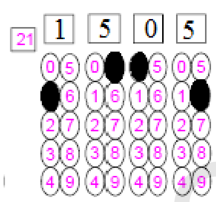
Model Grand Test
2023

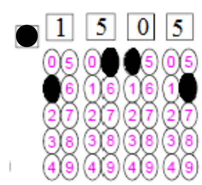
IMPORTANT INSTRUCTION:

1. Immediately fill in the Admission number on this page of the Test Booklet with **Blue/Black** Ball Point **Pen** only.
2. The candidates should not write their Admission Number anywhere (except in the specified space) on the Test Booklet/ Answer Sheet.
3. The test is of **3 hours** duration.
4. The Test Booklet consists of 90 questions. The maximum marks are **300.**
5. There are **three** parts in the question paper 1,2,3 consisting of **Physics, Chemistry** and **Mathematics** having **30 questions** in each subject and subject having **two sections.**

   (I) Section –I contains 20 **multiple choice** questions with only one correct option.
   Marking scheme: +4 for correct answer, 0 if not attempt and -1 in all other cases.
   (II) Section-II contains 10 **Numerical Value Type** questions. Attempt any 5 questions only,if more than 5 questions attempted, First 5 attempted questions will be considered.

- The Answer should be within **0 to 9999.** If the Answer is in **Decimal** then round off to the **nearest Integer** value (Example i,e. If answer is above **10** and less than **10.5** round off is **10** andIf answer is from **10.5** and less than **11** round off is **11).**
  To cancel any attempted question bubble on the question number box.
  For example: To cancel attempted question 21. Bubble on 21 as shown below





Question Answered for Marking **Question Cancelled for Marking**

Marking scheme: +4 for correct answer, 0 if **not attempt** and **-1** in all other cases.

6. Use **Blue / Black Point Pen only** for writing particulars / marking responses on the Answer Sheet. **Use of pencil is strictly prohibited.**
7. No candidate is allowed to carry any textual material, printed or written, bits of papers, mobile phone any electron device etc, except the Identity Card inside the examination hall.
8. Rough work is to be done on the space provided for this purpose in the Test Booklet only.
9. On completion of the test, the candidate must hand over the Answer Sheet to the invigilator on duty in the Hall. **However, the candidate are allowed to take away this Test Booklet with them.**
10. **Do not fold of make any stray marks on the Answer Sheet**

PHYSICS

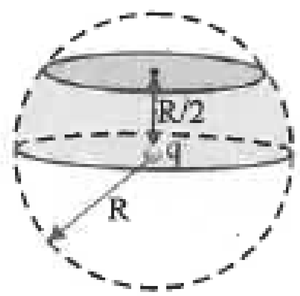
Max Marks: 100

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

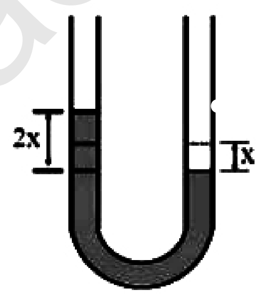
Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

1. Flux Passing through the shaded surface of a sphere when a point charge q is placed at the center is (radius of the sphere is R)



1) q / ε_0 2) $q / 2\varepsilon_0$ 3) $q / 4\varepsilon_0$ 4) Zero

2. 9 kg of mercury is poured into a glass U-tube with diameter of 1.2 cm. The mercury can flow without friction within the tube. Find the oscillation period. Density of mercury $=13.6 \times 10^3 kg / m^3$.



1)1.2s 2)3.4s 3)5.6s 4)7.8s

3. In a photo–emissive cell, with exciting wavelength λ, the maximum kinetic energy of the electron is K. If the exciting wavelength is changed to $3\lambda / 4$, the kinetic energy of the fastest emitted electron will be

1) $\frac{3K}{4}$ 2) $\frac{4K}{3}$ 3) Less than $\frac{4K}{3}$ 4) More than $\frac{4K}{3}$

4. STATEMNT-I :

If $W_{net} > 0$ then $\Delta KE > 0$ and $KE_{\text{final}} > KE_{\text{initial}}$.

Because

STATEMENT-II

If net force acting on the particle does positive work on it, then KE of the particle increases.

1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

2) Statement-1 is true, Statement-2 is True; Statement-2 is NOT a correct explanation fro

Statement-1.

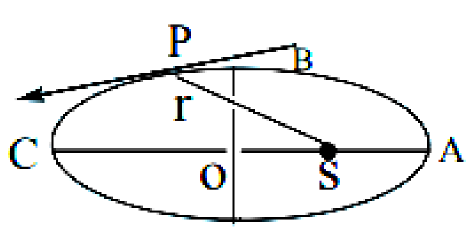
3) Statement-1 is True, Statement-2 is False.

4) Statement-1 is False, Statement-2 is True.

5. An object is placed Infront of a convex mirror at a distance of 50 cm. A plane mirror is introduced covering the lower half of the convex mirror. If the distance between the object and plane mirror is 30 cm, it is found that there is no parallax between the images formed by two mirrors. Radius of curvature of mirror will be

1) 12.5 cm 2) 25 cm 3) $\frac{50}{3} cm$ 4) 18 cm

6. A particle P of mass 2kg describes an elliptical orbit of semi-major axis $a = 5m$ and semi-minor axis b =4m due to an attractive force $\frac{\lambda}{r^2}$ towards a focus S where r is the distance of P from S and $\lambda = 40 Nm^2, Now,$



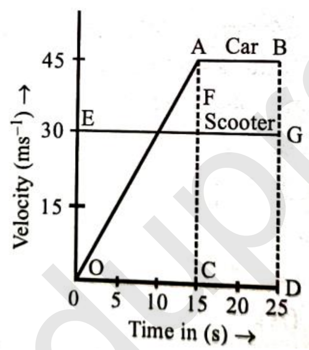
1) Its velocity at C is $2ms^{-1}$ 2) Its velocity at C is $4ms^{-1}$

3) Its velocity at A is $1ms^{-1}$ 4) Its velocity at A is $4ms^{-1}$

7. In hydrogen atom, if the difference in the energy of the electron in $n=2$ and $n=3$ orbits is E, the ionization energy of hydrogen atom is

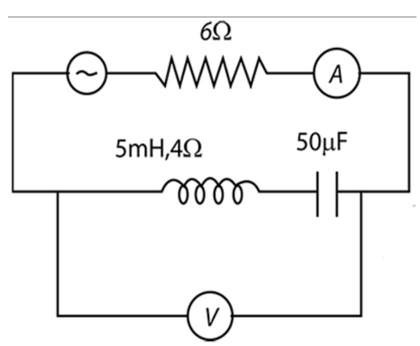
1) 13.2 E 2) 7.2 E 3) 5.6 E 4) 3.2 E

8. The velocity-time graphs of a car and a scooter are shown in the figure. (i) the difference between the distance travelled by the car and the scooter in 15s and (ii) the time at which the car will catch up with the scooter are, respectively



1) 337.5m and 25s 2) 225.5m and 10s
3) 112.5m and 22.5s 4) 150.5m and 15s

9. In the circuit shown in the figure, the ac source gives a voltage $V = 20\cos(2000t)$ Neglecting source resistance, the voltmeter and ammeter reading will be



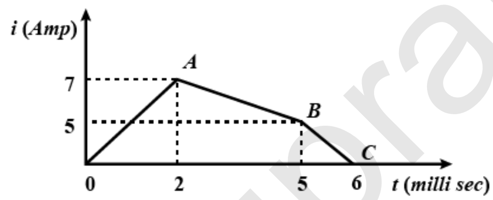
1) 0V,0.47 A 2) 1.68V, 0.47 A 3) 0 V, 1.4 A 4) 5.6 V, 1.4 A

10. A ball is thrown upward with an initial velocity v_0 from the surface of the earth. The motion of the ball is affected by a drag force equal to $m\gamma v^2$ (where m is mass of the ball, v is its instantaneous velocity and γ is a constant). Time taken by the ball to rise to its zenith is :

1) $\frac{1}{\sqrt{\gamma g}}\tan^{-1}\left(\sqrt{\frac{\gamma}{g}}v_0\right)$ 2) $\frac{1}{\sqrt{\gamma g}}\sin^{-1}\left(\sqrt{\frac{\gamma}{g}}v_0\right)$

3) $\frac{1}{\sqrt{\gamma g}}\ln\left(1+\sqrt{\frac{\gamma}{g}}v_0\right)$ 4) $\frac{1}{\sqrt{2\gamma g}}\tan^{-1}\left(\sqrt{\frac{2\gamma}{g}}v_0\right)$

11. The current through a 4.6 H inductor is shown in the following graph. The induced emf during the time interval $t=5$ milli–sec to 6 milli–sec will be



1) $10^3 V$ 2) $-23\times10^3 V$ 3) $23\times10^3 V$ 4) Zero

12. On the $x-$axis and at a distance x from the origin, the gravitational field due to a mass distribution is given by $\frac{Ax}{(x^2+a^2)^{3/2}}$ in the x-direction. The magnitude of gravitational potential on the x-axis at a distance x, taking its value to be zero at infinity, is :

1) $\frac{A}{(x^2+a^2)^{1/2}}$ 2) $\frac{A}{(x^2+a^2)^{3/2}}$ 3) $A(x^2+a^2)^{1/2}$ 4) $A(x^2+a^2)^{3/2}$

13. Given below are two statements:
Statement-I: The reactance of an circuit is zero. It is possible that the circuit contains a capacitor and an inductor.
Statement-II: In ac circuit, the average power delivered by the source never becomes zero.
In the light of the above statements, choose the correct answer from the options given below.

1) Both Statement I and Statement II are true.

2) Both Statement I and Statement II are false.

3) Statement I is true but Statement II is false.

4) Statement I is false but Statement II is true.

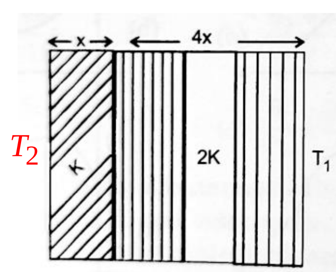
14. Potential energy as a function of r is given by $U = \frac{A}{r^{10}} - \frac{B}{r^5}$, where r is the interatomic distance, A and B are positive constants. The equilibrium distance between the two atoms will be :

1) $\left(\frac{A}{B}\right)^{\frac{1}{5}}$ 2) $\left(\frac{B}{A}\right)^{\frac{1}{5}}$ 3) $\left(\frac{2A}{B}\right)^{\frac{1}{5}}$ 4) $\left(\frac{B}{2A}\right)^{\frac{1}{5}}$

15. In a experiment to measure the internal resistance of a cell by a potentiometer, it is found that the balance point is at a length of 2 m when the cell is shunted by a 5Ω resistance and is at a length of 3 m when the cell is shunted by a 10Ω resistance, the internal resistance of the cell is

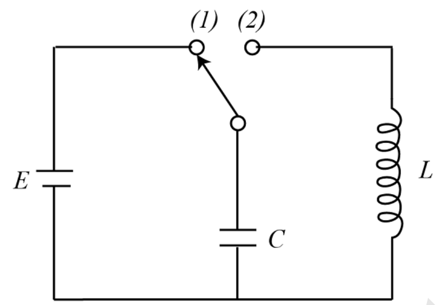
1) 1.5Ω 2) 10Ω 3) 15Ω 4) 1Ω

16. The temperature of the two outer surfaces of a composite slab, consisting of two materials having coefficients of thermal conductivity K and $2K$ and thickness x and $4x$, respectively, are T_2 and $T_1 (T_2 > T_1)$. The rate of heat transfer through the slab, in a steady state is $\left(\frac{A(T_2 - T_1)K}{x}\right) f$, with f equal to



1) $\frac{2}{3}$ 2) $\frac{1}{2}$ 3) 1 4) $\frac{1}{3}$

17. In the following electrical network at t < 0 (as given the figure), key is placed on (1) till the capacitor got fully charged. Key is placed on (2) at t = 0. Time when the energy in both the capacitor and the inductor will be same for the first time is



1) $\frac{\pi}{4}\sqrt{LC}$ 2) $\frac{3\pi}{4}\sqrt{LC}$ 3) $\frac{\pi}{3}\sqrt{LC}$ 4) $\frac{2\pi}{3}\sqrt{LC}$

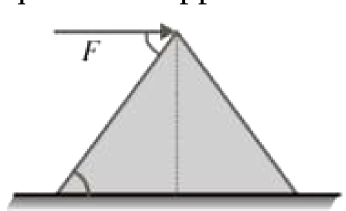
18. A mixture of hydrogen and oxygen has volume $500\,cm^3$, temperature 300K, pressure 400k Pa and mass 0.76g. The ratio of masses of oxygen to hydrogen will be

1) 3:8 2) 3:16 3) 16:3 4) 8:3

19. A luminous object is placed at a distance of 30 cm from the convex lens of focal length 20 cm. On the other side of the lens, at what distance from the lens a convex mirror of radius of curvature 10 cm be placed in order to have an upright image of the object coincident with it

1) 12 cm 2) 30 cm 3) 50 cm 4) 60 cm

20. A horizontal force F is applied at the top of an equilateral triangular block having mass m and side a as shown in figure. The minimum value of the coefficient of friction required to topple the block before translation will be



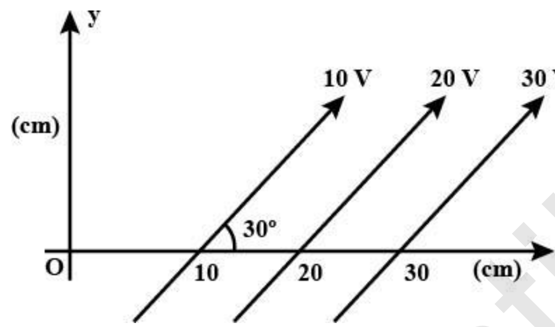
1) $\frac{2}{\sqrt{3}}$ 2) $\frac{1}{2}$ 3) $\frac{1}{\sqrt{3}}$ 4) $\frac{1}{3}$

(NUMERICAL VALUE TYPE)

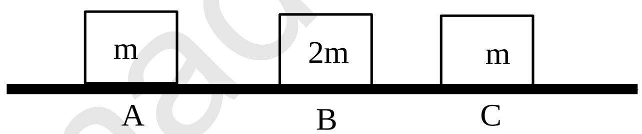
Section-II contains 10 Numerical Value Type questions. Attempt any 5 questions only. First 5 attempted questions will be considered if more than 5 questions attempted. The Answer should be within 0 to 9999. If the Answer is in Decimal then round off to the nearest Integer value (Example i,e. If answer is above 10 and less than 10.5 round off is 10 andIf answer is from 10.5 and less than 11 round off is 11).

Marking scheme: +4 for correct answer, 0 if not attempt and -1 in all other cases.

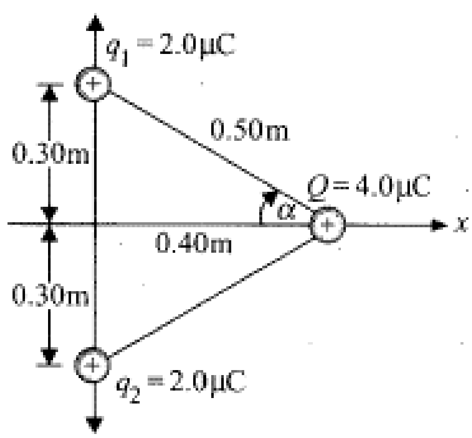
21. Equipotential surfaces are shown in figure. Then the electric field strength will be $4n\,Vm^{-1}$. The value of n is



22. Three objects A, B and C are kept in a straight line on a friction less horizontal surface. These have masses m, 2m and m respectively. The object A moves towards B with a speed 9m/s and makes an elastic collision with it. Thereafter B makes completely inelastic collision with C. All motions occur on the same straight line. Find the final speed (in m/s) of object C



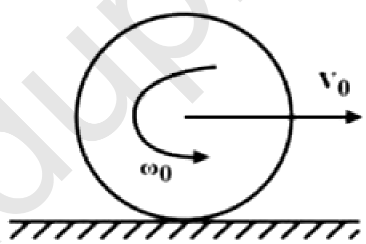
23. In figure, two equal positive point charges $q_1 = q_2 = 2.0\mu C$ interact with a third point charge $Q = 4.0\mu C$. The magnitude, as well as direction, of the net force on Q is $46\times10^{-n}N$. The value of n is



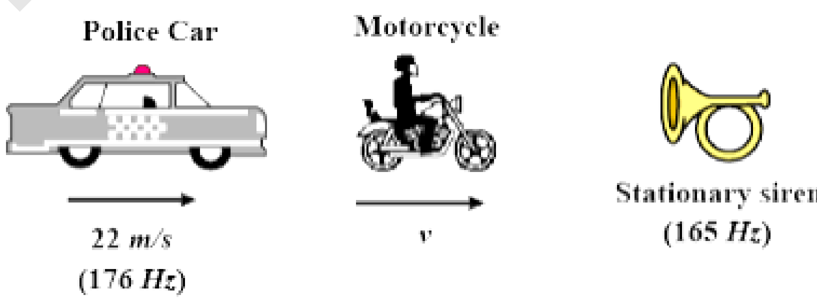
24. Two bodies of mass 1 kg and 2 kg move towards each other in mutually perpendicular direction with the velocities 3 m/s and 2 m/s respectively. If the bodies stick together after collision what will be the value of heat liberated in joule.

25. A wire density $9\times10^{3}\ kg/m^{3}$ is stretched between two clamps 1 m apart and is subjected to an extension of $4.9\times10^{-4} m$. The lowest frequency of transverse vibration in the wire is $\left(Y=9\times10^{10} N/m^{2}\right)\ \frac{x}{5} Hz$, the value of x is

26. A uniform solid sphere of radius R is placed on a rough horizontal surface and given a linear velocity v_0 and an angular velocity ω_0 as shown. If the angular velocity and the linear velocity both become zero simultaneously, then $xv_0 = 2\omega_0 R$, then x is ____.



27. A police car moving at 22 m/s, chases a motorcyclist. The police man sounds his horn at 176 Hz, while both of them move towards a stationary siren of frequency 165 Hz. Calculate the speed of the motorcycle, if it is given that he does not observes any beats (in m/s)



28. Two tall buildings are $200m$ apart. A ball must be thrown horizontally with a speed $(2N+2)m/s$ from the window $540m$ above the ground in one building, so that it will enter a window $50m$ above the ground in the other. Then the value of N is $(Take\ g = 9.8ms^{-2})$

29. On sounding tuning fork A with another tuning fork B of frequency 384 Hz, 6 beats are produced per second. After loading the prongs of A with wax and then sounding it again with B, 4 beats are produced per second. The frequency of the tuning fork A is $5 \times n\ Hz$. The value of n is

30. An iron wire of length 4m and diameter 2mm is loaded with a weight of 16kg. If the young's modulus 'Y' for iron is $2 \times 10^{11} Nm^{-2}$ then the increase in length of the wire is --- in mm $(g = 10\ ms^{2})$

CHEMISTRY

Max Marks: 100

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

31. Assertion: Azeotropic mixtures are formed only by non-ideal solutions and they may have boiling points either greater than both the components or less than both the components.

Reason: The composition of the vapour phase is same as that of the liquid of an azeotropic mixture.

1) Both Assertion (A) and Reason (R) are the true and Reason (R) is a correct explanation of Assertion (A).

2) Both Assertion (A) and Reason (R) are the true but Reason (R) is not a correct explanation of Assertion (A).

3) Assertion (A) is true and Reason (R) is false.

4) Assertion (A) is false and Reason (R) is true.

32. Statement :1 If V is molar volume of gas and gas obeys vander waals's equation, at high pressure the y- intercept of the plot PV (Y-axis) vs P(X-axis) is RT

Statement: 2 Compressibility factor of $He \& H_2$ is more than 1 in almost entire pressure range

1) Both statements are correct.

2) First statement is correct and second statement is wrong

3) First statement is wrong and second statement is correct

4) Both statements are wrong

33. Assertion: The enthalpy of physisorption is greater than chemisorption.

Reason: Molecule of adsorbate and adsorbent are held by vander Waal's forces in physisorption and by chemical bonds in chemisorption.

1) Both Assertion (A) and Reason (R) are the true and Reason (R) is a correct explanation of Assertion (A).

2) Both Assertion (A) and Reason (R) are the true but Reason (R) is not a correct explanation of Assertion (A).

3) Assertion (A) is true and Reason (R) is false.

4) Assertion (A) is false and Reason (R) is true.

34. $K_{sp}(BaSO_4)$ is 1.1×10^{-10}. In which case is $BaSO_4$ precipitated ?

1) 100 mL of $4\times10^{-3}M$ of $BaCl_2 + 300\,mL$ of $6.0\times10^{-4}M$ of Na_2SO_4

2) 100 mL of 4×10^{-4} M of $BaCl_2 + 300\,mL$ of $6\times10^{-8}M$ of Na_2SO_4

3) 100mL of 4×10^{-4} M of $BaCl_2 + 100\,mL$ of $6.0\times10^{-8}M$ of Na_2SO_4

4) In all cases

35. The e.m.f of a Daniell cell at 298K is E_1.

$$Zn\left|\begin{matrix} ZnSO_4 \\ (0.01M) \end{matrix}\right|\left|\begin{matrix} CuSO_4 \\ (1.0\,M) \end{matrix}\right|Cu$$

When the concentration of $ZnSO_4$ is 1.0 M and that of $CuSO_4$ is 0.01 M, the e.m.f. changed to E_2. What is the relationship between $E_1 \& E_2$?

1) $E_2 = 0 \neq E_1$ 2) $E_1 > E_2$ 3) $E_1 < E_2$ 4) $E_1 = E_2$

36. The correct sequence which shows decreasing order of the ionic radii of the elements is

1) $Na^+ > Mg^{2+} > Al^{3+} > O^{2-} > F^-$ 2) $Na^+ > F^- > Mg^{2+} > O^{2-} > Al^{3+}$

3) $O^{2-} > F^- > Na^+ > Mg^{2+} > Al^{3+}$ 4) $Al^{3+} > Mg^{2+} > Na^+ > F^- > O^{2-}$

37. Most acidic oxide among the following is

1) N_2O_5 2) P_2O_5 3) N_2O_4 4) As_2O_3

38. Which one of the following statements is correct?

1) $Ba(OH)_2 > Ca(OH)_2 > Mg(OH)_2 > Be(OH)_2$ [Thermal stability]

2) $BaSO_4 > SrSO_4 > CaSO_4 > MgSO_4$ [solubility in water]

3) $CaF_2 > MgF_2 > BaF_2 > BeF_2$ [solubility in water]

4) $BaCO_3 < SrCO_3 < CaCO_3 < MgCO_3$ [Thermal stability]

39. Which of the following order is/are incorrect regarding the property indicated against it?

(i) HF >HCl > HBr > HI : Dissociation energy

(ii) $Cl_2O_7 > Cl_2O_6 > ClO_2 > Cl_2O$: Acidic character

(iii) $SbCl_3 > SbCl_5$: Covalent character

(iv) MCl > MBr : Ionic character

1) (iii) only 2) (ii) only 3) (i) and (iii) 4) (ii) and (iv)

40. Regarding Hybridisation; Incorrect matching among following is _____.

1) Diborane (Boron) ___ sp^3 2) Silica (silicon) ___ sp^3

3) Diamond___ sp^2

4) Ortho silicate Anion (silicon) ___ sp^3

41. In the form of dichromate, Cr (VI) is a strong oxidizing agent in acidic medium but Mo (VI) is MoO_3 and W (VI) in WO_3 are not because ___________

(i) Cr (VI) is more stable than Mo (VI) and W (VI)

(ii) Mo (VI) and W (VI) are more stable than Cr (VI)

(iii) Higher oxidation states of heavier members of group-6 of transition series are more stable.

(iv) Lower oxidation states of heavier members of group-6 of transition series are more stable.

1) (i) and (ii) 2) (ii) and (iii) 3) (i) and (iv) 4) (ii) and (iv)

42. Correct IUPAC name of the compound is

$$CH_3CH_2-\underset{\displaystyle CH_3}{\underset{|}{CH}}-\underset{\displaystyle CHO}{\underset{|}{CH}}-CH_2CH_3$$

1) 2- methyl-3-ethylpentanal 2) 2,3-diethylbutanal

3) 2-ethyl-3-methylpentanal 4) 3-methyl-2-ethylpetanal

43. Assertion: $\left[Ti\left(H_2O\right)_6\right]^{3+}$ is coloured while $\left[Sc\left(H_2O\right)_6\right]^{3+}$ is colourless.

Reason: d – d transition is not possible in $\left[Sc\left(H_2O\right)_6\right]^{3+}$.

1) Both Assertion (A) and Reason (R) are the true and Reason (R) is a correct explanation of Assertion (A).

2) Both Assertion (A) and Reason (R) are the true but Reason (R) is not a correct explanation of Assertion (A).

3) Assertion (A) is true and Reason (R) is false.

4) Assertion (A) is false and Reason (R) is true.

44. Assertion(A) : In Friedel Carft's Acylation: more stable acetyl ion is $CH_3-C\equiv O^{\oplus}$ & not $CH_3-\overset{\oplus}{C}=O$

Reason(R) : Among resonating structures; negative charge on more electro negative atom is more stable than compared with positive charge on more electro negative atom.

1) A & R are correct but R is correct explanation to A.

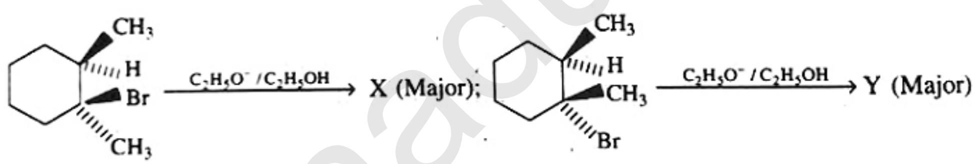
2) A & R are correct but R is not correct explanation to A.

3) A is correct and R is wrong

4) A is wrong and R is correct

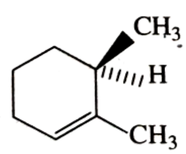
45. Which of the following is most acidic?

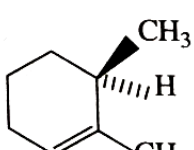
1) Benzyl alcohol
2) Cyclohexanol
3) Phenol
4) m-Chlorophenol

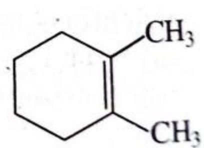
46. 

Correct statement among the following is

1) X and Y are same

2) X is 

3) Y is 

4) Y is 

47. Ethanoic acid can't be obtained by which of the following reaction?

(i) $C_2H_5Cl \xrightarrow[\text{(ii)}H_3O^+]{\text{(i)KCN}}$ (ii) $CH_3Cl \xrightarrow[\text{(ii)}H_3O^+]{\text{(i)AgCN}}$

(iii) $CH_3CH=CH_2 \xrightarrow[\text{heat}]{KMnO_4/H^+}$ (iv) $CH_3Br \xrightarrow[\text{(ii)}CO_2,\ \text{(iii)}H_3O^+]{\text{(i)Mg}}$

1) (iii) and (iv) 2) (i) and (ii) 3) (ii) and (iii) 4) (i) and (iv)

48. $X \xrightarrow[170^0 C]{conH_2SO_4} Y \xrightarrow[CCl_4]{Br_2} Q \xrightarrow[ii)\ NaNH_2,\ iii)\ H_2O]{i)\ alcKOH} R$ if X is ethyl alcohol correct statement(s) about R is/are

A) It decolourizes Bayer's reagent

B) It can give benzene when subjected to polymerization

C) It gives red precipitate with ammonical Cu_2Cl_2

D) It gives ethylene on hydrogenation in the presence of Lindlar's catalyst

1) A,B & C 2) B,C&D 3) A,B,C &D 4) A,B & D

49. Which of the following statements is false?

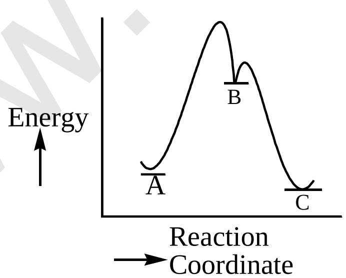
1) Artificial silk is derived from cellulose.

2) Nylon-66 is an example of elastomer.

3) The monomer in natural rubber is isoprene.

4) Both starch and cellulose are polymers of glucose.

50. Statement-1



Above graph is for nitration of benzene to form nitrobenzene reaction

Positions A & C are Aromatic ;

whereas B is Non Aromatic σ complex intermediate in the graph

Statement-2 During Electrophilic Substitution of Nitroso Benzene; the existing group is an ortho, para director for next incoming electrophile

1) Both Statements are true.

2) First statement is wrong and second statement is correct

3) First statement is correct and second statement is wrong

4) Both statements are wrong

(NUMERICAL VALUE TYPE)

Section-II contains 10 **Numerical Value Type** questions. Attempt any 5 questions only. First 5 attempted questions will be considered if more than 5 questions attempted. The Answer should be within **0 to 9999.** If the Answer is in **Decimal** then round off to the **nearest Integer** value (Example i,e. If answer is above **10** and less than **10.5** round off is **10** andIf answer is from **10.5** and less than **11** round off is **11).**

Marking scheme: +4 for correct answer, 0 if **not attempt** and **-1** in all other cases.

---

51. Convulsion is caused by the deficiency of vitamin B_x, then 'x' is

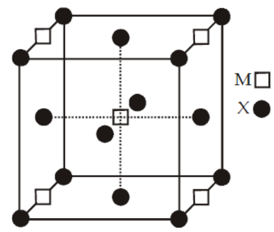
52. In the reaction $NaOH + Al(OH)_3 \rightarrow NaAlO_2 + H_2O$, the $\frac{M.wt}{Eq.wt}$ of $Al(OH)_3 is$ ____

53. The electrical properties and their respective SI units are given below. The correctly matched pairs are 'X' then X is ________

Electrical property	SI unit
Specific conductance	Sm^{-1}
Conductance	S
Equivalent conductance	$Sm^2 gequiv^{-1}$
Cell constant	m

54. For the equilibrium $AB_{(g)} \rightleftharpoons A_{(g)} + B_{(g)}$ at a given temperature $\frac{1}{3}$rd of AB is dissociated then $\frac{P}{K_P}$ will be numerically equal to _________

55. A compound M_pX_q has cubic close packing (ccp) arrangement of X. Its unit cell structure is shown below. The empirical formula of the compound is M_aX_b, then $a+b=$ ______



56. Heat of combustion of A(s) is -10 kcal mol^{-1} and that of B(s) is -15 kcal mol^{-1}. Mixture containing 3 moles A(s) and x moles B(s) on combustion released 105 kcal of heat. What is the value of x ?

57. Vapour pressure of benzene at 30°C is 121.8 mm Hg. When 15g of a non volatile solute is dissolved in 250 g of benzene its vapour pressure decreased to 120.2 mm Hg. The molecular weight of the solute (Considered as a dilute solution) [Nearest Integer]

(Mo. Wt. of solvent = 78)

58. The number of electrons with m = 0 in an atom with atomic number 33 is 5x. Then the value of x is _________

59. In a set of the given reactions, acetic acid yielded a product C.

$$CH_3COOH + PCl_5 \longrightarrow A \xrightarrow[\text{Anhyd.}AlCl_3]{C_6H_6} B \xrightarrow[\text{(ii) } H^+]{\text{(i) } C_2H_5MgBr\text{ / Ether}} C$$

In the Product C number of carbon atoms X and number of hydrogen atoms Y, then $\frac{Y-X}{2}=$ _____________

60. Select the number of species which are having fractional bond order, with paramagnetic nature $H_2^+, O_2, O_2^+, O_2^-, C_2, C_2^+, N_2, N_2^+, F_2, CN^-$

MATHEMATICS

Max Marks: 100

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

61. Let $f(x)=x^2+\int_0^x e^{-t} f(x-t)\,dt$ and $f(0)=0$. Then $\left[f(3)\right]$ where [.] represent greatest integer

1) 9 2) 18 3) 0 4) 12

62. If $f(x)=x^2+2bx+2c^2$ and $g(x)=-x^2-2cx+b^2$ are such that min $f(x)>\max g(x)$, then relation between b and c, is:

1) no relation 2) 0 < c < b/2 3) $|c|<\dfrac{|b|}{\sqrt{2}}$ 4) $|c|>\sqrt{2}|b|$

63. The solution of differential equation $2x^3ydy+\left(1-y^2\right)\left(x^2y^2+y^2-1\right)dx=0$ is:

1) $x^2y^2=(cx+1)\left(1-y^2\right)$ 2) $x^2y^2=(cx+1)\left(1+y^2\right)$

3) $x^2y^2=(cx-1)\left(1-y^2\right)$ 4) $x^2y^2=(cx-1)\left(1+y^2\right)$

64. Assertion(A): A function f satisfies the condition

$$f(x+T)=1+\left\{1-3f(x)+3(f(x))^2-(f(x))^3\right\}^{\frac{1}{3}}$$ (where T is a fixed positive number) is periodic with period 2T.

Reason(R): If $f(x+2T)=f(x)$, then period of $f(x)$ is 2T.

1) Both (A) & (R) are individually true & (R) is correct explanation of (A).

2) Both (A) & (R) are individually true but (R) is not the correct (proper) explanation of (A)

3) (A) is true but (R) is false.

4) (A) is false but (R) is true.

65. The differential equation for all family of lines which are at a unit distance from the origin is:

1) $\left(y-x\frac{dy}{dx}\right)^2=1-\left(\frac{dy}{dx}\right)^2$ 2) $\left(y+x\frac{dy}{dx}\right)^2=1+\left(\frac{dy}{dx}\right)^2$

3) $\left(y-x\frac{dy}{dx}\right)^2=1+\left(\frac{dy}{dx}\right)^2$ 4) $\left(y+x\frac{dy}{dx}\right)^2=1-\left(\frac{dy}{dx}\right)^2$

66. If $A=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 1\\ 0 & -2 & 4\end{bmatrix}, 6A^{-1}=A^2+cA+dI$,then (c, d) is

1) (-6,11) 2) (-11,6) 3) (11,6) 4) (6,11)

67. The value of the parameter 'a' such that the area bounded by $y=a^2x^2+ax+1$, coordinate axes and the line x = 1 attains its least value, is equal to

1) $-\frac{1}{4}$ 2) $-\frac{1}{2}$ 3) $-\frac{3}{4}$ 4) -1

68. If $\exp\left\{\left(\sin^2 x+\sin^4 x+\sin^6 x+...\text{upto}\infty\right)\ln 2\right\}$ satisfies the equation $x^2-17x+16=0$ then value of $\frac{2\cos x}{\sin x+2\cos x}\left(0<x<\pi/2\right)$ is

1) $\frac{1}{2}$ 2) $\frac{3}{2}$ 3) $\frac{5}{2}$ 4) $\frac{7}{2}$

69. $\int\frac{e^{x^2}\left(2x+x^3\right)}{\left(3+x^2\right)^2}dx$ is equal to:

1) $\frac{e^{x^2}}{\left(3+x^2\right)}+c$ 2) $\frac{1}{8}\frac{e^{x^2}}{\left(3+x^2\right)}+c$ 3) $\frac{1}{4}\frac{e^{x^2}}{\left(3+x^2\right)}+c$ 4) $\frac{1}{2}\frac{e^{x^2}}{\left(3+x^2\right)}+c$

70. The value of $\lim_{x\to 0}\left(\left[\frac{11x}{\sin x}\right]+\left[\frac{21\sin x}{x}\right]\right)$, where $[x]$ is the greatest integer less than or equal to x is

1) 32 2) 31 3) 11 4) 21

71. A five-digit number (having all different digits) is formed using the digits 1, 2, 3, 4, 5, 6, 7, 8 and 9. The probability that the formed number either begins or ends with an odd digit, is equal to:

1) $\frac{5}{6}$ 2) $\frac{1}{6}$ 3) $\frac{1}{3}$ 4) $\frac{2}{3}$

72. Assertion (A): The shortest distance between the skew lines $\frac{x-1}{2}=\frac{y-1}{-1}=\frac{z-0}{1}$ and $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+1}{2}$ is $\frac{10}{\sqrt{2}}$

Reason (R): Two lines are skew lines if there exists no plane passing through them .

1) Both (A) and (R) are individually True and (R) is the correct explanation of (A)

2) Both (A) and (R) are individually True and (R) is not the correct explanation of (A)

3) (A) is True but (R) is False

4) (A) is False but (R) is True

73. Locus of z if $\arg\left[z-(1+i)\right]=\begin{cases}\frac{3\pi}{4} \text{ when } |z|<|z-2| \\ \frac{-\pi}{4} \text{ when } |z|>|z-2|\end{cases}$ is:

1) Straight lines passing through (2, 0)

2) Lines passing through (2, 0) and (1, 1)

3) A line segment 4) A set of two rays

74. The value of c for which the conclusion of Lagrange's theorem holds for the function $f(x)=\sqrt{a^2-x^2}, a>1$ on the interval $[1,a]$ is

1) $\frac{a(a+1)}{2}$ 2) $\frac{1+a}{2}$ 3) $\sqrt{\frac{a(a+1)}{2}}$ 4) $\frac{a(a-1)}{2}$

75. The sum of the roots of the equation $2^{(33x-2)}+2^{(11x+2)}=2^{(22x+1)}+1$ is

1) $\frac{1}{11}$ 2) $\frac{2}{11}$ 3) $\frac{3}{11}$ 4) $\frac{4}{11}$

76. Assertion (A): If Rolle's theorem be applied in $f(x)$, then L.M.V. theorem is also applicable for $f(x)$.

Reason (R): Both Rolle's theorem & L.M.V. theorem can not be applied in $f(x)=|\sin|x||\text{in}\left[-\frac{\pi}{3},\frac{\pi}{3}\right]$.

The codes are given below, select the correct codes.

1) Both (A) & (R) are individually true & (R) is correct explanation of (A).

2) Both (A) & (R) are individually true but (R) is not the correct (proper) explanation of (A).

3) (A) is true but (R) is false.

4) (A) is false but (R) is true.

77. If A and B are two events such that $P(A)=\frac{4}{7},P(A\cap B)=\frac{3}{28}$ and the conditional probability $P\left(\frac{A}{A^c\cup B^c}\right)$ (where A^c denotes the compliment of the event A) is equal to λ, then the value of $\frac{13}{\lambda}$ is equal to

1) 25 2) 50 3) 26 4) 51

78. The angle between the lines whose direction cosines are given by the equations $l^2+m^2-n^2=0, l+m+n=0$ is

1) $\frac{\pi}{6}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{2}$

79. Consider the system of equations px + qy = 0, rx + sy = 0 where $p,q,r,s\in\{-1,1\}$

Statement-1: The probability that the system of equation has a unique solution is $\frac{1}{2}$.

Statement-2: The probability that the system of equations no solution is zero.

1) Statement-1 is true, statement-2 is true and ststement-2 is a correct explanation for statement-1

2) Statement-1 is true, statement-2 is true and ststement-2 is NOT the correct explanation for statement-1

3) Statement-1 is true, statement-2 is false

4) Statement-1 is false, statement-2 is true

80. If $A = \sin^2\theta + \cos^4\theta,$ then for all values of θ

1) $1 \le A \le 2$ 2) $\frac{3}{4} \le A \le 1$ 3) $\frac{13}{16} \le A \le 1$ 4) $\frac{3}{4} \le A \le \frac{13}{16}$

(NUMERICAL VALUE TYPE)

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Marking scheme: +4 for correct answer, 0 if **not attempt** and **-1** in all other cases.

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81. If $\int \frac{dx}{\sqrt{x} + \sqrt[3]{x}} = a\sqrt{x} + b\left(\sqrt[3]{x}\right) + c\left(\sqrt[6]{x}\right) + d\ln\left(\sqrt[6]{x} + 1\right) + e, e$ being an arbitrary constant, then the value of $|a + b + c + d|$ is

82. If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then k is equal to

83. Let C_1 be the graph of xy = 1 and the reflection of C_1 in the line y = 2x is C_2. If the equation of C_2 is expressed as $12x^2 + bxy + cy^2 + d = 0$, then bc =

84. If $\sin^4 x + \cos^4 y + 2 = 4\sin x \cos y, 0 \le x, y \le \pi/2$ then $\sin x + \cos y =$

85. If the area bounded by $y = \left||x^2| - 4|x| + 3\right|$ and the x-axis from x = 1 to x = 3 is $\frac{p}{q}$ (where, p & q are coprime), then the value of p + q is

86. The number of solutions of the equation $\cos\left(\pi\sqrt{x-4}\right)\cos\left(\pi\sqrt{x}\right)=1$ is

87. If $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n, \sum_{r=0}^{n}\left((r+1)^2\right)C_r = 2^{n-2}f(n)$ and if the roots of the equation f(x) = 0 are $\alpha \& \beta$, then the value of $\alpha^2+\beta^2$ is equal to (where C_r denotes ${}^nC_r$)

88. If the projections of a line segment on the $x, y,$ and z-axes in 3- dimensional space are 2,3 and 6 respectively, then the length of the line segment is

89. The probability of India winning a test match against Australia is $\frac{1}{4}$. Assuming the matches to be independent events, the probability that in a 7 match series India's second win occurs at 4^{th} test is P, then 256P is equal to

90. The distance of the point (2,3) from the line $4x-3y+26=0$ is same as its distance from the line $3x-4y+p=0$. The positive value of p is

KEY SHEET

PHYSICS

1	**3**	2	**2**	3	**4**	4	**1**	5	**2**
6	**4**	7	**2**	8	**3**	9	**4**	10	**1**
11	**3**	12	**1**	13	**3**	14	**3**	15	**2**
16	**4**	17	**1**	18	**3**	19	**3**	20	**3**
21	**50**	22	**4**	23	**2**	24	**4**	25	**175**
26	**5**	27	**22**	28	**9**	29	**78**	30	**1**

CHEMISTRY

31	**2**	32	**1**	33	**4**	34	**1**	35	**2**
36	**3**	37	**1**	38	**1**	39	**1**	40	**3**
41	**2**	42	**3**	43	**1**	44	**2**	45	**4**
46	**3**	47	**2**	48	**3**	49	**2**	50	**1**
51	**6**	52	**1**	53	**3**	54	**8**	55	**3**
56	**5**	57	**356**	58	**3**	59	**2**	60	**5**

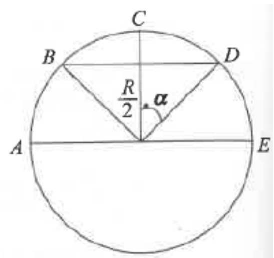
MATHEMATICS

61	**2**	62	**4**	63	**3**	64	**1**	65	**3**
66	**1**	67	**3**	68	**1**	69	**4**	70	**2**
71	**1**	72	**2**	73	**4**	74	**3**	75	**2**
76	**2**	77	**1**	78	**3**	79	**2**	80	**2**
81	**1**	82	**5**	83	**84**	84	**2**	85	**7**
86	**1**	87	**17**	88	**7**	89	**27**	90	**31**

SOLUTIONS

PHYSICS

1. $\alpha = 60°$. Solid angle subtended by BCD is



$\omega = 2\pi(1-\cos\alpha) = \pi$

Solid angle subtended by ABDE is

$\omega_{(ABCDE)} - \omega_{(BCD)} = 2\pi - \pi = \pi$

Hence, flux through ABDE is $\phi = \frac{q}{\varepsilon_0} \times \frac{\pi}{4\pi} = \frac{q}{4\varepsilon_0}$

2. $a = \frac{F_{net}}{m} = \frac{-2\rho Agx}{m} = -\omega^2 x$

$T = \frac{2\pi}{\omega}$

3. $K = \frac{hc}{\lambda} - \phi_0$ (i) and $K' = \frac{4hc}{3\lambda} - \phi_0$ (ii)

From Eq/s (i) & (ii), we get $\Rightarrow K' - K = \frac{4hc}{3\lambda} - \frac{hc}{\lambda}$

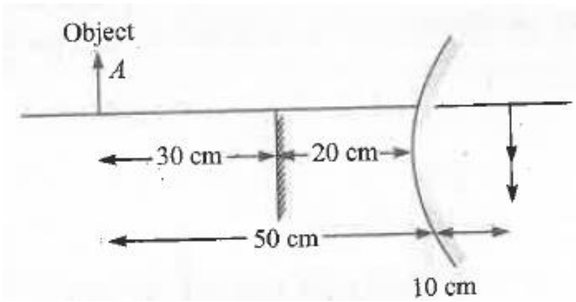
$K' - K = \frac{hc}{3\lambda}$

But from Eq. (i) $\frac{hc}{\lambda} = K + \phi_0 \therefore$ $K' - K = \frac{K}{3} + \frac{\phi_0}{3}$

$\Rightarrow$ $K' - K = \frac{4K}{3} + \frac{\phi_0}{3}$ Or $K' > \frac{4K}{3}$

4. Dependence of kinetic energy on net force

5. Since there is no parallax, it means that both images (By plane mirror and convex mirror) coinciding each other.



According to property of plane mirror it will form image at a distance of 30 cm behind it. Hence for convex mirror $u = -50\text{cm}, v = +10\text{cm}$

By using $\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \Rightarrow \frac{1}{f} = \frac{1}{+10} + \frac{1}{-50} = \frac{4}{50}$

$\Rightarrow f = \frac{25}{2}\text{cm} \Rightarrow R = 2f = 25\text{ cm}$

6. $F = \frac{-du}{dr} = \frac{\lambda}{r^2}$

Eccenricity $e = \sqrt{1 - \frac{b^2}{a^2}} = \frac{3}{5}$

so, SO = 3m, SA= 2m, SC = 8m

$U_A = -\frac{\lambda}{2} = -20J, U_c = -5J$

$r_A m V_A = r_c m V_c \; 2V_A = 8V_c$

$\frac{1}{2}mv_A^2 - 20 = \frac{1}{2}mv_c^2 - 5$

Solving we get $V_A = 4m/s$

7. Energy, $R = K\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$(K=constant)

$n_1 = 2$ and $n_2 = 3$. So $E = K\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = K\left[\frac{5}{36}\right]$

For removing an electron $n_1 = 1$ to $n_2 = \infty$

Energy $E_1 = K[1] = \frac{36}{5}E = 7.2E$

$\therefore$ Ionization energy $= 7.2E$

8. Using equation, $a = \frac{v-u}{t}$ and $S = ut + \frac{1}{2}at^2$

Distance travelled by car in 15 sec $= \frac{1}{2}\frac{(45)}{15}(15)^2 = \frac{675}{2}m$

Distance travelled by scooter in 15 seconds $= 30 \times 15 = 450$

$(\therefore \text{distance} = \text{speed} \times \text{time})$

Difference between distance travelled by car and scooter in

15 sec, 450-337.5 = 112.5m

Let car catches scooter in time t;

$\frac{675}{2} + 45(t-15) = 30t \Rightarrow 337.5 + 45t - 675 = 30t \Rightarrow 15t = 337.5$

$\Rightarrow t = 22.5\text{sec}$

9. $Z = \sqrt{(R)^2 + (X_L - X_C)^2};$

$R = 10\Omega, X_L = \omega L = 2000 \times 5 \times 10^{-3} = 10\Omega$

$X_C = \frac{1}{\omega C} = \frac{1}{2000 \times 50 \times 10^{-6}} = 10\Omega$ i.e $Z = 10\Omega$

Maximum current $i_0 = \frac{V_0}{Z} = \frac{20}{10} = 2\,A$

Hence $i_{rms} = \frac{2}{\sqrt{2}} = 1.4A$ And $V_{rms} = 4 \times 1.41 = 5.64V$

10. Net acceleration $\frac{dv}{dt} = a = -\left(g + \gamma v^2\right)$

Let time t required to rise to its zenith (v = 0) so,

$$\int_{v_0}^{0} \frac{-dv}{g + \gamma v^2} = \int_0^t dt \qquad \left[\text{for } H_{\max,} v = 0\right]$$

$$\therefore t = \frac{1}{\sqrt{\gamma g}} \tan^{-}\left(\frac{\sqrt{\gamma}}{\sqrt{g}} v_0\right)$$

11. Rate of decay of current between $t = 5$ ms to 6 ms

$$= \frac{di}{dt} = -\left(\text{slope of the line BC}\right) = -\left(\frac{5}{1 \times 10^{-3}}\right) = -5 \times 10^3 \,A/s$$

Hence induced emf $\varepsilon = -L\frac{di}{dt} = -4.6 \times \left(-5 \times 10^3\right) = 23 \times 10^3\,V$

12. Given : Gravitational field, $E_G = \frac{Ax}{(x^2 + a^2)^{3/2}}, V_\infty = 0$

$$\int_{V_\infty}^{V_x} dV = -\int_\infty^x \vec{E}_G . \vec{d}_x \Rightarrow V_x - V_\infty = -\int_\infty^x \frac{Ax}{(x^2 + a^2)^{3/2}} dx$$

$$\therefore V_x = \frac{A}{(x^2 + a^2)^{1/2}} - 0 = \frac{A}{(x^2 + a^2)^{1/2}}$$

13. $X = \left|X_C - X_L\right|$ and average power in ac circuit can be zero.

14. At equilibrium, $\vec{F}=0 \Rightarrow \frac{-du}{dr}=0 \Rightarrow \frac{-d}{dr}\left(\frac{A}{r^{10}}-\frac{B}{r^5}\right)=0$

$$\Rightarrow +\frac{A}{r^{11}}\times 10-\frac{5B}{r^6}=0 \quad \Rightarrow \frac{1}{r^6}\left[\frac{10A}{r^5}-\frac{5B}{1}\right]=0$$

$$\Rightarrow \frac{10A}{r^5}=5B \quad \Rightarrow r^5=\frac{10A}{5B} \quad \Rightarrow r=\left(\frac{2A}{B}\right)^{\frac{1}{5}}$$

15. In case of internal resistance measurement by potentiometer,

$$\frac{V_1}{V_2}=\frac{\ell_1}{\ell_2}=\frac{\left[ER_1/\left(R_1+r\right)\right]}{\left[ER_2/\left(R_1+r\right)\right]}=\frac{R_1\left(R_2+r\right)}{R_2\left(R_1+r\right)}$$

Here $\ell_1=2\text{m}, \ell_2=3\text{m}, R_1=5\Omega,$ and $R_2=10\Omega.$ So

$$\frac{2}{3}=\frac{5\left(10+r\right)}{10\left(5+r\right)} \qquad \text{or} \qquad r=10\Omega$$

16. The thermal resistance is given by

$$\frac{x}{KA}+\frac{4x}{2KA}=\frac{x}{KA}+\frac{2x}{KA}=\frac{3x}{KA}$$

Amount of heat flow per second,

$$\frac{dQ}{dt}=\frac{\Delta T}{\frac{3x}{KA}}=\frac{(T_2-T_1)KA}{3x}=\frac{1}{3}\left\{\frac{A(T_2-T_1)K}{x}\right\} \qquad \therefore f=\frac{1}{3}$$

17. When energy on both is same, means energy on capacitor is half of its maximum energy. $\frac{q^2}{2C}=\frac{1}{2}\frac{Q^2}{2C}\Rightarrow q=\frac{Q}{\sqrt{2}}$

$$\Rightarrow Q\cos\omega t=\frac{Q}{\sqrt{2}}\Rightarrow \cos\omega t=\frac{1}{\sqrt{2}}$$

$$\Rightarrow \omega t=\frac{\pi}{4}\Rightarrow t=\frac{\pi}{4\omega}=\frac{\pi}{4}\sqrt{LC}$$

18. From, ideal gas equation $PV = nRT$

$$\Rightarrow 400\times10^3\times500\times10^{-6} = n\left(\frac{25}{3}\right)(300) \Rightarrow n = \frac{2}{25}$$

Let n_1 & n_2 be no. of moles of hydrogen & oxygen respectively

Also, $n_1 + n_2 = n$

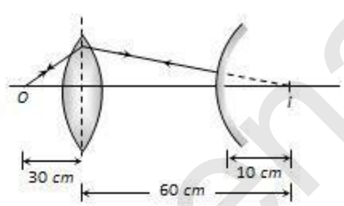
Using $n = \frac{m}{M}$ $\qquad \frac{m_1}{2} + \frac{m_2}{32} = \frac{2}{25}$

and $m_1 + m_2 = 0.76\,gm$ $\frac{m_2}{m_1} = \frac{16}{3}$

19. For lens $\mathrm{u} = 30\mathrm{cm}, \mathrm{f} = 20\mathrm{cm},$ hence by using

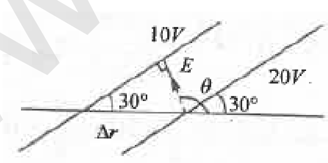
$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{+20} = \frac{1}{v} - \frac{1}{-30} \Rightarrow v = 60\,\mathrm{cm}$$

The final image will coincide the object, if light ray falls normally on convex mirror as shown. From figure, it could be seen clearly that separation between lens and mirror is 60 – 10 = 50 cm.



20. Equate torques about centre of mass

21. Using $dV = -\vec{E}.d\vec{r}$

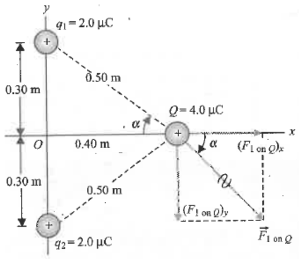


$$\Rightarrow \qquad \Delta V = -E.\Delta r\cos\theta \Rightarrow \qquad E = \frac{-\Delta V}{\Delta r\cos\theta}$$

$$\Rightarrow \qquad E = \frac{-\left(20-10\right)}{10\times10^{-2}\cos120^{\circ}} = \frac{-10}{10\times10\left(-\sin30^{\circ}\right)} = \frac{-10^{2}}{-1/2} = 200\text{V}/\text{m}$$

22. $2mv_2 = 3mv, 2\left(6\right) = 3v, v = 4m/s$

23.



$$F_{net} = 2\left|F_{31}\right|\cos\alpha = 2\times\frac{1}{4\pi\varepsilon_0}\times\frac{2\times4\times10^{-12}}{\left(0.5\right)^{2}}\times\frac{4}{5} = 0.46\text{N}$$

24. $m_1 = 1kg, m_2 = 2kg, u_1 = 3ms^{-1}, u_2 = 2ms^{-1}$

Initial momentum $= \sqrt{(m_1u_1)^2 + (m_2u_2)^2} = \sqrt{9+16} = 5\text{kg ms}^{-1}$

If combined velocity is $v(m_1 + m_2)v = 5, \qquad v = \frac{5}{3}\text{ms}^{-1}$

Heat liberated = loss in kinetic energy $= \frac{1}{2}m_1u_1^2\frac{1}{2}m_2u_2^2 - \frac{1}{2}(m_1+m_2)v^2 = \frac{13}{3}J$

25. For wire if

$M = \text{mass}, \rho = \text{density}, A = \text{Area of cross section}$

$V = \text{volume}, l = \text{length}, \Delta l = \text{change in length}$

Then mass per unit length $m = \frac{M}{l} = \frac{Al\rho}{l} = A\rho$

And Young's modulus of elasticity $Y = \frac{T/A}{\Delta l/l}$

$$\Rightarrow \qquad T = \frac{Y\Delta lA}{l}.$$

Hence lowest frequency of vibration

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2l}\sqrt{\frac{Y\left(\frac{\Delta l}{l}\right)A}{A\rho}} = \frac{1}{2l}\sqrt{\frac{Y\Delta l}{l\rho}}$$

$$\Rightarrow \quad n = \frac{1}{2\times 1}\sqrt{\frac{9\times 10^{10}\times 4.9\times 10^{-4}}{1\times 9\times 10^{3}}} = 35\text{ Hz}$$

26. $\vec{L}_{final} = 0, so\frac{2}{5}mR^2\omega_0 = mv_0R$

27. $n_1 =$ Frequency of the police car horn observer heard by motorcyclist

$n_2 =$ Frequency of the siren heard by the motorcyclist.

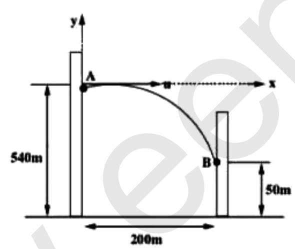
V = speed of motorcyclist.

$n_1 = \frac{330-V}{330-22}\times 176 \; n_2 = \frac{330+V}{330}\times 165 \therefore n_1 - n_2 = 0 \Rightarrow V = 22\,m/s$

28. Taking point A as origin and the axes as shown in for path A to B, we have R=200m.

H=540 – 50=490 m

For horizontal projection, we have $R = u\sqrt{\frac{2H}{g}}$



$\Rightarrow u = R\sqrt{\frac{g}{2H}} = 200\times\sqrt{\frac{9.8}{2\times 490}} = 20m/s \;\; N = 9$

29. Probable frequency of A is 390 Hz and 378 Hz and after loading the beats are decreasing from 6 to 4 so the original frequency of A will 390 Hz

30. $\Delta l = \frac{Fl}{Ay}$ =1

CHEMISTRY

31. Azeotropic mixtures and boiling points.

32. At high P,Z $Z = 1 + \frac{Pb}{RT} = \frac{PV}{RT}$

PV vs P, slope =b, intercept =RT

Vander waals Constant "a" values are negligible & neglected (for H_2 & He) in comparison with a values of other gases.

33. Assertion is false but Reason is true. The enthalpy of chemisorption is of the order of $40 - 400\,\mathrm{kJmol^{-1}}$ while for physical adsorption it is of the order of $20 - 40\,\mathrm{kJmol^{-1}}$.

34. Before mixing,

$\left[Ba^{+2}\right] = 4 \times 10^{-3} M \Rightarrow \left[SO_4^{-2}\right] = 6 \times 10^{-4} M$

after mixing,

$4 \times 10^{-3} \times 100 = M_2 \times 400$

$\left[Br^{+2}\right] = 10^{-3} M$

$6 \times 10^{-4} \times 300 = M_2 \times 400$

$\left[SO_4^{-2}\right] = 4.5 \times 10^{-4} M$

$IP = \left[Ba^{+2}\right]\left[SO_4^{-2}\right] = 4.5 \times 10^{-7}; Ksp < I.P.$

35. Cell reaction is, $\mathrm{Zn} + \mathrm{Cu}^{2+} \rightarrow \mathrm{Zn}^{2+} + \mathrm{Cu}$

$$\mathrm{E_{cell}} = \mathrm{E^0_{cell}} - \frac{\mathrm{RT}}{\mathrm{nF}} \ell\mathrm{n} \frac{\left[\mathrm{Zn}^{2+}\right]}{\left[\mathrm{Cu}^{2+}\right]}$$

Greater the factor $\left[\frac{\left(\mathrm{Zn}^{2+}\right)}{\left(\mathrm{Cu}^{2+}\right)}\right]$, less is the EMF

Hence $\mathrm{E_1} > \mathrm{E_2}$

36. More the magnitude of negative charge, more the size

37. Oxide in which central atom has higher charge and more electronegativity is more acidic, i.e.

$N_2O_5 > N_2O_4 > P_2O_5 > As_2O_3$

38. $BeSO_4$ to $BaSO_4$: Solubility decreases
$BeCO_3$ to $BaCO_3$: Thermal stability increases
$BeF_2 > MgF_2 > CaF_2 > SrF_2$: Solubility in water.

39. Metal halides with higher oxidation state are more covalent than the one in lower oxidation state.

40. Hybridisation of central atom

41. Oxidation power of Cr(V_1)

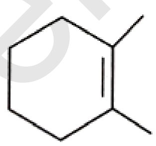
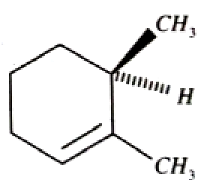
42. 2-ethyl-3-methylpentanal

43. Colour related to d-d transition.

44. $R-C\equiv O^{\oplus}$ octet filled structure. where as $R-\overset{\oplus}{C}=O$ has "C" with $6e^-$.

45. Acidic strength comparison

46. It is E_2 elimination and antiperiplanarity is required

Hence X is  and Y is 

47. i) Gives Propanoic acid

ii) Methyl isocyanide on hydrolysis give methyl amine

48.

$$\underset{(X)}{C_2H_5OH} \xrightarrow[170^0 c]{conc.H_2SO_4} \underset{(Y)}{CH_2=CH_2} \xrightarrow{Br_2/CCl_4}$$

$$\underset{Br}{\underset{|}{CH_2}}-\underset{Br}{\underset{|}{CH_2}} \xrightarrow[\Delta]{alc.KOH} \underset{\substack{Ethyne\\(R)}}{CH\equiv CH}$$

49. Nylon-66 is an example of first synthetic fibres produced from the simple molecules. It is prepared by condensation polymerization of adipic acid and hexamethylene diamine.

50. In graph $\Rightarrow$ Point B is Non aromatic, σ Complex, Arenonium ion intermediate. Nitroso is an o, p director.

51. Convulsion is caused by deficiency of vitamin B_6

52. $Al(OH)_3$ acts as acid and accepts one OH^- $\Rightarrow n-f=1$

53. Cell constant $= \ell / a$

Unit $= m / m^2 = m^{-1}$

54.

	AB	$\rightleftharpoons$	A	+	B	
at eq1	-1/3		1/3		1/3;	total moles =4/3

$$P_{AB} = \frac{2/3}{4/3} P = \frac{1}{2} P; \; P_A = P_B = \frac{1/3}{4/3} P = \frac{1}{4} P$$

$$K_P = \frac{P/4 \times P/4}{P/2} = \frac{P}{8}$$

55. No.of M atoms $= \frac{1}{4} \times 4 + \frac{1}{8} \times 8 = 1 + 1 = 2$

No.of X atoms $= \frac{1}{2} \times 6 + \frac{1}{8} \times 8 = 3 + 1 = 4$

So, formula $= M_2X_4 = MX_2$, a = 1 ; b = 2 ; a + b = 3.

56. $A + O_2 \rightarrow \Delta H_1 = -10 Kcal$

$B + O_2 \rightarrow \Delta H_2 = -15 Kcal$

$3A + xB = 105 \Rightarrow 3(10) + x(15) = 105 \Rightarrow x = 5$

57. From Raoult's law

$$\frac{P^\circ - P}{P^\circ} = \frac{w}{m} \times \frac{M}{W} \quad \frac{121.8 - 120.2}{121.8} = \frac{15}{m} \times \frac{78}{250}$$

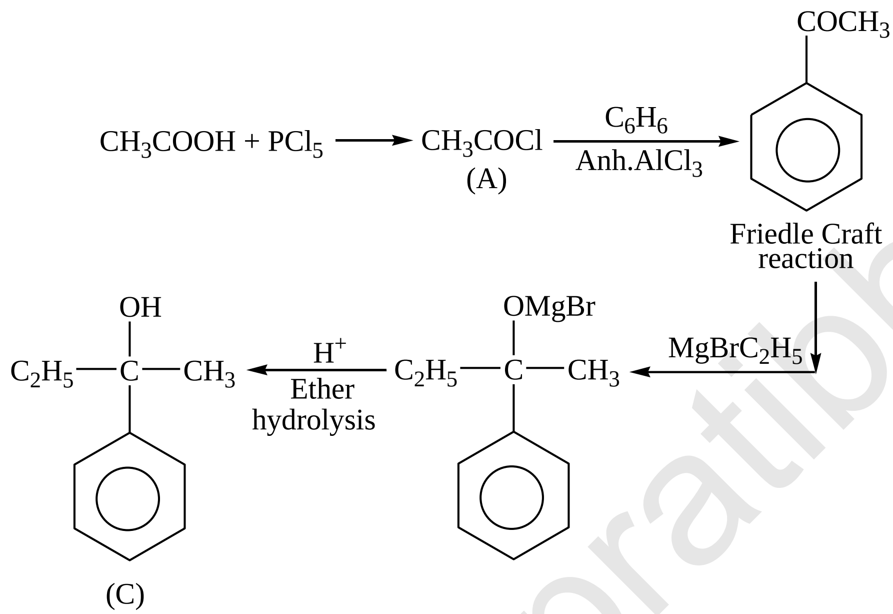
Or $m = \frac{15 \times 78}{250} \times \frac{121.8}{1.6} = 356.2$

58. Configuration $(Z=33)=1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2\ 3d^{10}\ 4p^3$

Each subshell has one orbital with m = 0

$\therefore$ Number of electrons $=2\times 7+1=15$

59.



60. $H_2^+, O_2^+, O_2^-, C_2^+, N_2^+$

MATHEMATICS

61. $f(x)=x^2+\int_0^x e^{-t}f(x-t)dt=x^2+\int_0^x e^{-(x-t)}f(t)dt=x^2+\int_0^x e^{-x}.e^t f(t)dt=x^2+e^{-x}\int_0^x e^t f(t)dt$

$f^1(x)=2x+e^{-x}.e^x f(x)-e^{-x}\int_0^x e^t f(t)dt=2x+f(x)-f(x)+x^2$

$f(x)=x^2+\dfrac{x^3}{3}+c\ f(0)=0\ f(x)=x^2+\dfrac{x^3}{3}$

$f(3)=9+9=18$

62. $f(x)=(x+b)^2+2c^2-b^2 \Rightarrow \min f(x)=2c^2-b^2$

Also $g(x)=-x^2-2cx+b^2=b^2+c^2-(x+c)^2 \Rightarrow \max g(x)=b^2+c^2$

As min $f(x)>\max g(x)$ we get $2c^2-b^2>b^2+c^2$

$\Rightarrow \quad c^2>2b^2 \Rightarrow |c|>\sqrt{2}|b|$

63. $2x^3ydy + (1-y^2)(x^2y^2 + y^2 - 1)dx = 0$

$\Rightarrow \frac{2y}{(1-y^2)^2}\frac{dy}{dx} + \frac{y^2}{1-y^2}\frac{1}{x} = \frac{1}{x^3}$

Put $\frac{y^2}{1-y^2} = u.$

Then $\frac{2y}{(1-y^2)^2}\frac{dy}{dx} = \frac{du}{dx}$

$\Rightarrow \frac{du}{dx} + \frac{u}{x} = \frac{1}{x^3} \Rightarrow$ Integrating factor $= e^{\int \frac{1}{x}dx} = e^{\ln x} = x$

Solution is $u.x = \int \frac{1}{x^2}dx + c \Rightarrow x^2y^2 = (cx-1)(1-y^2)$

64. $f(x+T) = 1 + \left\{1 - 3f(x) + 3(f(x))^2 - (f(x))^3\right\}^{1/3}$

$= 1 + \left\{(1-f(x))^3\right\}^{1/3}$

$f(x-T) = -f(x) + 2$ …(i)

$\Rightarrow f(x+T) + f(x) = 2$ …(ii)

$\therefore f(x+2T) + f(x+T) = 2$

$\therefore$ from (ii)-(i) we get

$f(x+2T) - f(x) = 0$

$\Rightarrow f(x+2T) = f(x)$

$\Rightarrow$ Period of $f(x)$ is 2T

Assertion (A) & Reason (R) both are true & Reason (R) is correct explanation of Assertion (A).

65. The equation of a straight line which is at a unit distance from the origin is

$x\cos\alpha + y\sin\alpha = 1 \to (1)$

Differentiating w.r.t. x

$\cos\alpha + \frac{dy}{dx}\sin\alpha = 0 \to (2)$

On eliminating α from (1) and (2), we get

$$\operatorname{cosec}\alpha\left(y - x\frac{dy}{dx}\right) = 1$$

$$\Rightarrow \left(y - x\frac{dy}{dx}\right) = \operatorname{cosec}\alpha \rightarrow (3)$$

Also, slope $= \dfrac{dy}{dx} = -\cot\alpha$ {using (2)}

$$\operatorname{cosec}\alpha = \sqrt{1 - \cot^2\alpha} = \sqrt{1 - \left(\frac{dy}{dx}\right)^2} \rightarrow (4)$$

From (3) and (4), we have $\left(y - \dfrac{xdy}{dx}\right)^2 = \left\{1 + \left(\dfrac{dy}{dx}\right)^2\right\}$

66. $6A^{-1} = A^2 + cA + dI$

$\Rightarrow \quad 6I = A^3 + cA^2 + dA$

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 5 \\ 0 & -10 & 14 \end{bmatrix}$$

and $A^3 = A^2A$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 5 \\ 0 & -10 & 14 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -11 & 19 \\ 0 & -38 & 46 \end{bmatrix}$$

Now,

$6I = A^3 + cA^2 + dA$

$6 = 1 + c + d,\ 0 = 19 + 5c + d$

$6 = -11 - c + d$

$6 = 46 + 14c + 4d,\ 0 = -38 - 10c - 2d$

$\Rightarrow d = 11,\ c = -6$

67. $a^2x^2 + ax + 1$ is positive for all real values of x.

$$\therefore \text{ Area } = \int_0^1 \left(a^2x^2 + ax + 1\right)dx$$

$$= \frac{a^2}{3} + \frac{a}{2} + 1 = \frac{1}{6}\left(2a^2 + 3a + 6\right)$$

$$\frac{1}{6}\left(2\left(a^2+\frac{3}{2}a+\frac{a}{16}\right)+6-\frac{18}{16}\right)$$

$$=\frac{1}{6}\left(2\left(a+\frac{3}{4}\right)^2+\frac{39}{8}\right)$$, which is minimum for $a=-\frac{3}{4}$

68. We have $\sin^2 x+\sin^4 x+\sin^6 x+\ldots$

$$=\frac{\sin^2}{1-\sin^2 x}=\tan^2 x$$

Therefore, $=\exp\left\{\left(\sin^2 x+\sin^4 x+\sin^6 x+\ldots\text{upto}\infty\right)\ln 2\right\}$

$$=\exp\left\{\tan^2 x\ln 2\right\}=\exp\left\{\ln 2^{\tan^2 x}\right\} \quad =2^{\tan^2 x}$$

As α satisfies the equation $x^2-17x+16=0$ we get

Since $0<x<\pi/2, \tan^2 x>0 \Rightarrow \alpha=2^{\tan^2 x}>1$. Therefore,

$2^{\tan^2 x}=16=2^4 \Rightarrow \tan^2 x=4 \Rightarrow \tan x=2 [\because \tan x>0]$

Thus, $\dfrac{2\cos x}{\sin x+2\cos x}=\dfrac{2}{\tan x+2}=\dfrac{2}{2+2}=\dfrac{1}{2}$

69. $$\frac{e^{x^2}\left(2x+x^3\right)}{\left(3+x^2\right)^2}=\frac{1}{2}e^{x^2}\left(\frac{1}{3+x^2}-\frac{1}{\left(3+x^2\right)^2}\right)2x$$

Hence, $\displaystyle\int\frac{e^{x^2}\left(2x+x^3\right)}{\left(3+x^2\right)}dx=\frac{1}{2}\frac{e^{x^2}}{\left(3+x^2\right)}+c$

70. Since $x>\sin x$, for $x>0$ and $\displaystyle\lim_{x\to 0+}\frac{x}{\sin x}=1$

so $\dfrac{11x}{\sin x}\to 11$ as $x\to 0+$ but $\dfrac{11x}{\sin x}>11$. Thus

$\left[\dfrac{11x}{\sin x}\right]=11$ for value $x\to 0+$. Similarly

$\dfrac{21\sin x}{x}\to 21$ as $x\to 0+$ but $20\le\dfrac{21\sin x}{x}<21 \left[\dfrac{21\sin x}{x}\right]=20$. Hence

$$\lim_{x\to 0+}\left(\left[\frac{11x}{\sin x}\right]+\left[\frac{21\sin x}{x}\right]\right)=31$$

Similarly $x<\sin x$ for $x<0$, so $\left[\frac{11x}{\sin x}\right]=11$

as $x\to 0-$ and $\left[21\frac{\sin x}{x}\right]=20$ as $x\to 0-$. Thus

$$\lim_{x\to 0-}\left(\left[\frac{11x}{\sin x}\right]+\left[\frac{21\sin x}{x}\right]\right)=31$$

71. Total formed numbers that begin with a odd digit $={}^{5}C_{1}.{}^{8}P_{4}=5(8)(7)(6)(5)$

Total formed numbers that end with a odd digit = $={}^{5}C_{1}.{}^{8}P_{4}=5(8)(7)(6)(5)$

Total formed number that begin with an odd digit and also end with an odd digit $={}^{5}C_{2}.2!.{}^{7}P_{3}=5.(4)(7)(6)(5)$

Thus total formed numbers that begin with an odd digit or end with an odd digits is equal to

5.7.6.60

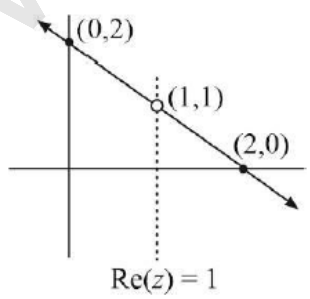
Total formed numbers $={}^{9}P_{5}=9.8.76.5$

Thus, required probability $=\frac{5}{6}$

72. Concept of skew lines

73. See figure, the given equation is written as

$$\arg\left[z-(1+i)\right]=\begin{cases}\frac{3\pi}{4}, \text{when } x\le 1\\ \frac{-\pi}{4}, \text{when } x>1\end{cases}$$



74. $$\frac{f(a)-f(1)}{a-1}=f'(c)=\frac{-c}{\sqrt{a^2-c^2}}$$

$$\frac{\sqrt{a^2-a^2}-\sqrt{a^2-1}}{a-1}=\frac{-c}{\sqrt{a^2-c^2}}$$

$$\Rightarrow \sqrt{\frac{a+1}{a-1}}=-\frac{c}{\sqrt{a^2-c^2}} \Rightarrow \quad \frac{a+1}{a-1}=\frac{c^2}{a^2-c^2}$$

$$\Rightarrow (a+1)\left(a^2-c^2\right)=c^2(a-1)$$

$$\Rightarrow c^2(-a-1+1-a)=-(a+1)a^2$$

$$\Rightarrow \quad c^2(2a)=(a+1)a^2 \Rightarrow \quad c^2=\frac{(a+1)a}{2}$$

Since $c\in(1,a)\, so\ c=\sqrt{\frac{(a+1)a}{2}}$

75. Let, $t=2^{11x}$

$$\Rightarrow \frac{\left(2^{11x}\right)^3}{2^2}+2^{11x}.2^2=\left(2^{11x}\right)^2.2+1$$

$$\frac{t^3}{4}+4t=2t^2+1$$

$$\Rightarrow t^3-8t^2+16t-4=0$$

Cubic it t has roots t_1,t_2,t_3

i.e. $t_1t_2t_3=4 \Rightarrow 2^{11x_1}.2^{11x_2}.2^{11x_3}=4$

$$\Rightarrow 2^{11(x_1+x_2+x_3)}=2^2$$

$$\Rightarrow 11(x_1+x_2+x_3)=2 \Rightarrow x_1+x_2+x_3=\frac{2}{11}$$

76. For Rolle's theorem & L.M.V theorem, the function $f(x)$ must be continuous & differentiable in the interval (a,b) if Rolle's theorem is applicable for $f(x)$ then L.M.V. theorem is applicable for $f(x)$

$\because f(x)=|\sin|x||\text{in}\left[\frac{-\pi}{3},\frac{\pi}{3}\right]$.is non differentiable at $x=0$

$\therefore$ Rolle's theorem & L.M.V theorem can not be applicable

for $f(x) = \left|\sin|x|\right| \forall x \in \left[\frac{-\pi}{3}, \frac{\pi}{3}\right]$.

77. $P\left(\frac{A}{(A^c \cup B^c)}\right) = \frac{P\left(A \cap (A \cap B)^c\right)}{P\left((A \cap B)^c\right)} = \frac{P\left(A \cap B^c\right)}{1 - P(A \cap B)}$

$$= \frac{P(A) - P(A \cap B)}{1 - P(A \cap B)} = \frac{\frac{4}{7} - \frac{3}{28}}{\frac{25}{28}} = \frac{13}{25} = \lambda$$

78. We are given $l^2 + m^2 - n^2 = 0$ and $l + m + n = 0$ also we have $l^2 + m^2 + n^2 = 1$

So that $2n^2 = 1 \Rightarrow n = \pm\frac{1}{\sqrt{2}}$ and $l + m = -n$

$\Rightarrow (l+m)^2 = n^2 = l^2 + m^2 \Rightarrow 2lm = 0$

$\Rightarrow$ Either $l = 0$ *or* $m = 0$ if $l = 0, m + n = 0 \Rightarrow \quad m = -n = \pm\frac{1}{\sqrt{2}}$

So the direction cosines of one of the lines are, $0, \pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}$ and if

$m = 0, l + n = 0 \quad \Rightarrow l = -n = \pm\frac{1}{\sqrt{2}}$ and the direction cosines of the other line are

$\pm\frac{1}{\sqrt{2}}, 0, \pm\frac{1}{\sqrt{2}}$.

Hence the required angle is $\frac{\pi}{3}$

79. We have

$$S = \left\{\begin{vmatrix}1 & -1\\1 & 1\end{vmatrix}, \begin{vmatrix}1 & 1\\-1 & 1\end{vmatrix}, \begin{vmatrix}-1 & 1\\-1 & -1\end{vmatrix}, \begin{vmatrix}-1 & -1\\1 & -1\end{vmatrix}, \begin{vmatrix}-1 & 1\\1 & 1\end{vmatrix}, \begin{vmatrix}1 & 1\\1 & -1\end{vmatrix}, \begin{vmatrix}1 & -1\\-1 & -1\end{vmatrix}, \begin{vmatrix}-1 & -1\\-1 & 1\end{vmatrix}, \begin{vmatrix}1 & 1\\-1 & -1\end{vmatrix}, \begin{vmatrix}-1 & -1\\1 & 1\end{vmatrix}\right.$$

$$\left.\begin{vmatrix}-1 & 1\\-1 & 1\end{vmatrix}, \begin{vmatrix}1 & -1\\1 & -1\end{vmatrix}, \begin{vmatrix}1 & 1\\1 & 1\end{vmatrix}, \begin{vmatrix}1 & -1\\-1 & 1\end{vmatrix}, \begin{vmatrix}-1 & 1\\1 & -1\end{vmatrix}, \begin{vmatrix}-1 & -1\\-1 & -1\end{vmatrix}\right\}$$

$\therefore n(S) = 16$

Now, number of favorable cases are given by ($\Delta \neq 0$, that is either $\Delta = 2$ or -2)

$$E=\left\{\begin{vmatrix}1&-1\\1&1\end{vmatrix},\begin{vmatrix}1&1\\-1&1\end{vmatrix},\begin{vmatrix}-1&1\\-1&-1\end{vmatrix},\begin{vmatrix}-1&-1\\1&-1\end{vmatrix},\begin{vmatrix}-1&1\\1&1\end{vmatrix},\begin{vmatrix}1&1\\1&-1\end{vmatrix},\begin{vmatrix}1&-1\\-1&-1\end{vmatrix},\begin{vmatrix}-1&-1\\-1&1\end{vmatrix}\right\}$$

Hence probability that the system of equations has a unique solution $=\frac{n(E)}{n(S)}=\frac{8}{16}=\frac{1}{2}$

80. $A=\sin^2\theta+\left(1-\sin^2\theta\right)^2$

$=1+\sin^2\theta\left(\sin^2\theta-1\right)$

$=1-\sin^2\theta\cos^2\theta\le 1$

Also $A=1-\left(\frac{1}{4}\right)\sin^2 2\theta\ge 1-\left(\frac{1}{4}\right)=\left(\frac{3}{4}\right)$. Hence $\frac{3}{4}\le A\le 1$

81. Let $x=u^6, dx=6u^5du$

$\int\frac{dx}{\sqrt{x}+\sqrt[3]{x}}=\int\frac{6u^5du}{u^3+u^2}=6\int\frac{u^3}{u+1}du$

$=6\int\left(u^2-u+1-\frac{1}{u+1}\right)du$

$=2u^3-3u^2+6u-6\ln(u+1)+e$

$=2\sqrt{x}-3\left(\sqrt[3]{x}\right)+6\left(\sqrt[6]{x}\right)-6\ln\left(\sqrt[6]{x}+1\right)+e$

$\therefore a=2, b=-3, c=6, d=-6\Rightarrow a+b+c+d=-1\Rightarrow|a+b+c+d|=1$

82. Any point on the first line is $(2r_1+1, 3r_1-1, 4r_1+1)$ and on the second line is $(r_2+3, 2r_2+k, r_2)$

The lines will intersect when

$2r_1+1=r_2+3, 3r_1-1=2r_2+k, 4r_1+1=r_2$

$\Rightarrow 2r_1-r_2=2$, $4r_1-r_2=-1$

$\Rightarrow r_1=-\frac{3}{2}, r_2=-5.$

and $k=3r_1-2r_2-1=-\frac{9}{2}+10-1=\frac{9}{2}.$

83. Any point on hyperbola xy = 1 (t, 1/t)

Now image of (t, 1/t) by the mirror y = 2x is

$$\frac{x-t}{2}=\frac{y-1/t}{-1}=-2\left(\frac{2t-1/t}{5}\right)$$

$$\Rightarrow 5x=\frac{4}{t}-3t \text{ and } 5y=4t+\frac{3}{t}$$

Now eliminating t from above two equations we get,

$12x^2-12y^2-7xy+25=0$

$\therefore bc=(-7)(-12)=84$

84. The given equation can be written as

$\sin^4 x+\cos^4 y+2-4\sin x\cos y=0$

$$\Rightarrow\left(\sin^2 x-1\right)^2+\left(\cos^2 y-1\right)^2+2\sin^2 x+2\cos^2 y-4\sin x\cos y=0$$

$$\Rightarrow\left(\sin^2 x-1\right)^2+\left(\cos^2 y-1\right)^2+2(\sin x-\cos y)^2=0$$

which is true if $\sin^2 x=1, \cos^2 y=1$ and $\sin x=\cos y$,

so $\sin x+\cos y=2$ as $0\le x, y\le \pi/2$.

85. $\text{Req.Area}=-\int_1^3\left(x^2-4x+3\right)dx=\frac{4}{3}=\frac{p}{q}\Rightarrow P+q=7$

86. Given equation is possible if $\cos\left(\pi\sqrt{x-4}\right)=1$ and $\cos\left(\pi\sqrt{x}\right)=1$

Since $x-4\ge 0\Rightarrow x\ge 4$ and $x\ge 0$

So $x=4$ is the only solution.

87. $(1+x)^n=\sum_{r=0}^{n} C_r x^r$

$\Rightarrow x(1+x)^n=\sum_{r=0}^{n} C_r x^{r+1}$

Differentiating w.r.t x we get

$xn(1+x)^{n-1}+(1+x)^n=\sum_{r=0}^{n}(r+1)C_r x^r$

Again multiplying both sides by x

$(1+x)^{n-1}(nx+1+x)x=\sum_{r=0}^{n}(r+1)C_r x^{r+1}$

Again differentiating w.r.t x we get,

$$\frac{d}{dx}\left((1+x)^{n-1}\left(nx^2+x^2+x\right)\right)=\sum_{r=0}^{n}\left((r+1)^2\right)C_r x^r$$

$$\sum_{r=0}^{n}\left((r+1)^2\right)C_r x^r=(1+x)^{n-1}(2nx+2x+1)+\left(nx^2+x^2+x\right)(n-1)(1+x)^{n-2}$$

Putting x = 1 on both sides, we get,

$$\sum_{r=0}^{n}\left((r+1)^2\right)C_r=2^{n-1}(2n+3)+(n+2).(n-1)2^{n-2}=2^{n-2}\left(n^2+5n+4\right)$$

Now, $f(x)=x^2+5x+4=(x+1)(x+4)\Rightarrow \alpha=-1,\beta=-4$

Hence, $\alpha^2+\beta^2=17$

88. Let r be the length of the line segment which makes angles α,β,γ respectively with x,y and $z-$ axes, then $r\cos\alpha=2, r\cos\beta=3, r\cos\gamma=6$

$\Rightarrow\ r^2\left(\cos^2\alpha+\cos^2\beta+\cos^2\gamma\right)=2^2+3^2+6^2=49$

$\Rightarrow\quad r=7$ as $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

89. 2^{nd} win in 4^{th} test $\Rightarrow$ In the first 3 matches there must be exactly 1 win so, required probability $={}^3C_1\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)$

$P=\frac{27}{256}$

90. We have

$$\left|\frac{4\times 2-3\times 3+26}{\sqrt{4^2+3^2}}\right|=\left|\frac{3\times 2-4\times 3+p}{\sqrt{4^2+3^2}}\right|$$

$\Rightarrow\quad \pm 25=6-12+p$

$\Rightarrow\quad \pm 25=6-12+p$

$\Rightarrow\quad p=\pm 25+6$

$\Rightarrow\quad p=31\ or-19.$