JEE-MAIN

MODEL GRAND TEST

IMPORTANT INSTRUCTIONS:

1. Immediately fill in the Admission number on this page of the Test Booklet with **Blue/Black** Ball Point Pen only.
2. The candidates should not write their Admission Number anywhere (except in the specified space) on the Test Booklet/ Answer Sheet.
3. The test is of **3 hours** duration.
4. The Test Booklet consists of 90 questions. The maximum marks are **300.**
5. There are **three** parts in the question paper 1,2,3 consisting of **Physics, Chemistry** and **Mathematics** having **30 questions** in each subject and subject having **two sections.**
   (I) Section –I contains 20 **multiple choice** questions with only one correct option.
   Marking scheme: +4 for correct answer, 0 if not attempt and -1 in all other cases.
   (II) Section-II contains 10 **Numerical Value Type** questions. Attempt any 5 questions only, if more than 5 questions attempted, First 5 attempted questions will be considered.

- The Answer should be within **0 to 9999.** If the Answer is in **Decimal** then round off to the **nearest Integer** value (Example i,e. If answer is above **10** and less than **10.5** round off is **10** and If answer is from **10.5** and less than **11** round off is **11).**
  To cancel any attempted question bubble on the question number box.
  For example: To cancel attempted question 21. Bubble on 21 as shown below

Question Answered for Marking **Question Cancelled for Marking**

Marking scheme: +4 for correct answer, 0 if **not attempt** and -1 in all other cases.

6. Use **Blue / Black Point Pen only** for writing particulars / marking responses on the Answer Sheet. **Use of pencil is strictly prohibited.**
7. No candidate is allowed to carry any textual material, printed or written, bits of papers, mobile phone any electron device etc, except the Identity Card inside the examination hall.
8. Rough work is to be done on the space provided for this purpose in the Test Booklet only.
9. On completion of the test, the candidate must hand over the Answer Sheet to the invigilator on duty in the Hall. **However, the candidate are allowed to take away this Test Booklet with them.**
10. **Do not fold of make any stray marks on the Answer Sheet**

PHYSICS

Section –A

(SINGLE CORRECT ANSWER TYPE)

1. A lead bullet penetrates into a solid object and melts, assuming that 40% of it's kinetic energy is used to heat it the intial speed of bullet is

   [intial temperature of bullet = 127^0C, melting point of the bullet is $=327^0C$]

   Latent heat of fusion of lead = $2.5\times10^4 Jkg^{-1}$

   Specific heat capacity of lead = $125J/kgK$

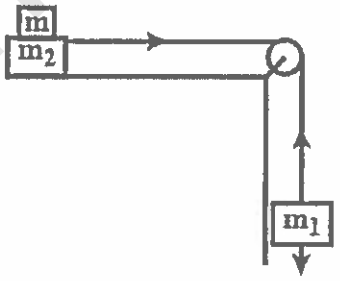
   (1) $125\,ms^{-1}$ (2) $500\,ms^{-1}$ (3) $250\,ms^{-1}$ (4) $600\,ms^{-1}$

2. For LED's to emit light in visible region of electromagnetic light. It should have energy band gap in the range of

   (1) 0.1 eV to 0.4 Ev (2) 0.5 eV to 0.8 eV

   (3) 0.9 eV to 1.6 eV (4) 1.7 eV to 3.0 eV

3. Two masses $m_1=5kg$ and $m_2=10kg$, connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15, the minimum weight "m" that should be put on top of m_2 to stop the motion is



   (1) 18.3 kg (2) 23.3 kg (3) 43.3 kg (4) 10.3 kg

4. Two objects of equal masses placed at certain distance from each other attracts each other with a force of F. If one-third mass of one object is transferred to the other object, then the new force will be

   (1) $\frac{2}{9}F$ (2) $\frac{16}{9}F$ (3) $\frac{8}{9}F$ (4) F

5. If a ball of steel $\left(d_b = 7.8 gcm^{-3}\right)$ attains a terminal velocity of $10 cm s^{-1}$ when falling in water $\left[\eta_{water} = 8.5 \times 10^{-4} Pa.s\right]$ then its terminal velocity in glycerine $\left[d_g = 1.2 gm\, xm^{-3}, \eta = 13.2 Pa.s\right]$ would be nearly

(1) $6.25 \times 10^{-4} cm s^{-1}$ (2) $6.45 \times 10^{-4} cm s^{-1}$

(3) $1.5 \times 10^{-5} cm s^{-1}$ (4) $1.6 \times 10^{-5} cm s^{-1}$

6. In a typical combustion engine the work done by a gas molecule is given by $W = \alpha^2 \beta e^{\frac{-\beta x^2}{KT}}$, where X is the displacement, K is the Boltzmann constant and "T" is the temperature. If α, β are constants, dimensions of "α" will be

(1) $\left[MLT^{-1}\right]$ (2) $\left[M^0 LT^0\right]$ (3) $\left[MLT^{-2}\right]$ (4) $\left[M^2 LT^{-2}\right]$

7. According to kinetic theory of gases,

A) The motion of the gas molecules freezes at $0^o C$

B) The mean free path of gas molecules decreases if the density of the molecules increased

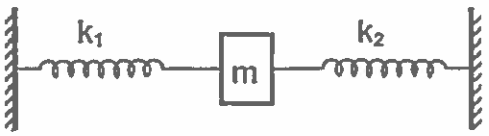
C) The mean free path of gas molecules increases if temperature is increased keeping pressure constant

D) Average kinetic energy per molecule per degree of freedom is $\frac{3}{2} K_B T$ (for monoatomic gases)

Choose the most appropriate answer from the options given below

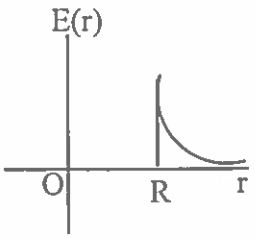
(1) A and C only (2) B and C only (3) A and B only (4) C and D only

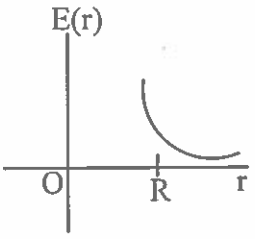
8. Two springs of force constant K_1 and K_2 are connected to a mass "m" as shown. The frequency of oscillation of the mass is "f". If both K_1 and K_2 are made four times their original values the frequency of oscillation becomes

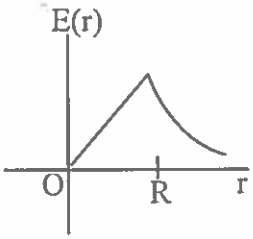


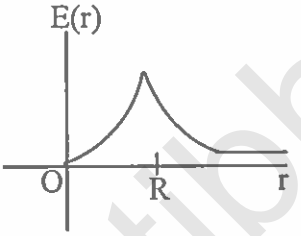
(1) $2f$ (2) $f/2$ (3) $f/4$ (4) $4f$

9. A thin spherical shell of radius "R" has charge "Q" spread uniformly over its surface which of teh following graphs most closely represents the electric field $E_{(r)}$ produced by the shell in the range $0 \le r \le \infty$, where "r" is the distance from the centre of teh shell?

(1) 

(2) 

(3) 

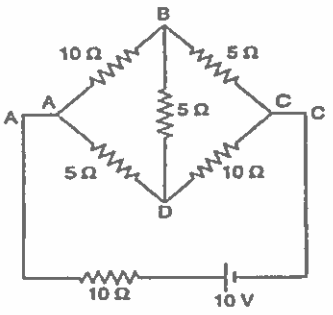
(4) 

10. A capacitor with capacitance $5\mu F$ is charged to $5\mu C$ if the plates are pulled apart to reduce the capacitance to $2\mu F$ how much work is done?

(1) $6.25\times10^{-6}J$ (2) $3.75\times10^{-6}J$

(3) $2.16\times10^{-6}J$ (4) $2.55\times10^{-6}J$

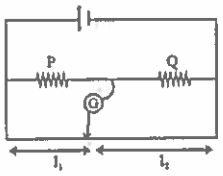
11. Determine the current in the branch AB of the network shown in figure



(1) $\frac{4}{17}A$ (2) $\frac{6}{17}A$

(3) $\frac{2}{17}A$ (4) $\frac{8}{17}A$

12. The meter bridge shown in balance position $\frac{P}{Q}=\frac{l_1}{l_2}$. If we now interchange the positions ofgalvanometer and cell, will the bridge work? If yes what will be balanced condition



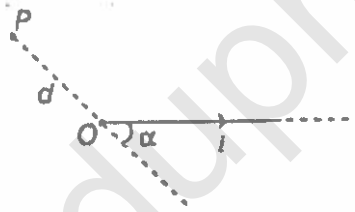
(1) yes, $\frac{P}{Q}=\frac{l_2-l_1}{l_2+l_1}$

(2) no, no null point

(3) yes, $\frac{P}{Q}=\frac{l_2}{l_1}$

(4) yes, $\frac{P}{Q}=\frac{l_1}{l_2}$

13. A steady current "i" flows through a wire with one end at O and the other end extending upto infinity as shown in the figure the magneticfield at a point "P" located at a distance "d" from "O" is



(1) $\frac{\mu_0 i}{4\pi d\cos\alpha}(1-\sin\alpha)$

(2) $\frac{\mu_0 i}{2\pi d\cos\alpha}(1-\sin\alpha)$

(3) $\frac{\mu_0 i}{4\pi d}$

(4) $\frac{\mu_0 i}{4\pi d\sin\alpha}(1-\cos\alpha)$

14. Assertion (A) : If is more difficult to push a magnet into a coil with more number of turns.
Reason (R) : The emf induced in a coil opposes the motion of a magnet when it is moved towards the coil.

(1) A is false, R is true

(2) Both A and R are true, R is correct explanation of A

(3) A is true, R is false

(4) Both A and R are true, R is not correct explanation of A

15. Photons of an electromagnetic radiation has an energy 11 KeV each. To which region of electromagnetic spectrum does it belong

(1) X-ray region (2) ultra violet region

(3) Infrared region (4) Visible region

16. A tansparent solid cylindrical rod has a refractive index of $\frac{2}{\sqrt{3}}$, it surrounded by air, A light ray is incident at the mid point of one end of the rod as shown in the figure. The incident angle θ for which the light ray grazes along the wall of the rod is

(1) $\sin^{-1}\left[\frac{\sqrt{3}}{2}\right]$ (2) $\sin^{-1}\left[\frac{2}{\sqrt{3}}\right]$ (3) $\sin^{-1}\left[\frac{1}{\sqrt{3}}\right]$ (4) $\sin^{-1}\left(1/2\right)$

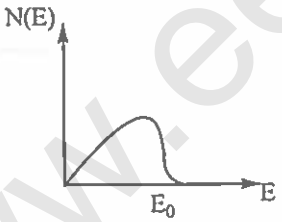
17. In double slit experiment, at a certain point on the screen, the path difference between the two interfering waves is $\frac{1}{8}$th of a wavelength. The ratio of the intensity of light at that point to that at the centre of a bright frindge is

(1) 0.853 (2) 0.672 (3) 0.568 (4) 0.760

18. If the kinetic energy of a free electron doubles, it's de- Broglie wavelength changes by the factor.

(1) 2 (2) $\frac{1}{2}$ (3) $\sqrt{2}$ (4) $\frac{1}{\sqrt{2}}$

19. The energy spectrum of β-particles [number N(E) as a function of β-energy E] emitted from a radioactive source is

(1)


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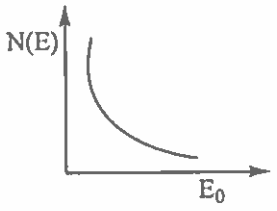

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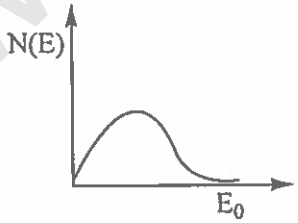

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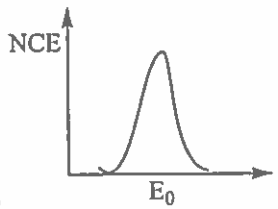
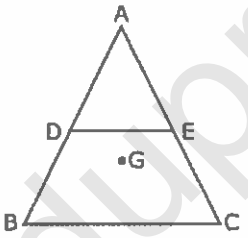


20. a goods train accelerating uniformely on a straight railway track, approaches an electric pole standing on the side of track, it's engine passes the pole with velocity "u" and the gaurd's room passes with velocity "V" then middle wagon of the train passes the pole with a velocity

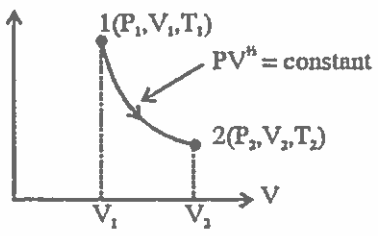
(1) $\frac{u+V}{2}$ (2) $\frac{1}{2}\sqrt{u^2+V^2}$ (3) $\sqrt{uV}$ (4) $\sqrt{\left[\frac{u^2+V^2}{2}\right]}$

Section-B
(NUMERICAL VALUE TYPE)

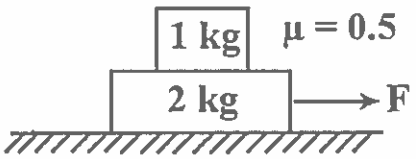
21. ABC is a plane lamina of the shape of an equilateral triangle DE are midpoints of AB and AC and G is the centroid of the lamina. Moment of inertia of the lamina about an axis passing through G and perpendicular to the plane ABC is I_0,. If part ADE is removed, the moment of inertia of the remaing part about same axis is $\frac{NI_0}{16}$ where N is an integer value of N is____



22. The radius of gyration of a cylindrical rod about an axis of rotation perpendicular to it's length and passing through the centre will be ____m. Given length of the rod is $10\sqrt{3}$ m
23. The force required to stretch a wire of cross section $1cm^2$ to double it's length will be (Young's modulus of the wire = $2\times10^{11}N/m^2$) _____$\times10^7 N$
24. Thermodynamic process is shown below on P – V diagram for one mole of an ideal gas. If $V_2 = 2V_1$ then the ratio of temperature $\frac{T_2}{T_1}$ is ($\sqrt{x}$) find the x value_____



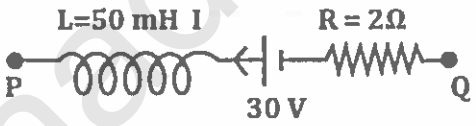
25. The coefficient of static fricition between two blocks is 0.5 and the table is smooth. The maximum horizontal force that can be applied to move the blocks together is _____ N (take $g = 10ms^{-2}$)



26. The energy dissipated by a resistor is 10 mJ in 1 s when an electric current of 2 mA flows through it. The resistance is _____Ω (Round off the nearest integer)

27. The value of aluminium susceptibility is 2.2×10^{-5} the percentage increase in the magnetic field if space with in a current carrying toroid is filled with aluminium is $\frac{X}{10^4}$ then the value of "X" is _____

28. A part of complete circuit is shown in the figure. At some instant, the value of I is 1A and it is decreasing at a rate of $10^2 As^{-1}$. The value of potential difference $V_P - V_Q$ (in volts) at that instant is _____



29. An antenna is placed in a dielectric medium of dielectric constant 6.25. if the maximum size of that antenna is 5.0 mm. It can radiate a signal of minimum frequency of _____ GHZ (given $\mu_r = 1$ for dielectric medium)

30. A tuning fork is vibrating at 250 HZ the length of the shortest closed organ pipe that will resonate with the running fork will be _____ cm (Speed of sound in air $340ms^{-1}$)

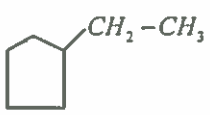
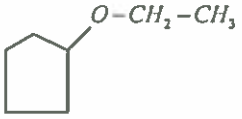
CHEMISTRY

Section –A

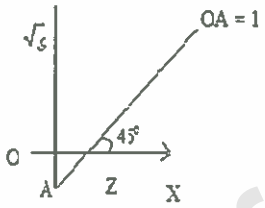
(SINGLE CORRECT ANSWER TYPE)

31.  $+Mg \xrightarrow[Ether]{dry} [A] \xrightarrow{Ethanol} P$

(major product)

1) 
2) 
3) 
4) 

32. The frequency of the characteristic x-ray of K_α line of metal tanget M' is 2500 cm^{-1} and the graph between $\sqrt{\vartheta}$ v_s 'Z' is as follows, then atomic number of M is



1) 49 2) 50 3) 51 4) 25

33. The correct order of Increasing $C-O$ bond length of CO, CO_3^{-2} & CO_2.

1) $CO_3^{-2} < CO_2 < CO$ 2) $CO_2 < CO_3^{-2} < CO$

3) $CO < CO_3^{-2} < CO_2$ 4) $CO < CO_2 < CO_3^{-2}$

34. Identify the pair in which the geometry of the spacious is T-shape and square planar respectively.

1) ClF_3, XeF_4 2) ICl_2^- and ICl_5 3) $XeOF_2$, $XeOF_4$ 4) ClF_3 and IO_4^-

35. The ground state energy in J, of hydrogen atom is – x. The minimum energy in J, required to promote on electron from n = 1 to n = 2 in He^+ is

1) $\frac{4x}{3}$ 2) $3x$ 3) $\frac{3x}{4}$ 4) $\frac{x}{3}$

36. Following four solutions are prepared by maxing different volumes of NaOH and HCl of different concentrations P^H of which one of them will be equal to 1.

1) $100\,ml\,\frac{M}{10}HCl + 100\,ml\,\frac{M}{10}NaOH$ 2) $75\,ml\,\frac{M}{5}HCl + 25\,ml\,\frac{M}{5}NaOH$

3) $60\,ml\,\frac{M}{10}HCl + 40\,ml\,\frac{M}{10}NaOH$ 4) $55\,ml\,\frac{M}{10}HCl + 45\,ml\,\frac{M}{10}NaOH$

37. $N_2 + O_2 \rightarrow 2NO \quad \Delta H = +Q_1$

$N_2 + 2O_2 \rightarrow N_2O_4 \quad \Delta H = -Q_2$

$NO + 1/2O_2 \rightarrow NO_2 \quad \Delta H = -Q_3$

$2NO_2 \rightarrow N_2O_4 \quad \Delta H = -Q_4$ then

1) $Q_2 = Q_1(-Q_3) + (-Q_4)$ 2) $-Q_2 = Q_1 - 2Q_3 - Q_4$

3) $Q_1 + Q_2 = (Q_3 + Q_4)$ 4) $Q_1 - Q_4 = Q_2 - Q_3$

38. For a reaction scheme $A \xrightarrow{K_1} B \xrightarrow{K_2} C$, if the Rate of formation of B is set to be zero then the concentration of B given by

1) $(K_1 + K_2)[A]$ 2) $\frac{K_1(A)}{K_2}$ 3) $K_1K_2[A]$ 4) $(K_2 - K_1)(A)$

39. For the given cell

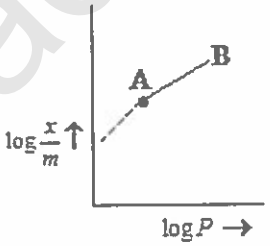
$Cu(s)\left|Cu^{+2}(C_1M)\right\|Cu^{+2}(C_2M)\left|Cu_{(s)}\right.$ change in Gibbs Energy (ΔG) is negative if

1) $C_1 = C_2$ 2) $C_2 = C_1/\sqrt{2}$ 3) $C_1 = 2C_2$ 4) $C_2 = \sqrt{2}C_1$

40. K_2HgI_4 is 40% ionised in aqous solutions. The value of it's van't hoff Factor (i) is

1) 1.8 2) 2.2 3) 1.6 4) 2.0

41. In freundlich adsorption Isotherm. Slope of AB line is



1) $\log\frac{1}{n} with (n<1)$ 2) $\frac{1}{n} with \left(\frac{1}{n} = 0 to 1\right)$

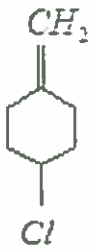
3) $\log n with (n>1)$ 4) n $with$ $(n, 0.1$ to $0.5)$

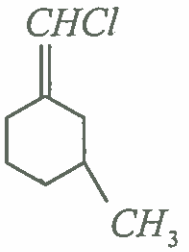
42. A solid having density of $9\times10^3 kg\ m^{-3}$ forms face centres cubic crystals of edge length $200\sqrt{2}pm$. What is the molar mass of the solid?

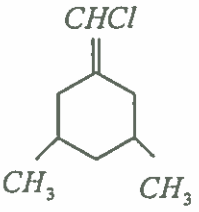
1) $0.0432\ kg\ mol^{-1}$ 2) $0.0216\ kg\ mol^{-1}$

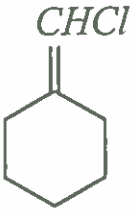
3) $0.4320\ kg\ mol^{-1}$ 4) $0.0305\ kg\ mol^{-1}$

43. Among the following compounds, geometrical isomerism is exhibited by

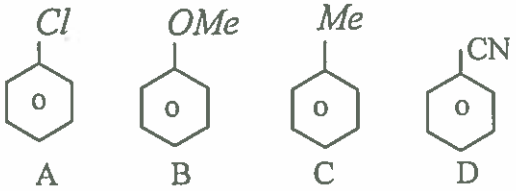
1) 

2) 

3) 

4) 

44. The Increasing order of Reactivity of the following compounds towards aromatic electrophonic substitution Reaction is



1) $D<B<A<C$ 2) $B<C<A<D$ 3) $A<B<C<D$ 4) $D<A<C<B$

45. The major product [R] in the following sequence of Reactions is

$$CH\equiv CH \xrightarrow[ii)\,CH_3-\underset{\displaystyle CH_3}{\underset{|}{CH}}-Br]{i)\,LINH_2\,/\,Either} P$$

$$P \xrightarrow[ii)\,NaBH_4]{HgSO_4\,/\,H_2SO_4} [Q]$$

$$(Q) \xrightarrow{H_2SO_4\ conc} [R]$$

1) $CH_2=\underset{\displaystyle CH_3}{\overset{}{C}}-CH_2-CH_3$

2) $\underset{\displaystyle CH_3}{\overset{CH_3}{\diagdown\!\diagup}}CH-CH=CH_2$

3) $\underset{\displaystyle CH_3}{\overset{CH_3}{\diagdown\!\diagup}}C=C\,(CH_3)2$

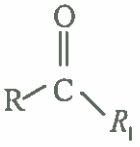
4) $\underset{\displaystyle CH_3}{\overset{CH_3}{\diagdown\!\diagup}}C=CH-CH_3$

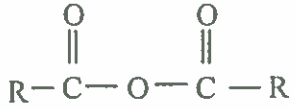
46. What is the work function of the metal if the light of wave length $4000A^0$ generates photo electrons of velocity $6\times10^5 ms^{-1}$?

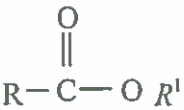
(mass of $e^- = 9\times10^{-31}, v = 3\times10^8 ms^{-1}, h = 6.6\times10^{-34} J\ 1ev = 1.602\times10^{-19} J$)

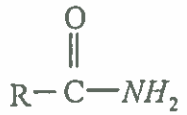
1) 0.9 ev 2) 4.0 ev 3) 2.1 ev 4) 3.1 ev

47. The correct order of their reactivity towards hydrolysis at room Temperature is

A) 

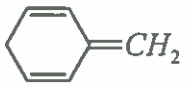
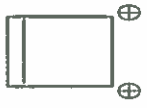
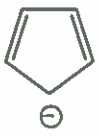
B) 

C) 

D) 

1) $D > A > B > C$ 2) $A > B > C > D$ 3) $A > C > B > D$ 4) $D > B > A > C$

48. Among the following the Aromatic compounds are

A)  B)  C)  D) 

Choose the correct answer from the following options.

1) B and C only 2) A and B only

3) A, B, C only 4) B, C, D only

49. Potassium permanganate on heating at 513 k gives a product which is

1) paramagnetic and colourless 2) diamagnetic and colourless

3) diamagnetic and green 4) paramagnetic and green

50. The number of Isomers possible for $\left[pt(en)(NO_2)_2\right]$ is -

1) 3 2) 2 3) 4 4) 1

Section-B
(NUMERICAL VALUE TYPE)

51. The P^H of the mixture of 25ml of 0.01M of CH_3COOH $\left(Ka\,of\,CH_3COOH = 5\times10^{-5}\right)$ and 25ml of 0.01M of $NaOH$ solutions is____

52. A container of just 10cc has 2.69×10^{20} gas molecules at O^0C what is the pressure Exerted _____

53. The volume (in ml) of 0.125M $AgNo_3$ required to quantitatively precipitate chloride ion 0.3 g of $\left[Co(NH_3)_6\right]cl_3$ is

$M\left(CO(NH_3)_6\right)cl_3 = 267.46\,gm$

$MAgNO_3 = 169.87\,gm$

54. Number of paramagnetic oxides among the following given oxides is _____

$Li_2O, CaO, Na_2O_2, KO_2, MgO, K_2O$

55. The Reaction of white phosphorous on Boiling with Alkali in Inert atmosphere resulted in the formation of product 'A' the Reaction of 1mole of 'A' with Excess of $AgNo_3$ in aqueous medium gives _____ moles of Ag.

56. A Xenon compound 'A' upon partial hydrolysis gives XeO_2F_2 the number of lone pair of electrons in compound 'A' is _____

57. The maximum prescribed concentration of copper in drinking water is _____ in PPM

58. The number of chairal carbon present in peptide $Ile - Arg - pro$, is _____

59. A copper plate of $200cm \times 10cm$ is to be plated with silver of 1 mm thickness on both the sides. Number of moles of silver required for the plating is _____ (density of $Ag = 10.8gml$)

60. $2MnO_4^- + bC_2O_4^{-2} + cH^{\oplus} \rightarrow xMn^{+2} + yCO_2 + ZH_2O$. If the above Equation is Balanced with integer co-efficients. The value of 'C' is __

MATHEMATICS

Section – A

(SINGLE CORRECT ANSWER TYPE)

61. If $x = \sec\theta - \cos\theta$ and $y = \sec^n\theta - \cos^n\theta$ and $(x^2+4)\left(\frac{dy}{dx}\right)^2 = K(y^2+4)$ then = ___

1) $\frac{1}{n^2}$ 2) 1 3) n 4) n^2

62. If p, q and r are three statements, then $p \rightarrow (q \rightarrow r) \leftrightarrow [(p \wedge q) \rightarrow r]$ is a

1) contingency 2) tautology 3) contradiction 4) None

63. The line $\frac{x}{k} = \frac{y}{2} = \frac{z}{-12}$ makes an isosceles triangle with planes $2x+y+3z-1=0$, $x+2y-3z-1=0$ then the value of k, is

1) 3 2) -2 3) 5 4) 0

64. The area of the region bounded by the straight lines $x=0, x=2$ and the curves $y=2^x, y=2x-x^2$ is

1) $\frac{3}{\log 2}-\frac{4}{3} sq.v$ 2) $\frac{3}{\log 2}+\frac{4}{3} sq.v$ 3) $\frac{3}{\log 2}-\frac{2}{3} sq.v$ 4) $\frac{3}{\log 2}-\frac{1}{2} sq.v$

65. The vector $\bar{c}$, directed along the internal bisector of the angle between the vectors $\bar{a}=7\bar{i}-4\bar{j}-4\bar{k}$ and $\bar{b}=-2\bar{i}-\bar{j}+2\bar{k}$ with $|\bar{c}|=5\sqrt{6}$ is _____

1) $\frac{5}{3}(\bar{i}-7\bar{j}+2\bar{k})$ 2) $\frac{5}{3}(5\bar{i}-5\bar{j}+2\bar{k})$ 3) $\frac{5}{3}(\bar{i}+7\bar{j}+2\bar{k})$ 4) $\frac{5}{3}(-5\bar{i}+5\bar{j}+2\bar{k})$

66. If $\underset{x\to 0}{Lt}\frac{xTan2x-2xTanx}{(1-Cos2x)^2}=\lambda$ then $8\lambda=$____

1) 1 2) 2 3) 6 4) 4

67. A straight line L with negative slope passes through the point (8,2) and cuts the positive coordinate axes at points P and Q. Find the absolute minimum value of OP + OQ as L varies, where 'O' is the origin

1) 18 2) -16 3) -14 4) 10

68. Tangents are drawn from any point on the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$ to the circle $x^2+y^2=9$. If the locus of the mid point of the chord of contact is $a(x^2+y^2)^2=bx^2-cy^2$, then the value of $a^2+b^2+c^2=$____

1) 7873 2) 7863 3) 7853 4) 8763

69. From the top of a light house 60 metres high with its base at the sea level, the angle of depression of a boat is 15^0. The distance of the boat from the foot of the light house is _____

1) $\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)60\ metres$ 2) $\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)60\ metres$ 3) $\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)^2 metres$ 4) $\frac{\sqrt{3}+1}{\sqrt{3}-1} metres$

70. The Pair of lines $lx^2+2(l+m)xy+my^2=0$ lies along two diameters of a circle and divides the circle into 4 sectors. If the area of bigger sector is 5 times the area of smaller sector, then $\frac{lm}{(l+m)^2}=$

1) $\frac{13}{12}$ 2) $\frac{11}{12}$ 3) $\frac{2}{\sqrt{3}}$ 4) $\frac{1}{2}$

71. If $\sqrt{1-c^2} = nc-1$ and $z = e^{i\theta}$ then $\frac{c}{2n}(1+nz)\left(1+\frac{n}{z}\right) =$

1) $1+2c\cos\theta$ 2) $1+c\cos\theta$ 3) $1+c\sin\theta$ 4) $1-2c\cos\theta$

72. If $a+b+c=3$ and $a>0, b>0, c>0$, then greatest value of $a^2b^3c^2$ is

1) $\frac{3^9.2^4}{7^7}$ 2) $\frac{3^{10}.2^4}{7^7}$ 3) $\frac{3^{11}.2^4}{7^7}$ 4) $\frac{3^{10}.2^4}{7^8}$

73. Number of non-negative integral solutions of equation $y^4+6xy^2-8x=0$ is equal to

1) 4 2) 3 3) 2 4) 1

74. For any positive integers m, n with $n \geq m$ and $\binom{n}{m} = n_{C_m}$ then $\binom{n}{m}+\binom{n-1}{m}+\binom{n-2}{m}+--+\binom{m}{m}$ is equal to

1) n_{C_m} 2) $n+1_{C_{m+1}}$ 3) $n_{C_{m+1}}$ 4) $n+1_{C_m}$

75. The number of positive integral solutions of the equation $\begin{vmatrix} x^3+1 & x^2y & x^2z \\ xy^2 & y^3+1 & y^2z \\ xz^2 & yz^2 & z^3+1 \end{vmatrix} = 30$ is

1) 0 2) 3 3) 6 4) 9

76. A pack of cards consists of 15 cards numbered 1 to 15. Three cards are drawn at random with replacement. Then the probability of getting two odd and one even numbered cards, is

1) $\frac{348}{1125}$ 2) $\frac{398}{1125}$ 3) $\frac{498}{1125}$ 4) $\frac{448}{1125}$

77. A bag contains $n+1$ coins. It is known that one of these coins shows heads on both sides, whereas the other coins are fair. One coin is selected at random and tossed. If the probability that toss results in heads is $\frac{7}{12}$, then value of 'n' is

1) 5 2) 4 3) 3 4) 2

78. The maximum of $(Sec^{-1}x)^2+(Cosec^{-1}x)^2$ is equal to

1) $\frac{5\pi^2}{4}$ 2) $\frac{\pi^2}{2}$ 3) π^2 4) $\frac{5\pi^2}{8}$

79. The tangent to the parabola $y=x^2-2x+8$ at $p(2, 8)$ touches the circle $x^2+y^2+18x+14y+\lambda=0$ at Q. The coordinates of the point Q are

1) (-7, -12) 2) $\left(\frac{-31}{5}, \frac{-42}{5}\right)$ 3) $(-9, -13)$ 4) $(-11, -16)$

80. If $\int \frac{x}{1+x+e^x}dx = px + q\log\left|1+x+e^x\right| + C$ where C is the constant of integration, then p – q = ____

1) 0 2) 1 3) 2 4) 3

Section-B

(NUMERICAL VALUE TYPE)

81. If $f(3x+2)+f(3x+29)=0 \ \forall x \in R$, then the period of $f(x)$ is _____

82. Line segments AC and BD are diameters of the circle of radius one. If $\angle BDC = 60^0$, the length of line segment AB is _____

83. In a shooting competition a man can score 5, 4, 3, 2 (or) 0 points for each shot. Then the number of different ways in which he can score 30 in seven shots is ____

84. The number of roots of the equation $x + 2Tanx = \frac{\pi}{2}$ in the interval $[0, 2\pi]$ is _____

85. If the straight line $y = x$ meets $y = f(x)$ at P, where $f(x)$ is a solution of the differential equation $\frac{dy}{dx} = \frac{x^2+xy}{x^2+y^2}$ such that $f(1) = 3$, then the value of $f'(x)$ at the point P is____

86. If the observations 1, 2, 3----- n occur with frequency n, n-1, n-2, ---1 respectively such that the mean of observations is $\frac{13}{3}$, then n is equal to ____

87. Let $f(x) = \max\{|x+1|, |x+2|, ..., |x+5|\}$. Then $\int_{-6}^{0} f(x)dx$ is equal to _____

88. Consider the system of equations $ax + y + bz = 0$, $bx + y + az = 0$ and $ax + by + abz = 0$ where $a, b, \in \{0,1,2,3,4\}$. The number of ordered pair (a, b) for which the system has non-trivial solution is____

89. If two points A and B lie on the curve $y = x^2$ such that $\overline{OA}.\hat{i} = 1$ and $\overline{OB}.\hat{j} = 4$ where 0 is the orgin and A, B lie in the 1st, 2nd quadrants respectively, then $\overline{OA}.\overline{OB}$ is equal to ____

90. The point (x, y) lies on the line $2x + 3y = 6$. The smallest value of the quantity $\sqrt{x^2+y^2}$ is m. Then the value of $\sqrt{13}m$ is____

KEY SHEET

PHYSICS

1	**2**	2	**4**	3	**2**	4	**3**	5	**1**
6	**2**	7	**2**	8	**1**	9	**1**	10	**2**
11	**1**	12	**1**	13	**4**	14	**2**	15	**1**
16	**3**	17	**1**	18	**4**	19	**3**	20	**4**
21	**11**	22	**5**	23	**2**	24	**2**	25	**15**
26	**2500**	27	**22**	28	**33**	29	**6**	30	**34**

CHEMISTRY

31	**3**	32	**3**	33	**4**	34	**1**	35	**2**
36	**2**	37	**2**	38	**2**	39	**4**	40	**1**
41	**2**	42	**4**	43	**2**	44	**4**	45	**4**
46	**3**	47	**2**	48	**1**	49	**4**	50	**1**
51	**8**	52	**1**	53	**27**	54	**1**	55	**4**
56	**1**	57	**3**	58	**4**	59	**40**	60	**16**

MATHEMATICS

61	**4**	62	**2**	63	**2**	64	**1**	65	**1**
66	**4**	67	**1**	68	**1**	69	**1**	70	**2**
71	**2**	72	**2**	73	**4**	74	**2**	75	**2**
76	**4**	77	**1**	78	**1**	79	**2**	80	**3**
81	**54**	82	**1**	83	**420**	84	**3**	85	**1**
86	**11**	87	**21**	88	**6**	89	**2**	90	**6**

SOLUTIONS AND HINTS

Physics

1. 40% KE = $ms\Delta T + mL \Rightarrow \left(\frac{40}{100}\right)\left(\frac{1}{2}mV^2\right) = ms\Delta T + mL$

$\frac{1}{5}V^2 = 200S + L \;\; ; \;\; \frac{1}{5}V^2 = (200\times125) + \left(2.5\times10^4\times5\right) \Rightarrow V = 500m/s$

2. Energy gap range is given by $E_g = \frac{hc}{\lambda}$ (visible region)

$\lambda = 4\times10^{-7}$ to $7\times10^{-7}m \;\; ; \;\; E_g \Rightarrow \{3.09eV\, to 1.75eV\}$

3. Given $m_1 = 5kg, m_2 = 10kg \;\; \mu = 0.15$

FBD for $m_1 \;\; ; \;\; m_1g - T = m_1a$

50 – T = 5a And T – 0.15 (m + 10)g = (10 + m)a

To stop it's motion (a = 0)

50 – (0.15) (m + 10) 10 = 0 $\therefore$ m = 23.3 kg

4. 

Let the masses are m and distance between them is I, then $F = \frac{Gm^2}{I^2}$

When 1/3rd mass is transferred to the other then masses will be $\frac{4m}{3}$ and $\frac{2m}{3}$. So new force will be

$$F' = \frac{G\frac{4m}{3}\times\frac{2m}{3}}{I^2} = \frac{8}{9}\frac{Gm^2}{I^2} = \frac{8}{9}F$$

5. $V = \frac{2r^2[d_b - d_l]g}{9\eta} \;\; ; \;\; V\alpha\frac{[d_b - d_l]}{9\eta}$

$\frac{V^1}{V} = \frac{d_b - d_g}{\eta_g}\times\frac{\eta_W}{d_b - d_w} \;\; ; \;\; \frac{V^1}{10} = \left[\frac{7.8-1.2}{13.2}\right]\times\left[\frac{8.5}{7.8-1}\right]$

$V^1 = 6.25\times10^{-4}cm/s$

6. $W = \alpha^2 \beta e^{\frac{-\beta x^2}{kT}}$

As exponents are dimensionless so, $\frac{\beta x^2}{Kt}$ should be dimensionless.

$$\therefore [\beta] = \frac{KT}{x^2} = \frac{ML^2T^{-2}}{L^2} = MT^{-2}$$

From the dimensional homogeneity $\alpha^2\beta$ should have dimensions of work

$$\therefore [\alpha^2\beta] = [ML^2T^{-2}] \qquad \Rightarrow \alpha^2 = \frac{ML^2T^{-2}}{MT^{-2}} \quad \Rightarrow \alpha = [M^0LT^0]$$

7. Kinetic theory of gases

8. Two springs are in parallel $K = K_1 + K_2$

$$\gamma = \frac{1}{2\pi}\sqrt{\frac{K_1 + K_2}{m}} \; ; \quad \gamma^1 = \frac{1}{2\pi}\sqrt{\frac{4K_1 + 4K_2}{m}} \; ; \quad \gamma^1 = 2\gamma \text{ or } f^1 = 2f$$

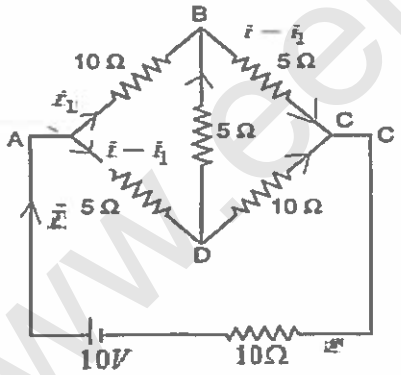
9. Concept of electric field

10. $U = U_f - U_i = \frac{q^2}{2}\left[\frac{1}{C_f} - \frac{1}{C_i}\right] = \frac{\left(5\times10^{-6}\right)^2}{2}\left[\frac{1}{2} - \frac{1}{5}\right] \times \frac{1}{10^{-6}}$

$$= \frac{25\times10^{-6}}{2}\left[\frac{3}{10}\right] \quad ; \quad U = 3.75\times10^{-6} J$$

11. ABD

100P



$$-10i_1 + 5(i - 2i_1) + 5(i - i_1) = 0$$

$2i - 5i_1$ (i)

$100p$ ADCEFA

$-5(i-i_1)10i_1+10-10i=0$

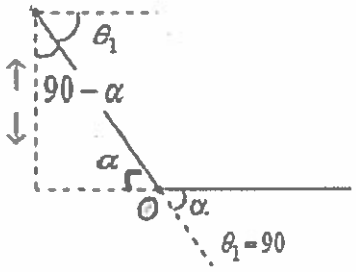
$5i_1+15i=10$ (ii)

From (i) and (ii)

Current in AB brance $\Rightarrow i_1=\frac{4}{17}$

12. yes, $\frac{P}{Q}=\frac{l_2-l_1}{l_2+l_1}$

13. $B=\frac{\mu_0 i}{4\pi r}(\sin\theta_1-\sin\theta_2)$



$\sin\alpha=\frac{r}{d}$

$r=d\sin\alpha$

$\theta_2=90-\alpha$

$B=\frac{\mu_0 i}{4\pi d\sin\alpha}\left[\sin 90-\sin(90-\alpha)\right]$

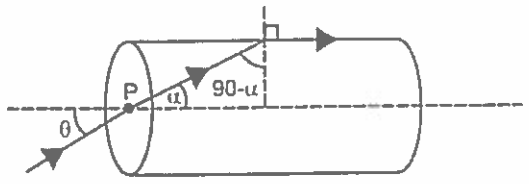
$B=\frac{\mu_0 i}{4\pi d\sin\alpha}\left[1-\cos\alpha\right]$

14. EMF induced in a coil.

15. $E=\frac{hc}{\lambda}$

$\lambda=\frac{hc}{E}=\frac{6.6\times10^{-34}\times3\times10^{8}}{11\times10^{3}\times1.6\times10^{-19}}=1.125\ A^0$

It belongs to X-ray region

16. 

$$n=\frac{\sin 90}{\sin(90-\alpha)} \Rightarrow n=\frac{1}{\cos\alpha}$$

Snells law at P

$$(1)\sin\theta=n(\sin\alpha)$$

$$\sin\theta=n\left[\frac{\sqrt{n^2-1}}{n}\right] \; ; \; \sin\theta=\sqrt{n^2-1}$$

$$\theta=\sin^{-1}\sqrt{\left(\frac{2}{\sqrt{3}}\right)^2-1}=\sin^{-1}\sqrt{\frac{4}{3}-1}=\sin^{-1}\sqrt{\frac{1}{3}} \; ; \; \theta=\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

17. Path difference $\Delta x=\frac{\lambda}{8}$

Phase difference $\Delta\phi=\frac{2\pi}{\lambda}(\Delta x)$

$$\Delta\phi=\frac{2\pi}{\lambda}\times\frac{\lambda}{8}=\frac{\pi}{4}$$

$$I=I_0\cos^2\left(\frac{\Delta\phi}{2}\right) \; ; \; \frac{I}{I_0}=\cos^2\left(\frac{\frac{\pi}{4}}{2}\right)=\cos^2\left(\pi/8\right)$$

$$\frac{I}{I_0}=0.853$$

18. $K.E=\frac{P^2}{2m}$; $P=\sqrt{2m(K.E)}$

$$\lambda=\frac{h}{\sqrt{2m(KE)}} \; ; \; \lambda\infty\frac{1}{\sqrt{(KE)}}$$

If KE is doubled wave length becomes $\frac{\lambda}{\sqrt{2}}$

$$\lambda^1=\frac{\lambda}{\sqrt{2}}$$

19. Energy of beta particles.

20. Let "s" be the distance between two ends of train, 'a' is constant acceleration

$$V^2 - u^2 = 2as, as = \frac{V^2 - u^2}{2}$$

Let V_C be velocity at midpoint $\therefore V_c^2 - u^2 = \frac{2as}{2}$

$$V_c^2 = \frac{V^2 - u^2}{2} + u^2 \quad ; \quad V_c = \sqrt{\frac{V^2 + u^2}{2}}$$

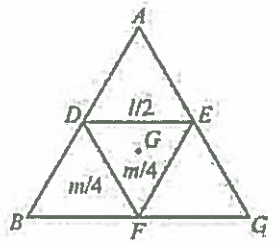
21. Moment of incrtia of lamina about an axis passing through centroid "G"

$$I_0 = Kml^2$$

Moment of incrtia of DEF

$$I_l \alpha \left(\frac{m}{4}\right)\left(\frac{1}{2}\right)^2 \quad ; \quad I_1 = \frac{I_0}{16}$$

Let $I_{ADE} = I_{BDF} = I_{EFG} = I_2 \ \therefore 3I_2 + I_1 = I_0$



$$\therefore 3I_2 + \frac{I_0}{16} = I_0 \quad ; \quad I_2 = \frac{5I_0}{16}$$

After removal of part ADE

$$I = 2I_2 + I_1 = 2\left[\frac{5I_0}{16}\right] + \frac{I_0}{16} \quad ; \quad I = \frac{11I_0}{16}$$

Given $I = \frac{NI_0}{16}$; N = 11

22. The moment of incrtia of a rod of mass m about on axix of perpendicular given by

$$I = \frac{ml^2}{12} \quad ; \quad I = mr^2 \text{ (general)}$$

$$r^2 = \frac{t^2}{12} \Rightarrow r = \frac{L}{\sqrt{12}} = \frac{l}{2\sqrt{3}} \quad ; \quad r = \frac{10\sqrt{3}}{2\sqrt{3}} = 5 \quad ; \quad r = 5$$

23. Force $F = YA\left(\frac{\Delta L}{L}\right)$; $F = (2\times10^{11})\times(10^{-4})\left(\frac{2L-L}{L}\right)$; $F = 2\times10^{7}N$

24. $PV^{1/2} = \text{constant}$

$PV = nRT$

$P \propto \frac{T}{V}$ $\therefore \frac{T}{V}\times V^{1/2} = \text{constant}$

$T \propto \sqrt{V}$; $\frac{T_2}{T_1} = \sqrt{\frac{V_2}{V_1}}$; $\frac{T_2}{T_1} = \sqrt{2}$

25. For block of mass 1 kg

$F = f_{(\max)}$

$(1)a = \mu N$

$(1)(a) = (0.5)(1)(10)$

$a = 5m/s^2$ $\therefore$ maximum horizontal force

$F_{\max} = ma = [2+1]5$; $F_{\max} = 15N$

26. $H = i^2Rt$

$10\times10^{-3} = \left(2\times10^{-3}\right)^2 R(1)$

$R = 2500\Omega$

27. $B = \mu_0\mu_r H = \mu_0(1+\chi)H$

$B = B_0[1+\chi]$; $B - B_0 = B_0\chi$

$\frac{B-B_0}{B_0} = \chi$; $\frac{B-B_0}{B_0}\times100 = 100\chi = 100\left[22\times10^{-5}\right]$

$= 22\times10^{-3} = 22\times10^{-4} = \frac{22}{10^4}$

Given $= \frac{\chi}{10^4}$; $\frac{B-B_0}{B_0}\times100 = \frac{22}{10^4}$

$X = 22$

28. $V_P - L\frac{di}{dt} - 30 + Ri = V_Q$

$V_P - V_Q = \left(50\times10^{-3}\times10^{2}\right) + 30 - 1\times2 = 5 + 30 - 2 \quad ; \quad V_P - V_Q = 33V$

29. $\lambda = 4l = 4\times5\times10^{-3} \quad ; \quad \lambda = 20\times10^{-3}m$

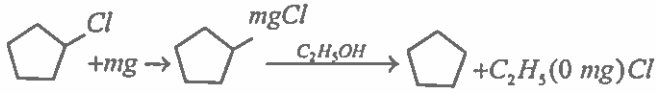
$v = \frac{C}{\sqrt{\mu_r \in_r}} \quad ; \quad f = \frac{C}{\lambda\sqrt{\mu_r \in_r}} = \frac{3\times10^{8}}{20\times10^{-3}\times\sqrt{6.25}} = 6\times10^{9}HZ$

$f = 6GHZ$

30. For closed pipe $f = \frac{nV}{4L} \quad ; \quad 250 = \frac{V}{4L}$

$L = 0.34m \quad ; \quad L = 34cm$

Chemistry

31. 

32. $\sqrt{v} = K_1(Z - K_2) \quad \Rightarrow \sqrt{v} = K_1Z - K_1K_2 \qquad \left[\left[\begin{matrix} m = K_1 = Tan45^0 = 1 \\ OA = 1 \quad K_2K_1 = -1 \end{matrix}\right.\right.$

$\Rightarrow y = mx + c \Rightarrow \sqrt{2500} = 1\times Z - 1 \quad \Rightarrow Z = 51$

33. Comparision of bond lengths.

34. $ClF_3 - 3B.P$ and $2l.P - X_3F_4 / SP^3d\,(T\ shape)$

$XeF_4 / SP^3d^2\ square\ planar$

35. $E_1 = \frac{-Xz^2}{n^2} = \frac{-X_4}{1} \quad ; \quad E_2 = \frac{-X4}{4} = -X$

$\Delta E = E_2 - E_1 = -X + 4X = 3X$

36. $N_aV_a \quad > \quad N_bV_b$

$7.5\times0.2 \quad 25\times0.2$

$15 \qquad 5$

$[H]^{+} = \frac{15-5}{100} = \frac{10}{100} = 10^{-1} \quad \therefore \quad P^H = 1$

37. $-Q_2 = Q_1 - 2Q_3 - Q_4$

$N_2 + O_2 \rightarrow 2NO \quad Q_1$

$2(NO + \frac{1}{2}O_2) \rightarrow 2NO_2 \quad -2Q_3$

$2NO_2 \rightarrow N_2O_4 \quad -Q_4$

$N_2 + 2O_2 \rightarrow N_2O_4 = -Q_2$

38. $A \xrightarrow{K_1} B \xrightarrow{K_2} C \; ; \; \frac{d(B)}{dt} = K_1(A) - K_2(B) = 0$

$K_1(A) = K_2(B) \Rightarrow [B] = \frac{K_1}{K_2}[A]$

39. $E^0_{cell} = 0 \Rightarrow KE_{cell} = \frac{-0.059}{2}\log\frac{C_1}{C_2}$

$\Delta G = -nFE_{cell} \Rightarrow \Delta G = -ve \quad (E_{cell} > 0) \; \therefore C_2 > C_1$

40. $i = \frac{1+(n-1)\alpha}{1} \Rightarrow i = \frac{1+(3-1)0.4}{1} = 1.8$

$K_2HgI_4 \rightarrow 2K^+ + HgI_4^{-2}$

41. $\frac{x}{M} = K.P^{1/n} \Rightarrow \log\frac{x}{M} = \log K + \frac{1}{n}\log P$

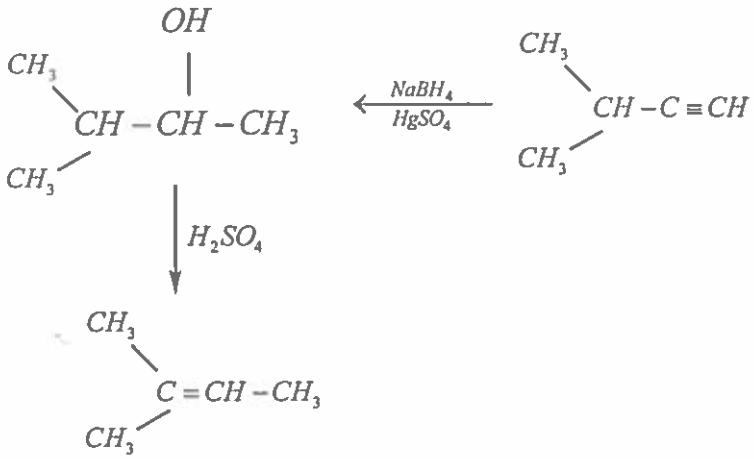
Hence slope is $\frac{1}{n}$ with $\left(\frac{1}{n} = 0 \; to \; 1\right) \; ; \; 0 < \frac{1}{n} < 1$

42. $\delta = \frac{Z \times M}{a^3 N_A} \Rightarrow 9 \times 10^3 = \frac{4 \times M}{\left(200\sqrt{2} \times 10^{-12}\right)^3 \times 6 \times 10^{23}} \; ; M = 0.0305 \; kg/mole$

43. Conditions of geometrical isomerism.

44. EDG is directly proportional to the reactivity of ESR

45. $CH \equiv CH \xrightarrow{LiNH_2} CH \equiv C^{\ominus} \xrightarrow{CH_3-CH(CH_3)-Br}$



46. $K.E = h\nu - \omega_o \quad \left(h\nu = \frac{c}{\lambda}\right)$

$\omega_o = h\nu - K.E \quad \left(K.E = \frac{1}{2}mv^2\right)$; $\omega_o = \frac{6.6\times10^{-34}\times3\times10^{8}}{4\times10^{3}\times10^{-10}} - \frac{1}{2}\times9\times10^{-31}\left(6\times10^{5}\right)^2$

$= 5\times10^{-9} - 1.62\times10^{-19} = 3.38\times10^{-19}\,J$

$1ev = 1.602\times10^{-19}\,J$

$? \quad = 3.38\times10^{-19}\,J \quad = 2.1ev$

47. Order of reactivity of carbonyl compounds.

48. Conditions of aromaticity.

49. $2KMnO_4 \xrightarrow{513K} K_2MnO_4 + MnO_2 + O_2$

MnO_4^{2-} Paramagnetic and green

50. Linkage isomers possible

$Pt(en)(NO_2)$; $Pt(en)(ONO)NO_2$; $Pt(en)(ONO)_2$

51. $P^H = 7 + \left(\frac{P^{Ka} + \log c}{2}\right) = 7 + \left(\frac{5 - \log 5 + \log\frac{0.01}{2}}{2}\right) = 7 + \left(\frac{5-3}{2}\right) \Rightarrow 8$

52. $PV = nRT \quad ; \quad P = \dfrac{nRT}{V} = \dfrac{\left(\dfrac{2.69\times10^{20}}{6.02\times10^{23}}\right)\times0.081\times273}{\dfrac{10}{1000}}$

$$P = \frac{2.69\times10^{-3}}{6.02\times10}\times0.821\times273\times1000 = \frac{2.69\times0.0821\times273}{602} = 1\,atm$$

53. $3AgNO_3 + \left[CO(NH_3)_6\right]Cl_3 \rightarrow 3\ mole\ AgCl$

$$\frac{0.3}{267.46}\times3 = 0.125\times V\times10^{-3}$$

$$V = \frac{0.3\times3\times1000}{267.46\times0.125} = 26.92\,ml$$

54. Presence of unpaired electrons.

55. $$P_4 + 3OH^- + 3H_2O \rightarrow PH_3 + 3H_2PO_2^-$$
$$H_2PO_2^- + 4Ag^+ + 2H_2O \rightarrow 4Ag + H_3PO_4 + 3H^{\oplus}$$

56. $XCF_6 + 2H_2O \rightarrow XCO_2F_2 + 4HF$

57. Allowable limits of metal ions in drinking water.

58. Given peptide has four carbon atoms with unsymmetrical configuration.

59. $V = 200\times10\times0.1 = 200\,cm^3$

$m = d\times v = 10.8\times200 = 2160m$

$108gm \rightarrow 1\,mole$

$2160 \rightarrow ?$

$\dfrac{2160}{108} = 20\,moles\ one\ side$

$Both\ side\ \ 20\times2 = 40\,moles$

60. Total of sixteen protons.

61. $\frac{dx}{d\theta} = \sec\theta\, Tan\theta + \sin\theta = Tan\theta(\sec\theta + \cos\theta)$

$\frac{dy}{d\theta} = n\sec^{n-1}\theta.Sec\theta Tan\theta - n\cos^{n-1}\theta(-\sin\theta)$

$= nTan\theta(Sec^n\theta + \cos^n\theta)$

$\therefore \frac{dy}{dx} = \frac{n(\sec^n\theta + \cos^n\theta)}{(\sec\theta + \cos\theta)}$

$\left(\frac{dy}{dx}\right)^2 = \frac{n^2(\sec^n\theta + \cos^n\theta)^2}{(\sec\theta + \cos\theta)^2}$

$= \frac{n^2\left[(\sec^n\theta - \cos^n\theta)^2 + 4\right]}{(\sec\theta - \cos\theta)^2 + 4} = \frac{n^2(y^2+4)}{x^2+4}$

62. $p \to (q \to r) \equiv \sim p \vee (q \to r)$

$\equiv \sim p \vee (\sim q \vee r)$

$\equiv p \wedge q \to r$, It is a tautalogy

63. The given line will be parallel to one of the bisector planes of the given planes.

$\therefore \frac{2x + y + 3z - 1}{\sqrt{14}} = \pm\frac{x + 2y - 3z - 1}{\sqrt{14}}$

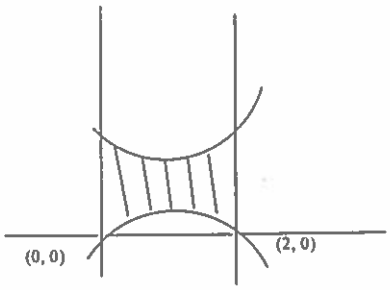
$3x + 3y - 2 = 0 \quad x - y + 6z = 0$

$\because \frac{x}{K} = \frac{y}{2} = \frac{z}{-12}$ will be parallel to the plane $x - y + 6z = 0$

$\frac{1}{\frac{1}{k}} = \frac{-1}{1/2} = \frac{6}{-1/12}$

K=-2

64. Area= $= \int_0^2 \left(2^x - 2x + x^2\right) dx = \frac{3}{\log 2} - \frac{4}{3} sq.v.$



65. Required vetor is

$$= \lambda\left(\frac{\bar{a}}{|\bar{a}|} + \frac{\bar{b}}{|\bar{b}|}\right) = \frac{\lambda}{9}\left(\bar{i} - 7\bar{j} + 2\bar{k}\right)$$

$$|\bar{c}| = \pm\frac{\lambda}{9}\sqrt{54} \quad \Rightarrow 5\sqrt{6} = \pm\frac{\lambda}{9}\sqrt{54} \Rightarrow \lambda = \pm 15$$

$$\bar{c} = \pm\frac{5}{3}\left(\bar{i} - 7\bar{j} + 2\bar{k}\right)$$

66. $\underset{x\to 0}{Lt} \dfrac{x\left(\dfrac{2Tanx}{1-Tan^2x}\right) - 2xTanx}{(2\sin^2 x)^2} = \underset{x\to 0}{Lt} \dfrac{2xTan^3x}{4\sin^4 x(1-Tan^2x)}$

$$= \frac{1}{2}\underset{x\to 0}{Lt}\left(\frac{x^4}{\sin^4 x} \times \frac{Tan^3x}{x^3} \times \frac{1}{1-Tan^2x}\right)$$

$$= \frac{1}{2} = \lambda \Rightarrow 8\lambda = 4$$

67. $y - 2 = m(x-8), m < 0$

$$y - 2 = mx - 8m$$

$$mx - y = 8m - 2$$

$$\frac{x}{\frac{8m-2}{m}} + \frac{y}{2-8m} = 1$$

$$\because P = \left(\frac{8m-2}{m}, 0\right) = \left(8 - \frac{2}{m}, 0\right), Q = (0, 2-8m)$$

$$\therefore OP + OQ = 8 - \frac{2}{m} + 2 - 8m + 10 - 8m - \frac{2}{m}$$

Let $f(m)=10-8m-\frac{2}{m}$

$f^1(m)=-8+\frac{2}{m^2}$

for min. value $f^1(m)=0$

$-8+\frac{2}{m^2}=0\Rightarrow m^2=\frac{1}{4}$

$m=\pm 1/2$

$m=-\frac{1}{2}(m<0)$

Min value=18

68. Let $P(3Sec\theta,2Tan\theta)$ be any point on $\frac{x^2}{9}-\frac{y^2}{4}=1$

chord of contact of P with respect to $x^2+y^2=9$ is $(3Sec\theta)x+(2Tan\theta)y=9\Rightarrow(1)$

Let $Q(x_1\,y_1)$ be the mid point of the chord of contact.

$\therefore Eq:-$ of mid point of chord is $xx_1+yy_1=x_1^2+y_1^2\rightarrow(2)$

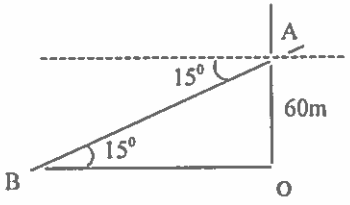
comparing (1) and (2) $\rightarrow\frac{3\sec\theta}{x_1}=\frac{2Tan\theta}{y_1}=\frac{9}{x_1^2+y_1^2}$

$Sec\theta=\frac{9x_1}{3(x_1^2+y_1^2)}\quad Tan\theta=\frac{9y_1}{2(x_1^2+y_1^2)}\quad\therefore\sec^2\theta-Tan^2\theta=-1$

$\frac{x^2}{9}-\frac{y^2}{4}=\frac{(x^2+y^2)^2}{81}\Rightarrow 4(x^2+y^2)^2=36x^2-81y^2$

a = 4, b = 36, c = -81

69. $Tan15^0=\frac{60}{OB}\Rightarrow OB=60\cot15^0=60\cot(45^0-30^0)=60\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)metres$



70. $A+B=180^0 \Rightarrow A+5A=$

$180^0 \Rightarrow A=30^0 \quad \therefore Tan30^0=\frac{2\sqrt{(l+m)^2-lm}}{l+m}\left(Tan\theta=\frac{2\sqrt{h^2-ab}}{a+b}\right)$

$(l+m)^2=12\left[(l+m)^2-lm\right]\Rightarrow 12lm=11(l+m)^2$

$\Rightarrow \frac{lm}{(l+m)^2}=\frac{11}{12}$

71. $1-c^2=n^2c^2-2nc+1 \Rightarrow 2nc=c^2(n^2+1)\Rightarrow \frac{c}{2n}=\frac{1}{n^2+1}$

$\therefore \frac{c}{2n}(1+nz)\left(1+\frac{n}{z}\right)=\frac{c}{2n}\left[1+n^2+n\left(z+\frac{1}{z}\right)\right]$

$=\frac{1}{n^2+1}\left[1+n^2+2nCos\theta\right]=1+\left(\frac{2n}{n^2+1}\right)\cos\theta=1+cCos\theta$

72. $\frac{2.\frac{a}{2}+3.\frac{b}{3}+2.\frac{c}{2}}{7}\geq\left\{\left(\frac{a}{2}\right)^2\left(\frac{b}{3}\right)^3\left(\frac{c}{2}\right)^2\right\}^{1/7}$

$\left[\begin{array}{l}Taking\ A.M.and\ G.M.of\\ 7\ numbers\ \frac{a}{2},\frac{a}{2},\frac{b}{3},\frac{b}{3},\frac{c}{2},\frac{c}{2}\end{array}\right]$

$\frac{3}{7}\geq\left(\frac{a^2b^3c^2}{2^2.3^3.2^2}\right)^{1/7}\Rightarrow a^2.b^3.c^2\leq\frac{3^{10}.2^4}{7^7}$

73. $x=\frac{y^4}{8-6y^2}$ $\because x$ is non negative,

$\frac{y^4}{8-6y^2}\geq 0\Rightarrow 8-6y^2>0\Rightarrow y^2<\frac{4}{3}$

i.e., y=0, 1 ($\therefore y$ is non negative)

$\therefore y=0\Rightarrow x=0$

$y=1\Rightarrow x=\frac{1}{2}\notin I$

only one solution.

74. Coeff.of $x^m in(1+x)^n + Coeff.of x^m in(1+x)^{n-1} +$

Coeff.of $x^m\, in(1+x)^{n-2} + - - - -$

$=$Coeff.of $x^m in\left[(1+x)^n + (1+x)^{n-1} + - - - + (1+x)^m\right]$

$= coeff.of\ x^m\, in(1+x)^m\ \left[1+(1+x)+(1+x)^2 + - - - + (1+x)^{n-m}\right]$

$= Coeff.of\ x^m\, in(1+n)^m\left[\dfrac{(1+x)^{n-m+1}-1}{(1+x)-1}\right]$

$= Co.eff.of x^m in\left[\dfrac{(1+x)^{n+1}-(1+x)^m}{x}\right]$

$= Co.eff.of x^{m+1} in\left[(1+x)^{n+1}-(1+x)^m\right]$

$= n+1C_{m+1} - 0 = n+1C_{m+1}$

75. $\begin{vmatrix} x^3 & x^2y & x^2z \\ xy^2 & y^3+1 & y^2z \\ xz^2 & yz^2 & z^3+1 \end{vmatrix} + \begin{vmatrix} 1 & x^2y & x^2z \\ 0 & y^3+1 & y^2z \\ 0 & yz^2 & z^3+1 \end{vmatrix}$

$= x\begin{vmatrix} x^2 & x^2y & x^2z \\ y^2 & y^3+1 & y^2z \\ x^2 & yz^2 & z^3+1 \end{vmatrix} + (y^3+1)(z^3+1) - y^3z^3$

$C_2 \to C_2 - y_{C_1}$ and $C_3 \to C_3 - z_{C_1}$

$= x\begin{vmatrix} x^2 & 0 & 0 \\ y^2 & 1 & 0 \\ z^2 & 0 & 1 \end{vmatrix} + y^3z^3 + z^3 + y^3 + 1 - y^3z^3 = x^3 + y^3 + z^3 + 1$

$\therefore x^3 + y^3 + z^3 + 1 = 30 \Rightarrow x^3 + y^3 + z^3 = 29$

Solutions are (3, 1, 1)(1,3,1) (1,1,3)

76. $= P(E \cap 0 \cap 0) + P(0 \cap E \cap 0) + P(0 \cap 0 \cap E)$

$= P(E).P(0).P(0) + P(0)P(E)P(0) + P(0)P(0)P(E)$

$= \dfrac{7}{15}.\dfrac{8}{15}.\dfrac{8}{15} + \dfrac{8}{15}.\dfrac{7}{15}.\dfrac{8}{15} +$

$\dfrac{8}{15}.\dfrac{8}{15}.\dfrac{7}{15} = \dfrac{3\times 7\times 8^2}{15^2} = \dfrac{448}{1125}$

77. $P(A)=P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)$

$$\frac{7}{12}=\frac{1}{n+1}\times 1+\frac{n}{n+1}\times\frac{1}{2}\Rightarrow n=5$$

78. Let $I=(\sec^{-1}x)^2+(\operatorname{cosec}^{-1}x)^2$

$$I=(Sec^{-1}x+\operatorname{cosec}^{-1}x)^2-2\sec^{-1}x\left(\frac{\pi}{2}-\sec^{-1}x\right)$$

$$=\frac{\pi}{4}+2(Sec^{-1}x)^2-\pi\sec^{-1}x$$

$$=\frac{\pi^2}{4}+2\left[\left(\sec^{-1}x-\frac{\pi}{4}\right)^2-\frac{\pi^2}{16}\right]$$

$$=\frac{\pi^2}{4}-\frac{\pi^2}{8}+2\left[\frac{9\pi^2}{16}\right]$$

$$=\frac{\pi^2}{8}+\frac{9\pi^2}{8}=\frac{10\pi^2}{8}=\frac{5\pi^2}{4}$$

79. The eq. of tangent at P(2, 8) is $x(2)-1(x+2)-\frac{1}{2}(y+8)+8=0\ \ 2x-y+4=0\rightarrow$ (1)

$$\therefore m=2\quad \frac{-1}{m}=\frac{-1}{2}$$

we know that, Eq: of normal at a Q passes through centre of the circle

$$C=(-9,-7)\ \frac{-1}{m}=\frac{-1}{2}$$

Eq. of normal is

$$y+7=\frac{-1}{2}(x+9)\Rightarrow x+2y+23=0\rightarrow(2)$$

Solving (1) & (2) we get $Q=\left(\frac{-31}{5},\frac{-42}{5}\right)$

80. $$\int\frac{(1+x+e^x)-(1+e^x)}{1+x+e^x}dx=\left(1-\frac{1+e^x}{1+x+e^x}\right)dx$$

$$=x-\log\left((1+x+e^x)\right)+C\ \therefore p=1\quad q=-1\ ;\ p-q=1+1=2$$

81. $f(3x+2)+f(3x+29)=0 \rightarrow (1)$

Replacing x by $x+9$, we get

$f(3(x+9)+2)+f(3(x+9)+29)=0$

$f(3x+29)+f(3x+56)=0 \rightarrow (2)$

$from(1)\&(2)$

$\Rightarrow f(3x+2)=f(3x+56)$

Put $3x+2=y \Rightarrow f(y)=f(y+54)$

$\therefore$ Period of $f(x)$ is 54

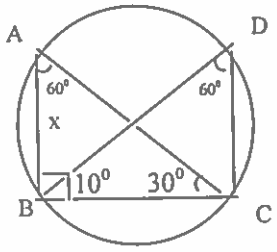
82. $\angle A=\angle D=60$

$AC=2(1)=2$

$\angle ABC=90^0 \quad \angle ACB=30^0$

$Sin30=\frac{AB}{AC}=\frac{x}{2}$

$\frac{1}{2}=\frac{x}{2}\Rightarrow x=1$



83. Number of ways of making 30 in 7 shots is, Coefficient of $x^{30}in\left[x^0+x^2+x^3+x^4+x^5\right]^7$ Co-efficient of

$x^{30}in\left[(1+x^2+x^3)+x^4(1+x)\right]^7 = co.eff\ of\ x^{30}in[7_{C_0}.x^{28}(1+x)^7+7_{C_1}$

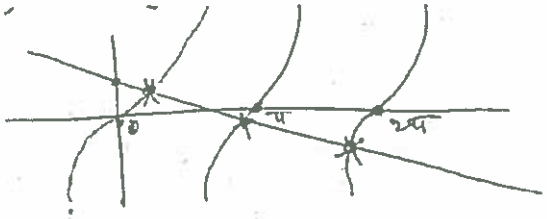
$x^{24}(1+x)^6.(1+x^2+x^3)+7_{C_2}.$

$x^{20}(1+x)^5.(1+x^2+x^3)_1^2 \_\_\_\_]$

$=7_{C_0}.7_{C_5}+7_{C_1}(6_{C_6}+6_{C_4}+6_{C_3})+7_{C_2}(5_{C_1}+2)$

=21+252+147=420

84. Let $y = Tanx \Rightarrow x + 2y = \frac{\pi}{2} \Rightarrow y = \frac{\pi}{4} - \frac{x}{2}$



$$y = \frac{\pi}{4} - \frac{x}{2}$$

No. of P.I. = 3

So, No. of solutions = 3

85. $\therefore y = f(x) \Rightarrow \frac{dy}{dx} = f^1(x) \quad \therefore f^1(x) = \frac{x^2 + xy}{x^2 + y^2}$

$$f^1(x) = \frac{x^2 + x^2}{x^2 + x^2} = \frac{2x^2}{2x^2} = 1 \quad (\because y = x)$$

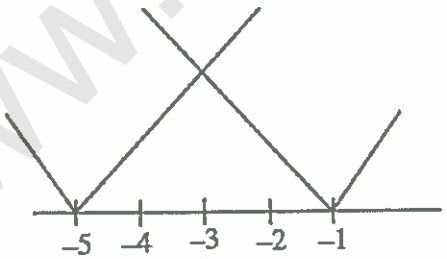
86. $\text{Mean} = \frac{\sum fi\,xi}{\sum fi} = \frac{\sum_{r=1}^{n} r(n+1-r)}{\sum fi} = \frac{13}{3}$

$$\Rightarrow \frac{(n+1).\frac{n(n+1)}{2} - \frac{n(n+1)(2n+1)}{6}}{\frac{n(n+1)}{2}} = \frac{13}{3}$$

$$\Rightarrow (n+1) - \left(\frac{2n+1)}{3}\right) = \frac{13}{3} \Rightarrow \frac{n+2}{3} = \frac{13}{3}$$

$n = 11$

87. $f(x) = \max\{|x+1|, |x+2|, |x+3|, |x+4|, |x+5|\}.$



$$\int_{-6}^{0} f(x)dx = \int_{-6}^{-3} |x+1|dx + \int_{-3}^{0} |x+5|dx = -\int_{-6}^{-3} (x+1)dx + \int_{-3}^{0} (x+5)dx$$

$$= -\left[\frac{x^2}{2}+x\right]_{-6}^{-3} + \left[\frac{x^2}{2}+5x\right]_{-3}^{0} = -\left[\left(\frac{9}{2}-3\right)-(18-6)\right]+\left[0-\left(\frac{9}{2}-15\right)\right]$$

$$= -\left[\frac{3}{2}-12\right]+\frac{21}{2} = \frac{21}{2}+\frac{21}{2} = 21$$

88. $\begin{vmatrix} a & 1 & b \\ b & 1 & a \\ a & b & ab \end{vmatrix} = 0 \Rightarrow (a-b)(a-b^2) = 0$

$$\Rightarrow a = b \ (or) \ a = b^2$$

$(0,0),(1,1),(2,2),(3,3),(4,4)$: $(4,2)$

No. of ordered pairs =6

89. Let $A(\alpha,\alpha^2), B(\beta,\beta^2)$ where $\alpha > 0, \beta < 0$

$\therefore \overline{OA} = \alpha\bar{i} + \alpha^2\bar{j}$ $\quad \overline{OB} = \beta\bar{i} + \beta^2\bar{j}$

$\overline{OA}.i = \alpha$ $\quad \overline{OB}.\bar{j} = \beta^2$

$1 = \alpha$ $\quad 4 = \beta^2 \Rightarrow \beta = \pm 2$

$\beta = -2 (\because \beta < 0)$

$\therefore \overline{OA} = \bar{i} + \bar{j}$ $\quad \overline{OB} = -2\bar{i} + 4\bar{j}$

$\overline{OA}.\overline{OB} = -2 + 4 = 2$

90. Let $x = r\cos\theta, \ y = r\sin\theta, \ r = \sqrt{x^2+y^2}$

$2r\cos\theta + 3r\sin\theta = 6 \Rightarrow r = \frac{6}{2\cos\theta + 3\sin\theta}$

$\because r$ is min : when $2\cos\theta + 3\sin\theta$ must be max $\therefore r_{min} = \frac{6}{\sqrt{13}} = m \Rightarrow \sqrt{13}m = 6$