marking**



**Question Cancelled for Marking**

**Marking scheme:** +4 for correct answer, 0 if **not attempt** and **-1** in all other cases.

6. Use **Blue / Black Point Pen only** for writing particulars / marking responses on the Answer Sheet. **Use of pencil is strictly prohibited.**
7. No candidate is allowed to carry any textual material, printed or written, bits of papers, mobile phone any electron device etc, except the Identity Card inside the examination hall.
8. Rough work is to be done on the space provided for this purpose in the Test Booklet only.
9. On completion of the test, the candidate must hand over the Answer Sheet to the invigilator on duty in the Hall. **However, the candidate are allowed to take away this Test Booklet with them.**
10. **Do not fold of make any stray marks on the Answer Sheet**

# PHYSICS

## Section –A

### (SINGLE CORRECT ANSWER TYPE)

1. Identify the correct statements from the following

A. Work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket is negative

B. Work done by gravitational force in lifting a bucket out of a well by a rope tied to the bucket is negative

C. Work done by friction on a body sliding down an inclined plane is positive

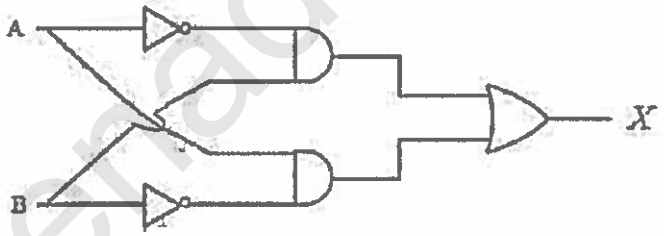
D. Work done by an applied force on a body moving on a rough horizontal plane with uniform velocity in zero

E. Work done by the air resistance on an oscillating pendulum in negative

Choose the correct answer from the options given below

1) B and D only
2) B, D and E only
3) B and E only
4) A and C only

19. For the given logic gates combination, the correct truth table will be




1)

| A | B | X |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

2)

| A | B | X |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

3)

| A | B | X |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

4)

| A | B | X |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

3. The modulation index for an A.M wave having maximum and minimum peak – to peak Voltages of 14 mV and 6mV respectively is

1) 0.2
2) 0.4
3) 0.6
4) 1.4

4. An object moves at a constant speed along a circular path in a horizontal plane with center at the origin . When the object is at $x=+2$ m, its velocity is $-4\hat{j}\ m/s$ . The object's velocity (v) and acceleration (a) at $x=-2$ m will be

1) $v=4\hat{j}\ m/s, a=8\hat{i}\ m/s^2$ 2) $v=-4\hat{j}\ m/s, a=8\hat{i}\ m/s^2$

3) $v=-4\hat{i}\ m/s, a=-8\hat{j}\ m/s^2$ 4) $v=4\hat{i}\ m/s, a=8\hat{j}\ m/s^2$

5. At $300K$, the rms speed of oxygen molecules is $\sqrt{\frac{\alpha+5}{\alpha}}$ times to that of its average speed in the gas. Then the value of $\alpha$ will be (used $\pi=\frac{22}{7}$)

1) 32 2) 27 3) 24 4) 28

6. The electric current in a circular coil of four turns produces a magnetic induction 32 T at its center. The coil is unwound and is rewound into a circular coil of single turn, the magnetic induction at the center of the coil by the same current will be:

1) 4 T 2) 16 T 3) 8T 4) 2T

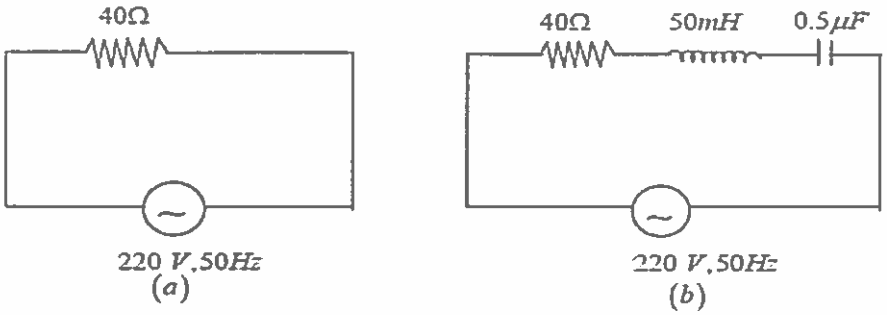
7. Substance A has atomic mass number 16 and half-life of 1 day. Another substance B has atomic mass number 32 and half-life of $\frac{1}{2}$ day. If both A and B simultaneously start undergo radio activity at the same time with initial mass 320g each, how many total atoms of A and B combined would be left after 2 days.

1) $3.38\times10^{24}$ 2) $1.69\times10^{24}$ 3) $6.76\times10^{24}$ 4) $6.76\times10^{23}$

8. The time taken by an object to slide down $45^0$ rough inclined plane is n times as it takes to slide down a perfectly smooth $45^0$ incline plane. The coefficient of kinetic friction between the object and the incline plane is.

1) $\sqrt{\frac{1}{1-n^2}}$ 2) $1+\frac{1}{n^2}$ 3) $\sqrt{1-\frac{1}{n^2}}$ 4) $1-\frac{1}{n^2}$

9. For the given figures, choose the correct options




1) The rms current in circuit (b) can never be larger than that in (a)

2) The rms current in circuit (b) can be larger than that in (a)

3) The rms current in figure (a) is always equal to that in figure (b)

4) At resonance, current in (b) is less than that in (a)

10. A scientist is observing a bacteria through a compound microscope. For better analysis and to improve its resolving power he should. (Select the best option)

1) Decrease the focal length of the eye piece.

2) Increase the refractive index of the medium between the object and objective lens.

3) Decrease the diameter of the objective lens.

4) Increase the wave length of the light.

11. The equation of a circle is given by $x^2 + y^2 = a^2$, where $a$ is the radius. If the equation is modified to change the origin other than $(0,0)$, then find out the correct dimensions of A and B in a new equation: $(x - At)^2 + \left(y - \frac{t}{B}\right)^2 = a^2$. The dimensions of t is given as [$T^{-1}$]

1) $A = [LT], B = \left[L^{-1}T^{-1}\right]$

2) $A = \left[L^{-1}T^{-1}\right], B = [LT]$

3) $A = \left[L^{-1}T^{-1}\right], B = \left[LT^{-1}\right]$

4) $A = \left[L^{-1}T\right], B = \left[LT^{-1}\right]$

12. Given below are two statements:

Statement I: Electromagnetic waves are not deflected by electric and magnetic field.

Statement II: The amplitude of electric filed and the magnetic field in electromagnetic waves are related toeach other as $E_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} B_0$.

In the light of the above statements, choose the correct answer from the options given below

1) Statement I is false but statement II is true

2) Statement I is true but statement II is false

3) Both Statement I and statement II are false

4) Both Statement I and statement II are true

13. A fully loaded boeing aircraft has a mass of $5.4\times10^5 kg$. Its total wing area is 500 $m^2$. It is in level flight with a speed of 1080 km/h. If the density of air $\rho$ is 1.2 $kg\ m^{-3}$, the fractional increases in the speed of the air on the upper surface of the wing relative to the lower surface in percentage will be $\left(g = 10\ m/s^2\right)$

1) 6 2) 8 3) 10 4) 16

14. A square loop of area 25 $cm^2$ has a resistance of 10 $\Omega$. The loop is placed in uniform magnetic field of magnitude 40.0 T. The plane of loop is perpendicular to the magnetic field. The work done in pulling the loop out of the magnetic field slowly and uniformly in 1.0 sec, will be

1) $2.5\times10^{-3} J$ 2) $5\times10^{-3} J$ 3) $1.0\times10^{-3} J$ 4) $1.0\times10^{-4} J$

15. The ratio of de- Broglie wavelength of an $\alpha$ particle and a proton accelerated from rest by the same potential is $\frac{1}{\sqrt{m}}$, the value of m is

1) 2 2) 16 3) 8 4) 4

16. The time period of satellite of earth is 24 hours. If the separation between the earth and the satellite is decreased to one fourth of the previous value, then its new time period will become

1) 6 hours 2) 12 hours 3) 4 hours 4) 3 hours

17. With the help of potentiometer, we can determine the value of emf of a given cell. The Sensitively of the potentiometer is

A. Directly proportional to the length of the potentiometer wire

B. Directly proportional to the potential gradient of the wire

C. inversely proportional to the potential gradient of the wire

D. inversely proportional to the length of the potentiometer wire

Choose the correct option for the above statements

1) A and C only 2) B and D only 3) C only 4) A only

18. Heat energy of 184 kJ is given to ice of mass 600 g at $-12^0C$. Specific heat of ice is $2222.3J\ kg^{-1^0}C^{-1}$ and latent heat of ice in $336kJ/kg^{-1}$

A. Final temperature of system will be O°C

B. Final temperature of system will be greater than $0^0C$

C. The final system will have a mixture of ice and water in theratio of 5:1

D. The final system will have a mixture of ice and water in theratio of 1:5

E. The final system will have water only

Choose the correct answer from the options given below:

1) A and C only 2) A and E only 3) B and D only 4) A and D only

19. A point charge $2\times10^{-2}$ is moved from P to S in a uniform electric field of $30NC^{-1}$ along Positive x-axis. If coordinates of P and S are (1,2,0) m and (0,0,0) m respectively, the work done by electric field will be

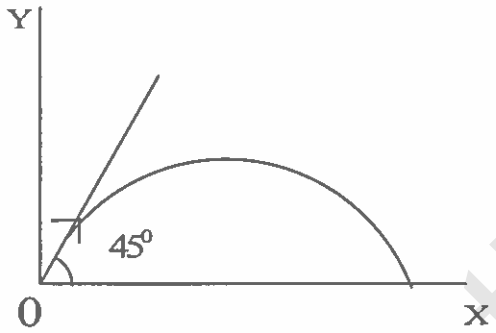
1) – 1200 Mj 2) 600 mJ 3) -600 mJ 4) 1200 mJ

20. A force acts for 20 s on a body of mass 20 kg starting from rest, after which the force ceases and then body describes 50 m inthe next 10 s. The value of force will be :

1) 5 N 2) 40 N 3) 20 N 4) 10 N

## Section-B
## (NUMERICAL VALUE TYPE)

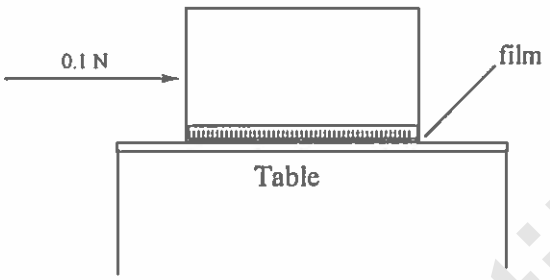
21. A particle of mass 100g is projected at time $t = 0$ with a speed 20 $ms^{-1}$ at an angle $45^0$ to the horizontal as given in the figure . The magnitude of the angular momentum of the particle about the staringpoint at time t=2 s is found to be $\sqrt{K}$ $kg\ m^2 / s$ .The value of K is _____$\left(Take\ g = 10ms^{-2}\right)$




22. For a charged spherical ball, electrostatic potential inside the ball varies with r as $V = 2ar^2 + b$ Here, a and b are constant and r is the distance from the center. The volume charge density inside the ball is $-\lambda a\varepsilon$ .The value of $\lambda$ is _____($\varepsilon$ = permittivity of the medium)

23. In an experiment of measuring the refractive index of a glass slab using travelling microscope in physics lab , a student measures real thickness of the glass slab as 5.25 mm and apparent thickness of the glass slab as 5.00 mm . Travelling microscope has 20 divisions in one cm on main scale and 50 divisions on Vernier scale is equal to 49 divisions on main scale .The estimated uncertainty in the measurement of refractive index of the slab is $\frac{x}{10} \times 10^{-3}$ , where x is _____

24. A metal block of base area $0.20\,m^2$ is placed on a table , as shown in figure. A liquid film of thickness 0.25 mm is inserted between the block and the table. The block is pushed by a horizontal force of 0.1 N and moves with a constant speed .If the viscosity of the liquid is $5.0\times10^{-3}$ Pl, the speed of block is_____$\times10^{-3}\ m/s$




25. a particle of mass 250 g executes a simple harmonic motion under a periodic force $F=(-25x)$N. The particle attains a maximum speed of 4 m/s during its oscillation . The amplitude of the motion is _____cm.

26. when two resistances $R_1$ and $R_2$ connected in series and introduced into the left gap of a meter bridge and a resistance of 10 Ω is introduced into the right gap, a null point is found at 60 cm from left side. When $R_1$ and $R_2$ are connected in parallel and introduced into the left gap, a resistance of 3Ω is introduced into the right -gap to get null point at 40 cm from left end .The product of $R_1R_2$ is____$\Omega^2$

27. A car is moving on a circular path of radius 600 m such that the magnitudes of the tangential acceleration and centripetal acceleration are equal .The time taken by the car to complete first quarter of revolution ,if it is moving with an initial speed of 54 km/hr is $t\left(1-e^{-\pi/2}\right)s$. .The value of t is ____

28. An inductor of inductance $2\,\mu$H is connected in series with a resistance , a variable capacitor and an AC source of frequency 7 kHz. The value of capacitance for which maximum current is drawn into the circuit is $\frac{1}{x}F$ , where the value of $x$ is _____ (Take $\pi=\frac{22}{7}$)

29. A null point is found at 200 cm in potentiometer when cell in secondary circuit is shunted by $5\Omega$. When a resistance of $15\Omega$ is used for shunting, null point moves to 300cm. The internal resistance of the cell is _____$\Omega$.

30. Unpolarised light is incident on the boundary between two dielectric media, whose dielectric constants are 2.8 (medium-1) and 6.8 (medium -2), respectively. To satisfy the condition, so that the reflected and refracted rays are perpendicular to each other, the angle of incidence should be $\tan^{-1}\left(1+\frac{10}{\theta}\right)^{\frac{1}{2}}$ the value of $\theta$ is _____ (Given for dielectric media $\mu_r = 1$)

## CHEMISTRY

### Section –A

### (SINGLE CORRECT ANSWER TYPE)

31. Match List I with List II

| List I | List II |
|---|---|
| A. Van't Hoff factor,i | I. Cryoscopic constant |
| B. Kf | II. Isotonic solutions |
| C. solutions with same Osmotic pressure | III. Normal molar mass<br>Abnormal molar mass |
| D. Azeotropes | IV. Solution with same composition of vapour above it |

Choose the correct answer from the options given below

1) A – III, B –II, C – I, D – IV
2) A – III, B –I, C – II, D – IV
3) A – I, B –III, C – II, D – IV
4) A – III, B –I, C – IV, D – II

32. When the hydrocarbon A undergoes combustion in the presence of air, it requires 9.5 equivalents of oxygen and produces 3 equivalents of water. What is the molecular formula of A?

1) $C_6H_6$
2) $C_8H_6$
3) $C_9H_6$
4) $C_9H_9$

33. Which of the following relations are correct?

(A) $\Delta U = q + p\Delta V$ (B) $\Delta G = \Delta H - T\Delta S$

(C) $\Delta S = \frac{q_{rev}}{T}$ (D) $\Delta H = \Delta U - \Delta nRT$

Chose the most appropriate answer from the option given below

10) B and D only 2) A and B only 3) C and D only 4) B and C only

34. A solution of $CrO_5$ in amyl alcohol has a _____ colour.

1) Green 2) Yellow 3) Blue 4) Orange-Red

35. The set of correct statements is:

(i) Manganese exhibits +7 oxidation state in its oxide.

(ii) Ruthenium and Osmium exhibit +8 oxidation intheir oxide.

(iii) Sc shows +4 oxidation sate nature which is oxidizing innature.

(iv) Cr shows oxidizing nature in +6 oxidation state.

1) (i) and (iii) 2) (ii), (iii) and (iv)

3) (i), (ii) and (iv) 4) (ii) and (iii)

36. The concentration of dissolved oxygen in water for growth of fish should be more than X ppm and Biochemical Oxygen Demand in clean water should be less than Y ppm. X and Y in ppm are respectively.

1) $\begin{matrix} X & Y \\ 6 & 5 \end{matrix}$ 2) $\begin{matrix} X & Y \\ 4 & 15 \end{matrix}$ 3) $\begin{matrix} X & Y \\ 6 & 12 \end{matrix}$ 4) $\begin{matrix} X & Y \\ 4 & 8 \end{matrix}$

37. Correct order of spin only magnetic moment of the following complex ions is (Given At.no. $Fe = 26, Co = 27$)

1) $[FeF_6]^{-3} > [Co(C_2O_4)_3]^{3-} > [CoF_6]^{3-}$

2) $[Co(C_2O_4)_3]^{3-} > [CoF_6]^{3-} > [FeF_6]^{-3}$

3) $[CoF_6]^{3-} > [FeF_6]^{-3} > [Co(C_2O_4)_3]^{3-}$

4) $[FeF_6]^{-3} > [CoF_6]^{3-} > [Co(C_2O_4)_3]^{3-}$

38. The one giving maximum number of isomeric alkenes on dehydrohalogenation reaction is (excluding rearrangement)

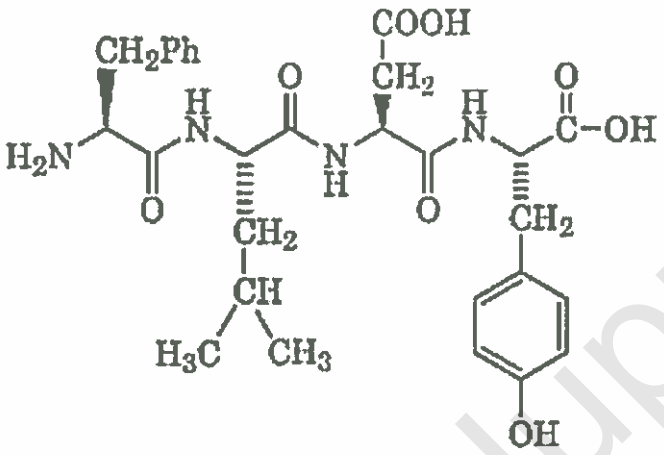
1) 2 – Bromopropane

2) 2 – Bromopentane

3) 1 – Bromo – 2 – methylbutane

4) 2 – Bromo – 3, 3 – dimethylpentane

39. Following tetrapeptide can be represented as




(F, L, D, Y, I, Q, P are one letter codes for amino acids)

1) YQLF 2) FLDY 3) FIQY 4) PLDY

40. Match List IwithList II

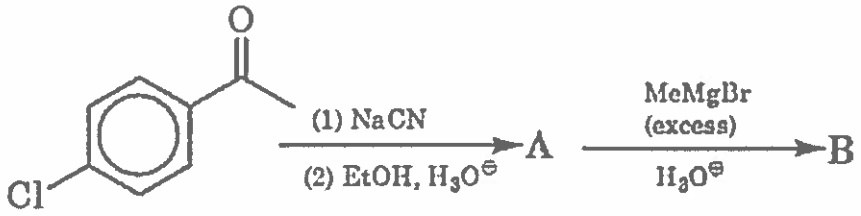
| List I | List II |
|---|---|
| A. Elastomeric polymer | I. Urea formaldehyde resin |
| B. Fibre Polymer | II. Polystyrene |
| C. Thermosetting Polymer | III. Polyester |
| D. Thermoplastic Polymer | IV. Neoprene |

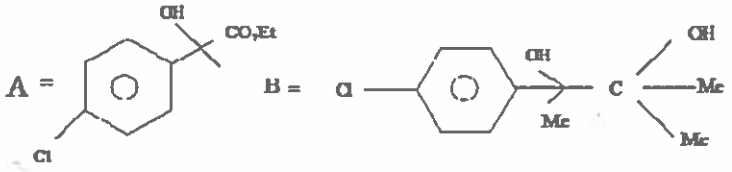
Choose the correct answer from the options given below:

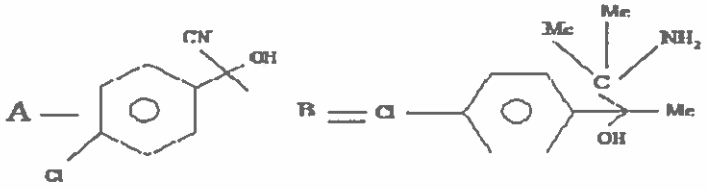
1) A – IV, B –I, C – III, D – II 2) A – II, B –I, C – IV, D – III

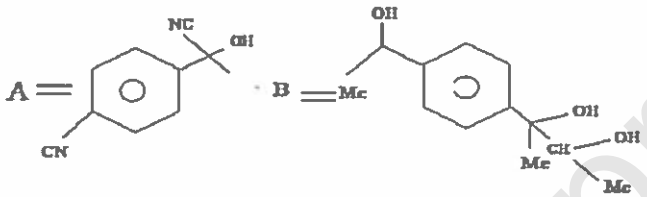
3) A – IV, B –III, C – I, D – II 4) A – II, B –III, C – I, D – IV

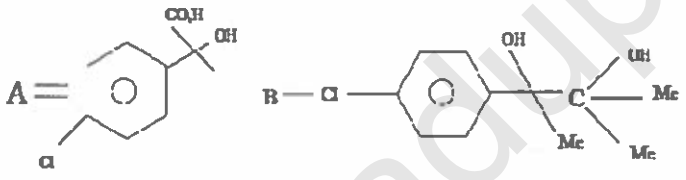
41. Find out the major products from the following reaction sequence.




1) 


2) 


3) 


4) 


42. Match List IwithList II

| List I | List II |
|---|---|
| A. Osmosis | I. Solvent molecules pass through semi permeable Membrane towards solvent side |
| B. Reverse osmosis | II. Movement of charged colloidal particles under the influence of applied electric potential towards oppositely charged electrodes. |
| C. Electro osmosis | III. Solvent molecules pass through semi permeable membrane towards solution side. |
| D. Electrophoresis | IV. Dispersion medium move in an electric field. |

Choose the correct answer from the options given below

1) A – I, B –III, C – IV, D – II

2) A – III, B –I, C – IV, D – II

3) A – I, B –III, C – II, D – IV

4) A – III, B –I, C – II, D – IV

43. Reaction of propanamide with $Br_2$ / KOH(aq) produces:

1) Ethylnitrile 2) Propanenitrile

3) Propylamine 4) Ethylamine

44. A doctor prescribed the drug Equanil to a patient. The patient was likely to have symptoms of which disease?

1) Depression and hypertension 2) Anxiety and stress

3) Hyperacidity 4) Stomach ulcers

45. According to MO theory the bond orders for $O_2^{2-}$, CO and $NO^+$ respectively, are

1) 1,3 and 3 2) 2,3 and 3 3) 1,2 and 3 4) 1,3 and 2

46. Given below are two statements:

Statements I : The decrease in first ionization enthalpyfrom B to Al is much larger than that from Al to Ga.

Statements II : The d orbitals in Ga are completely filled.

In the light of the above statements, choose the most appropriate answers from the options given below

1) Statement I is incorrect but statement II is correct

2) Both the statements I and II are incorrect

3) Both the statements I and II are correct

4) Statement I is correct but statement II is incorrect

47. The major component of which of the following ore is sulphide based mineral?

1) Calamine 2) Sphalerite 3) Malachite 4) Siderite

48. Given below are two statements:

Statements I : Nickel is being used as the catalyst for producingsyn gas and edible fats.

Statements II : Silicon forms both electron rich and electrondeficient hydrides.

In the light of the above statements, choose the most appropriate answer from the options given below:

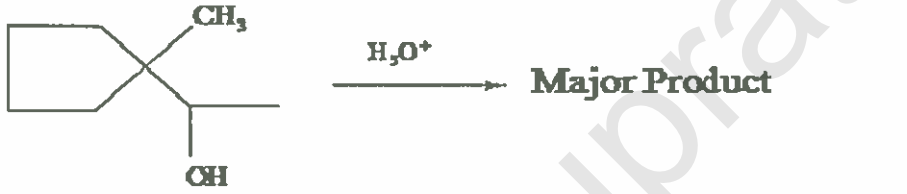
1) Statement I is incorrect but statement II is correct

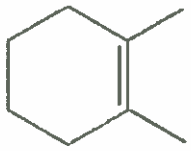
2) Both the statements I and II are correct

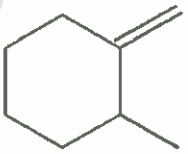
3) Statement I is correct but statement II is incorrect

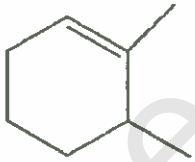
4) Both the statements I and II are incorrect

49. Find the major product for the following reaction.




1) 

2) 

3) 

4) 

50. An indicator 'X' is used for studying the effect of variation in concentration of iodide on the rate of reaction of iodide ion with $H_2O_2$ at room temp. The indicator 'X' forms blue colored complex with compound 'A' present in the solution. The indicator 'X' and compound 'A' respectively are

1) Starch and iodine

2) Methyl orange and $H_2O_2$

3) Starch and $H_2O_2$

4) Methyl orange and iodine

## Section-B

## (NUMERICAL VALUE TYPE)

51. The volume of $HCl$, containing $73\,gL^{-1}$, required to completely neutralise $NaOH$ obtained by reacting 0.69 g of metallic sodium with water, is _____ ml. ( Nearest integer)

(Given: Molar Masses of $Na, Cl, O, H$ are 23,35.5,16 and $1\,g\,mol^{-1}$ respectively).

52. For the conversion of compound $A \rightarrow B$, the rate constant of the reaction was found to be $4.6\times10^{-5}\,L\,mol^{-1}s^{-1}$. The order of the reaction is _____

53. The equilibrium constant for the reaction $Zn(s)+Sn^{2+}(aq) \rightleftharpoons Zn^{2+}(aq)+Sn(s)\,is\,1\times10^{20}\,at\,298\,K$. The magnitude of standard electrode potential of $Sn/Sn^{2+}$ if $E^{o}_{Zn^{2+}/Zn} = -0.76$V. is _____ $\times10^{-2}$ V (Nearest integer). Given: $\frac{2.303RT}{F} = 0.059V$

54. At 298 K

$N_2(g)+3H_2(g) \rightleftharpoons 2NH_3(g), K_1 = 4\times10^{5}$

$N_2(g)+O_2(g) \rightleftharpoons 2NO(g), K_2 = 1.6\times10^{12}$

$H_2(g)+\frac{1}{2}O_2(g) \rightleftharpoons H_2O(g), K_3 = 1.0\times10^{-13}$

Based on above equilibria, the equilibrium constant of the reaction,

$2NH_3(g)+\frac{5}{2}O_2(g) \rightleftharpoons 2NO(g)+3H_2O(g)\,is$ _____ $\times10^{-33}$.(Nearest integer)

55. When 0.01 mol of an organic compound containing 60% carbon was burnt completely, 4.4 g of $CO_2$ was produced. The molar mass of compound is _____ $g\,mol^{-1}$. (Nearest integer)

56. Total number of acidic oxides among

$N_2O_3, NO_2, N_2O, Cl_2O_7, SO_2, CO, CaO, Na_2O$ and $NO$ is _____

57. On heating, $LiNO_3$ gives how many compounds among the following?

_____ $Li_2O, N_2, O_2, LiNO_2, NO_2$

58. The denticity of the ligand present in the Fehling's reagent is _____

59. A metal M forms hexagonal close-packed structure. The total number of voids in 0.02 mol of it is _____ $\times 10^{21}$ (Nearest integer). (Given $N_A = 6.02 \times 10^{23}$)

60. Assume that the radius of the first Bohr orbit of hydrogen atom is $0.6\,A°$. The radius of the third Bohr orbit of $He^+$ is _____ Picometer. (Nearest integer)

## MATHEMATICS

### Section – A

### (SINGLE CORRECT ANSWER TYPE)

61. The value of the integral $\int_{1/2}^{2} \frac{\tan^{-1} x}{x} dx$ is equal to

1) $\pi \log_e 2$ 2) $\frac{\pi}{4} \log_e 2$ 3) $\frac{\pi}{2} \log_e 2$ 4) $\frac{1}{2} \log_e 2$

62. Let f and g be twice differentiable functions on $\mathbb{R}$ such that

$f''(x) = g''(x) + 6x \;;\; f'(1) = 4g'(1) - 3 = 9 \;;\; f(2) = 3g(2) = 12.$

Then which of the following is NOT true?

1) $|f'(x) - g'(x)| < 6 \Rightarrow -1 < x < 1$ 2) $g(-2) - f(-2) = 20$

3) If $-1 < x < 2$, then $|f(x) - g(x)| < 8$

4) There exists $x_0 \in (1, 3/2)$ such that $f(x_0) = g(x_0)$

63. The set of all values of $\lambda$ for which the equation $\cos^2 2x - 2\sin^4 x - 2\cos^2 x = \lambda$ has a real solution x, is

1) $\left[-2,-\frac{3}{2}\right]$ 2) $\left[-\frac{3}{2},-1\right]$ 3) $[-2,-1]$ 4) $\left[-1,-\frac{1}{2}\right]$

64. Consider a function $f:\mathbb{N}\to\mathbb{R}$, satisfying

$f(1)+2f(2)+3f(3)+.....+xf(x)=x(x+1)f(x);x\geq 2 with f(1)=1.$

Then $\frac{1}{f(2022)}+\frac{1}{f(2028)}$ is equal to

1) 8100 2) 8000 3) 8200 4) 8400

65. If $\vec{a}=\hat{i}+2\hat{k},\vec{b}=\hat{i}+\hat{j}+\hat{k},\vec{c}=7\hat{i}-3\hat{j}+4\hat{k},\vec{r}\times\vec{b}+\vec{b}\times\vec{c}=\vec{0}$ and $\vec{r}.\vec{a}=0$ .

Then $\vec{r}.\vec{c}$ is equal to

1) 36 2) 34 3) 30 4) 32

66. The plane $2x-y+z=4$ intersects the line segment joining the points $A(a,-2,4)$ and $B(2,b,-3)$ at the point C in the ratio 2:1 and the distance of the point C from the origin is $\sqrt{5}$. If $ab<0$ and P is the point $(a-b,b,2b-a)$ then $CP^2$ is equal to

1) $\frac{17}{3}$ 2) $\frac{97}{3}$ 3) $\frac{73}{3}$ 4) $\frac{16}{3}$

67. The area of the region $A=\left\{(x,y):|\cos x-\sin x|\leq y\leq \sin x,0\leq x\leq\frac{\pi}{2}\right\}$ is

1) $\sqrt{5}+2\sqrt{2}-4.5$ 2) $\frac{3}{\sqrt{5}}-\frac{3}{\sqrt{2}}+1$ 3) $\sqrt{5}-2\sqrt{2}+1$ 4) $1-\frac{3}{\sqrt{2}}+\frac{4}{\sqrt{5}}$

68. If the tangent at a point P on the parabola $y^2=3x$ is parallel to the line $x+2y=1$ and the tangents at the points Q and R on the ellipse $\frac{x^2}{4}+\frac{y^2}{1}=1$ are perpendicular to the line $x-y=2$, then the area of the triangle PQR is

1) $5\sqrt{3}$ 2) $\frac{3}{2}\sqrt{5}$ 3) $\frac{9}{\sqrt{5}}$ 4) $3\sqrt{5}$

69. Let $y=y(x)$ be the solution of the differential equation $x(\log_e x)\frac{dy}{dx}+y=x^2\log_e x,(x>1)$ . If $y(2)=2$ then $y(e)$ is equal to

1) $\frac{1+e^2}{4}$ 2) $\frac{4+e^2}{4}$ 3) $\frac{2+e^2}{2}$ 4) $\frac{1+e^2}{2}$

70. The value of the integral $\int_1^2\left(\frac{t^4+1}{t^6+1}\right)dt$ is

1) $\tan^{-1}2-\frac{1}{3}\tan^{-1}8+\frac{\pi}{3}$ 2) $\tan^{-1}\frac{1}{2}+\frac{1}{3}\tan^{-1}8-\frac{\pi}{3}$

3) $\tan^{-1}2+\frac{1}{3}\tan^{-1}8-\frac{\pi}{3}$ 4) $\tan^{-1}\frac{1}{2}-\frac{1}{3}\tan^{-1}8+\frac{\pi}{3}$

71. Let R be a relation defined on $\mathbb{N}$ as $aRb$ if $2a+3b$ is a multiple of 5, $a,b\in\mathbb{N}$. Then R is

1) an equivalence relation 2) transitive but not symmetric

3) not reflexive 4) symmetric but not transitive

72. The statement $B\Rightarrow((\sim A)\vee B)$ is not equivalent to

1) $B\Rightarrow(A\Rightarrow B)$ 2) $A\Rightarrow(A\Leftrightarrow B)$

3) $B\Rightarrow((\sim A)\Rightarrow B)$ 4) $A\Rightarrow((\sim A)\Rightarrow B)$

73. The number of 3-digit numbers, that are divisible by either 3 or 4 but not divisible by 48, is

1) 507 2) 432 3) 400 4) 472

74. Let $S=\{w_1,w_2,....\}$ be the sample space associated to a random experiment. Let $P(w_n)=\frac{P(w_{n-1})}{2},n\geq 2.$ , Let $A=\{2k+3l;k,l\in\mathbb{N}\}$ and $B=\{w_n:n\in A\}$. Then $P(B)$ isequal to

1) $\frac{3}{32}$ 2) $\frac{1}{32}$ 3) $\frac{3}{64}$ 4) $\frac{1}{16}$

75. Let K be the sum of the coefficients of the odd powers of $x$ in the expansion of $(1+x)^{99}$. Let $a$ bethe middle term in the expansion of $\left(2+\frac{1}{\sqrt{2}}\right)^{200}$. If $\frac{^{200}C_{99}K}{a}=\frac{2^l m}{n}$, where m and n are odd numbers, then the ordered pair $(l,n)$ is

1) $(50,51)$ 2) $(50,101)$ 3) $(51,99)$ 4) $(51,101)$

76. The letters of the word OUGHT are written in all possible ways and these words are arranged as in a dictionary, in a series. Then the serial number of the word TOUGH is

1) 89 2) 86 3) 84 4) 79

77. The set of all values of $t \in \mathbb{R}$ ,for which the matrix

$$\begin{bmatrix} e^t & e^{-t}(\sin t-2\cos t) & e^{-t}(-2\sin t-\cos t) \\ e^t & e^{-t}(2\sin t+\cos t) & e^{-t}(\sin t-2\cos t) \\ e^t & e^{-t}\cos t & e^{-t}\sin t \end{bmatrix}$$ *is invertible, is*

1) $\left\{k\pi+\frac{\pi}{4}, k\in\mathbb{Z}\right\}$ 2) $\mathbb{R}$

3) $\left\{(2k+1)\frac{\pi}{2}, k\in\mathbb{Z}\right\}$ 4) $\{k\pi, k\in\mathbb{Z}\}$

78. The shortest distance between the lines $\frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}$ and $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}$ is

1) $3\sqrt{3}$ 2) $4\sqrt{3}$ 3) $2\sqrt{3}$ 4) $5\sqrt{3}$

79. If the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z+3}{1}$ and $\frac{x-a}{2}=\frac{y+2}{3}=\frac{z-3}{1}$ intersect at the point P, then the distance of the point P from the plane $z=a$ is :

1) 28 2) 16 3) 22 4) 10

80. Let $\vec{a}=4\hat{i}+3\hat{j},\vec{b}=3\hat{i}-4\hat{j}+5\hat{k}$. If $\vec{c}$ is a vector such that $\vec{c}.\left(\vec{a}\times\vec{b}\right)+25=0$, $\vec{c}.\left(\hat{i}+\hat{j}+\hat{k}\right)=4$, andprojection of $\vec{c}$ on $\vec{a}$ is 1, then the projection of $\vec{c}$ on $\vec{b}$ equals

1) $\frac{1}{\sqrt{2}}$ 2) $\frac{3}{\sqrt{2}}$ 3) $\frac{5}{\sqrt{2}}$ 4) $\frac{1}{5}$

## Section-B

## (NUMERICAL VALUE TYPE)

81. Let $\alpha_1,\alpha_2,.....,\alpha_7$ be the roots of the equation $x^7+3x^5-13x^3-15x=0$ and $|\alpha_1|\geq|\alpha_2|\geq....\geq|\alpha_7|$. Then $\alpha_1\alpha_2-\alpha_3\alpha_4+\alpha_5\alpha_6$ is equal to _____

82. The total number of 4-digit number whose greatest common divisor with 54 is 2, is _____.

83. Let $\alpha=8-14i$, $A=\left\{z\in C:\frac{\alpha z-\bar{\alpha}\bar{z}}{z^2-\left(\bar{z}\right)^2-112i}=1\right\}$ and $B=\left\{z\in C:|z+3i|=4\right\}$. Then $\sum_{z\in A\cap B}\left(\mathrm{Re}\,z-\mathrm{Im}\,z\right)$ is equal to _____.

84. Let $X=\{11,12,13,.....,40,41\}$ and $Y=\{61,62,63,.....,90,91\}$ be the two sets of observations. If $\bar{x}$ and $\bar{y}$ are their respective means and $\sigma^2$ is the variance of all the observations in $X\cup Y$, then $\left|\bar{x}+\bar{y}-\sigma^2\right|$ is equal to _____.

85. Let $\{a_k\}$ and $\{b_k\}$, $k\in N$, be two G.P.s with common ratios $r_1$ and $r_2$ respectively such that $a_1=b_1=4$ and $r_1<r_2$. Let $c_k=a_k+b_k, k\in N$. If $c_2=5$ and $c_3=\frac{13}{4}$ then $\sum_{k=1}^{\infty}c_k-\left(12a_6+8b_4\right)$ is equal to _____.

86. Let $a_1=b_1=1$ and $a_n=a_{n-1}+(n-1), b_n=b_{n-1}+a_{n-1}, \forall n\geq 2$. If $S=\sum_{n=1}^{10}\frac{b_n}{2^n}$ and $T=\sum_{n=1}^{8}\frac{n}{2^{n-1}}$, then $2^7(2S-T)$ is equal to _____.

87. A triangle is formed by the tangents at the point $(2,2)$ on the curves $y^2 = 2x$ and $x^2 + y^2 = 4x$, and the line $x + y + 2 = 0$. If r is the radius of its circumcircle, then $r^2$ is equal to _____.

88. Let A be a symmetric matrix such that $|A| = 2$ *and* $\begin{bmatrix} 2 & 1 \\ 3 & \frac{3}{2} \end{bmatrix} A = \begin{bmatrix} 1 & 2 \\ \alpha & \beta \end{bmatrix}$. If the sum of the diagonal elements of A is s, then $\frac{\beta s}{\alpha^2}$ is equal to _____.

89. If the equation of the normal to the curve $y = \frac{x-a}{(x+b)(x-2)}$ at the point $(1,-3)$ is $x - 4y = 13$, then the value of $a + b$ is equal to _____.

90. A circle with center $(2,3)$ and radius 4 intersects the line $x + y = 3$ at the points P and Q. If the tangents at P and Q intersect at the point $S(\alpha, \beta)$, then $4\alpha - 7\beta$ is equal to _____.

# KEY SHEET

## PHYSICS

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1) | **1** | 2) | **3** | 3) | **2** | 4) | **1** | 5) | **4** |
| 6) | **4** | 7) | **1** | 8) | **4** | 9) | **1** | 10) | **2** |
| 11) | **1** | 12) | **2** | 13) | **3** | 14) | **3** | 15) | **3** |
| 16) | **4** | 17) | **1** | 18) | **4** | 19) | **1** | 20) | **3** |
| 21) | **800** | 22) | **12** | 23) | **41** | 24) | **25** | 25) | **40** |
| 26) | **30** | 27) | **40** | 28) | **3872** | 29) | **5** | 30) | **7** |

## CHEMISTRY

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 31) | **2** | 32) | **2** | 33) | **4** | 34) | **3** | 35) | **3** |
| 36) | **1** | 37) | **4** | 38) | **2** | 39) | **2** | 40) | **3** |
| 41) | **1** | 42) | **2** | 43) | **4** | 44) | **1** | 45) | **1** |
| 46) | **3** | 47) | **2** | 48) | **3** | 49) | **1** | 50) | **1** |
| 51) | **15** | 52) | **2** | 53) | **17** | 54) | **4** | 55) | **200** |
| 56) | **4** | 57) | **3** | 58) | **4** | 59) | **36** | 60) | **270** |

## MATHEMATICS

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 61) | **3** | 62) | **3** | 63) | **2** | 64) | **1** | 65) | **2** |
| 66) | **1** | 67) | **3** | 68) | **4** | 69) | **2** | 70) | **3** |
| 71) | **1** | 72) | **2** | 73) | **2** | 74) | **3** | 75) | **2** |
| 76) | **1** | 77) | **2** | 78) | **2** | 79) | **1** | 80) | **3** |
| 81) | **9** | 82) | **3000** | 83) | **14** | 84) | **603** | 85) | **9** |
| 86) | **461** | 87) | **10** | 88) | **5** | 89) | **4** | 90) | **11** |

# SOLUTIONS AND HINTS

## PHYSICS

1. In lifting bucket

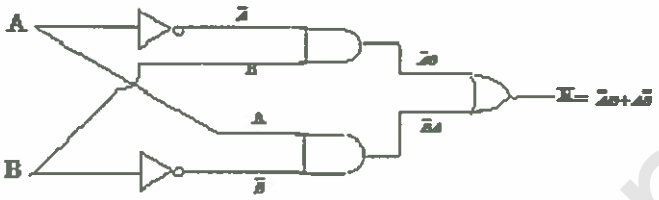
   Man does positive work

   Gravitational force does negative work

   Friction does negative work on block sliding down

   Work done by applied force is non-zero due to friction force.

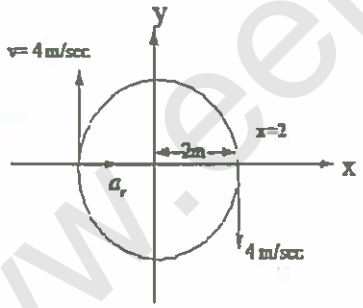
   Air resistance does negative work on oscillating pendulum as it acts opposite to direction of motion.

2. 


   Exclusively OR gate or $XOR$ gate

3. Modulation index, $m = \frac{A_{max} - A_{min}}{A_{max} + A_{min}} \Rightarrow m = \frac{14-6}{14+6} = \frac{8}{20} = 0.4$

   Hence correct Answer is (2)

4. 


   $a_r = \frac{v^2}{R} = \frac{4^2}{2} = 8$

5. $\sqrt{\frac{3RT}{M}} = \sqrt{\frac{\alpha+5}{\alpha}}\sqrt{\frac{8RT}{\pi M}}\ 3 = \left(\frac{\alpha+5}{\alpha}\right)\frac{8}{\left(\frac{22}{7}\right)} \Rightarrow \frac{\alpha+5}{\alpha} = \frac{33}{28} \Rightarrow \alpha = 28$

6. As $B = \frac{\mu_0 Ni}{2R} = \frac{\mu_0 i\pi}{(2\pi R)N}N^2 = \frac{\mu_0 i\pi}{l}N^2 \quad B = B_o N^2 \quad 32 = B_o \times (4)^2$

   $B_o = 2T$

7. $$N_A + N_B = \left[\frac{\frac{320}{2^2}}{16} + \frac{\frac{320}{2^4}}{32}\right] N_A = \left(5 + \frac{5}{8}\right) \times N_A = 33.879 \times 10^{23} = 3.38 \times 10^{24}$$

8. $$l = \frac{1}{2} g (\sin\theta - \mu\cos\theta) t_1^2 = \frac{g}{2\sqrt{2}} (1-\mu) t_1^2 \ldots\ldots\ldots (i)$$

$$l = \frac{1}{2} g \sin\theta t_2^2 = \frac{g}{2\sqrt{2}} t_2^2 \ldots\ldots\ldots (ii)$$

From equation (i) & (ii)

$$(1-\mu) t_1^2 = t_2^2 \; as \; t_1 = n t_2, \text{ So } n^2 = \frac{1}{1-\mu} \Rightarrow \mu = 1 - \frac{1}{n^2}$$

9. At resonance RMS current in both is same

In other cases, RMS in (a) is more than (b)

10. $R.P = \frac{\mu \sin\alpha}{0.61\lambda}$ for compound microscope

Resolving power will increase by increasing refractive index of the medium between the object and objective lens, by increasing diameter of the objective lens and by decreasing the wave length of the light .a

11. Given $[t] = T^{-1}$

$$[At] = [x]\left[\frac{t}{B}\right] = [y]$$

$$[A] = LT \; [B] = T^{-1} L^{-1}$$

12. EM waves do not have charge as they are photons hence not deflected by electric and magnetic fields

$E_0 = B_0 C \; E_0 = \frac{B_0}{\sqrt{\mu_0 \in_0}}$ Hence statement II is false

13. $\Delta P \times A = mg$

$$\Delta P \times 500 = 5.4 \times 10^5 \times 10$$

$$\Delta P = 108 \times 10^2 \, pa$$

$$\frac{1}{2} \rho \left(V_2^2 - V_1^2\right) = \left(P_1 - P_2\right)$$

$$\frac{1}{2}(1.2)(V_2+V_1)(V_2-V_1)=108\times10^2$$

But $\frac{V_2+V_1}{2}=V_{avg}=1080\frac{km}{hr}=1080\times\frac{5}{18}m/s=300m/s$

$$(1.2)300(V_2-V_1)=108\times10^2$$

$$V_2-V_1=30m/s$$

Fractional change$=\frac{V_2-V_1}{V_{avg}}=\frac{30}{300}=\frac{1}{10}$

$$\therefore \%increase=\frac{1}{10}\times100=10\%$$

14. Work done by agent

$$=Power\times t=i^2R\times t=\left(\frac{BlV}{R}\right)^2Rt=\frac{B^2l^2V^2t}{R}=1\times10^{-3}J$$

15. $$\frac{\lambda\alpha}{\lambda p}=\frac{\frac{h}{\sqrt{2m_\alpha q_\alpha V}}}{\frac{h}{\sqrt{2m_p q_p V}}}=\sqrt{\frac{m_p}{m_\alpha}\frac{q_p}{q_\alpha}}=\sqrt{\frac{m}{4m}\frac{e}{2e}}=\frac{1}{\sqrt{8}}=\frac{1}{\sqrt{m}}\ m=8$$

16. $$T^2\alpha R^3\left(\frac{T}{24}\right)^2=\left(\frac{R}{\frac{4}{R}}\right)^3\ T=3hr$$

17. $$K=\frac{V_A-V_B}{l}$$

Sensitivity of potentiometer wire $\alpha\ \frac{1}{K}\ \alpha\ l$

18. $Q_1=ms\Delta t=0.6\times2222.3\times12=16000.56J\approx16KJ$

For complete melting

$Q_{required}=mL=0.6\times336=201.6KJ$

All ice do not melt

Mass of ice melted $=\frac{184KJ-16KJ}{336KJ}=0.5kg$

Final mixture has 0.5kg water and 0.1 kg ice at $0^0c$

19. Work done, $W = \vec{F}.\vec{S}$ where $\vec{F} = q\vec{E} = 2\times10^{-2}\times30\hat{i} = 0.6\hat{i}$

And $\vec{S} = \vec{r_2} - \vec{r_1} = 0 - \left(\hat{i} + 2\hat{j}\right) = -\left(\hat{i} + 2\hat{j}\right)$

So $W = \vec{F}.\vec{S} = 0.6\hat{i}.\left(-\hat{i} - 2\hat{j}\right) = -0.6J = -600mJ$

20. Let say applied force is F then Impulse, J = F $\Delta t = mu \Rightarrow u = \dfrac{F\times20}{20} = F$

as $s = ut \Rightarrow 50 = F\times10 \Rightarrow F = 5N$

21. $m = 100\ g = 0.1\ kg$

After time 't', position vector of particle is

$\vec{r} = x\hat{i} + y\hat{j}$ $\vec{r} = (u\cos\theta)t\ \hat{i} + \left(u\sin\theta t - \dfrac{1}{2}gt^2\right)\hat{j}$

Velocity of particle is

$\vec{V} = V_x\hat{i} + V_y\hat{j}$ $\vec{V} = u\cos\theta\hat{i} + (u\sin\theta - gt)\hat{j}$

Angular momentum of particle

$\vec{L} = m\left(\vec{r}\times\vec{v}\right)$ $\vec{L} = m\left[u^2\sin\theta\cos\theta t - u^2\sin\theta\cos\theta t + \dfrac{1}{2}gt^2u\cos\theta\right]\hat{k}$

$\left|\vec{L}\right| = \dfrac{1}{2}mgt^2u\cos\theta$ $\left|\vec{L}\right| = \dfrac{1}{2}\times0.1\times10\times4\times20\times\dfrac{1}{\sqrt{2}} = \dfrac{40}{\sqrt{2}} = 20\sqrt{2}$

$\left|\vec{L}\right| = \sqrt{800}\ kg\ m^2/s$

22. $V = 2ar^2 + b$

Electric field $E = -\dfrac{dV}{dr} = -4ar$

From gauss law $\oint\vec{E}.\vec{ds} = \dfrac{q}{\epsilon_0}$

$E4\pi r^2 = \dfrac{\rho\left(\frac{4}{3}\pi r^3\right)}{\epsilon_0}$ $E = \dfrac{\rho r}{3\epsilon_0}$

$-4ar = \dfrac{\rho r}{3\epsilon_0}$ $\Rightarrow \rho = -12a\epsilon_0$

23. Refractive index of slab $\mu=\frac{\text{Real thicknessof slab}}{\text{Apparent thickness of slab}}$

$\mu=\frac{d_r}{d_a}=\frac{5.25}{5.00}=1.05\ \frac{\Delta\mu}{\mu}=\frac{\Delta d_r}{d_r}+\frac{\Delta d_a}{d_a}$

Least count error in measurement of thickness is

L.C=1M.S.D-1V.S.D $=\frac{1}{20}\text{cm}-\frac{49}{50}\times\frac{1}{20}\text{cm}=\frac{1}{1000}cm$

L.C = 0.01 mm

$\frac{\Delta\mu}{\mu}=\frac{0.01}{5.25}+\frac{0.01}{5.00}\ \Delta\mu=(0.0039)(1.05)=0.0041\ \Delta\mu=\frac{41}{10}\times10^{-3}$

24. From Newton's formula $F=\eta A\frac{dv}{dx}\ 0.1=5\times10^{-3}\times\frac{(0.20)v}{25\times10^{-5}}\ v=25\times10^{-3}m/s$

25. Given, F = -25 x

Compare with F = - Kx

K = 25 N/m

Angular frequency of oscillation $\omega=\sqrt{\frac{K}{M}}$

$\omega=\sqrt{\frac{25}{0.25}}=10$ rad/s

Maximum speed of particle $V_{max}=A\omega$

$4=A(10)$

$A=0.4m=40\ cm$

26. Balancing condition in meter bridge is $\frac{R}{S}=\frac{l}{100-l}$

R = Resistance in left gap

S = Resistance in right gap

l = balancing length

When $R,R_2$ are in series, $l=60\ cm, S=10\ \Omega$

$\frac{R_1+R_2}{S}=\frac{60}{40}\ \frac{R_1+R_2}{10}=\frac{3}{2}\ R_1+R_2=15$ ..... (i)

When $R,R_2$ are in parallel, $l=40\ cm, S=3\ \Omega$ $\frac{(R_1R_2/R_1+R_2)}{3}=\frac{40}{60}\ \frac{R_1+R_2}{10}=\frac{3}{2}$

$R_1R_2=2(R_1+R_2)\quad \therefore R_1R_2=2(15)=30\Omega^2$

27. as $a_r=a_t\Rightarrow R\omega^2=R\alpha$

$\Rightarrow \alpha=\frac{d\omega}{dt}=\omega^2\Rightarrow\frac{d\omega}{\omega^2}=dt$

$\Rightarrow \int_{\omega_0}^{\omega}\frac{d\omega}{\omega^2}=t$ , where $\omega_0=\frac{v_0}{R}=\frac{15}{600}=\frac{1}{40}$ Rad/sec

$\frac{1}{\omega_0}-\frac{1}{\omega}=t\Rightarrow\omega=\frac{1}{40-t}=\frac{dq}{dt}$

$\Rightarrow \int_0^{\pi/2}d\theta=\int_0^{t}\frac{dt}{40-t}\Rightarrow\frac{\pi}{2}=\left[-\ln(40-t)\right]_0^t$

$\Rightarrow\frac{\pi}{2}=\ln\frac{40}{40-t}\Rightarrow 1-\frac{t}{40}=e^{-\pi/2}\Rightarrow t=40\left(1-e^{-\pi/2}\right)$

28. For maximum currenct, $X_C=X_L$

$\Rightarrow\omega=\frac{1}{\sqrt{LC}}\Rightarrow C=\frac{1}{\omega^2L}$

$\Rightarrow C=\frac{1}{\left(2\times\frac{22}{7}\times7\times10^3\right)^2\times2\times10^{-6}}$

$=\frac{1}{4\times22^2\times2}=\frac{1}{3872}$

29. as $r=R\left(\frac{l_0}{l}-1\right)$ When $R=5\Omega, l=200\ cm$

So $r=5\left(\frac{l_0}{200}-1\right)$ ..... (i)

When $R=15\Omega, l=300\,cm$ So $r=15\left(\frac{l_0}{300}-1\right)$ ..... (ii)

From equation (i) and (ii)

$$5\left(\frac{l_0}{200}-1\right)=15\left(\frac{l_0}{300}-1\right)\Rightarrow\frac{l_0}{200}-1=\frac{3l_0}{300}-3\Rightarrow\frac{l_0}{100}-\frac{l_0}{200}=2\Rightarrow\frac{l_0}{200}=2$$

So from Eqn (i)

$$r=5\left(\frac{l_0}{200}-1\right)=5(2-1)=5\Omega$$

30. Angle of incidence should be equal to Brewester's angle $(\theta_B)$. as

$$\theta_B=\tan^{-1}\left(\frac{\mu_2}{\mu_1}\right)$$

As $C^1=\frac{1}{\sqrt{\mu_0\in_r}}=\frac{C}{\mu}$

$$\Rightarrow\frac{C}{\mu_1}=\frac{1}{\sqrt{\mu_0K_1\in_0}}\quad and\quad\frac{C}{\mu_2}=\frac{1}{\sqrt{\mu_0K_2\in_0}}$$

$$\Rightarrow\frac{\mu_2}{\mu_1}=\sqrt{\frac{K_2}{K_1}}\Rightarrow\theta_B=\tan^{-1}\left(\frac{6.8}{2.8}\right)^{1/2}$$

$$\theta_B=\tan^{-1}\left(1+\frac{4}{2.8}\right)^{1/2}=\tan^{-1}\left(1+\frac{10}{7}\right)^{1/2}$$

## CHEMISTRY

31. A. Van't Hoff factor, i $\rightarrow \frac{\text{Normal molar mass}}{\text{Abnormal molar mass}}$

B. Kf $\rightarrow$ I. Cryoscopic constant

C. solutions with same $\rightarrow$ II. Isotonic solutions , Osmotic pressure

D. Azeotropes $\rightarrow$ IV. Solution with same composition of vapour above it

32. $$CxHy + \left(x + \frac{y}{4}\right)O_2 \rightarrow xCO_2 + \frac{y}{2}H_2O$$

9.5 3

$$x + \frac{y}{4} = 9.5 \qquad y = 6$$

$$4x = 38 - 6 = 32$$

$$x = \frac{32}{4} = 8$$

$\therefore Hydrocorbon\ is\ C_8H_6$

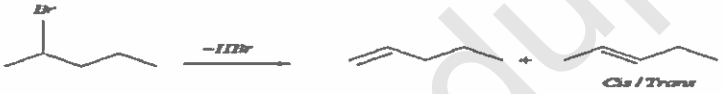
33. $\Delta G = \Delta H - T\Delta S, \Delta S = \frac{q_{rev}}{T}$ *are correct*

34. $CrO_5$ in amyl alcohol exhibits blue colour

35. 1,2,4 Statements are correct.

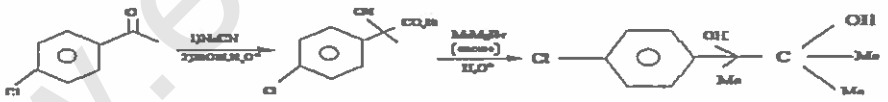
36. $X = 6, Y = 5$

37. $[FeF_6]^{-3} > [CoF_6]^{3-} > [Co(C_2O_4)_3]^{3-}$

38. 

39. Based on Hydrolysis $(F, L, D, Y)$

40. Elastomeric polymer – Neoprene, Fibre Polymer - Polyester
Thermosetting Polymer- Urea formaldehyde resin
Thermoplastic Polymer- Polystyrene

41. 

42. A. Osmosis → III. Solvent molecules pass through semi permeable membrane towardssolution side.
B. Reverse osmosis → I. Solvent molecules pass through semi permeable Membrane towards solvent side
C. Electro osmosis → IV. Dispersion medium move in an electric field.
D. Electrophoresis → II. Movement of charged colloidal particles under the influence of applied electric potential towards oppositely charged electrodes

43. $CH_3CH_2CONH_2 + Br_2/KOH \rightarrow CH_3CH_2NH_2$

44. drug Equanil is used for Depression and hyertension

45. Bond order of $O_2^{2-}$ is 1 Bond order of CO is 3

Bond order of $NO^+$ is 3

46. Both the statements I and II are correct

47. Calamine- $ZnCO_3$

Sphalerite - $ZnS$

Malachite - $CuCO_3.Cu(OH)_2$

Siderite - $FeCO_3$

48. Statement I is correct but statement II is incorrect

Silicon can form only electron precise hydride

49. 

50. Iodide ion is oxidized to $I_2$ Which gives blue color with starch indicator

51. $Na + H_2O \longrightarrow NaOH + \frac{1}{2}H_2, \frac{0.69}{23} = 0.03 \quad 0.03$

Milli equivalent of NaOH = milli equivalent of HCl

$0.03 \times 10^3 = 2 \times V(\text{ml})$ V=15ml

52. Unit of rate constant is $L mol^{-1} s^{-1}$

53. $Zn(s) + Sn^{+2}(aq) \rightleftarrows Zn^{2+}(aq) + Sn(s)$ $K_c = 10^{20}$

Ecell = 0 at equilibrium $E°cell = \frac{0.059}{2} \log 10^{20}$

$E°Sn^{2+}/sn - E°Zn^{+2}/zn = 0.59$ $E°Sn/Sn^{+2} = 17 \times 10^{-2}$V

54. $K = \frac{1}{K_1} \times (K_3)^3 \times K_2 = 4 \times 10^{-33}$

55. 0.01 mol compound = 0.1 mol carbon

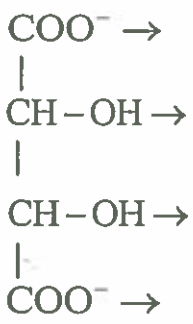
1 mol compound = 10 mol carbon = 120 gram

60 gm carbon = 100 gm compound

120 gm carbon = 200 gm

56. $N_2O_3, NO_2, Cl_2O_7, SO_2$ are acidic

57. $2LiNO_3 \longrightarrow Li_2O + 2NO_2 + \frac{1}{2}O_2$

58.


59. mol of h.c.p = 0.02

T.V $= 2 \times 0.02 = 0.04$

O.V $= 0.02$

Total void in mol $= 0.06$

No of void $= 0.06 \times 6.02 \times 10^{23} = 36 \times 10^{21}$

60. $r = 0.6\frac{n^2}{z}A^0 = 270\,\text{pm}$ n = 3, z = 2

## MATHEMATICS

61. $I = \int_{1/2}^{2} \frac{\tan^{-1} x}{x} dx \quad Put\ x = \frac{1}{t}$

$I = \int_{1/2}^{2} \tan^{-1}\left(\frac{1}{t}\right)\left(\frac{1}{t}\right) dt$

$\therefore 2I = \int_{1/2}^{2} \frac{\pi}{2}\frac{dt}{t} \Rightarrow I = \frac{\pi}{2}\ln 2$

62. $f''(x) - g''(x) = 6x \Rightarrow f'(x) - g'(x) = 3x^2 + C$

$x = 1 \Rightarrow 9 - 3 = 3 + C \therefore C = 3$

$f'(x) - g'(x) = 3x^2 + 3$

$f(x) - g(x) = x^3 + 3x + C_1$

$x=2\Rightarrow 12-4=8+6+C_1\Rightarrow C_1=-6$

$h(x)=f(x)-g(x)=x^3+3x-6$

$h'(x)=3x^2+3>0$

$for -1<x<2\Rightarrow h(-1)<h(x)<h(z)\Rightarrow h(x)<8$

63. $\cos^2 2x-2\sin^4 x-2\cos^2 x=\lambda$

$\Rightarrow \cos^2 2x-\frac{1}{2}(1-\cos 2x)^2-(1+\cos 2x)=\lambda$

$\Rightarrow 2\cos^2 2x-(1+\cos^2 2x-2\cos 2x)-2-2\cos 2x=2\lambda$

$\Rightarrow \cos^2 2x-3=2\lambda$

$\therefore \lambda\in\left[-\frac{3}{2},-1\right]$

64. $f(1)+2f(2)+.....+xf(x)=x(x+1)f(x)$

$f(1)+2f(2)=2.3f(2)\Rightarrow f(2)=\frac{1}{4}$

$f(1)+2f(2)+3f(3)=3.4f(3)\Rightarrow f(3)=\frac{1}{6}$

$\because f(2)=\frac{1}{2.2},f(3)=\frac{1}{2.3},.....f(n)=\frac{1}{2.n}$

$\frac{1}{f(2022)}+\frac{1}{f(2028)}=2\times 2022+2\times 2028=8100$

65. given $\vec{r}\times\vec{b}+\vec{b}\times\vec{c}=\vec{0}$

$\Rightarrow \vec{r}\times\vec{b}-\vec{c}\times\vec{b}=\vec{0}\Rightarrow(\vec{r}-\vec{c})\times\vec{b}=\vec{0}$

$\Rightarrow(\vec{r}-\vec{c})=\lambda\vec{b}\Rightarrow\vec{r}=\vec{c}-\lambda\vec{b}$

$\Rightarrow \vec{r}.\vec{a}=\vec{c}.\vec{a}-\lambda\vec{b}.\vec{a}=0\Rightarrow\lambda=5$

$\therefore \vec{r}.\vec{c}=\vec{c}.\vec{c}-5\vec{b}.\vec{c}=34$

66. $2x-y+z=4$, $A(a,-2,4)$ $B(2,b,-3)$ Let $C(x,y,z)$

$x=\frac{4+a}{3},y=\frac{2b-2}{3},z=\frac{-6+4}{3}=\frac{-2}{3}$

$\Rightarrow 8+2a-2b+2-2=12\Rightarrow 2a-2b=4\Rightarrow a-b=2$

$OC=\sqrt{5}\Rightarrow\frac{(4+a)^2}{9}+\frac{(2b-2)^2}{9}+\frac{4}{9}=5$

$a^2+8a+16+4a^2-24a+36+4=45\qquad(\because b=a-2)$

$\Rightarrow 5a^2-16a+11=0$

$\Rightarrow(5a-11)(a-1)=0$

$\Rightarrow a=1\ \because ab<0\ \&\ b=-1$

$C\left(\frac{5}{3},-\frac{4}{3},-\frac{2}{3}\right)P(2,-1,-3)$

$CP^2=\left(2-\frac{5}{3}\right)^2+\left(\frac{4}{3}-1\right)^2+\left(3-\frac{2}{3}\right)^2\Rightarrow CP^2=\frac{1}{9}+\frac{1}{9}+\frac{49}{9}=\frac{51}{9}=\frac{17}{3}$

67. $A=\int_{\theta}^{\pi/2}\left(\sin x-|\cos x-\sin x|\right)dx, where\ \tan\theta=\frac{1}{2}$

$=\int_{\theta}^{\pi/4}(\cos x-\sin x-\sin x)dx+\int_{\pi/4}^{\pi/2}\left(\sin x-(\sin x-\cos x)\right)dx$

$=2\cos x+\sin x]_{\theta}^{\pi/4}+\sin x]_{\pi/4}^{\pi/2}=\sqrt{5}+1-2\sqrt{2}$

68. Slope of the tangent at $P(t)$ on the parabola $y^2=3x$ is parallel to the line

$x+2y=1$ is $\frac{1}{t}=\frac{-1}{2}\Rightarrow P=\left(at^2,2at\right)=(3,-3)$

Slope the tangents at the points Q and R on the ellipse $\frac{x^2}{4}+\frac{y^2}{1}=1$ are

perpendicular to the line $x-y=2$ is $m=-1\Rightarrow c^2=a^2m^2+b^2\Rightarrow c=\pm\sqrt{5}$

$\Rightarrow Q=\left(\frac{4}{\sqrt{5}},\frac{1}{\sqrt{5}}\right)and\ Q=\left(\frac{-4}{\sqrt{5}},\frac{-1}{\sqrt{5}}\right)$

Area of $\Delta PQR=3\sqrt{5}$ Squints.

69. $x\ln x\frac{dy}{dx}+y=x^2\ln x$

$\frac{dy}{dx}+\frac{y}{x\ln x}=x\qquad\therefore I.F=e^{\int\frac{1}{\sin x}dx}=e^{\ln|\ln x|}=\ln x$

$Sol\ is\quad y(\ln x)=\int x.\ln x dx$

$y\ln x=\ln x\left(\frac{x^2}{2}\right)-\frac{x^2}{4}+c$

$\because x=2,y=2\Rightarrow 2\ln 2=\ln 2(2)-1+c\Rightarrow c=1$

$For\ x=e,\quad y=\frac{e^2}{2}-\frac{e^2}{4}+1\Rightarrow y(e)=1+\frac{e^2}{4}$

70. $I=\int_1^2\frac{t^4+1}{t^6+1}dt=\int_1^2\frac{\left(t^4-t^2+1+t^2\right)}{\left(t^6+1\right)}dt=\int_1^2\frac{t^4-t^2+1}{\left(t^6+1\right)}+\frac{\left(t^2+1\right)}{\left(t^6+1\right)}dt=\int_1^2\frac{dt}{t^2+1}+\frac{1}{3}\int_1^2\frac{3t^2dt}{\left(t^3\right)^2+1}$

$=\tan^{-1}t]_1^2+\frac{1}{3}\tan^{-1}t^3]_1^2=\left(\tan^{-1}2-\tan^{-1}(1)\right)+\frac{1}{3}\left(\tan^{-1}(2)-\tan^{-1}(1)\right)$

$=\tan^{-1}2+\frac{1}{3}\tan^{-1}8-\frac{\pi}{3}$

71. Let $(a,b) \in R$

$f(a,b) = 2a + 3b$

For Reflexive

$f(a,a) = 2a + 3a = 5a$ *i,e divisible by* $5 \Rightarrow (a,a) \in R$

For symmetric

$f(b,a) = 2b + 3a = 5a + 5b - (2a + 3b)$

Divisible Divisible

by 5 . by 5

$f(b,a)$ is divisible by $5 \Rightarrow (b,a) \in R$

For transitive

$f(a,b) = 2a + 3b$ is divisible by 5

$f(b,c) = 2b + 3c$ is divisible by 5

So, $2a + 3c$ is divisible by $5 \Rightarrow (a,c) \in R$

72.

| A | $\sim A$ | B | $(\sim AvB)$ | $B \Rightarrow (\sim AvB)$ | $A \Rightarrow B$ | $B \Rightarrow (A \Rightarrow B)$ |
|---|---|---|---|---|---|---|
| T | F | T | T | T | T | T |
| F | T | F | T | T | T | T |
| T | F | F | F | T | F | T |
| F | T | T | T | T | T | T |

73. No's divisible by 3 = 300,No's divisible by 4 =225

No's divisible by 12 = 75,No's divisible by 48 = 18

Total no's =300+225-75-18,Total no's=432

74. Let $P(w_1) = \alpha, P(w_2) = \frac{\alpha}{2}, P(w_3) = \frac{\alpha}{2^2} \ldots\ldots\ldots$

$\Sigma P(w_i) = \alpha\left(1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} \ldots\ldots\ldots\right) = 2\alpha = 1 \Rightarrow \alpha = \frac{1}{2}$

$A = \{5,6,7,8,\ldots..\} \Rightarrow P(B) = \frac{1}{2^5} + \frac{1}{2^6} + \frac{1}{2^7} \ldots\ldots\ldots = \frac{3}{64}$

75. $k=\frac{f(1)-f(-1)}{2}=\frac{2^{99}}{2}=2^{98}$

$a={}^{200}C_{100}(2)^{100}\left(\frac{1}{\sqrt{2}}\right)^{100}={}^{200}C_{100}2^{50}$

$\frac{{}^{200}C_{99}.2^{98}}{{}^{200}C_{100}.2^{50}}=\frac{2^l m}{n}\Rightarrow\frac{100}{200-100+1}.2^{48}=\frac{2^l m}{n}$

$\Rightarrow\frac{100}{101}.2^{48}=\frac{2^l m}{n}$

$\Rightarrow\frac{25.2^{50}}{101}=\frac{2^l m}{n}\Rightarrow l=50,n=101$

76. Given words in alphabetical order GHOTU

Words starting withG____ 24

Words starting withH____ 24

Words starting withO____ 24

Words starting withTG____ 6

Words starting withTH ____ 6

Words starting withT O G ____ 2

Words starting withT O H ____ 2

Words starting withT O U G H _ 1

Rank of the word is 89

77. $e^t.e^{-t}.e^{-t}\begin{vmatrix}1 & \sin t-2\cos t & -2\sin t-\cos t\\ 1 & 2\sin t+\cos t & \sin t-2\cos t\\ 1 & \cos t & \sin t\end{vmatrix}\neq 0$

$1\left(2\sin^2 t+\sin t\cos t-\cos t\sin t+2\cos^2 t\right)$

$-1\left(\sin^2 t-2\sin t\cos t+2\sin t\cos t+\cos^2 t\right)$

$+1\left(\sin^2 t-4\sin t\cos t+4\cos^2 t+4\sin^2 t+\cos^2 t+4\sin t\cos t\right)\neq 0$

$=2-1+5=6\neq 0\ \ \forall t\in R$

78. Given lines $\frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}$ and $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}$

$$\overline{b_1}\times\overline{b_2}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -7 & 5\\ 2 & 1 & -3\end{vmatrix}=\hat{i}(16)-\hat{j}(-16)+\hat{k}(16)$$

$$\left(\overline{a_2}-\overline{a_1}\right).\left(\overline{b_1}\times\overline{b_2}\right)=\left(10\hat{j}+2\hat{k}\right)\left(\hat{i}(16)+\hat{j}(16)+\hat{k}(16)\right)$$

$$=16\times12$$

$$\therefore S.D=\frac{16\times12}{16\sqrt{3}}$$

79. Given lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z+3}{1}$ and $\frac{x-a}{2}=\frac{y+2}{3}=\frac{z-3}{1}$

Point of intersection be P

$$(\lambda+1,2\lambda+2,\lambda-3)=(2k+a,3k-2,k+3)$$

$$\lambda+1=2k+a;2\lambda+2=3k-2;\lambda-3=k+3$$

$$\Rightarrow 2k+12+2=3k-2\Rightarrow k=16,\lambda=22$$

$$\Rightarrow a=-9,P=(23,46,19)$$

Distance from P to z + 9=0 is 28

80. $$\overline{a}\times\overline{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & 3 & 0\\ 3 & -4 & 5\end{vmatrix}=15\hat{i}-20\hat{j}-25\hat{k}\Rightarrow\left|\overline{a}\times\overline{b}\right|^2=1250$$

$$\overline{c}=x\overline{a}+y\overline{b}+Z\left(\overline{a}\times\overline{b}\right)$$

$$\overline{c}.\overline{a}=5\Rightarrow5=x\left|\overline{a}\right|^2=25x\therefore x=\frac{1}{5}$$

$$c.(a\times b)=z\left|\overline{a}\times\overline{b}\right|^2=1250z=-25\Rightarrow z=-\frac{1}{50}$$

$$\overline{c}=\frac{4}{5}\hat{i}+\frac{3}{5}\hat{j}+y\overline{b}+-\frac{1}{50}\left(15\hat{i}-20\hat{j}-25\hat{k}\right)$$

$$\overline{c}=\left(\frac{4}{5}+3y-\frac{3}{10}\right)\hat{i}+\left(\frac{3}{5}-4y+\frac{z}{5}\right)\hat{j}+\left(5y+\frac{1}{z}\right)\hat{k}$$

$$c\left(\hat{i}+\hat{j}+\hat{k}\right)=4\Rightarrow\frac{1}{2}+3y+1-4y+5y+\frac{1}{z}=4\Rightarrow4y=2\Rightarrow y=\frac{1}{2}$$

$$\therefore\frac{\overline{c}.\overline{b}}{\left|\overline{b}\right|}=y\left|\overline{b}\right|=\frac{5}{\sqrt{2}}$$

81. $x=0, x^6+3x^4-13x^2-15=0$ Let $x^2=t \Rightarrow t^3+3t^2-13t-15=0$

$\Rightarrow x^2=t=-1,3,-5 \Rightarrow x=-i\sqrt{5}, i\sqrt{5}, \sqrt{3}, -\sqrt{3}, -i, -i$

$\Rightarrow \alpha_1=-i\sqrt{5}, \alpha_2=i\sqrt{5}, \alpha_3=\sqrt{3}, \alpha_4=-\sqrt{3}, \alpha_5=-i, \alpha_6=-i$

$\Rightarrow \alpha_1\alpha_2-\alpha_3\alpha_4+\alpha_5\alpha_6=5+3+1=9$

82. $54=2.3^3$

N be the 4-digit number which should be divisible by2 but not by 3

number of numbers divisible 2=9000/2=4500

number of numbers divisible by 6= 9000/6=1500

Required 4-digit number=number of numbers divisible 2

– number of numbers divisible by 6 =4500-1500=3000

83. Let $z=x+iy$ given $\alpha=8-14i$ $\alpha z-\bar{\alpha}\bar{z}=z^2-\bar{z}^2-112i$

$z(\alpha-z)-\bar{z}\left(\bar{\alpha}-\bar{z}\right)=-112i$ $Im\left(z(\alpha-z)\right)=-\frac{112}{2}$

$y(8-x)-x(14+y)=-56 \Rightarrow 8y-xy-14x-xy=-56 \Rightarrow xy+7x-4y-28=0$

$(x-4)(y+7)=0 \ldots\ldots (1)$ $\quad x^2+(y+3)^2=16 \ldots\ldots (2)$

Solving (1) and (2) we will get $(0,-7)$ and $(4,-3)$

$\sum_{z\in A\cap B}\left(\mathrm{Re}(z)-\mathrm{Im}(z)\right)=4+10=14$

84. $\bar{x}=\frac{\frac{31}{2}(11+41)}{31}=\frac{1}{2}\times 52=26$ $\quad \bar{y}=\frac{\frac{31}{2}(61+91)}{31}=\frac{1}{2}\times 152=76$

$\sigma^2=\frac{\sum x_i^2+\sum y_i^2}{62}-\left(\frac{\sum x_i^2+\sum y_i^2}{62}\right)^2=\frac{\left(11^2+12^2+\ldots.+41^2\right)+\left(61^2+62^2+\ldots.+91^2\right)}{62}-(51)^2$

$=\frac{\left(\frac{41.42.83}{63}-\frac{10.11.21}{6}\right)+\left(\frac{91.92.183}{6}-\frac{60.61.121}{6}\right)}{62}-(51)^2$

$=\frac{(41.7.83-11.35)+(91.46.61-10.61.121)}{62}-(51)^2$

$=\frac{23436+181536}{62}-(51)^2=3306-2601=705$

$\bar{x}+\bar{y}-\sigma^2=26+76-705=-603$

85. $a_1 = b_1 = 4; \quad c_2 = a_1 r_1 + b_1 r_2;\ c_2 = 4(r_1 + r_2) = 5$

$c_3 = 4\left(r_1^2 + r_2^2\right) = \frac{13}{4}$

$(r_1 + r_2)^2 - 2r_1 r_2 = \frac{13}{16} \Rightarrow \frac{25}{16} - \frac{13}{16} = 2r_1 r_2$

$\Rightarrow 2r_1 r_2 = \frac{12}{16} = \frac{3}{4} \Rightarrow r_1 = \frac{1}{2}, r_2 = \frac{3}{4}$

$\sum_{k=1}^{\infty} C_k = 4\left(1 + \frac{1}{2} + \frac{1}{2^2} + \ldots.\right) + 4\left(1 + \frac{3}{4} + \left(\frac{3}{4}\right)^2 + \ldots\right)$

$= 8 + 4 \times 4 = 24$

$Ans = 24 - 15 = 9$

86. $S = \sum_{n=1}^{10} \frac{b_n}{2^n} = \frac{b_1}{2} + \frac{b_2}{2^2} + \ldots\ldots. + \frac{b_{10}}{2^{10}}$

$\frac{S}{2} = \frac{b_1}{2^2} + \ldots. + \frac{b_7}{2^{10}} + \frac{b_{10}}{2^{11}}$ *on subtracting*

$\frac{S}{2} = \frac{b_1}{2} + \frac{b_2 - b_1}{2^2} + \frac{b_3 - b_2}{2^3} + \ldots\ldots. + \frac{b_{10} - b_9}{2^{10}} - \frac{b_{10}}{2^{11}}$

$\frac{S}{2} = \frac{1}{2} + \frac{a_1}{2^2} + \frac{a_2}{2^3} + \ldots + \frac{a_9}{2^{10}} - \frac{b_{10}}{2^{11}}$

$\frac{S}{4} = \frac{1}{4} + \frac{a_1}{2^3} + \frac{a_2}{2^4} + \ldots\ldots + \frac{a_8}{2^{10}} + \frac{a_9}{2^{11}} - \frac{b_{10}}{2^{12}}$ *on subtracting*

$\frac{S}{4} = \frac{1}{4} + \frac{a_2 - a_1}{2^3} + \frac{a_3 - a_2}{2^4} + \ldots.. + \frac{a_9 - a_8}{2^{10}} - \frac{b_{10}}{2^{11}} - \frac{a_9}{2^{11}} + \frac{b_{10}}{2^{12}}$

$S = 1 + \frac{a_2 - a_1}{2} + \frac{a_3 - a_2}{2^2} + \ldots. + \frac{a_9 - a_8}{2^8} - \frac{a_9}{2^9} - \frac{b_{10}}{2^{10}}$

$S = 1 + \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \ldots. + \frac{8}{2^8} - \frac{a_9}{2^9} - \frac{b_{10}}{2^{10}}$

$T = 1 + \frac{2}{2} + \frac{3}{2^2} + \ldots. + \frac{8}{2^7}$

$2S = 2 + \left(1 + \frac{2}{2^1} + \frac{3}{2^2} + \ldots. + \frac{8}{2^7}\right) - \frac{a_9}{2^8} - \frac{b_{10}}{2^9}$

$2S - T = 2 - \frac{a_9}{2^8} - \frac{b_{10}}{2^9}$

$2^7(2S - T) = 2^8 - \frac{a_9}{2} - \frac{b_{10}}{4} \ldots\ldots\ldots\ldots (1)$

$a_n - a_{n-1=n-1}$ $\quad a_1 = 1, a_2 = 2, a_3 = 4$

$a_k = ak^2 + bk + c$

$a + b + c = 1$

$4a + 2b + c = 2$

$9a + 3b + c = 4$

$a = \frac{1}{2}, b = -\frac{1}{2}, c = 1$

$a_9 = 37$

$b_k = ak^3 + bk^2 + ck + d$

$a + b + c + d = 1$

$8a + 4b + 2c + d = 2$

$27a + 9b + 3c + d = 4$

$64a + 16b + 4c + d = 8$

$b_k = \frac{k^3}{6} - \frac{k^2}{2} + \frac{4}{3}k$

$b_{10} = 130$

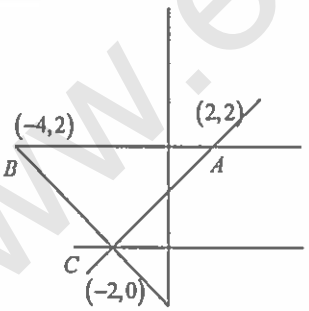
Using $a_9$ & $b_{10}$ in equation ....(1)

We get S=461

87. $2y = x + 2$ *for* $y^2 = 2x$

$2x + 2y = 2(x + 2)$ *for* $x^2 + y^2 = 4x$

$y = 2 \qquad x + y + 2 = 0$




To find circumcenter Find the perpendicular bisectors of $AB$ & $AC$

For $AB$ if is $x = -1$

For $BC$

$(x+4)^2+(y-2)^2=(x+2)^2+y^2$

$8x-4y+20=4x+4\Rightarrow 4x-4y+16=0\Rightarrow x-y+4=0$

$\therefore Centre(-1,3)\quad Radius\ r^2=10$

88. $|A|=2$ $\quad\begin{bmatrix}2 & 1\\ 3 & \frac{3}{2}\end{bmatrix}A=\begin{bmatrix}1 & 2\\ \alpha & \beta\end{bmatrix}$

*Det.on both sides* $\Rightarrow \beta-2\alpha=0$

*Let* $A=\begin{bmatrix}a & b\\ b & c\end{bmatrix}\quad ac-b^2=2.....(i)$

$\begin{bmatrix}2 & 1\\ 3 & \frac{3}{2}\end{bmatrix}\begin{bmatrix}a & b\\ b & c\end{bmatrix}=\begin{bmatrix}1 & 2\\ \alpha & 2\alpha\end{bmatrix}$

$2a+b=1.....(ii)$ *and* $2b+c=2.....(iii)$

$3a+\frac{3b}{2}=\alpha\Rightarrow 3b+\frac{3c}{2}=2\alpha\Rightarrow 6a+3b=3b+\frac{3c}{2}$

$c=4a.....(iv)$ $\quad from\ (i)\&(ii)a=\frac{3}{4}$

$b=-\frac{1}{2}.c=3S=a+c=\frac{15}{4},$

$\alpha=\frac{9}{4}+\left(-\frac{3}{4}\right)=\frac{6}{4}=\frac{3}{2}\Rightarrow\beta=3\ \ \therefore\frac{\beta S}{\alpha^2}=5$

89. $(1,-3)$ lie on $y=\frac{x-a}{(x+b)(x-2)}\Rightarrow a+3b+2=0------I$

Slope of the tangent$=-4=\left(\frac{dy}{dx}\right)_{x=1}$

$\Rightarrow -4=\frac{(1+b)3-(1-a)(b)}{(1+b)^2}$

$\Rightarrow -4(1+b)^2=3(1+b)(1-a)-----II$

*solve I and II*

$\Rightarrow b=-3,a=7\Rightarrow a+b=4$

90. The given line is the polar of $S(\alpha,\beta)$ w.r.t given circle.

Circle : $x^2+y^2-4x-6y-3=0$

Chord of contact : $\alpha x+\beta y-2(x+\alpha)-3(y+\beta)-3=0$

$\Rightarrow(\alpha-2)x+(\beta-3)y-(2\alpha+3\beta+3)=0$

But equation of chord of contact is $x+y-3=0$

Comparing the coefficients

$$\frac{\alpha-2}{1}=\frac{\beta-3}{1}=\frac{-(2\alpha+3\beta+3)}{3}\Rightarrow\alpha=-6,\beta=-5\Rightarrow 4\alpha-7\beta=11$$