JEE-MAIN

MODEL GRAND TEST

IMPORTANT INSTRUCTIONS:

1. Immediately fill in the Admission number on this page of the Test Booklet with **Blue/Black** Ball Point Pen only.
2. The candidates should not write their Admission Number anywhere (except in the specified space) on the Test Booklet/ Answer Sheet.
3. The test is of **3 hours** duration.
4. The Test Booklet consists of 90 questions. The maximum marks are **300.**
5. There are **three** parts in the question paper 1,2,3 consisting of **Physics, Chemistry** and **Mathematics** having **30 questions** in each subject and subject having **two sections.**
   (I) Section –I contains 20 **multiple choice** questions with only one correct option.
   Marking scheme: +4 for correct answer, 0 if not attempt and -1 in all other cases.
   (II) Section-II contains 10 **Numerical Value Type** questions. Attempt any 5 questions only, if more than 5 questions attempted, First 5 attempted questions will be considered.

- The Answer should be within **0 to 9999.** If the Answer is in **Decimal** then round off to the **nearest Integer** value (Example i,e. If answer is above **10** and less than **10.5** round off is **10** and If answer is from **10.5** and less than **11** round off is **11).**
  To cancel any attempted question bubble on the question number box.
  For example: To cancel attempted question 21. Bubble on 21 as shown below

Question Answered for Marking **Question Cancelled for Marking**

Marking scheme: +4 for correct answer, 0 if **not attempt** and **-1** in all other cases.

6. Use **Blue / Black Point Pen only** for writing particulars / marking responses on the Answer Sheet. **Use of pencil is strictly prohibited.**
7. No candidate is allowed to carry any textual material, printed or written, bits of papers, mobile phone any electron device etc, except the Identity Card inside the examination hall.
8. Rough work is to be done on the space provided for this purpose in the Test Booklet only.
9. On completion of the test, the candidate must hand over the Answer Sheet to the invigilator on duty in the Hall. **However, the candidate are allowed to take away this Test Booklet with them.**
10. **Do not fold of make any stray marks on the Answer Sheet**

PHYSICS

Section –A

(SINGLE CORRECT ANSWER TYPE)

1. In YDSE Violet light of wavelength $4200\,\overset{0}{A}$ is used. Find the slit separation in setup so that third minimum is obtained on screen at an angle 30^0 (nearly)

1) $1\mu m$ 2) $2\mu m$ 3) $3\mu m$ 4) $4\mu m$

2. Due to a point isotropic source of sound, loudness at a point is 40dB. If sound velocity is 330 m/s and air density is 15/11 kg/m^3. Find the pressure amplitude at this point

1) $0.006N/m^2$ 2) $0.009N/m^2$ 3) $0.003\ N/m^2$ 4) $0.001\ N/m^2$

3. A very tall cylindrical vessel with an ideal gas of molar mass M, filled in it is placed in a region of uniform gravity 'g'. In this container gas temperature varies in a manner such that density of gas remain constant throughout. Find the temperature gradient dT/dh, in container.

1) $\frac{-MR}{g}$ 2) $\frac{-g}{MR}$ 3) $\frac{-Mg}{R}$ 4) $\frac{-R}{Mg}$

4. Two cylinders A and B, fitted with pistons, contain equal amounts of an ideal diatomic gas at 300K. The piston of A is free to move while that of B is held fixed. Same amount of heat is given to the gas in each cylinder. If rise in temperature of gas in cylinder A is 30K then find the rise in temperature of the gas in cylinder B.

1) 32 K 2) 42 K 3) 52 K 4) 62 K

5. A black hole is an object whose gravitational field is so strong that even light cannot escape from it. To what approximate radius would earth (mass $=5.98\times10^{24}kg$) have to be compressed to be a black hole?

1) $10^{-9}m$ 2) $10^{-6}m$ 3) $10^{-2}m$ 4) $10^{2}m$

6. A bar magnet is oscillating in the Earth's magnetic field with a period T. What happens to its period and motion if its mass is quadrupled?

1) Motion remains simple harmonic with time period $= T/2$

2) Motion remains S.H.M with time period $= 2T$

3) Motion remains S.H.M with time period $= 4T$

4) Motion remains S.H.M and period remains nearly constant.

7. A small block starts sliding down an inclined plane forming an angle α with the horizontal. The friction coefficient depends on the distance x covered by block as $\mu = bx$ where b is a constant. Find its maximum speed during sliding

1) $\sqrt{\frac{b\cos\alpha}{g\sin^2\alpha}}$ 2) $\sqrt{\frac{b\sin\alpha}{g\cos^2\alpha}}$ 3) $\sqrt{\frac{g\sin^2\alpha}{b\cos\alpha}}$ 4) $\sqrt{\frac{g\cos^2\alpha}{b\sin\alpha}}$

8. Two concentric uniformly charged shells of radii a and b(b>a) with equal surface charge densities σ coulomb$/m^2$. Find the electric potential at common centre of the shells?

1) $\frac{\sigma}{2\varepsilon_0}(a+b)$ 2) $\frac{\sigma}{2\varepsilon_0}(a-b)$ 3) $\frac{\sigma}{\varepsilon_0}(a-b)$ 4) $\frac{\sigma}{\varepsilon_0}(a+b)$

9. In a parallel L and C connections, $C = 4\mu F$ and maximum potential difference across capacitor is 1.5 V and maximum current in circuit is 50mA. Determine the oscillation frequency of circuit?

1) $1.66\ kHz$ 2) $1.33\ kHz$ 3) $1.63\ kHz$ 4) $1.39\ kHz$

10. A body of mass m is lifted from ground by a force which varies with height h as $F=2(1-bh)mg$. Find work done by this force over the first half of the ascent of body.

1) $\frac{2}{3}\frac{mg}{b}$ 2) $\frac{3}{2}\frac{mg}{b}$ 3) $\frac{3}{4}\frac{mg}{b}$ 4) $\frac{4}{3}\frac{mg}{b}$

11. On a smooth horizontal plane a thin uniform rod of length ℓ is placed. One end of rod is given an impulse in direction normal to it. Find the displacement of centre of mass of rod by the time it completes one rotation.

1) $\frac{\pi\ell}{3}$ 2) $\frac{\pi\ell}{6}$ 3) $\frac{\pi\ell}{9}$ 4) $\frac{\pi\ell}{2}$

12. Assertion: A body that is a good radiator is also a good absorber of radiation at a given wave length

Reason: According to kirchhoff's law the absorptivity of a body is equal to its emissivity at a given wave length

1) Both Assertion & reason are correct and reason is correct explanation of Assertion

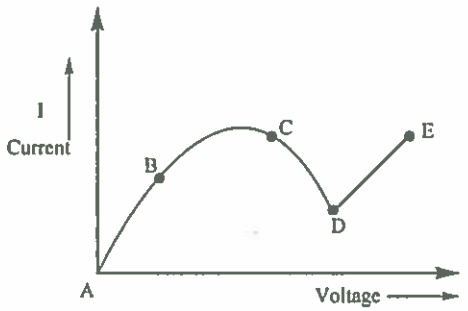
2) Both Assertion & Reason are correct and Reason is not correct explanation of Assertion

3) Assertion is true reason is false 4) Assertion & Reason are false

13. Two cylindrical conductors with equal cross sections and different resistivity ρ_1 and ρ_2 are put end to end. Find the charge at the boundary of the conductors if a current I flows from 1 to 2.

1) $\frac{I\varepsilon_0}{\rho_2-\rho_1}$ 2) $\frac{I\varepsilon_0}{\rho_1-\rho_2}$ 3) $(\rho_1-\rho_2)I\varepsilon_0$ 4) $(\rho_2-\rho_1)I\varepsilon_0$

14. From the graph between current (I) and voltage (V) shown below, identify the portion corresponding to negative resistance



1) CD 2) DE 3) AB 4) BC

15. An object is placed infront of a glass slab $(\mu=1.5)$ of thickness 6cm at a distance 28cm from it. Other face of glass slab is silvered. Find the position of final image of object from silvered face of slab

1) 15cm 2) 24cm 3) 6cm 4) 30cm

16. A satellite is orbiting around earth in orbit very close to earth surface in equatorial orbit with a rotation period 1.5hrs. If at an instant it is vertically above a point p on earth surface, find the time after which it will again come above p (if it is moving from east to west).

1) 1.1 hr 2) 1.2hrs 3) 1.3hrs 4) 1.4hrs

17. A piece of gold weighing 36 gm in air, weighs 34gm in water. If there is some copper impurity in gold find the amount of copper in it given specific gravity of gold is 19.3 and that of copper is 8.9

1) 2.225 grams 2) 3.325 grams 3) 4.425 grams 4) 5.525 grams

18. A Capacitor C is charged by a battery of EMF V volts and battery is disconnected. If now capacitor plates are pulled apart to increase the separation to 3d/2 from d and a dielectric constant $k=3$ is filled between the plates. Find final energy stored in this capacitor. Take $C=10\mu F$ and $V=20$ Volt.

1) $1\mu J$ 2) $1nJ$ 3) $1mJ$ 4) $1pJ$

19. Human body excretes certain materials by a law similar to radioactivity. If technetium is injected in some form in a human body, the body excretes half the amount in 24 Hrs. A patient is given an injection containing a radio isotope Tc^{99}. This isotope has half life of 6 hours. Find the time after which the activity of Tc inside body of patient reduces to half.

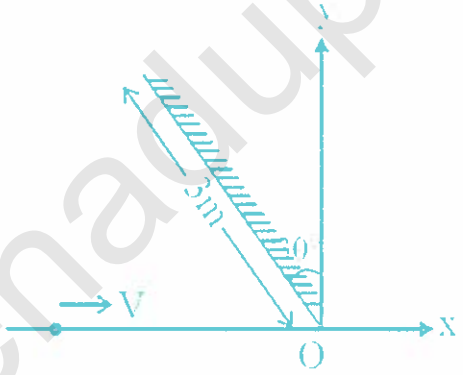
1) 2.4 hours 2) 4.8 hours 3) 7.2 hours 4) 9.6 hours

20. Two identical boats A and B start from a point in a river with speed V relative to water in mutually perpendicular paths, A along the river current and B perpendicular to river current. If they move equal distances from initial point and return back then find ratio of time taken by the boats A and B in round trips. Velocity of river current is 0.8V.

1) $\frac{10}{3}$ 2) $\frac{9}{3}$ 3) $\frac{5}{3}$ 4) $\frac{4}{3}$

Section-B
(NUMERICAL VALUE TYPE)

21. A plane mirror is placed with its plane at an angle 30° with the y-axis. Plane of the mirror is perpendicular to the xy-plane and the length of the mirror is 3 m. An insect moves along x-axis starting from a distant point towards the mirror, with speed v = 2 cm/s. The duration of the time in seconds for which the insect can see its own image in the mirror is:



22. The percentage increase in the speed of transverse waves produced in a stretched string if the tension is increased by 4% will be ____%

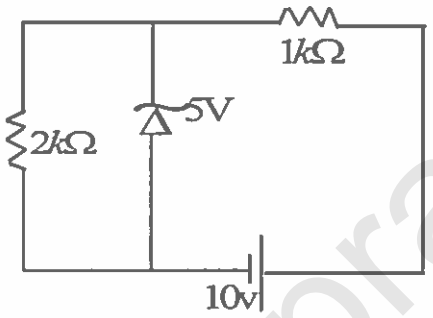
23. The peak electric field produced by the radiation coming from the 8W bulb at a distance of 10m is $\frac{X}{10}\sqrt{\frac{\mu_0 C}{\pi}}\frac{V}{m}$ .the efficiency of the bulb is 10% and if is a point source. The value of X is ____

24. A small bob tied at one end of a thin string of length 1m is describing a vertical circle so that maximum and minimum tension in the string are in the

ratio 5:1. The velocity of the bob at the highest position is ____m/s $\left(\text{Take } g = 10m/s^2\right)$

25. A uniform metallic wire is elongated by 0.04m when subjected to a linear force F. The elongation, if its length and diameter is doubled and subjected to the same force will be _____ cm

26. In connection with the circuit drawn below, the value of current flowing through $2k\Omega$ resistor is ___$\times 10^{-4}\,A$



27. A force $\vec{F} = 4\hat{i} + 3\hat{j} + 4\hat{k}$ is applied on an intersection point $x=2$ plane and $x-$axis. The magnitude of torque of this force about a point (2,3,4) is____ (Round off to the nearest integer)

28. In an electrical circuit, a battery is connected to pass 20C of charge through it in a certain given time. The potential difference between two plates of the battery is maintained at 15V. The work done by the battery is ____J.

29. If the highest frequency modulating a carrier is 5kHz, then the number of AM broadcast stations accommodated in a 90kHz bandwidth are _____

30. An unpolarized light beam is incident on the polarizer of a polarization experiment and the intensity of light beam emerging form the analyzer is measured as 100 Lumens, Now, if the analyzer is rotated around the horizontal axis (direction of light) by 30^0 in clockwise direction, the intensity of emerging light will be _____Lumens

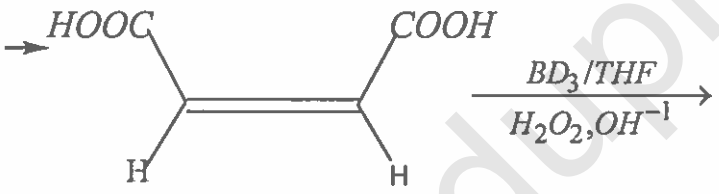
CHEMISTRY

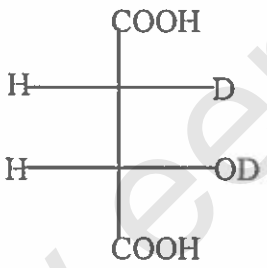
Section –A

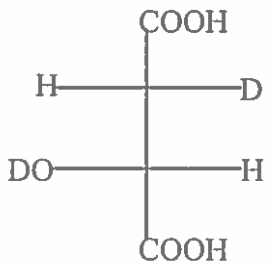
(SINGLE CORRECT ANSWER TYPE)

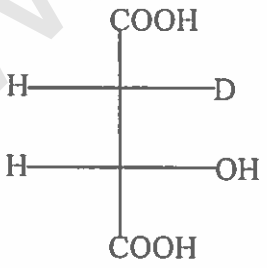
31. 20 ml of a solution containing $FeSO_4$ and $Fe_2(SO_4)_3$ is acidified with con H_2SO_4 and reduced with Zn; This solution required 30ml of $\frac{N}{10}K_2Cr_2O_7$ for oxidation. Before reduction with Zn, 20ml of the same solution required 20ml of $\frac{N}{10}K_2Cr_2O_7$ for oxidation. The molarities of $FeSO_4$ and $Fe_2(SO_4)_3$ respectively are

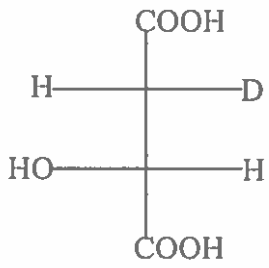
1) 0.1 , 0.05 2) 0.05, 0.025 3) 0.1 , 0.1 4) 0.1 , 0.025

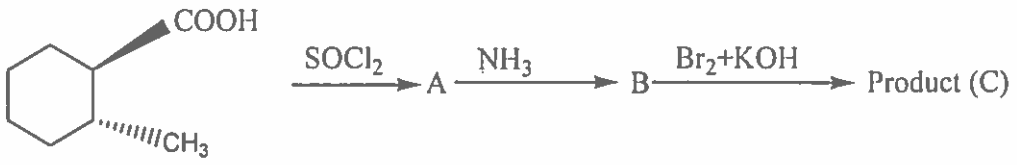
32. 

1) 

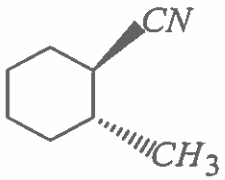
2) 

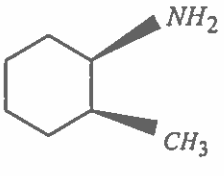
3) 

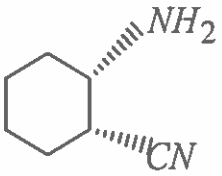
4) 

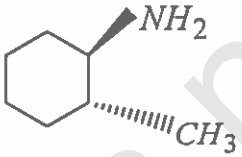
33. 

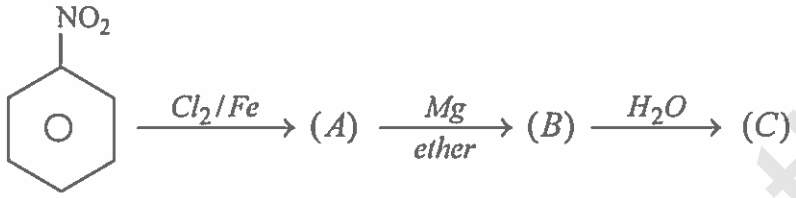
The product C is

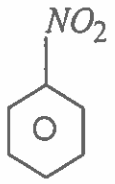
1) 

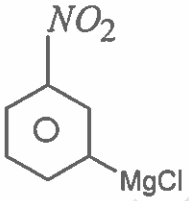
2) 

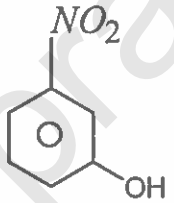
3) 

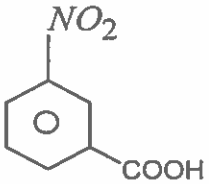
4) 

34. 

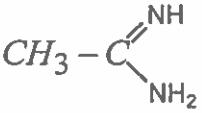
1) 

2) 

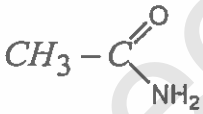
3) 

4) 

35. The correct order of basicity of the following compounds is

a) 

b) CH_3NH_2 c) $(CH_3)_2NH$

d) 

e) CH_3CN

1) $b > a > c > d > e$ 2) $a > c > b > d > e$

3) $c > a > b > e > d$ 4) $a > c > b > e > d$

36. Assertion A: Sulphanilic acid exists as zwitter ion.

Reason R. Sulphanilic acid is soluble in NaOH but not in HCl.

1) if both A and R are correct and R is the correct reason of A.

2) if both A and R are correct and R is not the correct reason of A.

3) if both A is correct and R is wrong

4) if A is wrong and R is correct

37. Match the following drug with their therapeutic action

i) Ranitidine	a) Antidepressant
ii) Nardil	b) Anti biotic
iii) Chloramphenicol	c) Antihistamines
iv) Dimetane	d) Antacid
	e) Analgesic

1) $i-d; ii-c; iii-a; iv-e$ 2) $i-d; ii-a; iii-b; iv-c$

3) $i-e; ii-a; iii-c; iv-d$ 4) $i-a; ii-c; iii-b; iv-e$

38. Hydrogen like ion has the wavelength difference between the first line of Balmer and Lyman series equal to 33.4 nm. The atomic number of that ion is

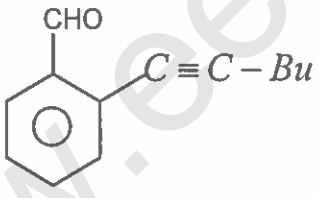
1) 4 2) 3 3) 2 4) 5

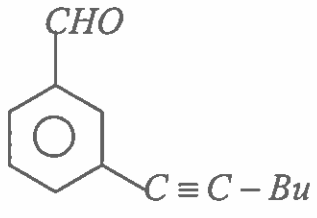
39. Compressibility factor of He as a real gas is

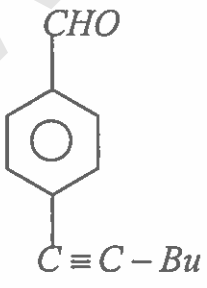
1) $1+\frac{Pb}{R}$ 2) $1-\frac{a}{RTV}$ 3) $1+\frac{Pb}{RT}$ 4) $\frac{RTV}{1-a}$

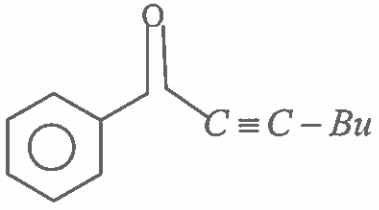
40. $Bu-C\equiv CH \xrightarrow{NaNH_2} A \xrightarrow[H_2O]{ph-CHO} B \xrightarrow{MnO_2} C$

Compound 'C' of the reaction is

1) 

2) 

3) 

4) 

41. Which of the following statements is False.

1) Two sucrose solutions of same molality prepared in different solvents will have the same freezing point depression.

2) Osmotic pressure of solution =MRT where M is Molarity of solution (for non electrolytic salts)

3) Correct order of osmotic pressure for 0.01 M aqueous solution of each compound is $BaCl_2 > KCl > CH_3COOH >$ urea. (At a given temperature)

4) Raoult's law states that the vapour pressure of a non volatile; non electrolyte compounds over a solution is proportional to its mole fraction

42. Mark incorrect statement regarding NH_4CN solution.

1)solution can act an buffer solution

2)For the solution $K_h = \frac{K\omega}{K_a.K_b}$

3)Solution involves only anionic hydrolysis.

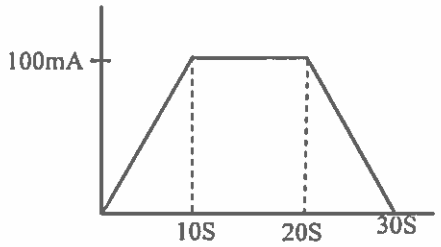
4)Degree of hydrolysis is equal to $(K_h)\frac{1}{2}$ approximately

43. $S_{(s)} + \frac{3}{2}O_{2(g)} \rightarrow SO_{3_{(g)}} + 2x\ kcal$; $SO_{2_{(g)}} + \frac{1}{2}O_{2(g)} \rightarrow SO_{3(g)} + y\,kcal$

Calculate heat of formation of $SO_{2(g)}$

1) $y-2x$ 2) $2x+y$ 3) $x+y$ 4) $2x-y$

44. In a Cu-Voltammeter , mass deposited in 30 seconds is 100grams. Carefully analyse the current – time graph shown below and identify incorrect statement



1) Electrochemical equivalent for cu is 50

2) A constant current of 66.667mA would also discharge the same amount in the same time

3) 33.33 grams got discharged in 10 seconds

4) 50 grams got discharged in 15 seconds.

45. The edge length of $LiCl$ is $4.0A^{\circ}$. It is in FCC structure, so radius of anion is
1) $2A^{\circ}$ 2) $1.414A^{\circ}$ 3) $1.732A^{\circ}$ 4) $1A^{\circ}$

46. Assertion (A): Some metals like platinum and palladium. can be used as storage media for hydrogen.
Reason (R): Platinum and palladium can absorb large volumes of hydrogen.
1) Assertion and reason both are correct and reason is correct explanation of assertion.
2) Assertion and reason both are wrong statements.
3) Assertion is correct statement both reason is wrong statement.
4) Assertion is wrong statement but reason is correct statement.

47. The common features among the species CN^{-}, CO , NO^{+} is/ are
1) Isoelectronic and weak field ligands
2) Bond order three and Isoelectronic
3) Bond order three and weak field ligands
4) Bond order two and π - acceptors

48. Assertion : Ethanol is a weaker acid than phenol.
Reason : Sodium ethoxide may be prepared by the reaction of ethanol with aqueous NaOH.
1)Assertion and reason both are correct and reason is correct explanation of assertion.
2) Assertion and reason both are wrong statements.
3) Assertion is correct statement but reason is wrong statement.
4) Assertion is wrong statement but reason is correct statement.

49. Select Complex which has all the following characteristics.
(i) Paramagnetic
(ii) No unpaired electrons in t_2g orbital
(iii) Exist in geometrical isomeric form.
1) $K_4\left[Fe(NO_2)_2(CN)_4\right]$ 2) $\left[Zn(NH_3)_4(en)\right]Cl_2$
3) $\left[Co(en)_3\right]Cl_3$ 4) $\left[Ni(NH_3)_2(en)_2\right]SO_4$

50. If 60% of a first order reaction was completed in 60 minutes, 50% of same reaction would be completed in approximately

1) 45 minutes 2) 60 minutes 3) 40 minutes 4) 50 minutes

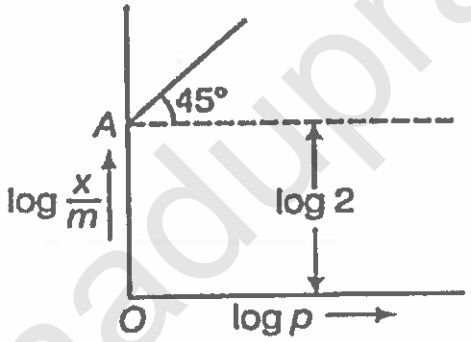
Section-B

(NUMERICAL VALUE TYPE)

51. Find the number of planar species from the following

CO_3^{-2} , $COCl_2$, SiO_4^{-4} , C_3O_2 , HCO_3^- , CS_3^{-2} and C_3S_2

52. A graph between $\log\left(\frac{x}{m}\right)$ and $\log p$ is a straight line at angle of 45° with intercept OA as shown in figure. $\left(\frac{x}{m}\right)$ at a pressure of 2 atm will be _____



53. During estimation of Nitrogen present in organic compound by kjeldahl's method, the ammonia evolved from 0.5g of organic compound is neutralised by 10ml 1M H_2SO_4. The % of Nitrogen in organic compound is _____

54. How many non radio active alkaline earth metals will give flame test

55. How many of the following can be used as flux in metallurgical procedure for removing acidic impurities Silica, NaCl, $CaCO_3$, Na_2SO_3 and P_2O_5

56. How many of the following are (+) vely charged colloids. Haemoglobin, starch , charcoal, clay, acidic dye stuffs, Al_2O_3, XH_2O, As_2S_3, Cu and CdS

57. The number of reducing sugars among the following is _____

Glucose, fructose, sucrose, starch, cellulose galactose, lactose and maltose

58. How many of the following polymers contain ester linkages :

Nylon, glyptal, Dacron, PVC and Buna-S polystyrene.

59. $PCl_3 + H_2O \rightarrow X + HCl.X$ is an oxoacid of phosphorus. X on strong heating at $200^\circ C$ gives Y and Z. Sum of oxidation numbers of phosphorous in Y and Z is _____

60. How many compounds will liberate chlorine gas from oxidation of HCl

MnO_2, $KMnO_4$, PbO_2, pb_3O_4, O_3, $K_2Cr_2O_7$ and $NaOCl$

MATHEMATICS

Section – A

(SINGLE CORRECT ANSWER TYPE)

61. Let $a = \{\ln(1+e)\} + \{\ln(1+e^2)\} + \{\ln(1+e^4)\} + \{\ln(1+e^8)\} + \ldots$ to ∞, where $\{x\}$ denotes fractional part function, then the value of e^a is

1) $\frac{e}{e-1}$ 2) $\frac{1}{e-1}$

3) $\frac{e}{e+1}$ 4) $\frac{1}{e+1}$

62. $\left(4\cos^2 9^0 - 3\right)\left(4\cos^2 27^0 - 3\right)\left(4\cos^2 81^0 - 3\right)\left(4\cos^2 243^0 - 3\right) =$

1) 1 2) $\tan 9^0$

3) $\tan 27^0$ 4) $\cot 9^0$

63. Let $f(x) = x + \sin x$ and $g(x) = (x + \sin x)\, e^{\sin\frac{x}{2}}$.

Statement I: $\lim\limits_{x\to\infty} \dfrac{f(x)}{g(x)} = \lim\limits_{x\to\infty} \dfrac{f'(x)}{g'(x)}$

Statement II: $\lim\limits_{x\to\infty} \dfrac{f'(x)}{g'(x)} = 0$

1)Statement-I is true; Statement –II is true; Statement –II is a correct explanation for Statement –I.

2) Statement-I is true; Statement –II is true; Statement –II is not a correct explanation for Statement –I.

3) Statement-I is false ; Statement –II is true.

4) Statement-I is true ; Statement –II is false.

64. If the two straight lines $\dfrac{x-\cos^2\alpha}{\cos^2\alpha} = \dfrac{y-\cos^2\beta}{\cos^2\beta} = \dfrac{z-\cos^2\gamma}{\cos^2\gamma}$ and $\dfrac{x-\sin^2\alpha}{1} = \dfrac{y-\sin^2\beta}{2} = \dfrac{z-\sin^2\gamma}{3}$ are coplanar, then the equation of the plane containing both the lines is

1) $6x-6y+2z=1$ 2) $x+y-z=0$

3) $x+4y-3z=1$ 4) $x-2y+z=0$

65. The statement 'If it rains tomorrow, then if I get paid, I will go to Paris.' is equivalent to

1)If neither it rains tomorrow nor I get paid, then I will not go to Paris.

2) Either it rains tomorrow and I get paid or I will not go to Paris.

3) It does not rain tomorrow or I do not get paid or I will go to Paris.

4) If it rains tomorrow or I get paid, then I will go to Paris.

66. An urn contains 5 white balls and 3 black balls. Balls are drawn from the urn one by one without replacement. Let E_1, E_2 and E_3 denote the events 'white ball on the 6th draw', '2nd black on the 3rd draw' and 'exactly 3 white balls in the first 6 draws' respectively, then which of the following is NOT CORRECT?

1) $P(E_1)=\frac{5}{8}$

2) $P(E_1' \cap E_2 \cap E_3)=\frac{1}{28}$

3) $P(E_1|E_2)=\frac{4}{5}$

4) $P(E_1 \cap E_2|E_3)=\frac{2}{5}$

67. Let $f:[0,\infty)\to[0,\infty)$ be a function satisfying $f(x)=xe^{-f(x)}\forall x\in[0,\infty)$, then which of the following is NOT CORRECT?

1) $f(x)$ is strictly increasing.

2) $\lim_{x\to\infty} f(x)=\infty$

3) $\lim_{x\to\infty} \frac{f(x)}{\ell nx}=\frac{1}{e}$

4) $f(x)$ is onto function.

68. Let $A=\begin{bmatrix}1&1&1\\1&2&2\\1&2&3\end{bmatrix}$ and $B=\begin{bmatrix}1&0&0\\1&1&0\\1&1&1\end{bmatrix}$, then

Statement I: There exists a matrix C such that $A=BC$, But $A\neq CB$

Statement II: $\det(B)\neq 0$ and $AB\neq BA$.

1) Statement-I is true; Statement –II is true; Statement –II is a correct explanation for Statement –I.

2) Statement-I is true; Statement –II is true; Statement –II is not a correct explanation for Statement –I.

3) Statement-I is false ; Statement –II is true.

4) Statement-I is true ; Statement –II is false.

69. Let all the 2022 zeroes of the polynomial $P(x) = a_{2022}x^{2022} + a_{2021}x^{2021} + \ldots + a_1x + a_0$, where $a_{2022}, a_{2021}, \ldots, a_0 \in R$ *and* $a_{2022} \neq 0$, be real, distinct and less than 1. For any such polynomial, consider the function $f(x) = a_{2022}\frac{e^{2022x}}{2022} + a_{2021}\frac{e^{2021x}}{2021} + \ldots + a_1e^x + a_0x$, then

1) $f'(x)$ has 2022 real zeroes in $(-\infty, 0)$

2) $f''(x)$ has no real zero in $(0, \infty)$

3) f is decreasing in $(0, \infty)$

4) f is increasing in $(0, \infty)$

70. PQ is a variable chord of the ellipse $\frac{x^2}{4} + y^2 = 1$ that passes through the centre of the ellipse. Let $R(\alpha, \beta), \alpha, \beta \geq -1$, be a point on the line $x + 2y = 4$, such that P, Q and R are non-collinear. If P, Q, R are chosen such that the area of ΔPQR is maximum, then

1) $PQ = 2$

2) $PQ = \sqrt{\frac{26}{5}}$

3) Area of ΔPQR is $\sqrt{10}$

4) Area of ΔPQR is $\sqrt{26}$

71. A ray starts from the point A(2,6), gets reflected first by the line $2x - y = 3$ at B and then by the line $3x + y = 2$ at C and finally passes through A again. A circle S having centre at D touches the lines AB, BC and CA such that A and D are on the opposite sides of BC. Then the coordinates of point D are

1) $(1, -1)$ 2) $\left(\frac{3}{2}, \frac{-1}{2}\right)$ 3) $\left(\frac{5}{3}, \frac{-2}{3}\right)$ 4) $\left(\frac{7}{6}, \frac{-5}{6}\right)$

72. Let $f : R \to R$ be a function satisfying the differential equation $\frac{dy}{dx} + 2xy = \sin^2 x$ *and* $f(0) = 0$. Then which of the following is correct ?

1) f is an even function.

2) $\lim_{x \to \infty} f(x) = \frac{1}{2}$

3) $2f(x) = e^x$ has at least one real solution.

4) $f(4) < \frac{1}{2}$

73. The number of 5 digit natural numbers divisible by 12 and having digits in the set{0,1, 2,…,6} only is

1) 1029 2) 1372 3) 2058 4) 4116

74. Let $\alpha, \beta, where \alpha < \beta$, be the roots of the quadratic equation $x^2 - 4x + 2 = 0$. For each $n \in N$, define $S_n = \alpha^n + \beta^{n-1} + \alpha^{n-2} + \beta^{n-3}$. Then which of the following is correct ?

1) $\frac{S_{2024} + S_{2021}}{S_{2023}} = 7$ 2) $\frac{S_{2024} + 8\,S_{2021}}{S_{2022}} = 14$

3) $\frac{S_{2024} - 3\,S_{2023}}{S_{2022} - S_{2021}} = 3$ 4) $\frac{S_{2024} - 6\,S_{2022}}{S_{2023} - 2\,S_{2021}} = 3$

75. Let $z = i^{{}^{64}C_0} + i^{{}^{64}C_1} + i^{{}^{64}C_2} + ... + i^{{}^{64}C_{64}}$, where $i = \sqrt{-1}$, be a complex number. Then which of the following is correct ?

1) $\mathrm{Re}\left[(z-63)^{2022}\right] > 0$ 2) $\mathrm{Im}\left[(z-60)^{2022}\right] = 0$

3) $\mathrm{Re}\left[(z-61)^{2022}\right] < 0$ 4) $\mathrm{Im}\left[(z-59)^{2022}\right] > 0$

76. The area of the region

$\left\{(x,y) : 0 \le y \le \ell n\left(1+\sqrt{1-x^2}\right) - \sqrt{1-x^2} - \ell n\, x,\ 0 < x \le 1\right\}$ is

1) $\frac{\pi}{4}$ 2) $\frac{1}{e} + \frac{\pi}{4}$ 3) $\ell n\sqrt{2} + \frac{\pi}{4}$ 4) $e - \frac{\pi}{4} - 1$

77. Let $f, g : (1,\infty) \to R$ such that $f'(x) = \frac{\sqrt{\sqrt{x} + \sqrt{x-1}}}{1+\sqrt{x}}$ and

$g'(x) = \frac{\sqrt{\sqrt{x} - \sqrt{x-1}}}{1+\sqrt{x}}$ $\forall x > 1$ and $f(4) = -g(4)$. Then which of the following is NOT CORRECT ?

1) $f(9) = \frac{8\sqrt{2}}{3} - g(9)$ 2) $f(x) + g(x) > \frac{-8}{3}$ $\forall x > 1$

3) $\lim_{x \to 1^+} (f(x) + 2) = \lim_{x \to 1^+} \frac{2 - 3g(x)}{3}$

4) $(f'(x))^2 + (g'(x))^2 < \lambda$ for some $\lambda \in R$

78. The radius of the largest circle that touches the parabola $y^2 = x$ at $(1,1)$ such that no part of the circle in the first quadrant lies outside (above) the parabola is

1) $\frac{\sqrt{5}}{2}$ 2) $\sqrt{5}$ 3) $\sqrt{3}$ 4) $2\sqrt{3}$

79. Let ABCD be a square in the x y plane with vertex A in the first quadrant. A line intersects the lines BA, AD, DC and BC at points P(-2,4) , Q(0,2), R(1,1) and S(2,0) respectively. Then the vertex A lies on the line

1) $x + y = 4$ 2) $2x + y = 4$ 3) $x + 2y = 7$ 4) $3x + 2y = 8$

80. Area of the region bounded by the curve $2\sin^{-1} y = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$ and the x-axis is

1) $\frac{\pi}{4}$ sq. units 2) $\frac{\pi}{3}$ sq. units 3) $\frac{\pi}{2}$ sq. units 4) π sq. units

Section-B
(NUMERICAL VALUE TYPE)

81. Let $f:\left[-\sqrt{\frac{13}{3}},\sqrt{\frac{13}{3}}\right] \to R$ be a function such that

$$f(x) = \begin{vmatrix} x & \sqrt{19-x^2} & -1 \\ -\sqrt{3} & \sqrt{x^2+1} & \sqrt{3}x \\ \sqrt{15-x^2} & -1 & \sqrt{13-3x^2} \end{vmatrix} \forall\, x \in \text{dom}(f)$$. If the absolute maximum value of the function f is M, then the greatest integer less than or equal to M is ____

82. The number of relations on the set A = {1,2,3,4} that are reflexive and symmetric but not transitive is ___

83. Three children go to an ice-cream parlour, where three flavours of ice-creams are available. A child can have either two scoops of same flavour or one scoop each of two distinct flavours. In how many ways all three children can choose ice-creams so that each flavour is taken by at least one child?

84. Let a, b, c, d be four distinct non-zero numbers in A.P. If 9 is added to one of the numbers, so that the numbers form a G.P., then the value of $|a+b+c+d|$ is ___.

85. The sum of all positive real solutions to the equation

$2\cos 2x\left(\cos 2x-\cos\frac{36\pi^2}{x}\right)=\cos 4x-1$ is $k\pi$, then the value of k is ___.

86. Let $f(x)=[\tan^{-1}\left(\sin^{-1}x\right)+\sin^{-1}\left(\tan^{-1}x\right)]$, where [x] denotes the greatest integer function, then the number of integers in the range of f is

87. Let the sum of the coefficients of all the integral powers of x in the binomial expansion of $\left(\frac{4x+1}{2\sqrt{x}+2\sqrt[4]{x}+1}-\frac{2x\sqrt{x}+2}{x-\sqrt{x}+1}\right)^{10}$ be S, then the remainder when S^{2017} is divided by 7 is

88. Let $P\left(\alpha,\frac{1}{\alpha}\right)$, where $0<\alpha<1$, be any point on the hyperbola xy = 1. Tangent to the hyperbola at P intersects the coordinate axes at Q and R and normal at P intersects the coordinate axes at M and N (Q and M lie on x-axis). If ΔOQR and ΔOMN have equal area, then $(QR)^4$ is equal to (Where O is origin)

89. The mean of a data consisting of five distinct integers is also an integer. If their variance lies in the interval (5,6), then their mean deviation about the mean is equal to

90. Consider the following functions:

$$f_0(x)=\begin{cases} x^{4/3}\sin\frac{1}{x}, & x\neq 0 \\ 0, & x=0 \end{cases}, f_1(x)=\begin{cases} \int_0^x x\left[\frac{1}{x}\right]dx, & x\neq 0 \\ 0, & x=0 \end{cases}, f_2(x)=\left|6^x-3^x-2^x+1\right| \text{ and}$$

$f_3(x)=\{\sin x\}+[\tan x]+[\sec x]$,

where [x] denotes G.I.F and {x} denotes F.P.F. Then the sum of all the values of $k\in\{0,1,2,3\}$ for which f_k is differentiable at $x=0$ is____

KEY SHEET

PHYSICS

1	**2**	2	**3**	3	**3**	4	**2**	5	**3**
6	**2**	7	**3**	8	**4**	9	**2**	10	**3**
11	**1**	12	**1**	13	**4**	14	**1**	15	**4**
16	**4**	17	**1**	18	**3**	19	**2**	20	**3**
21	**300**	22	**2**	23	**2**	24	**5**	25	**2**
26	**25**	27	**20**	28	**300**	29	**9**	30	**75**

CHEMISTRY

31	**4**	32	**3**	33	**4**	34	**1**	35	**2**
36	**3**	37	**2**	38	**1**	39	**3**	40	**4**
41	**1**	42	**3**	43	**1**	44	**3**	45	**2**
46	**1**	47	**2**	48	**3**	49	**4**	50	**1**
51	**6**	52	**4**	53	**56**	54	**3**	55	**1**
56	**2**	57	**5**	58	**2**	59	**2**	60	**7**

MATHEMATICS

61	**1**	62	**1**	63	**3**	64	**4**	65	**3**
66	**4**	67	**3**	68	**1**	69	**2**	70	**2**
71	**1**	72	**4**	73	**2**	74	**2**	75	**3**
76	**1**	77	**3**	78	**2**	79	**4**	80	**1**
81	**72**	82	**49**	83	**138**	84	**14**	85	**39**
86	**4**	87	**6**	88	**128**	89	**2**	90	**6**

SOLUTIONS AND HINTS

PHYSICS

1. Path diff $= S_2P - S_1P = d\sin\theta$ $\Delta = (2N+1)\frac{\lambda}{2}$

2. $Lin(dB) = 10\log\left(\frac{I}{I_0}\right)$ $I = \frac{(\Delta P_0)^2}{2\rho V}$

3. $dp = -dh\rho g$ $p = \frac{\rho RT}{M}, dp = \frac{\rho R}{M}dT$ $\frac{dT}{dh} = \frac{-Mg}{R}$

4. $\Delta Q = nC_p\Delta T_1 \rightarrow (1)$
   $\Delta Q = nC_v\Delta R_2 \rightarrow (2)$
   $(1) = (2)$ $\Delta T_2 = 42K$

5. For be coming black hole $Ve > C\sqrt{\frac{2GM}{r}} \geq 3\times10^8 m/s$

6. $T = 2\pi\sqrt{\frac{I}{MB}}$

7. $a = \sin g\alpha - \mu g\cos\alpha$ $a = V\frac{dx}{dx}$ $V_{\max} = \sqrt{\frac{g\sin^2\alpha}{b\cos\alpha}}$ $V = \sqrt{2gx\sin\alpha - bgx^2\cos\alpha}$

   $a = 0$ $\mu = bx$
   $g\sin\alpha - \mu g\cos\alpha = 0$
   $\left(x = \frac{\tan\alpha}{b}\right)$

8. Potential at common centre is

   $V_C = \frac{Kq_A}{a} + \frac{Kq_B}{b}$
   $= \frac{\sigma}{\varepsilon_0}(a+b)$

9. $\frac{1}{2}C V_m^2 = \frac{1}{2}LI_m^2, \omega = \frac{1}{\sqrt{LC}} = 1.33KHZ$

10. $W=\int_0^{H/2} Fdy \quad ; \quad F-mg=ma \quad V\frac{dv}{dy}=g(1-2by) \quad H=\frac{1}{b}$

11. For linear motion $J=MV_c$ For angular motion $J\left(\frac{\ell}{2}\right)=\frac{M\ell^2}{12}.\omega_c$

$t=\frac{2\pi}{\omega_c} \quad S=V_c^t=\frac{\pi\ell}{3}$

12. Applications of Kirchhoff's law

13. By Gause law on closed surface $E_2S-E_1S=\frac{q_B}{\varepsilon_0}$ E=ρJ = P$\frac{I}{A}$

14. Concept of negative resistance

15. Position of final image

16. $t_1=\frac{\theta}{\omega e}=\frac{2\pi-\theta}{\omega s}\frac{2\pi}{\omega_c+\omega_s}=\frac{2\pi}{\frac{2\pi}{24}+\frac{2\pi}{1.5}}=1.4hr$

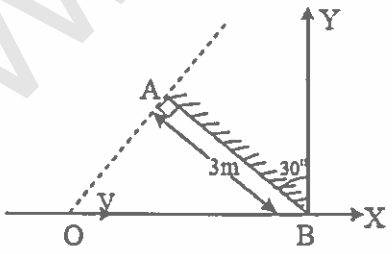
17. $V=\frac{x}{8.9\rho_\omega}+\frac{36-x}{19.3\rho_\omega} \quad V\rho_\omega g=2g$

18. $C=\frac{\varepsilon_0 A}{d} \quad V_f=\frac{q^2}{2C}$

19. $T_{1/2}=\frac{t_1t_2}{t_1+t_2}=4.8h$

20. $t_A=\frac{\ell}{v+u}+\frac{\ell}{v-u} \quad t_B=\frac{\ell}{\sqrt{v^2-u^2}}+\frac{\ell}{\sqrt{v^2}-u^2} \quad \frac{t_A}{t_B}=\frac{5}{3}$

21.



$\therefore 2\times t=\left(\frac{3}{\cos 60^0}\right)\times 100\,\text{t} = 300$ seconds

22. $V = \sqrt{\frac{T}{\mu}}$

23. $I = \frac{P}{4\pi r^2} = \frac{1}{2}\varepsilon_0 E_0{}^2 C$

24. Conceptual

25. $Y = \frac{F/A}{x/L}$

26. $i = \frac{5}{2000} = 25mA$

27. $\vec{\tau} = \left(\vec{r_2} - \vec{r_1}\right) \times \vec{F}$

28. $W = \Delta qV$

29. $B.W = 2fm = 10KHZ$ No. of channel $= \frac{90KHZ}{10KHZ} = 9$

30. $I_R = I_0 \cos^2\theta$

CHEMISTRY

31. number of moles of $FeSO_4 = a$ number of moles of $Fe_2(SO_4)_3 = b$

$a + 2b = 3$ $a = 2$

32. Syn – addition –of cis alkene gives Meso compounds (or) erythro racemate

33. Acid $\xrightarrow{SOCl_2}$ acid chloride $\xrightarrow[-HCl]{NH_3}$ acid amide $\longrightarrow$ 1^0 amine

34. Hoffmann Bromamide reaction involving syn migration

A = Meta Chloro Nitrobenzene

B = Grignard reagent

C = Nitro benzene

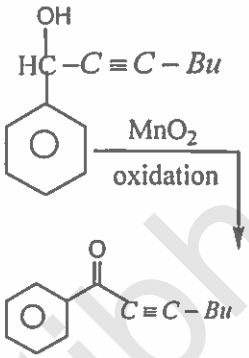
35. More the lone pair donor – more the basic nature.

36. Statement A is correct but R is wrong be+ cause sulphanilic acid is soluble in NaOH as well as in HCl

37. Drugs and their therapeutic action.

38. $\lambda_1 = \frac{36}{5RZ^2}; \lambda_2 = \frac{4}{3RZ^2}\ \Delta\lambda = \lambda_1 - \lambda_2\ 334 = \left(\frac{36}{5} - \frac{4}{3}\right)\frac{1}{R_H} \times \frac{1}{Z^2}\ Z = 4$

39. In vander waal's equation a is neglected $p(V-b) = RT$ Z is (+)ve

40. $Bu - C \equiv CH \xrightarrow{NaNH_2} Bu - C \equiv C - Na \xrightarrow[H_2O]{ph-CHO}$  $\xrightarrow[\text{oxidation}]{MnO_2}$

41. k_f changes with Solvent.

42. Solution undergoes both cationic and anionic hydrolysis. because it is the salt of ω.A and ω.B

43. $S + O_2 \rightarrow SO_2; \Delta H_f = ?$ .....(3) $\Delta H_f = \Delta H_1 - \Delta H_2 = -2x - (-y) = y - 2x$

44. Faraday's 1st law 50g will be discharged in 15 seconds

45. $\sqrt{2}a = 4r\ \sqrt{2}(4) = 4r\ r = 1.414$

46. Adsorption of hydrogen by some metals

47. Ligands and their electric structure

48. Acidic nature of alcohols and phenols

49. $3d^8 = t_{2g}^6 e_g^2$, $[M\ a_2(AA)_2]^{\pm n}$ Paramagnetic

2 G.I Possible

50. $\frac{K \times 60}{K \times t} = \frac{\log(100/40)}{\log(100/50)}$; $\frac{60}{t} = \frac{4}{3}$

Therefore, time t = 45 min

51. SiO_4^{-4} is tetrahedral

52. $\frac{x}{m} = kp^{\frac{1}{n}}$ slope$= \frac{1}{n} = 1$ So n=1 $\log k = \log 2$ So k=2 $\frac{X}{m} = 4$

53. $\% N = \frac{1.4(m.eq\ of\ NH_3)}{\omega_{0.C}} \frac{1.4 \times 10 \times 2}{0.5} = 56$

54. Three : Ca, Sr and Ba

55. $CaCO_3$ is basic and hence can be used as acidic flux

56. Haemoglobin, $Al_2O_3.XH_2O$

57. sucrose and polysaccharides are non reducing sugars

58. Glyptal, Dacron , PMMA

59. $PCl_3 + H_2O \rightarrow H_3PO_3 + HCl$; $H_3PO_3 \xrightarrow{200^0 C} H_3PO_4 + PH_3$

$5-3=2$

60. MnO_2, $KMnO_4$, PbO_2, O_3, $K_2Cr_2O_7$ and $NaOCl$

MATHEMATICS

61. $\because \ln e^k < \ln\left(1+e^k\right) < \ln\left(e^{k+1}\right) \forall k \in N$

$\therefore \left\{\ln\left(1+e^k\right)\right\} = \ln(1+e^k) - k = \ln\left(1+\frac{1}{e^k}\right) \forall\ k \in N$

$\therefore a = \ln\left(1+\frac{1}{e}\right) + \ln\left(1+\frac{1}{e^2}\right) + \ln\left(1+\frac{1}{e^4}\right) + \ldots$

$= \ln\left(\left(1+\frac{1}{e}\right)\left(1+\frac{1}{e^2}\right)\left(1+\frac{1}{e^4}\right)\ldots\right) = \ln\left(1+\frac{1}{e}+\frac{1}{e^2}+\frac{1}{e^3}+\ldots\right)$

$= \ln\left(\frac{1}{1-1/e}\right) = \ln\left(\frac{e}{e-1}\right)$

62. $\therefore\ 4\cos^2\theta - 3 = \dfrac{4\cos^3\theta - 3\cos\theta}{\cos\theta} = \dfrac{\cos 3\theta}{\cos\theta}$

$$\frac{\cos 27^0}{\cos 9^0}\cdot\frac{\cos 81^0}{\cos 27^0}\cdot\frac{\cos 243^0}{\cos 81^0}\cdot\frac{\cos 729^0}{\cos 243^0} = \frac{\cos 729^0}{\cos 9^0} = 1$$

63. $\lim\limits_{x\to\infty}\dfrac{f(x)}{g(x)}$ does not exist.

$$\lim_{x\to\infty}\frac{f'(x)}{g'(x)} = \lim_{x\to\infty}\frac{1+\cos x}{(1+\cos x)e^{\sin\frac{x}{2}} + (x+\sin x)e^{\sin\frac{x}{2}}\frac{1}{2}\cos\frac{x}{2}}$$

$$= \lim_{x\to\infty}\frac{4\cos\frac{x}{2}}{e^{\sin\frac{x}{2}}\left(4\cos\frac{x}{2} + x + \sin x\right)} = 0$$

64. First line passes through (0,0,0). Again $(\cos^2\alpha, \cos^2\beta, \cos^2\gamma)$ and $(\sin^2\alpha, \sin^2\beta, \sin^2\gamma)$ lie in the plane and so their mid point $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$ also lies in the plane. Hence equation of the plane is

$$\begin{vmatrix} x-0 & y-0 & z-0 \\ \frac{1}{2}-0 & \frac{1}{2}-0 & \frac{1}{2}-0 \\ 1 & 2 & 3 \end{vmatrix} = 0 \quad \Rightarrow x - 2y + z = 0$$

65. Let p: It rains tomorrow, q: I get paid and r: I will go to Paris. Then the given statement is $p \to (q \to r)$.

$$p \to (q \to r) \equiv p \to (\sim q \vee r) \equiv \sim p \vee (\sim q \vee r)$$

66. $P(E_1 \cap E_2 | E_3) = \dfrac{n(E_1 \cap E_2 \cap E_3)}{n(E_3)} = \dfrac{2\times 2}{{}^6C_3} = \dfrac{1}{5}$

67. Let y = f(x), then $x = ye^y$ $\lim\limits_{x\to\infty}\frac{f(x)}{\ell nx} = \lim\limits_{y\to\infty}\frac{y}{\ell n(ye^y)} = \lim\limits_{y\to\infty}\frac{y}{\ell ny + y} = 1$

68. $\because \det(B) \neq 0$

$\therefore A = BC \Leftrightarrow C = B^{-1}A$

$A = CB \Leftrightarrow C = AB^{-1}$

$B^{-1}A = AB^{-1} \quad \Leftrightarrow B\left(B^{-1}A\right) = BA\,B^{-1} \quad \Leftrightarrow AB = BA$

69. $\therefore P(x)$ has 2022 zeros in $(-\infty,1)$ $\therefore$ using Rolle's theorem $P'(x)$ also has all zeros in $(-\infty,1)$. Now $f'(x) = P(e^x)$ and $f''(x) = P'(e^x)e^x$ but $e^x > 1 \forall x \in (0,\infty)$ $\therefore f'(x)$ and $f''(x)$ have no zero in $(0,\infty)$

70. Let $P = (2\cos\theta, \sin\theta), Q = (-2\cos\theta, -\sin\theta)$ and $R = (4-2\beta, \beta), \beta \in \left[-1, \frac{5}{2}\right]$

then area of $\Delta PQR = \frac{1}{2}\left|\begin{vmatrix} 2\cos\theta & \sin\theta & 1 \\ -2\cos\theta & -\sin\theta & 1 \\ 4-2\beta & \beta & 1 \end{vmatrix}\right|$

$= \left|-2\beta\cos\theta + (4-2\beta)\sin\theta\right| \le \sqrt{(2\beta)^2 + (4-2\beta)^2} = 2\sqrt{2}\sqrt{(\beta-1)^2+1}$

$\le 2\sqrt{10}$ (taking $\beta = -1$).

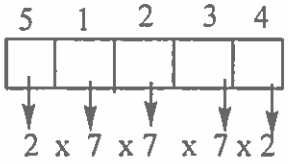
$\therefore P = \left(\frac{2}{\sqrt{10}}, \frac{3}{\sqrt{10}}\right), Q = \left(\frac{-2}{\sqrt{10}}, \frac{-3}{\sqrt{10}}\right).$

71. The given lines are external bisectors of angles ABC and ACB respectively and so D is the point of intersection of the given lines.

72. Integrating factor $= e^{x^2}$

$\therefore f(x) = e^{-x^2}\int_0^x e^{x^2}\sin^2 x\,dx < e^{-x^2}\int_0^x e^{x^2}.x\,dx \;\; \forall\, x > 0$

$\therefore f(x) < \frac{1}{2} \forall x > 0$

73. 

Fill the digits in the given order

74. $S_{n+2} - 4S_{n+1} + 2S_n = 0 \ \forall \ n \in N$

so, $S_{2024} = 4S_{2023} - 2S_{2022}$ and $S_{2023} = 4S_{2022} - 2\,S_{2021}$

75. $z = 61 + 2i$

76. $y = \ell n\left(1+\sqrt{1-x^2}\right) - \sqrt{1-x^2} - \ell nx$

$\frac{dy}{dx} = -\frac{\sqrt{1-x^2}}{x}$ $\therefore$ required area $= \int_0^1 y.1\,dx = [yx]_0^1 - \int_0^1 \frac{dy}{dx}.x\,dx$

$= 0 + \int_0^1 \sqrt{1-x^2}\,dx = \frac{\pi}{4}$

77. Let $h(x) = f(x) + g(x) = \int \frac{\sqrt{\sqrt{x}+\sqrt{x-1}} + \sqrt{\sqrt{x}-\sqrt{x-1}}}{1+\sqrt{x}}dx$, then $h(4) = 0$

put $t = \sqrt{\sqrt{x}+\sqrt{x-1}} + \sqrt{\sqrt{x}-\sqrt{x-1}}$, then $t^2 = 2\sqrt{x} + 2$

$\therefore h(x) = 2\int (t^2 - 2)dt = 2\left(\frac{t^3}{3} - 2t\right) + C \therefore h(x) = \frac{4\sqrt{2}}{3}\sqrt{\sqrt{x}+1}\left(\sqrt{x}-2\right)$

78. The largest circle will pass through the vertex of the parabola.

79. Let slope of AB be m, then equations of sides AB, BC, CD, DA are

$y - 4 = m(x+2), y - 0 = \frac{-1}{m}(x-2), y - 1 = m(x-1)$ and $y - 2 = \frac{-1}{m}(x-0)$

respectively. $AD = AB \Rightarrow \frac{|(2m+4)-(1-m)|}{\sqrt{1+m^2}} = \frac{|2m-2|}{\sqrt{1+m^2}}$

$\Rightarrow m = -5 \ or \ \frac{-1}{5}$ For $m = -\frac{1}{5}$, $A = \left(\frac{4}{13}, \frac{46}{13}\right)$

80. Putting $x=\sin\theta, \theta\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

$$y=\begin{cases}-\sqrt{1-x^2}, -1\le x\le\frac{-1}{\sqrt{2}}\\ x,\ \frac{-1}{\sqrt{2}}\le x\le\frac{1}{\sqrt{2}}\\ \sqrt{1-x^2}, \frac{1}{\sqrt{2}}\le x\le 1\end{cases}$$ $\therefore$ required area $=\left(\frac{\pi}{8}\times 1^2\right)=\frac{\pi}{4}$ sq.units.

81. Take columns of f as $\vec{a},\vec{b},\vec{c}$ respectively and use $\vec{a}.\vec{b}\times\vec{c}\le\left|\vec{a}\right|\left|\vec{b}\right|\left|\vec{c}\right|$.

so $f(x)\le\sqrt{x^2+3+15-x^2}\times\sqrt{19-x^2+x^2+1+1}\times\sqrt{1+3x^2+13-3x^2}$

$=42\sqrt{3}=f(\sqrt{3})$.

82. The number of relations that are reflexive and symmetric $=2^{\frac{4^2-4}{2}}=2^6=64$

The number of equivalence relations on the set A $=1+4+3+6+1=15$

$\therefore$ No. of relations that are reflexive and symmetric but not transitive

$=64-15=49$

83 $6^3-{}^3C_1.3^3+{}^3C_2.1^3=138$

84. Let $b=a+x, c=a+2x, d=a+3x$. 9 must be added to b or c.

Let $a, a+x+9, a+2x, a+3x$ be in G.P., then

$$\frac{a+x+9}{a}=\frac{a+2x}{a+x+9}=\frac{a+3x}{a+2x}$$

$\Rightarrow x=3, a=-8 \quad \therefore |a+b+c+d|=14$

85. $2\cos^2 2x-2\cos 2x\cos\frac{36\pi^2}{x}=-2\sin^2 2x$

$$\Rightarrow\cos\left(2x+\frac{36\pi^2}{x}\right)+\cos\left(2x-\frac{36\pi^2}{x}\right)=2$$

$\Rightarrow 2x+\frac{36\pi^2}{x}=2m\pi \text{ and } 2x-\frac{36\pi^2}{x}=2n\pi$

$\Rightarrow 2x=(m+n)\pi=p\pi, \text{and}\frac{36\pi^2}{x}=(m-n)\pi=q\pi,$

where $p=m+n$ and $q=m-n$

$\therefore 2x.\frac{36\pi^2}{x}=pq\pi^2 \therefore x\in\{\pi,2\pi,3\pi,6\pi,9\pi,18\pi\}.$

86. Range of $f=\{-2,-1,0,1\}$

87. $\therefore\left(\frac{4x+1}{2\sqrt{x}+2\sqrt[4]{x}+1}-\frac{2x\sqrt{x}+2}{x-\sqrt{x}+1}\right)^{10}=\left(2\sqrt{x}-2\sqrt[4]{x}+1-2\left(\sqrt{x}+1\right)\right)^{10}$

$=\left(1+2\sqrt[4]{x}\right)^{10}=\sum_{r=0}^{10}{}^{10}C_r.2^r x^{r/4}$

$\therefore S={}^{10}C_0.2^0+{}^{10}C_4.2^4+{}^{10}C_8.2^8=7k-1,k\in 1$

$\therefore S^{2017}=(7k-1)^{2017}=7m+(-1)^{2017},m\in 1 \therefore$ Remainder = 6.

88. Equation of tangent to the hyperbola at P is $\frac{x.\frac{1}{\alpha}+y\alpha}{2}=1$

$\therefore Q=(2\alpha,0) and\ R=\left(0,\frac{2}{\alpha}\right)$ Equation of normal to the hyperbola at P is

$\frac{x-\alpha}{\frac{1}{\alpha}}=\frac{y-\frac{1}{\alpha}}{\alpha} \therefore M=\left(\frac{\alpha^4-1}{\alpha^3},0\right) \text{and } N=\left(0,\frac{1-\alpha^4}{\alpha}\right.$

Now area of ΔOQR = area of $\Delta OMN \Rightarrow \frac{1}{2}.2\alpha.\frac{2}{\alpha}=\frac{1}{2}\left|\frac{a^4-1}{\alpha^3}.\frac{1-\alpha^4}{\alpha}\right|=2$

$\Rightarrow\alpha=\sqrt{\sqrt{2}-1}$

89. The variance is minimum when the data consists of five consecutive integers and increases when the integers are scattered away from the mean. So the integers can be taken to be $-3,-2,0,2,3$.

90. All the four functions are differentiable at $x=0$.