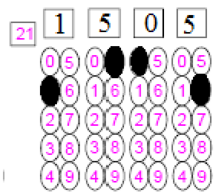
Model Grand Test

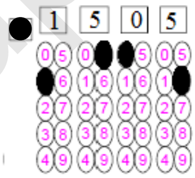
2023

IMPORTANT INSTRUCTION:

1. Immediately fill in the Admission number on this page of the Test Booklet with **Blue/Black** Ball Point **Pen** only.
2. The candidates should not write their Admission Number anywhere (except in the specified space) on the Test Booklet/ Answer Sheet.
3. The test is of **3 hours** duration.
4. The Test Booklet consists of 90 questions. The maximum marks are **300.**
5. There are **three** parts in the question paper 1,2,3 consisting of **Physics, Chemistry** and **Mathematics** having **30 questions** in each subject and subject having **two sections.**
   (I) Section –I contains 20 **multiple choice** questions with only one correct option.
   Marking scheme: +4 for correct answer, 0 if not attempt and -1 in all other cases.
   (II) Section-II contains 10 **Numerical Value Type** questions. Attempt any 5 questions only,if more than 5 questions attempted, First 5 attempted questions will be considered.

■ The Answer should be within **0 to 9999.** If the Answer is in **Decimal** then round off to the **nearest Integer** value (Example i,e. If answer is above **10** and less than **10.5** round off is **10** andIf answer is from **10.5** and less than **11** round off is **11).**
   To cancel any attempted question bubble on the question number box.
   For example: To cancel attempted question 21. Bubble on 21 as shown below





Question Answered for Marking **Question Cancelled for Marking**

Marking scheme: **+4** for correct answer, 0 if **not attempt** and **-1** in all other cases.

6. Use **Blue / Black Point Pen only** for writing particulars / marking responses on the Answer Sheet. **Use of pencil is strictly prohibited.**
7. No candidate is allowed to carry any textual material, printed or written, bits of papers, mobile phone any electron device etc, except the Identity Card inside the examination hall.
8. Rough work is to be done on the space provided for this purpose in the Test Booklet only.
9. On completion of the test, the candidate must hand over the Answer Sheet to the invigilator on duty in the Hall. **However, the candidate are allowed to take away this Test Booklet with them.**
10. **Do not fold of make any stray marks on the Answer Sheet**

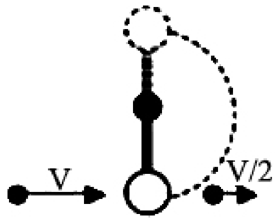
PHYSICS **Max Marks: 100**

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

1. A bullet of mass m and speed v hits a pendulum bob of mass M at time t_1, and passes completely through the bob. The bullet emerges at time t_2 with a speed of v/2. The pendulum bob is suspended by a stiff rod of length l and negligible mass. After the collision, the bob can barely swing through a complete vertical circle. At time t_3, the bob reaches the highest position. What quantities are conserved in this process?



1) Total kinetic energy of the bob and bullet during the time interval $\Delta t = t_2 - t_1$.

2) Total momentum of the bob and the bullet during the time interval $\Delta t = t_2 - t_1$.

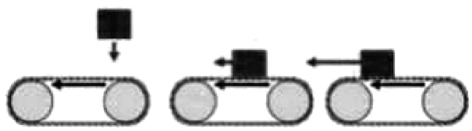
3) Total mechanical energy of the bob and the bullet during the time interval $t_3 - t_1$.

4) Momentum of the bob after t_2

2. A soap bubble (surface tension $= T$) is charged to a maximum surface density of charge $= \sigma$. When it is just going to burst? Its radius R is given by

1) $R = \dfrac{\sigma^2}{8\varepsilon_0 T}$ 2) $R = 8\varepsilon_0 \dfrac{T}{\sigma^2}$ 3) $R = \dfrac{\sigma}{\sqrt{8\varepsilon_0 T}}$ 4) $R = \dfrac{\sqrt{8\varepsilon_0 T}}{\sigma}$

3. A 300 kg crate is dropped vertically onto a conveyor belt that is moving at 1.20 m/s. A motor maintains the belts's constant speed. The belt initially slides under the crate with a coefficient of friction of 0.400. After a short time, the crate is moving at the speed of the belt. During the period in which the crate is being accelerated, find the work done by the motor which drives the belt

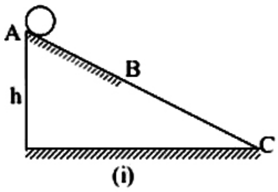


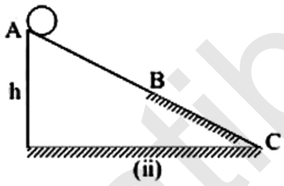
1) 432 J 2) 216 J 3) 108 J 4) 54 J

4. In Young's double–slit experiment, the slits are 0.5 mm apart and the interference is observed on a screen at a distance of 100 cm from the slits. It is found that the ninth bright fringe is at a distance of 7.5 mm from the second dark fringe from the center of the fringe pattern. The wavelength of the light used is

1) $5000\overset{o}{A}$ 2) $\frac{5000}{7}\overset{o}{A}$ 3) $2500\overset{o}{A}$ 4) $\frac{2500}{7}\overset{o}{A}$

5. In both the figures, all other factors are same, except that in figure (i) AB is sufficiently rough and BC is smooth while in figure (ii) AB is smooth and BC is sufficiently rough. Kinetic energy of the ball on reaching the bottom (if AB=BC)





1) is same in both the cases 2) is greater in case (i)
3) is greater in case (ii) 4) information insufficient

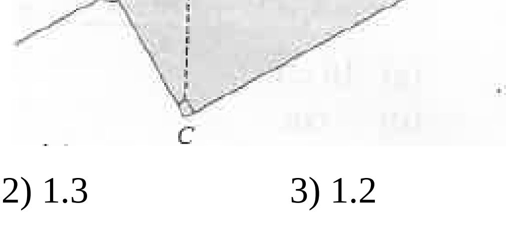
6. Half life of a radio–active substance is 20 minutes. The time between 20% and 80% decay will be

1) 20 minutes 2) 40 minutes 3) 30 minutes 4) 25 minutes

7. In a typical combustion engine the work done by a gas molecule is given by $W = \alpha^2 \beta e^{\beta x^2/kT}$, where x is the displacement, k is the Boltzmann constant and T is the temperature, If α and β are constants, dimensions of α will be

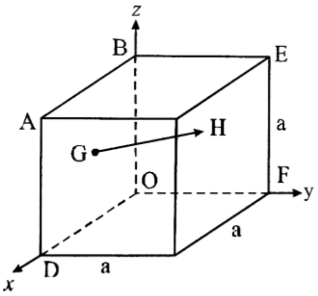
1) $\left[MLT^{-1}\right]$ 2) $\left[M^0LT^0\right]$ 3) $\left[MLT^{-2}\right]$ 4) $\left[M^2LT^{-2}\right]$

8. ACB is right–angled prism with other angles as 60° and 30°. Refractive index of the prism is 1.5. AB has thin layer of liquid on it as shown. Light falls normally on the face AC. For total internal reflections, maximum refractive index of the liquid is (Nearly)



1) 1.4 2) 1.3 3) 1.2 4) 1.6

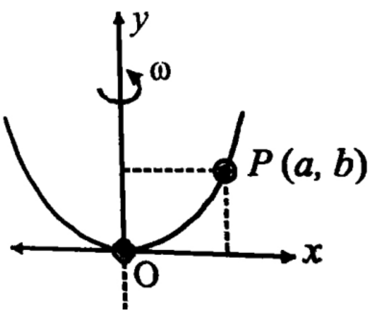
9. In the cube of side 'a' shown in the figure, the vector from the central point of the face ABOD to the central point of the face BEFO will be:



1) $\frac{1}{2}a\left(\hat{k}-\hat{i}\right)$ 2) $\frac{1}{2}a\left(\hat{i}-\hat{k}\right)$ 3) $\frac{1}{2}a\left(\hat{j}-\hat{i}\right)$ 4) $\frac{1}{2}a\left(\hat{j}-\hat{k}\right)$

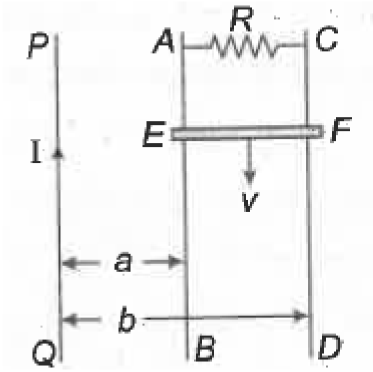
10. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.
Assertion A: The photoelectric effect does not take place, if the energy of the incident radiation is less than the work function of metal.
Reason R: Kinetic energy of the photoelectrons is zero, if the energy of the incident radiation is equal to the work function of a metal.
In the light of the above statements, choose the most appropriate answer from the options given below.
1) Both A and R are correct and R is the correct explanation of A
2) Both A and R are correct but R is not the correct explanation of A
3) A is correct but R is not correct
4) A is not correct but R is correct

11. A bead of mass m stays at point $P(a,b)$ on a wire bent in the shape of a parabola $y=4Cx^2$ and rotating with angular speed ω (see figure). The value of ω is (neglect friction):



1) $2\sqrt{2gC}$ 2) $2\sqrt{gC}$ 3) $\sqrt{\frac{2gC}{ab}}$ 4) $\sqrt{\frac{2g}{C}}$

12. PQ is an infinite current carrying conductor. AB and CD are smooth conducting rods on which a conductor EF moves with constant velocity v as shown. The force needed to maintain constant speed of EF is



1) $\frac{1}{vR}\left[\frac{\mu_0 Iv}{2\pi}\ln\left(\frac{b}{a}\right)\right]^2$
2) $\left[\frac{\mu_0 Iv}{2\pi}\ln\left(\frac{a}{b}\right)\right]^2\frac{1}{vR}$
3) $\left[\frac{\mu_0 Iv}{2\pi}\ln\left(\frac{b}{a}\right)\right]^2\frac{v}{R}$
4) $\frac{v}{R}\left[\frac{\mu_0 Iv}{2\pi}\ln\left(\frac{a}{b}\right)\right]^2$

13. A body of mass $m = 10^{-2}$ kg is moving in a medium and experiences a frictional force $F = -kv^2$. Its initial speed is $v_0 = 10ms^{-1}$. If after 10s, its energy is $\frac{1}{8}mv_0^2$, the value of k will be:

1) 10^{-4}kg m^{-1} 2) 10^{-1}kg m$^{-1}s^{-1}$ 3) 10^{-3}kg m^{-1} 4) 10^{-3}kg s^{-1}

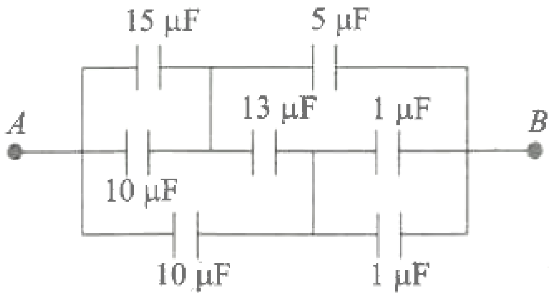
14. Two concentric coils, each of radius equal to $2\pi cm$, are placed at right angles to each other. Currents of 3A and 4A, respectively, are flowing through the two coils. The magnetic induction, in $Wb\,m^{-2}$, at the center of the coils will be $\left[\mu_0 = 4\pi \times 10^{-7} Wb\left(Am^{-1}\right)\right]$

1) 5×10^{-5} 2) 7×10^{-5} 3) 12×10^{-5} 4) 10^{-5}

15. A fluid is flowing through a horizontal pipe of varying cross-section, with speed $v\,\text{ms}^{-1}$ at a point where the pressure is P Pascal. At another point where pressure is $\frac{P}{2}$ Pascal its speed is $V\,\text{ms}^{-1}$. If the density of the fluid is ρ kg m^{-3} and the flow is streamline, then V is equal to :

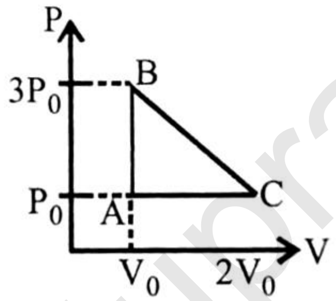
1) $\sqrt{\frac{P}{\rho}+v}$ 2) $\sqrt{\frac{2P}{\rho}+v^2}$ 3) $\sqrt{\frac{P}{2\rho}+v^2}$ 4) $\sqrt{\frac{P}{\rho}+v^2}$

16. Find the equivalent capacitance across A and B



1) $\frac{35}{6}\mu F$ 2) $\frac{25}{6}\mu F$ 3) $15\mu F$ 4) None of these

17. One mole of an ideal monoatomic gas is taken along the path ABCA as shown in the PV diagram. The maximum temperature attained by the gas along the path BC is given by



1) $\frac{25}{8}\frac{P_0V_0}{R}$ 2) $\frac{25}{4}\frac{P_0V_0}{R}$ 3) $\frac{25}{16}\frac{P_0V_0}{R}$ 4) $\frac{5}{8}\frac{P_0V_0}{R}$

18. The ratio of radii of nuclei ${}_{13}Al^{27}$ & ${}_{52}X^{A}$ is 3 : 5. The number of neutrons in the nuclei of X will be

1) 52 2) 73 3) 125 4) 13

19. STATEMENT-I

If the amplitude of a simple harmonic oscillator is double, its total energy becomes four times. because

STATEMENT-II

The total energy is directly proportional to the square of the amplitude ofvibration of the harmonic oscillator.

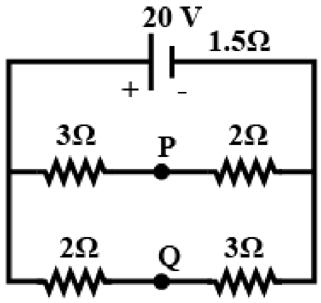
1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

2) Statement-1 is True, Statement2 is True; Statement-2 is NOT a correct explanation for Statement-1.

3) Statement-1 is True, Statement-2 is False.

4) Statement-1 is False, Statement-2 is True.

20. If in the circuit shown below, the internal resistance of the battery is 1.5Ω and V_P and V_Q are the potentials at P and Q respectively, what is the potential difference between the points P and Q



1) Zero
2) $4\ volt\ (V_P > V_Q)$
3) $4\ volt\ (V_Q > V_P)$
4) $2.5\ volt\ (V_Q > V_P)$

(NUMERICAL VALUE TYPE)

Section-II contains 10 **Numerical Value Type** questions. Attempt any 5 questions only. First 5 attempted questions will be considered if more than 5 questions attempted. The Answer should be within **0 to 9999.** If the Answer is in **Decimal** then round off to the **nearest Integer** value (Example i,e. If answer is above **10** and less than **10.5** round off is **10** andIf answer is from **10.5** and less than **11** round off is **11**).

Marking scheme: +4 for correct answer, 0 if **not attempt** and **-1** in all other cases.

21. A soap bubble of radius r is blown up to form a bubble of radius 2r under isothermal conditions. If σ is the surface tension of soap solution, the energy spent in doing so is $x\left(4\pi\sigma r^2\right)$.Find x

22. What is the power output of ${}_{92}U^{235}$ reactor if it takes 30 days to use up 2 kg of fuel and if each fission gives $142 MeV$ of usable energy? Avogadro's number $= 6.2 \times 10^{26}$ per kilomole. (In megawatt)

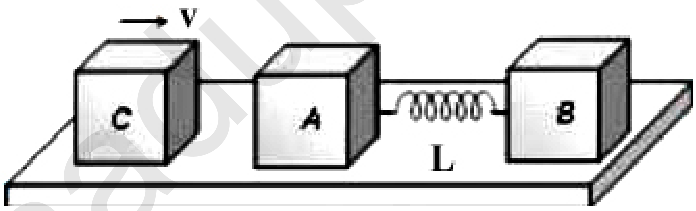
23. A circular platform is mounted on a frictionless vertical axle, its radius R = 2m and its moment of inertia about the axle is $200 kgm^2$. It is initially at rest. A 50kg man stands on the edge of the platform and begins to walk along the edge at the speed of 1m/s relative to the ground. Time taken by the man to complete one revolution is $k\pi$ sec. Find 'k'

24. An ammeter is obtained by shunting a 30Ω galvanometer with a 30Ω resistance. What additional shunt should be connected across it to double the range (in Ω)?

25. A vernier caliper (Non standard) is constructed which has one main scale division of length 1mm and 20 vernier scale divisions measure 18 mm. In an experiment, zero of vernier scale lies between 0.4 and 0.5 cm on main scale and 2nd division of vernier scale coincides with a main scale division (when jaws of vernier are in closed position). When a cylinder whose diameter is to be measured is kept between the jaws of vernier caliper, the zero of vernier scale lies between 2.4 and 2.5 cm on main scale and 12th division of vernier scale coincide with a main scale division. The diameter of cylinder in cm is

26. Two slits in Young's interference experiment have width in the ratio 1 : 4. The ratio of intensity at the maxima and minima in their interference is ___

27. Two blocks A and B each of mass m are connected by a massless spring of natural length L and spring constant K. The blocks are initially resting on a smooth horizontal floor with the spring at its natural length as shown in figure. A third identical block C also of mass m moves on the floor with a speed v along the line joining A and B and collides with A. Then the kinetic energy of the $A-B$ system at maximum compression of the spring is mv^2/a, the value of a =(assume the collisions are elastic)



28. Find the ratio between total acceleration of the electron in singly ionized helium atom and hydrogen atom (both in ground state).

29. A cubical block of wood of edge $3cm$ floats in water. The lower surface of the cube just touches the free and of a vertical spring fixed at the bottom of the pot. The maximum weight that can be put on the block without wetting it is (P) Newton. Then the value of $2P$ is (Spring constant = 50 N/m, density of wood $=800kg/m^3, g=10m/s^2$)

30. Two long current carrying conductors are placed parallel to each other at a distance of 8 cm between them. The magnitude of magnetic field produced at mid–point between the two conductors due to current flowing in them is $300\mu T$. The equal current flowing in the two conductors is in (A)

CHEMISTRY

Max Marks: 100

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

31. 50ml H_2O_2 is completely oxidized by 10 ml, 0.2 N $KMnO_4$ solution in acidic medium. Incorrect value for the strength of hydrogen peroxide is

1) 0.68 gm/lit 2) $\frac{M}{50}$ 3) $\frac{N}{25}$ 4) 33.4% (w/v)

32. In a reaction $A \rightarrow$ products, when start is made from $8.0 \times 10^{-2} M$ of A, half-life is found to be 120 minutes. For the initial concentration $4.0 \times 10^{-2} M$, the half-life of the reaction becomes 240 minutes. The order of the reaction is:

1) Zero 2) One 3) Two 4) 0.5

33. $\log \frac{K_p}{K_c} + \log RT = 0$ is true relationship for the following reaction

1) $PCl_5 \rightleftharpoons PCl_3 + Cl_2$
2) $2SO_2 + O_2 \rightleftharpoons 2SO_3$
3) $N_2 + 3H_2 \rightleftharpoons 2NH_3$
4) both (2) & (3)

34. Potassium has a bcc structure with nearest neighbour distance $4.52 \overset{0}{A}$. Its atomic weight is 39. Its density $\left(in\ kg\ m^{-3}\right)$ will be

1) 454 2) 804 3) 852 4) 910

35. Ionisation energy of He^+ is $19.6 \times 10^{-18} J\ atom^{-1}$.The energy of the first stationary state of Li^{2+} is

1) $-4.41 \times 10^{-18} J.atom^{-1}$
2) $-4.41 \times 10^{-17} J.atom^{-1}$
3) $-44.1 \times 10^{-16} J.atom^{-1}$
4) $-8.72 \times 10^{-18} J.atom^{-1}$

36. A colloidal solution is subjected to an electric field. The particles move towards anode. The coagulation of same sol is studied using $NaCl, BaCl_2$ & $AlCl_3$ solutions. The order of their coagulation power should be

1) $NaCl > BaCl_2 > AlCl_3$
2) $BaCl_2 > AlCl_3 > NaCl$
3) $AlCl_3 > BaCl_2 > NaCl$
4) $BaCl_2 > NaCl > AlCl_3$

37. Which one of the following is an example of disproportionation reaction

1) White $P_4 + NaOH + H_2O$ 2) Halogen (Except F_2) with KOH

3) Sulphur $+ NaOH$ 4) All the above

38. Which of the following on thermal decomposition gives oxygen gas?

1) Ag_2O 2) Pb_3O_4 3) PbO_2 4) All of these

39. Assertion (A): Both $Be \& A\ell$ Exhibit similar properties in their compounds.

Assertion (R) : Due to almost same polarizing power of cations ; Some pairs of elements like $Be \& A\ell$ are diagonally related in their properties of compounds.

1) A & R are true & R is correct Explanation of A

2) A & R are true & R is not correct Explanation of A

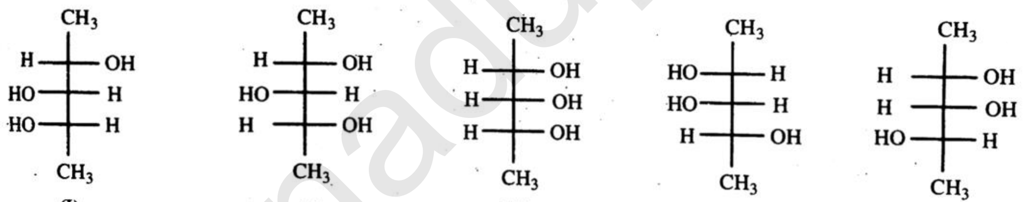
3) A is correct and R is wrong

4) A is wrong and R is correct

40. Which is a planar molecule?

1) XeO_4 2) XeF_4 3) $XeOF_4$ 4) XeO_2F_2

41. Which structure(s) represent(s) diastereomer(s) of I?



(I) (II) (III) (IV) (V)

1) V 2) III 3) IV 4) All

42. Assertion: Cuprous ion $\left(Cu^{+}\right)$ has unpaired electrons while cupric $\left(Cu^{++}\right)$ does not.

Reason: Cuprous ion $\left(Cu^{+}\right)$ is colourless whereas cupric ion $\left(Cu^{++}\right)$ is blue in the aqueous solution.

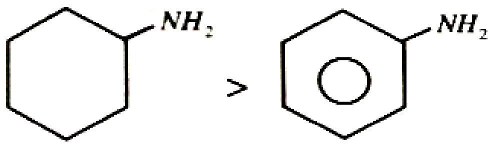
1) Both Assertion (A) and Reason (R) are the true and Reason (R) is a correct explanation of Assertion (A).

2) Both Assertion (A) and Reason (R) are the true but Reason (R) is not a correct explanation of Assertion (A).

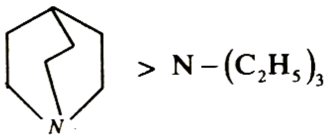
3) Assertion (A) is true and Reason (R) is false.

4) Assertion (A) is false and Reason (R) is true.

43. Which of the following order(s) is correct

1)  (basic nature)

2) $HCOOH > CH_3COOH$ (Acidic nature)

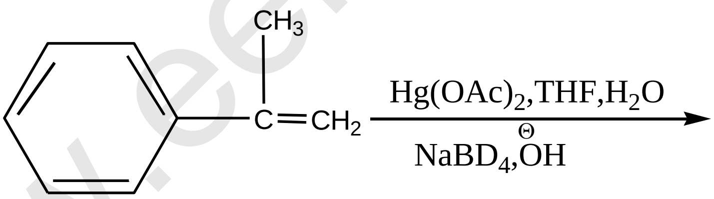
3)  (Basic nature)

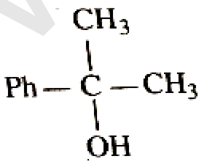
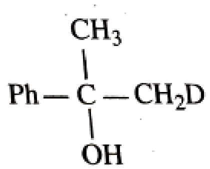
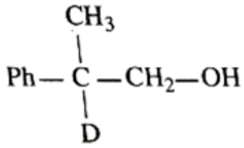
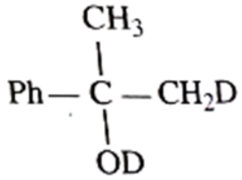
4) All the above

44. Match the columns

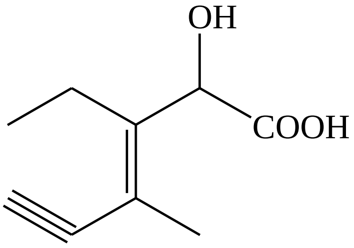
	Column – I		Column – II
A)	$C_2H_6 \xrightarrow{Cl_2/\text{UV light}} C_2H_5Cl$	p)	Finkelstein reaction
B)	$C_6H_5NH_2 \xrightarrow[273-278K]{NaNO_2+HCl/Cu_2Cl_2} C_6H_5Cl$	q)	Free radical substitution
C)	$CH_3Cl + NaI \rightarrow CH_3I + NaCl$	r)	Swarts reaction
D)	$CH_3 - Br + AgF \rightarrow CH_3F + AgBr$	s)	Sandmeyer's reaction

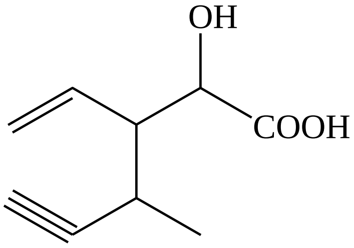
1) A – q; B – s; C – p; D – r
2) A – q; B – r; C – p; D – s
3) A – r; B – p; C – s; D – q
4) A – s; B – r; C – p; D – q

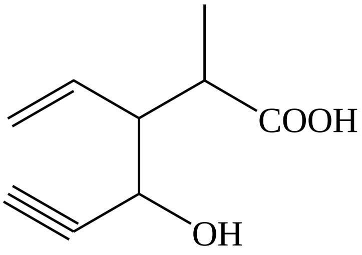
45.  Major Product:

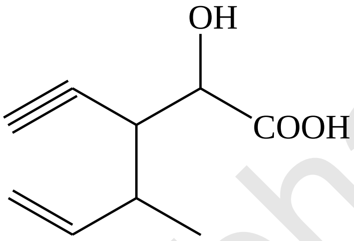
1) 
2) 
3) 
4) 

46. Structure of the compound whose IUPAC name is 3-ethyl-2-hydroxy-4-methylhex-3-en-5-ynoic acid is:

1) 

2) 

3) 

4) 

47. Assertion(A) : During E^2 Elimination of Ethylidene Bromide to form Ethyne; $NaNH_2$ is a better dehydrohalogenating agent than alcoholic KOH .

Reason (R) : OH^- is more basic than NH_2^-

1) A & R are correct but R is correct explanation to A.

2) A & R are correct but R is not correct explanation to A

3) A is correct and R is wrong

4) A is wrong and R is correct

48. Which of the following amines can be prepared by Gabriel method?

(i) $CH_3CH_2NH_2$ (ii) $(CH_3)_2CHNH_2$

(iii) $(CH_3)_3CNH_2$ (iv) $C_6H_5NH_2$

1) (i) and (iii) 2) (ii) and (iv) 3) (i), (ii) and (iii) 4) (i) and (ii)

49. Arrange the following compounds in the order of their reactivity towards general electrophilic substitution.

i) Chlorobenzene ii) Nitrobenzene iii) Benzene

iv) Phenol v) Toluene

1) iv $>$ v $>$ iii $>$ i $>$ ii 2) ii $> i >$ iii $> v >$ iv

3) $iv > v >$ iii $> ii > i$ 4) $v > iv >$ iii $> i > ii$

50. Assertion: At isoelectric point, the amino acids group does not migrate under the influence of electric field.

Reason: Only at isoelectric point, amino acid exists as a zwitterion.

1) Both Assertion (A) and Reason (R) are the true and Reason (R) is a correct explanation of Assertion (A).

2) Both Assertion (A) and Reason (R) are the true but Reason (R) is not a correct explanation of Assertion (A).

3) Assertion (A) is true and Reason (R) is false.

4) Assertion (A) is false and Reason (R) is true.

(NUMERICAL VALUE TYPE)

Section-II contains 10 Numerical Value Type questions. Attempt any 5 questions only. First 5 attempted questions will be considered if more than 5 questions attempted. The Answer should be within 0 to 9999. If the Answer is in Decimal then round off to the nearest Integer value (Example i,e. If answer is above 10 and less than 10.5 round off is 10 andIf answer is from 10.5 and less than 11 round off is 11).

Marking scheme: +4 for correct answer, 0 if not attempt and -1 in all other cases.

---

51. 2.0g of polybasic organic acid (Molecular mass = 600) required 100mL of a $\frac{M}{6} NaOH$ solution for complete neutralization. Find the basicity of acid.

52. How many of the following oxidizing reaction of $KMnO_4$ occurs in acidic medium?

(i) $Fe^{2+}(green)$ is converted to $Fe^{3+}(yellow)$

(ii) Iodide is converted to iodate.

(iii) Thiosulphate oxidized to sulphate.

(iv) Nitrite is oxidized to nitrate.

53. Two flasks A and B have equal volumes. A is maintained at 300 K and B at 600 K. while A contains H_2 gas, B has an equal mass of CH_4 gas. Assuming ideal behavior for the both gases find the ratio of $(u_{av})_A : (u_{av})_B$.

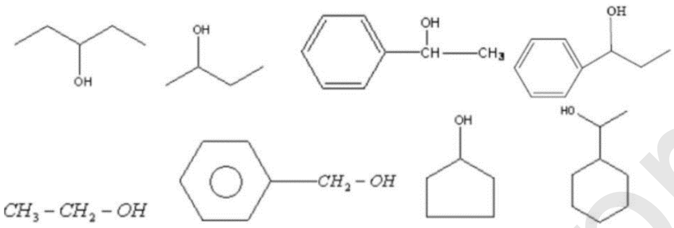
54. The energies of activation for forward and reverse reactions for are $A_2 + B_2 \rightleftharpoons 2AB$ $180\,kJ\,mol^{-1}$ and $200\,kJ\,mol^{-1}$ respectively. The presence of a catalyst lowers the activation energy of both (forward and reverse) reactions by $100\,kJ\,mol^{-1}$. The enthalpy change (magnitude only) of the reaction $(A_2 + B_2 \rightarrow 2AB)$ in the presence of a catalyst will be (in kJ mol^{-1})

55. pH of a mixture of HA and A^- buffer is 5. K_b of $A^- = 10^{-10}$. Hence $[HA]/[A^-]$ will be _____ $\times 10^{-1}$

56. The total number of possible coordination isomers for the complex compound $\left[\mathrm{Cu^{II}}\left(\mathrm{NH_3}\right)_4\right]\left[\mathrm{Pt^{II}Cl_4}\right]$

57. Calculate equilibrium constant (in natural number multiple of 10^{-80}) when sodium reduces Aluminium oxide to aluminium at 298K. ΔG_f^0 of $Na_2O_{(s)}$ at $298K = -377\ KJ\ mole^{-1}$ and ΔG_f^0 of $A\ell_2O_3$ at $298K = -1582\ KJ\ mole^{-1}$

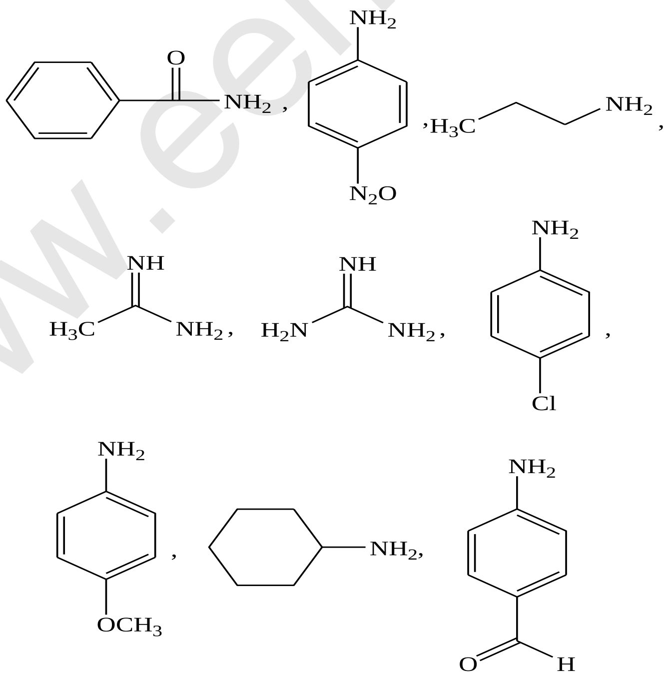
58. Find out number of alcohols that can give positive iodoform test



59. The number of species involving sp^3d hybridization are

$XeF_2, SCl_4, PCl_5, SF_4, ClF_3, ICl_2^-, XeF_4, XeOF_2$......

60. Identify how many of the given compounds are more basic than aniline?



MATHEMATICS
Max Marks: 100

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

61. If $f(x)^2 f\left(\frac{1-x}{1+x}\right)=x^3, x\neq -1,1$ and $f(x)\neq 0$, then $\{f(-2)\}=$ __________

Here {x}represents fractional part of x

1) 2/3 2) 1/3 3) 1/2 4) 0

62. The value of $\int \frac{\left(\tan^{-1}(\sin x+1)\right)\cos x}{\left(3+2\sin x-\cos^2 x\right)}dx$ is (where c is the constant integration)

1) $\tan^{-1}(\sin x)+c$
2) $\left(\tan^{-1}(\sin x)\right)^2+c$
3) $\frac{\left(\tan^{-1}(\sin x+1)\right)^2}{2}+c$
4) $\frac{\left(\tan^{-1}(\sin x)\right)^2}{2}+c$

63. Suppose A, B, C are angles of a triangle, and let

$\Delta=\begin{vmatrix} e^{2iA} & e^{-iC} & e^{-iB} \\ e^{-iC} & e^{2iB} & e^{-iA} \\ e^{-iB} & e^{-iA} & e^{2iC} \end{vmatrix}$ where $e^{i\theta}=\cos\theta+i\sin\theta$ and $i^2=-1$

Then value of Δ is

1) -1 2) -4 3) 0 4) 4

64. The sum of n terms of the series $\frac{5}{1.2}.\frac{1}{3}+\frac{7}{2.3}.\frac{1}{3^2}+\frac{9}{3.4}.\frac{1}{3^3}+\frac{11}{4.5}.\frac{1}{3^4}+....$ is:

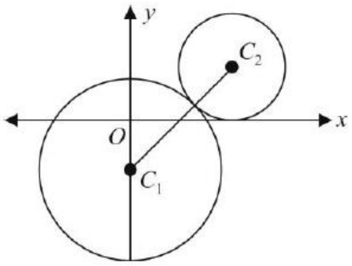
1) $1-\frac{1}{n+1}.\frac{1}{3^n}$ 2) $1+\frac{1}{n+1}.\frac{1}{3^n}$ 3) $1-\frac{1}{n-1}.\frac{1}{3^{n+1}}$ 4) 1

65. If the system of equations

$\lambda x_1+x_2+x_3=1, x_1+\lambda x_2+x_3=1, x_1+x_2+\lambda x_3=1$ is inconsistent, then λ equals

1) 5 2) -2/3 3) -3 4) -2

66. In the given figure, the equation of the large circle is $x^2+y^2+4y-5=0$ and the distance between centres is 4. Then the equation of smaller circle is:



1) $\left(x-\sqrt{7}\right)^2+\left(y-1\right)^2=1$ 2) $\left(x+\sqrt{7}\right)^2+\left(y-1\right)^2=1$

3) $x^2+y^2=2\sqrt{7}x+2y$ 4) $\left(x+\sqrt{7}\right)^2+\left(y+1\right)^2=1$

67. $\sin^{-1}\frac{1}{\sqrt{2}}+\sin^{-1}\frac{\sqrt{2}-1}{\sqrt{6}}+\sin^{-1}\frac{\sqrt{3}-\sqrt{2}}{\sqrt{12}}+\sin^{-1}\frac{2-\sqrt{3}}{\sqrt{20}}+\sin^{-1}\frac{\sqrt{5}-2}{\sqrt{30}}+...\infty$

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{8}$ 4) $\frac{\pi}{12}$

68. If the area included between the two parabolas $y^2=4a\left(x+a\right)$ and $y^2=4b\left(b-x\right)$ is $\frac{16}{3}$, then $\left(a,b>0\right)$

1) AM and GM of a and b are equal $\forall a,b>0$

2) AM and GM of a and b are reciprocals of each other $\forall a,b>0$

3) $\sqrt{\frac{a^2+b^2}{2}}=a+b$ 4) $a+b=ab$

69. Let $y=\frac{1}{\mu^2+\mu-2}$ where $\mu=\frac{1}{x-1}$ Then y is discontinuous only at $x=$

1) 1,2 2) 1,-2 3) 1,1/2,2 4) $-1,\frac{-1}{2},-2$

70. The point $\left(-2m,m+1\right)$ is an interior point of the smaller region bounded by circle $x^2+y^2=4$ and the parabola $y^2=4x$ then:

1) $-1<m<-5+2\sqrt{6}$ 2) $-1<m<\frac{3}{5}$

3) 0 < m < 4 4) $-5-2\sqrt{6}<m<1$

71. If $y=\tan^{-1}\frac{1}{1+x+x^2}+\tan^{-1}\frac{1}{x^2+3x+3}+\tan^{-1}\frac{1}{x^2+5x+7}+\tan^{-1}\frac{1}{x^2+7x+13}+...n$ terms, then $y'(0)$ is equal to

1) $-1/(n^2+1)$ 2) $-n^2/(n^2+1)$ 3) $n^2/(n^2+1)$ 4) $\frac{1}{(n^2+1)}$

72. The complex number z = 1 + i is rotated through an angle $\frac{3\pi}{2}$ in anticlockwise direction about the origin and stretched by additional $\sqrt{2}\,unit$, then the new complex number is:

1) $-\sqrt{2}-\sqrt{2}i$ 2) $\sqrt{2}-\sqrt{2}i$ 3) $2-\sqrt{2}i$ 4) $2(1-i)$

73. The sum of the intercepts of a tangent to $\sqrt{x}+\sqrt{y}=\sqrt{a}, a>0$ upon the coordinate axes is

1) $2a$ 2) a 3) $a/2$ 4) $\sqrt{a}$

74. Let P and Q are the two distinct points on a circle which has centre at C(4, 3) and which pass through origin. If OC is perpendicular to both the line segment CP and CQ, then the set (P, Q) is:

1) $(1,7),(7,1)$ 2) $(1,7),(7,-1)$ 3) $(7,1),(1,-7)$ 4) $(-7,1),(7,-1)$

75. The distance of that point on $y=x^4+3x^2+2x$ which is nearest to the line $y=2x-1$ is

1) $\frac{4}{\sqrt{5}}$ 2) $\frac{3}{\sqrt{5}}$ 3) $\frac{2}{\sqrt{5}}$ 4) $\frac{1}{\sqrt{5}}$

76. A bag contains a mixed lot of red and blue balls. If two balls are drawn at random, the probability of drawing two red balls is five times the probability of drawing two blue balls and the probability of drawing one ball of each colour is six times the probability of drawing two blue balls. Then, the total number of red and blue balls in the bag is

1) 9 2) 12 3) 14 4) 15

77. The line L has intercepts a and b on the coordinate axes. The coordinate axes are rotated through a fixed angle, keeping the origin fixed. If p and q are the intercepts of the line L on the new axes, then $\frac{1}{a^2}-\frac{1}{p^2}+\frac{1}{b^2}-\frac{1}{q^2}$ is equal to

1) -1 2) 0 3) 1 4) 4

78. The consecutive odd integers whose sum is 45^2-21^2 are

1) 43, 45, ……., 75
2) 43, 45, ……., 79
3) 43, 45, ……., 85
4) 43, 45, ……., 89

79. The value of a so that the volume of the parallelepiped formed by $i+aj+k,\ j+ak$ and $ai+k$ becomes minimum is

1) -3 2) 3 3) $\frac{1}{\sqrt{3}}$ 4) $\sqrt{3}$

80. Priya and Padmini solve a problem on "How to count without actually counting" given in the text book as follows:
Find the number of ways in which a selection of 5 books can be done out of 3 physics, 3 chemistry and 3 mathematics books of taking at least one book of each subject, books of same subject being different.
Priya's Solutions:
One book of each subject can be taken in ${}^3C_1$ ways balance 2 books from remaining 6 books can be taken in ${}^6C_2$ ways hence total no.of ways using fundamental principle of counting $={}^3C_1.{}^3C_1.{}^3C_1.{}^6C_2$.
Padmini's Solutions:
Number of ways in which atleast one Physics book can be selected = $(2^3$-1) perpendicularly one chemistry in = $(2^3$-1) ways and one mathematics in = $(2^3$-1) ways.
$\therefore$ Total ways =$(2^3$-1) $(2^3$-1) $(2^3$-1)
Which one of the following options is correct?
1) Priya is correct and Padmini is wrong.
2) Padmini is correct and Priya is wrong.
3) Both Priya and Padmini are wrong and their answers are higher than the actual answer.
4) Both Priya and Padmini are wrong and their answers are lower than the actual answer.

(NUMERICAL VALUE TYPE)

Section-II contains 10 **Numerical Value Type** questions. Attempt any 5 questions only. First 5 attempted questions will be considered if more than 5 questions attempted. The Answer should be within **0 to 9999.** If the Answer is in **Decimal** then round off to the **nearest Integer** value (Example i,e. If answer is above **10** and less than **10.5** round off is **10** andIf answer is from **10.5** and less than **11** round off is **11**).

Marking scheme: +4 for correct answer, 0 if **not attempt** and **-1** in all other cases.

81. The integer n for which $\lim\limits_{x\to 0}\dfrac{(\cos x-1)(\cos x-e^{x})}{x^{n}}$ is a finite non-zero number is

82. Let f(x) is a continuous function for all real values of x and satisfies $\int_{0}^{x} f(t)dt=\int_{x}^{1} t^{2}.f(t)dt+\dfrac{x^{16}}{8}+\dfrac{x^{6}}{3}+a$ then the value of -840a is equal to:

83. $\sec^{2}\left(\tan^{-1}2\right)+\cos ec^{2}\left(\cot^{-1}3\right)$ is equal to

84. The value of $\lim\limits_{n\to\infty}\sum\limits_{r=1}^{r=4n}\dfrac{\sqrt{n}}{\sqrt{r}\left(3\sqrt{r}+4\sqrt{n}\right)^{2}}$ = K. Then 50K =_______

85. The number of integral values of a for which the equation $\cos 2x+a\sin x=2a-7$ possesses solution is

86. For $n\in N$, in the expansion of $\left(\sqrt[4]{x^{-3}}+a\sqrt[4]{x^{5}}\right)^{n}$, the sum of all the binomial coefficients lies between 200 and 400. Also, the term independent of x is 448, then the value of a is

87. The distance between the point $(-1,-5,-10)$ and the point of intersection of the line $\dfrac{x-2}{3}=\dfrac{y+1}{4}=\dfrac{z-2}{12}$ with the plane $x-y+z=5$ is

88. Digit at the unit place of the sum of $(1!)^{2}+(2!)^{2}+(3!)^{2}..........+(2023!)^{2}$ is

89. If the point (3,4) lies on the locus of the point of intersection of the lines $x\cos\alpha+y\sin\alpha=a$ and $x\sin\alpha-y\cos\alpha=b(\alpha$ is a variable$)$, The point (a,b) lies on the line $3x-4y=0$ then $|a+b|$ is equal to

90. Tangents are drawn from the point (α,β) to the hyperbola $3x^{2}-2y^{2}=6$ and are inclined at angles $\theta \& \phi$ to the x-axis. If $(\tan\theta)(\tan\phi)=2$, then the value of $2\alpha^{2}-\beta^{2}$ is

KEY SHEET

PHYSICS

1	**2**	2	**2**	3	**1**	4	**1**	5	**2**
6	**2**	7	**2**	8	**2**	9	**3**	10	**2**
11	**1**	12	**1**	13	**1**	14	**1**	15	**4**
16	**1**	17	**1**	18	**2**	19	**1**	20	**4**
21	**6**	22	**45**	23	**2**	24	**15**	25	**2**
26	**9**	27	**4**	28	**8**	29	**1**	30	**30**

CHEMISTRY

31	**4**	32	**3**	33	**2**	34	**4**	35	**2**
36	**3**	37	**4**	38	**4**	39	**1**	40	**2**
41	**2**	42	**4**	43	**4**	44	**1**	45	**2**
46	**1**	47	**3**	48	**4**	49	**1**	50	**3**
51	**5**	52	**2**	53	**2**	54	**20**	55	**1**
56	**4**	57	**9**	58	**4**	59	**7**	60	**5**

MATHEMATICS

61	**1**	62	**3**	63	**2**	64	**1**	65	**4**
66	**1**	67	**2**	68	**2**	69	**3**	70	**1**
71	**2**	72	**4**	73	**2**	74	**2**	75	**4**
76	**1**	77	**2**	78	**4**	79	**3**	80	**3**
81	**3**	82	**167**	83	**15**	84	**5**	85	**5**
86	**2**	87	**13**	88	**7**	89	**7**	90	**7**

SOLUTIONS

PHYSICS

1. During Collision Kinetic Energy Is Not Conserved.

2. The pressure due to surface tension $=\frac{4T}{R}$

The pressure due to electrostatic forces $=\frac{\sigma^2}{2\varepsilon_0}$

Just before the bubble bursts.

$$\frac{4T}{R}=\frac{\sigma^2}{2\varepsilon_0} \quad \text{or} \quad R=\frac{8T\varepsilon_0}{\sigma^2}$$

3. Work done by the friction on block is $= -216\text{J}$

Therefore the work done by the motor is $=2\times 216J=432J$

4. $y_9=$ position of 9th bright fringe $=9\left(\frac{\lambda D}{d}\right)$

$y_2=$ position of 2nd dark fringe $=\left(2-\frac{1}{2}\right)\frac{\lambda D}{d}=\frac{3}{2}\frac{\lambda D}{d}$

$$y_9-y_2=7.5\text{ mm}\Rightarrow\frac{\lambda D}{d}\left(9-\frac{3}{2}\right)=7.5\times10^{-3}$$

$$\therefore \quad \lambda=\left(7.5\times10^{-3}\right)\left(\frac{2}{15}\right)\left(\frac{0.5\times10^{-3}}{100\times10^{-2}}\right)$$

$$=(75)\left(\frac{2}{15}\right)(5)\left(10^{-4-4}\right)=50\times10^{-8}\text{m} \qquad =5000\,\mathring{A}$$

5. In cae (i) work done by friction is zero, while in case (ii) it is non-zero as it will forward slip (may be for some time) on BC.

6. Here $T_{1/2}=20$ minutes; we know $\frac{N}{N_0}=\left(\frac{1}{2}\right)^{t/T_{1/2}}$

For 20% decay $\frac{N}{N_0}=\frac{80}{100}=\left(\frac{1}{2}\right)^{t_1/20}$ (i)

For 80% decay $\frac{N}{N_0}=\frac{20}{100}=\left(\frac{1}{2}\right)^{t_2/20}$ ....(ii)

Dividing (ii) by (i)

$\frac{1}{4}=\left(\frac{1}{2}\right)^{\frac{(t_2-t_1)}{20}}$; On solving we get $t_2-t_1=40$ min.

7. $W=\alpha^2\beta e^{-\frac{\beta x^2}{kT}}$

As exponents are dimensionless, so, $\frac{\beta x^2}{kT}$ should be dimensionless.

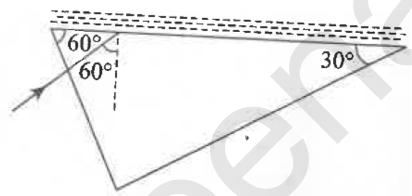
$[\beta]=\left[\frac{kT}{x^2}\right]=\frac{ML^2T^{-2}}{L^2}=MT^{-2}$

From the dimensional homogeneity, $\alpha^2\beta$ should have dimension of work.

$\therefore\left[\alpha^2\beta\right]=ML^2T^{-2}\Rightarrow\left[\alpha^2\right]=\frac{ML^2T^{-2}}{MT^{-2}}\Rightarrow[\alpha]=M^0LT^0$

8. Clearly, $i_c\le 60°$

So, maximum possible value of i_c is $60°$.



Now, ${}_1\mu_g=\frac{1}{\sin i_c}\ \frac{\mu_g}{\mu_1}=\frac{1}{\sin i_c}$ Or $\mu_1=\mu_g\sin i_c$

$=1.5\sin 60°=1.5\times\frac{\sqrt{3}}{2}=1.5\times 0.8666=1.299=1.3$

9. From figure, $\vec{r}_G=\frac{a}{2}\hat{i}+\frac{a}{2}\hat{k}$

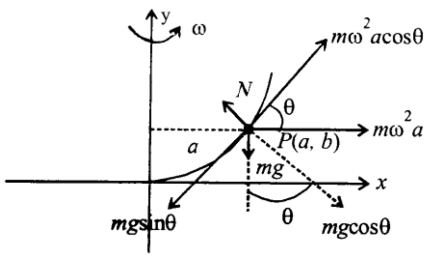
$\vec{r}_H=\frac{a}{2}\hat{j}+\frac{a}{2}\hat{k}\ \therefore\vec{r}_H-\vec{r}_G=\left(\frac{a}{2}\hat{j}+\frac{a}{2}\hat{k}\right)-\left(\frac{a}{2}\hat{i}+\frac{a}{2}\hat{k}\right)=\frac{a}{2}\left(\hat{j}-\hat{i}\right)$

10. To free electron from the metal surface minimum energy required is equal to the work function of that metal.So Assertion A, is correct

$h\nu = W_0 + +K.E_{max}$ If $h\nu = W_0 \Rightarrow K.E_{max} = 0$

Hence reason R, is correct, But R is not the correct explanation of A.

11.



$y = 4Cx^2 \Rightarrow \dfrac{dy}{dx} = \tan\theta = 8Cx$ At P, $\tan\theta = 8Ca$

For steady circular motion

$$m\omega^2 a\cos\theta = mg\sin\theta \Rightarrow \omega = \sqrt{\frac{g\tan\theta}{a}}$$

$$\therefore \omega = \sqrt{\frac{g\times 8aC}{a}} = 2\sqrt{2gC}$$

12. Induced emf $\int_0^b Bvdx = \int_a^b \dfrac{\mu_0 I}{2\pi x} vdx \Rightarrow$ induced emf, $E = \dfrac{\mu_0 Iv}{2\pi}\ln\left(\dfrac{b}{a}\right)$

$\Rightarrow$ power dissipated $= \dfrac{E^2}{R}$

Also, power $= F.V \Rightarrow F = \dfrac{E^2}{vR}$ $F = \left[\dfrac{\mu_0 Iv}{2\pi}\ln\left(\dfrac{b}{a}\right)\right]^2 \dfrac{1}{vR}$

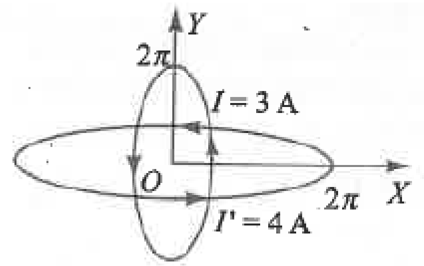
13. Let V_f is the final speed of the body. From questions,

$\dfrac{1}{2}mV_f^2 = \dfrac{1}{8}mV_0^2$ $\Rightarrow V_f = \dfrac{V_0}{2} = 5m/s$

$F = m\left(\dfrac{dV}{dt}\right) = -kV^2$ $\therefore (10^{-2})\dfrac{dV}{dt} = -kV^2$

$$\int_{10}^{5}\frac{dV}{V^2} = -100K\int_0^{10} dt\ \frac{1}{5} - \frac{1}{10} = 100K(10) \quad or,\ K = 10^{-4}kgm^{-1}$$

14.



$$B_x = \frac{\mu_0}{2}\frac{I}{2\pi\times10^{-2}} = \frac{\mu_0}{4\pi}3\times10^2 = 3\times10^{-5}T$$

$$B_y = \frac{\mu_0}{2}\frac{I'}{2\pi\times10^{-2}} = 4\times10^{-5}T$$

$$B_{net} = \sqrt{B_x^2 + B_y^2} = \sqrt{25\times10^{-10}} = 5\times10^{-5}T$$

15. Using Bernoulli's equation

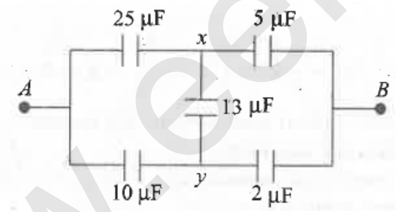
$$P_1 + \frac{1}{2}\rho\upsilon_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho\upsilon_2^2 + \rho g h_2$$

For horizontal pipe, $h_1 = 0$ and $h_2 = 0$ and taking

$P_1 = P, P_2 = \frac{P}{2}$, we get $\Rightarrow P + \frac{1}{2}\rho\upsilon^2 = \frac{P}{2} + \frac{1}{2}\rho V^2$

$$\Rightarrow \frac{p}{2} + \frac{1}{2}\rho\upsilon^2 = \frac{1}{2}\rho V^2 \Rightarrow V = \sqrt{\upsilon^2 + \frac{P}{\rho}}$$

16. Circuit can be redrawn as follows:



Now, $V_x = V_y$, $C_{eq} = \left(\frac{5\times25}{5+25}\right) + \left(\frac{10\times2}{10+2}\right) = \frac{35}{6} = \frac{35}{6}\mu F$

17. Equation of the BC is given by two point straight line equation

$$\Rightarrow \frac{P - P_0}{V - 2V_0} = \frac{3P_0 - P_0}{V_0 - 2V_0} \Rightarrow P = P_0 - \frac{2P_0}{V_0}(V - 2V_0)$$

using PV = nRT

Temperature, $T = \dfrac{P_0V - \dfrac{2p_0V^2}{V_0} + 4P_0V}{1\times R}$

$(\therefore n = 1\text{mole given})$

$T = \dfrac{P_0}{R}\left[5V - \dfrac{2V^2}{V_0}\right], \dfrac{dT}{dV} = 0 \Rightarrow 5 - \dfrac{4V}{V_0} = 0 \Rightarrow V = \dfrac{5}{4}V_0$

$T = \dfrac{P_0}{R}\left[5\times\dfrac{5V_0}{4} - \dfrac{2}{V_0}\times\dfrac{25}{16}V_0^2\right] = \dfrac{25}{8}\dfrac{P_0V_0}{R}$

18. $\mathrm{r} \propto \mathrm{A}^{1/3} \Rightarrow \dfrac{\mathrm{r}_1}{\mathrm{r}_2} = \left(\dfrac{\mathrm{A}_1}{\mathrm{A}_2}\right)^{1/3}$

$\Rightarrow \dfrac{3}{5} = \left(\dfrac{27}{\mathrm{A}}\right)^{1/3} \Rightarrow \dfrac{27}{125} = \dfrac{27}{\mathrm{A}} \Rightarrow \mathrm{A} = 125$

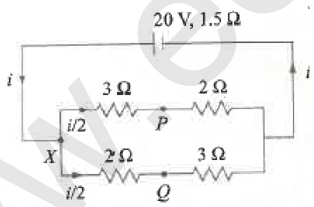
Number of nuclei in atom $\mathrm{X} = \mathrm{A} - 52 = 125 - 52 = 73$.

19. Total energy,

$E = \dfrac{1}{2}m\omega^2a^2$ i.e., $E \propto a^2$, $\dfrac{E'}{E} = \left(\dfrac{2a}{a}\right)^2$ $\quad E' = 4E.$

20. $\mathrm{R}_{eq} = \dfrac{5}{2}\Omega$ $\mathrm{i} = \dfrac{20}{\dfrac{5}{2}+1.5} = 5\mathrm{A}$

Potential difference between X and P,



$\mathrm{V_x} - \mathrm{V_P} = \left(\dfrac{5}{2}\right)\times 3 = 7.5\mathrm{V}$ $\qquad$ (i)

$\mathrm{V_x} - \mathrm{V_Q} = \dfrac{5}{2}\times 2 = 5\mathrm{V}$ $\qquad$ (ii)

On solving (i) & (ii) $\mathrm{V_P} - \mathrm{V_Q} = -2.5\text{volt}$; $\mathrm{V_Q} > \mathrm{V_P}$.

21. Surface area of bubble of radius $r = 4\pi r^2$.Surface are of bubble of radius $2r = 4\pi(2r)^2 = 16\pi r^2$.Therefore, increases in surface area $= 16\pi r^2 - 4\pi r^2 = 12\pi r^2$ .since a bubble has two surface, the total increase in surface are $= 24\pi r^2$

Energy spent = work done $= 24\pi\sigma r^2$

22. Number of atoms in 2 kg fuel $\frac{2}{235} \times 6.02 \times 10^{26} = 5.12 \times 10^{24}$

Fission rate = number of atoms fissioned in one second

$$= \frac{5.12 \times 10^{24}}{30 \times 24 \times 60 \times 60} = 1.975 \times 10^{18} \mathrm{s}^{-1}$$

Each fission gives 185 MeV. Hence, energy obtained in one second,

$$\mathrm{E} = 185 \times 1.975 \times 10^{18} \mathrm{MeV\ s}^{-1}$$
$$= 142 \times 1.975 \times 10^{18} \times 1.6 \times 10^{-19} \times 10^{6} \mathrm{J\ s}^{-1} = 44.87 \mathrm{MW}$$

23. MI of plat form $= 200 kg\ m^2$

MI of man $= mR^2 = 200 kgm^2$

For system (plat form + man) by using COAM

$$Ip\omega_0 = mvR\ \omega_0 = \frac{50 \times 1 \times 2}{200} = \frac{1}{2} rad / s$$

Angular velocity of man w.r.t platform $= \frac{V}{R} + \omega_0$

$$\frac{1}{2} + \frac{1}{2} = 1 rad / \sec = \frac{2\pi\, rad}{1 rad / \sec} = 2\pi \mathrm{\ s}$$

Time taken by the man to complete one revolution

24. Total range is doubled, i.e., $4\mathrm{I_g}$

Now shunt required is

$$\mathrm{S} = \frac{\mathrm{I_g}}{4\mathrm{I_g} - \mathrm{I_g}} \times \mathrm{G} = 10\Omega \quad \frac{1}{30} + \frac{1}{\mathrm{x}} = \frac{1}{10} \text{ or } \quad \mathrm{x} = 15\Omega$$

25. Diameter $= 2.4 + 2\times LC - (0.4 + 2\times LC) = 2\text{cm}$

26. The intensity of the wave is proportional to the area of the slit. Thus we can use the intensities $I_1 \& I_2$ from the slits on screen are in ratio

$$\frac{I_1}{I_2} = \frac{b_1\ell}{b_2\ell} = \frac{b_1}{b_2} = \frac{1}{4}$$

If $a_1 \& a_2$ are the amplitudes of the waves, $\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \frac{1}{4} \Rightarrow \frac{a_1}{a_2} = \frac{1}{2}$

The ratio of maximum to minimum intensity is given as

$$\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)} = \frac{(1+2)^2}{(1-2)^2} = \frac{9}{1}$$

27. Let the velocity acquired by A and B be V, then

$mv = mV + mV \Rightarrow V = \frac{v}{2}$, Also $\frac{1}{2}mv^2 = \frac{1}{2}mV^2 + \frac{1}{2}mV^2 + \frac{1}{2}kx^2$

Where x is the maximum compression of the spring. On solving the above equations, we get $x = v\left(\frac{m}{2k}\right)^{1/2}$

At maximum compression, kinetic energy of the

$A-B\text{system} = \frac{1}{2}mV^2 + \frac{1}{2}mV^2 = mV^2 = \frac{mv^2}{4}$

28. Acceleration $a \propto \frac{V^2}{r}$

Where $V \propto \frac{Z}{n} \& r \propto \frac{n^2}{Z} \Rightarrow a \propto \frac{Z^3}{n^4}$ $\therefore$ both are in ground state i.e n = 1

So, $a \propto Z^3$ $\frac{a_{He^+}}{a_H} = \left(\frac{Z_{He^+}}{Z_H}\right)^3 = \left(\frac{2}{1}\right)^3 = 8$

29. Let a =edge of the block $= 3cm$ $x =$ height of the block above the water surface in floating condition. From F.B.D., we have

$\rho a^3 g = \rho_w a^2 (a - x) g$ ...(1)

$$\rho a^3 g + W = kx + \rho_w a^3 g \qquad ...(2)$$

From eqn. (1), we get $x = a\left(1 - \frac{\rho}{\rho_w}\right) = 3(1-0.8) = 0.6cm$

From eqn. (2). we get

$$W = kx + \left(\rho_w - \rho\right)a^3 g = 50\times 0.006 + (1000-800)\times(3\times 10^{-2})^3 \times 10$$

$$= 0.354 = (20) \text{ Newtons}$$

30. $B = \frac{\mu_0 i}{\pi r}$ $i = \frac{\pi r B}{\mu_0} = \frac{\pi\times 4\times 10^{-2}\times 300\times 10^{6}}{4\pi\times 10^{-7}} = 30A$

30 A in opposite direction

CHEMISTRY

31. Eq. $H_2O_2 =$ eq. $KMnO_4$

$50\times N = 10\times 0.2$

$N = \frac{0.2}{5} = \frac{1}{25}; M = \frac{N}{2} = \frac{1}{50}$

$M = \frac{S}{M.wt.} \Rightarrow S = \frac{1}{50}\times 34 = 0.68g/L$

32. $\frac{(t_{1/2})_1}{(t_{1/2})_2} = \left(\frac{a_2}{a_1}\right)^{n-1}; \frac{120}{240} = \left(\frac{4\times 10^{-2}}{8\times 10^{-2}}\right)^{n-1}; n = 2$

33. $\log\frac{K_p}{K_c} + \log RT = 0 \Rightarrow \left(\frac{K_p}{K_C}\right) = (RT)^{-1} \Rightarrow \Delta ng = -1$

34. For bcc, $d = \frac{\sqrt{3}}{2}a$ or $a = \frac{2d}{\sqrt{3}} = \frac{2\times 4.52}{1.732} = 5.219\overset{0}{A} = 522\,pm$

$$\rho = \frac{z\times M}{a^3\times N_A\times 10^{-30}} = \frac{2\times 39}{(522)^3\times\left(6.023\times 10^{23}\right)\times 10^{-30}} = 0.91g/cm^3 = 910\,kg\ m^{-3}$$

35. $\frac{E_1}{E_2} = \frac{Z_1^2}{Z_2^2}\times\frac{n_2^2}{n_1^2}$

36. As colloidal particles move towards anode so these particles are negatively charged and coagulated by cations of electrolyte.

According to Hardy Schulze rule,

Coagulation power $\propto$ charge of ion

$\therefore$ Order of coagulation power is $Al^{3+} > Ba^{2+} > Na^{+}$

37. With non-metals like $Cl_2, Br_2, I_2, P_4, S_8$; sodium hydroxide gives disproportionated products.

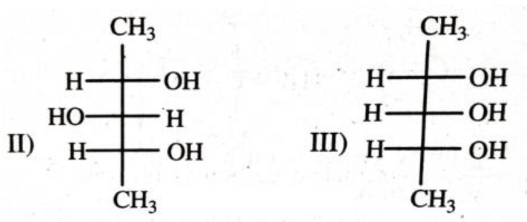
38. $2Ag_2O(s) \rightarrow 4Ag(s) + O_2(g)$ $\quad$ $2Pb_3O_4(s) \rightarrow 6PbO(s) + O_2(g)$

$2PbO_2(s) \rightarrow 2PbO(s) + O_2(g)$

39. $(Be; A\ell)(Li; Mg)(B, Si)$ Diagonally related, in properties. Pairs

40. As all 5 atoms of the molecule are in same plain of XeF_4

41.



42. Paramagnetism is due to the presence of unpaired electrons.

43. a) lone pair of electrons is not involved in resonance

b) +I α basic nature

c) Cyclic compounds are more basic then acylic camp

44. Alkyl iodides are often prepared by the reaction of alkyl chlorides/bromides with NaI in dry acetone. This reaction is known as Finkelstein reaction.

$R - X + NaI \rightarrow R - I + NaX$

$X = Cl, Br$

NaCl or NaBr thus formed is precipitated in dry acetone.

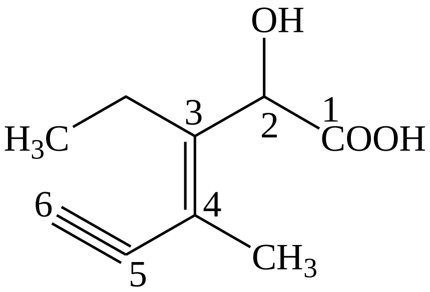
It facilitates the forward reaction according to lechatelier's principle. The synthesis of alkyl fluorides is best accomplished by heating an alkyl chloride/ bromide in the presence of a metallic fluoride such as AgF, Hg_2F_2, CoF_2 or SbF_3. The reaction is termed as Swarts reaction.

$H_3C - Br + AgF \rightarrow H_3C - F + AgBr$

45.

$$Ph - \underset{\displaystyle OH}{\overset{\displaystyle CH_3}{\underset{|}{\overset{|}{C}}}} - CH_2D$$

46.



47. $NaNH_2$ is a stronger base than KOH. Intermediate vinyl Bromide needs stronger Base $NaNH_2$ for better yield.

48. For the preparation of Me_3CNH_2, the required alkylhalide is Me_3CX which will react with potassium phthalmide, a strong base, to form alkene rather than substituted product. For preparing $C_6H_5NH_2, C_6H_5Cl$ will be starting halide in which Cl is non-reactive.

49. Phenol (OH) group is strongly activating and electron releasing group, so it prefers elctrophilic substitution.

50. Molecule having both +ve charge is known as zwitter ion.

51. no.of equivalents of = no. of equivalents of
acid base

$$\frac{2}{\frac{600}{n}} = \frac{1}{6} \times \frac{100}{1000} \qquad n = 5$$

52. i) $5Fe^{2+} + MnO_4^- + 8H^+ \rightarrow Mn^{2+} + 4H_2O + 5Fe^{3+}$

iv) $5NO_2^- + 2MnO_4^- + 6H^+ \rightarrow 2Mn^{2+} + 5NO_3^- + 3H_2O$

53. $$\frac{(u_{av})A}{(u_{av})_B} = \sqrt{\frac{T_A}{M_A} \times \frac{M_B}{T_B}} = \sqrt{\frac{300}{600} \times \frac{16}{2}} = 2$$

54. Presence of catalyst does not affect enthalpy change of reaction

$\Delta H_R = E_f - E_b = 180 - 200 = -20 kJ/mol$

55. $$pH = 5 = pKa + \log\frac{[S]}{[A]}$$

$$pKa + pKb = 14 \Rightarrow pKa = 14 - 10 = 4$$

$$5 = 4 + \log\frac{[S]}{[A]} \Rightarrow \frac{[A^-]}{[H_A]} = 10$$

56. $\left[Cu(NH_3)_3Cl\right]\left[Pt(NH_3)Cl_3\right]$

$\left[Cu(NH_3)Cl_3\right]\left[Pt(NH_3)_3Cl\right]$

$\left[CuCl_4\right]\left[Pt(NH_3)_4\right]$

$\&\left[Cu(NH_3)_4\right]\left[PtCl_4\right]$

The isomers $\left[Cu(NH_3)_2Cl_2\right]\left[Pt(NH_3)_2Cl_2\right]$ does not exist due to both parts being neutral

57. The given reaction is

$6Na+Al_2O_3 \rightleftharpoons 3Na_2O+2Al$

So ΔG of this reaction

$3(-377)-(-1582)=451kJ$

We know that $\Delta G=-2.303$ RT log k

$\log k=79.04 \Rightarrow k=9\times 10^{-80}$

58. Serial numbers of the four alcohols are : 2, 3, 5, 8

59. $XeF_2, SCl_4, PCl_5, SF_4, ClF_3, ICl_2^-, XeOF_2$

60. Basic strength comparison.

MATHEMATICS

61. Replacing x by $\frac{1-x}{1+x}$ in the equation

$\left(f(x)\right)^2 f\left(\frac{1-x}{1+x}\right)=x^3$ ....(i)

We have $\left(f\left(\frac{1-x}{1+x}\right)\right)^2 f(x)=\left(\frac{1-x}{1+x}\right)^3$ ...(ii)

Solving (i) and (ii), we have

$$\left(\frac{x^3}{\left(f(x)\right)^2}\right)^2 f(x)=\left(\frac{1-x}{1+x}\right)^3 \Rightarrow \frac{\left(f(x)\right)^3}{x^6}=\left(\frac{1+x}{1-x}\right)^3 \Rightarrow f(x)=x^2\left(\frac{1+x}{1-x}\right)$$

$f(-2)=4\left(\frac{-1}{3}\right)=-\frac{4}{3}$

$\{f(-2)\}=-\frac{4}{3}-\left[-\frac{4}{3}\right]=-\frac{4}{3}+2=\frac{2}{3}$

62. $$\frac{\left(\tan^{-1}(\sin x+1)\right)^2}{2}+c$$

Let, $I=\int \frac{\tan^{-1}(\sin x+1)\cos x}{3+2\sin x-\left(1-\sin^2 x\right)}dx$

Substituting, $1+\sin x=t \Rightarrow \cos dx=dt$

$$I=\int \frac{\tan^{-1}(t)dt}{2+(t-1)^2+2(t-1)}=\int \frac{\tan^{-1}(t)dt}{1+t^2}=\frac{\left(\tan^{-1}(t)\right)^2}{2}+c$$

$$I=\frac{\left(\tan^{-1}(\sin x+1)\right)^2}{2}+c$$

63. Taking e^{iA}, common from R, e from R_2 and e^{iC} from R_3, we get

$$\Delta=e^{i(A+B+C)}\Delta_1$$

where

$$\Delta_1=\begin{vmatrix} e^{iA} & e^{-i(A+C)} & e^{-i(A+B)} \\ e^{-i(B+C)} & e^{iB} & e^{-i(A+B)} \\ e^{-i(B+C)} & e^{-i(A+C)} & e^{iC} \end{vmatrix}$$

But A+ B+ C $=\pi$, so that $e^{i(A+B+C)}=e^{i\pi}$

$=\cos\pi+i\sin\pi=-1.$ Also,

A+C $=\pi-B \Rightarrow e^{-i(A+C)}=e^{-\pi i}e^{iB}=-e^{iB}$

Thus,

$$\Delta_1=\begin{vmatrix} e^{iA} & -e^{iB} & -e^{iC} \\ -e^{iA} & e^{iB} & -e^{iC} \\ -e^{iA} & -e^{iB} & e^{iC} \end{vmatrix}=e^{i(A+B+C)}\begin{vmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{vmatrix}$$

Using $C_1 \to C_1+C_2$, we get

$$\Delta_1=(-1)\begin{vmatrix} 0 & -1 & -1 \\ 0 & 1 & -1 \\ -2 & -1 & 1 \end{vmatrix}=(-1)(-2)(2)=4$$

Therefore , $\Delta=(-1)\Delta_1=-4$

64. $$t_n=\frac{2n+3}{n(n+1)}.\frac{1}{3^n}$$

Therefore,

$$t_n = \frac{2n+3}{n(n+1)}.\frac{1}{3^n} = \left(\frac{2(n+1)+1}{n(n+1)}\right).\frac{1}{3^n} = \left(\frac{2}{n} + \frac{1}{n(n+1)}\right)\frac{1}{3^n}$$

$$= \left(\frac{2}{n} + \frac{(n+1)-n}{n(n+1)}\right)\frac{1}{3^n} = \left(\frac{2}{n} + \frac{1}{n} - \frac{1}{n+1}\right)\frac{1}{3^n}$$

$$t_n = \left(\frac{3}{n} - \frac{1}{n+1}\right).\frac{1}{3^n} = \frac{1}{n}.\frac{1}{3^{n-1}} - \frac{1}{n+1}.\frac{1}{3^n}$$

$$S_n = \sum_{n=1} t_n = 1 - \frac{1}{n+1}.\frac{1}{3^n}$$

65. Let $\Delta = \begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = \begin{vmatrix} \lambda+2 & 1 & 1 \\ \lambda+2 & \lambda & 1 \\ \lambda+2 & 1 & \lambda \end{vmatrix} \left[C_1 \to C_1 + C_2 + C_3\right]$

$$= (\lambda+2)\begin{vmatrix} 1 & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = \left|\begin{vmatrix} 1 & 0 & 0 \\ 1 & \lambda-1 & 0 \\ 1 & 0 & \lambda-1 \end{vmatrix}\right|$$

$\left[\text{using } C_2 \to C_2 - C_1 \text{ and } C_3 \to C_2 - C_1\right] = (\lambda+2)(\lambda-1)^2$

If $\Delta = 0$, then $\lambda = -2$ or $\lambda = 1$

But when $\lambda = 1$, the system of equation becomes $x_1 + x_2 + x_3 = 1$ which has infinite number of solutions. When $\lambda = -2$, by adding three equations, we obtain $0 = 3$ and thus, the system of equations is inconsistent.

66. Since $x^2 + y^2 + 4y - 5 = 0$

Centre is $C_1(0,-2)$ and Radius $r_1 = \sqrt{4+5} = 3$

Let $C_2(h,k)$ be the centre of the smaller circle and its radius r_2.

Since, $C_1C_2 = 4 \Rightarrow \sqrt{h^2 + (k+2)^2} = 3 + r_2 = 4 \Rightarrow r_2 = 1$

But $k = r_2 = 1$

Hence, using equation (1), we have

$4 = \sqrt{h^2 + (1+2)^2} \Rightarrow 16 = h^2 + 9$ $\qquad \therefore h = \pm\sqrt{7}$

Since, h > 0

Hence, required circle is $\left(x - \sqrt{7}\right)^2 + (y-1)^2 = 2$

67.

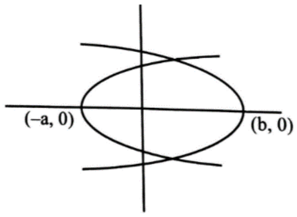
$$= \sum_1^{\infty} \sin^{-1} \frac{\sqrt{n} - \sqrt{n-1}}{\sqrt{n(n+1)}} = \sum_1^{\infty} \sin^{-1}\left[\frac{1}{\sqrt{n}}\sqrt{\frac{n}{n+1}} - \frac{1}{\sqrt{n+1}}\sqrt{\frac{n-1}{n}}\right]$$

$$=\sum_{1}^{\infty}\sin^{-1}\left[\frac{1}{\sqrt{n}}\sqrt{1-\frac{1}{n+1}}-\frac{1}{\sqrt{n+1}}\sqrt{1-\frac{1}{n}}\right]$$

$$=\underset{n\to\infty}{Lt}\sum_{1}^{n}\left[\sin^{-1}\frac{1}{\sqrt{n}}-\sin^{-1}\frac{1}{\sqrt{n+1}}\right]$$

$$=\underset{n\to\infty}{Lt}\left[\frac{\pi}{2}-\sin^{-1}\frac{1}{\sqrt{n+1}}\right]=\frac{\pi}{2}\text{ (by telescopic)}$$

68. $\frac{16}{3}=2\int_{-a}^{b-a}2\sqrt{a}\sqrt{x+a}\,dx+4\int_{b-a}^{b}\sqrt{b}\sqrt{b-x}\,dx$



$$\Rightarrow\frac{8}{3}\sqrt{ab}(a+b)=\frac{16}{3}\Rightarrow\left(\frac{a+b}{2}\right)\sqrt{ab}=1$$

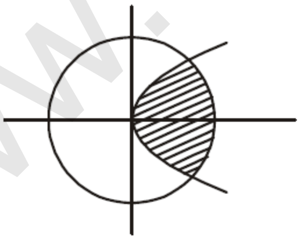
69. $\mu=\phi(x)=\frac{1}{x-1}$ is discontinuous at $x=1$,

$y=f(\mu)=\frac{1}{\mu^1+\mu-2}=\frac{1}{(\mu+2)(\mu-1)}$ is discontinuous at μ

$=-2,\mu=1.$ If $\mu=-2$ then $-2=\frac{1}{x-1}$

$\Rightarrow x=1/2.$ If $\mu=1$ then $1=\frac{1}{x-1}\Rightarrow x=2.$ Hence the composite function is discontinuous only at $x=1,1/2,2.$

70. The point (–2k, k + 1) is the interior point of the circle and parabola



So $(2k)^2+(k+1)^2-4<0$

$\Rightarrow 4k^2+k^2+2k+1-4<0\Rightarrow 5k^2+2k-3<0$

$(k+1)\left(k-\frac{3}{5}\right)<0\Rightarrow k\in\left(-1,\frac{3}{5}\right)\to(1)$

Now, $(k+1)^2-4(-2k)<0$

$\Rightarrow k^2+2k+1+8k<0 \Rightarrow k^2+10k+1<0$

$k \in \left(-5-2\sqrt{6}, -5+2\sqrt{6}\right) \to (2)$

So from (1) & (2) $k \in \left(-1, -5+2\sqrt{6}\right)$

71. $y=\tan^{-1}\frac{1}{1+x+x^2}+\tan^{-1}\frac{1}{x^2+3x+3}+\ldots+\text{upto } n \text{ terms}$

$=\tan^{-1}\frac{(x+1)-x}{1+x(x+1)}+\tan^{-1}\frac{(x+2)-(x+1)}{1+(x+1)(x+2)}+\ldots n \text{ terms}$

$=\tan^{-1}(x+1)-\tan^{-1}x+\tan^{-1}(x+2)-\tan^{-1}(x+1)+\ldots+$

$\tan^{-1}(x+n)-\tan^{-1}(x+(n-1))$

$=\tan^{-1}(x+n)-\tan^{-1}x.$

$y'(x)=\frac{1}{1+(x+n)^2}-\frac{1}{1+x^2}$

$\Rightarrow \ y'(0)=\frac{1}{1+n^2}-1=-\frac{n^2}{1+n^2}.$

72. If z_1 be the new complex number then $|z_1|=|z|+\sqrt{2}=2\sqrt{2}$

Also

$\frac{z_1}{z}=\frac{|z_1|}{|z|}e^{i3\pi/2} \Rightarrow z_1=z.2\left(\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2}\right)=2(1+i)(0-i)=-2i+2=2(1-i)$

73. $\frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{y}}\frac{dy}{dx}=0 \quad \Rightarrow \frac{dy}{dx}=-\frac{\sqrt{y}}{\sqrt{x}}.$

Equation of tangent at any point (x,y) of the curve is $Y-y=-\frac{\sqrt{y}}{\sqrt{x}}(X-x)$

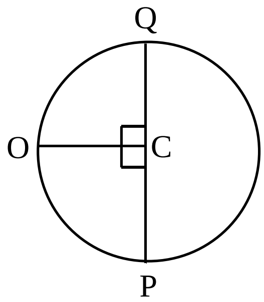
.So intercepts of $X-$axis and $Y-$axis are $x+\sqrt{xy}$ and $y+\sqrt{xy}$.

Therefore, the sum of intercepts $=x+y+2\sqrt{xy}=(\sqrt{x}+\sqrt{y})^2=(\sqrt{a})^2=a$

74. Distance CP = CQ = OC = 5 units

Slope of $OC=\frac{3}{4}$

Slope of $PQ=\frac{-4}{3} \Rightarrow \tan\theta=-\frac{4}{3}$



Co-ordinates of point P and Q are $\left(4+5\cos\theta,3+5\sin\theta\right)$ and $\left(4-5\cos\theta,3-5\sin\theta\right)$

= (4 – 3, 3 + 4) and (4 + 3, 3 – 4)

(1, 7) and (7, –1)

75. Distance of any point $\left(x,y\right)$ from $y=2x-1$ is: $\left|\dfrac{y-2x+1}{\sqrt{5}}\right|$.

If $\left(x,y\right)$ is on $y=x^4+3x^2+2x$ then this distance is $S=\dfrac{x^4+3x^2+1}{\sqrt{5}}$

$$\frac{dS}{dx}=\frac{4x^3+6x}{\sqrt{5}} \Rightarrow \frac{ds}{dx}=0 \Rightarrow x=0.$$

Als , $S^{'}\left(x\right)<0$ for $x<0$ and $S^{'}\left(x\right)>0$ for $x>0.$

Thus S is minimum when $x=0,$ and min. S is $\dfrac{1}{\sqrt{5}}$.

76. Let, a = number of red balls.

b = number of blue balls,

p_1 = probability of drawing two red balls

$$=\frac{{}^{a}C_2}{{}^{a+b}C_2}=\frac{a\left(a-1\right)}{\left(a+b\right)\left(a+b-1\right)}$$

$p_2=$ probability of drawing two blue balls $=\dfrac{{}^{b}C_2}{{}^{a+b}C_2}=\dfrac{b\left(b-1\right)}{\left(a+b\right)\left(a+b-1\right)}$

$p_3=$ probability of drawing one red and one blue ball

$$=\frac{{}^{a}C_1.{}^{b}C_1}{{}^{a+b}C_2}=\frac{2ab}{\left(a+b\right)\left(a+b-1\right)}$$

Given that $p_1=5p_2 \,\&\, p_3=6p_2$

$\Leftrightarrow a\left(a-1\right)=5b\left(b-1\right)$ and 2ab = 6b (b – 1)

$\Rightarrow a=6, b=3 \Rightarrow$ Total number of balls = 9

77. Equation of the line L in the two coordinate system is $\frac{x}{a}+\frac{y}{b}=1, \frac{X}{p}+\frac{Y}{q}=1$

Where (X,Y) are the new coordinates of a point (x,y) when the axes are rotated through a fixed angle, keeping the origin fixed. As the length of the perpendicular from the origin has not changed

$$\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}=\frac{1}{\sqrt{\frac{1}{p^2}+\frac{1}{q^2}}} \Rightarrow \frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}+\frac{1}{q^2} \text{ or } \frac{1}{a^2}-\frac{1}{p^2}+\frac{1}{b^2}-\frac{1}{q^2}=0.$$

78. Let n consecutive odd integers are

2m + 1, 2m + 3, 2m + 5, ……, 2m + 2n – 1

Given that,

$$45^2-21^2=(2m+1)+(2m+3)+(2m+5)+.......+(2m+2n-1)$$

$$=2mn+(1+3+5.....+(2n-1))=2mn+n^2=m^2+2mn+n^2-m^2$$

$$\Rightarrow 45^2-21^2=(m+n)^2-m^2$$

$$\Rightarrow m+n=45 \,\&\, m=21$$

$$\Rightarrow n=24 \,\&\, m=21$$

Hence, the numbers are

43, 45, ……, 89

79. Volume of the parallelepiped is

$$V(a)=\begin{vmatrix}1 & a & 1\\ 0 & 1 & a\\ a & 0 & 1\end{vmatrix}=a^3-a+1\, V'(a)=3a^2-1=0 \text{ if } a=\pm\frac{1}{\sqrt{3}}$$

$$V''(a)=6a>0 \text{ if } a=\frac{1}{\sqrt{3}}$$

Thus $V(a)$ is minimum when $a=\frac{1}{\sqrt{3}}$.

80. Number of ways $={}^{9}C_5$ - no.of ways in which she selects 5 from only of two categories.

$$={}^{9}C_5-3.{}^{6}C_5=126-18=108$$

Alternatively: Number of ways 1 1 3 or 2 2 1 i.e.,

$$3.{}^{3}C_1.{}^{3}C_1.{}^{3}C_3+3.{}^{3}C_2.{}^{3}C_2.{}^{3}C_1=27+81=108.$$

81. $=\left(\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-...\right)-1\right)\left[\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-...\right)-\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}..\right)\right]$

$=x^2\left(-\frac{1}{2}+\frac{x^2}{4!}...\right)\left(-x-x^2-\frac{x^3}{3!}...\right)$

$=x^3\left(-\frac{1}{2}+\frac{x^2}{4!}...\right)\left(-1-x-\frac{x^2}{3!}...\right)$

$\Rightarrow \frac{(\cos x-1)(\cos x-e^x)}{x^3}=\left(\begin{array}{c}\frac{1}{2}+\text{terms containig } x\\ \text{and higher powers } of\ x\end{array}\right)$

82. $\int_0^x f(t)dt=\int_x^1 t^2.f(t)dt+\frac{x^{16}}{8}+\frac{x^6}{3}+a\rightarrow(1)$

For x = 1, $\int_0^1 f(t)dt=0+\frac{1}{8}+\frac{1}{3}+a=\frac{11}{24}+a$

Diff. both sides of (1) w. r. t. x we get;

$f(x)=0-x^2f(x)+2x^{15}+2x^5$

$\Rightarrow 2\int_0^1\frac{x^{15}+x^5}{1+x^2}dx=\frac{11}{24}+a\Rightarrow 2\int_0^1\left(x^{13}-x^{11}+x^9-x^7+x^5\right)dx=\frac{11}{24}+a$

$\Rightarrow 2\left(\frac{1}{14}-\frac{1}{12}+\frac{1}{10}-\frac{1}{8}+\frac{1}{6}\right)=\frac{11}{24}+a\Rightarrow a=-\frac{167}{840}$

83. The given expression is equal to

$1+\tan^2\left(\tan^{-1}2\right)+1+\cot^2\left(\cot^{-1}3\right)$

$=1+4+1+9=15.$

84.

$\lim_{n\to\infty}\sum_{r=1}^{r=4n}\left(\frac{1}{\sqrt{r/n}\left(3\sqrt{\frac{r}{n}}+4\right)^2}\right).\frac{1}{n}$

$=\int_0^4\frac{1}{\sqrt{x}\left(3\sqrt{x}+4\right)^2}dx$

Put, $3\sqrt{x}+4=t$

$\frac{3}{2\sqrt{x}}dx=dt\Rightarrow\frac{1}{\sqrt{x}}dx=\frac{2}{3}dt$

When x = 0 then t = 4
When x = 4 then t = 10

$$= \frac{2}{3}\int_4^{10} \frac{1}{t^2} dt = \frac{2}{3}\left(-\frac{1}{t}\right)_4^{10} = -\frac{2}{3}\left(\frac{1}{10} - \frac{1}{4}\right) = \frac{1}{10}$$

85. The given equation can be written as $1 - 2\sin^2 x + a\sin x = 2a - 7$

$$\Rightarrow \quad 2\sin^2 x - a \sin x + 2a - 8 = 0$$

$$\Rightarrow \quad \sin x = \frac{a \pm \sqrt{a^2 - 8(2a-8)}}{4} = \frac{a \pm (a-8)}{4}$$

$\Rightarrow \sin x = \dfrac{a-4}{2}$ which is possible if $-1 \le \dfrac{a-4}{2} \le 1$ or $2 \le a \le 6$.

so the required values of a are 2,3,4,5,6 and hence the required number is 5.

86. $200 < {}^nC_0 + {}^nC_1 + ... + {}^nC_n < 400$

$\Rightarrow 200 < 2^n < 400$

$\Rightarrow n = 8$

$$T_{r+1} = {}^8C_r\left(\sqrt[4]{x^{-3}}\right)^{8-r}\left(a\sqrt[4]{x^5}\right)^r$$

$\Rightarrow T_{r+1} = {}^8C_r a^r x^{2r-6}$

For this term to be independent of x,

$2r - 6 = 0 \Rightarrow r = 3$

$T_4 = 8C_3 a^3 \Rightarrow 448 = 56a^3 \Rightarrow a^3 = 8$

$\Rightarrow a = 2$

87. Any point on the given line is $(3r+2, 4r-1, 12r+2)$ which lies on the given plane is $3r + 2 - (4r-1) + 12r + 2 = 5 \Rightarrow r = 0$

So the point of intersection of the line and the plane is (2, -1, 2) and the required distance is

$$\sqrt{(2+1)^2 + (-1+5)^2 + (2+10)^2} = 13$$

88. Given the sum $S = (1!)^2 + (2!)^2 + (3!)^2 + ... + (2023!)^2$

Since the digit at units place is zero in $n!$ for $n \ge 5$

Hence, $S = (1)^2 + (2)^2 + (6)^2 + (24)^2 +$ numbers having zero at units place = 617 + all other numbers having zero at units place.

89. Squaring and adding the given equations of the lines we get

$x^2+y^2=a^2+b^2$ as the locus of the point of intersection of these lines.

Since (3,4) lies on the locus, we get

$9+16=a^2+b^2$ i.e $a^2+b^2=25$ (i)

Also, (a,b) lies on $3x-4y=0$

so $3a-4b=0$ $\Rightarrow b=(3/4)a$ (ii)

From (i) , $a^2+(9/16)a^2=25 \Rightarrow a^2=16$

So that $|a+b|^2=(7/4)^2a^2=49$ $|a+b|=7$

90. Given hyperbola is $3x^2-2y^2=6$ or $\frac{x^2}{2}-\frac{y^2}{3}=1$

Slope from of tangent is $y=mx\pm\sqrt{a^2m^2-b^2}$ or $(y-mx)^2=a^2m^2-b^2$

Tangent from the point (α,β) is given by,

$(\beta-m\alpha)^2=2m^2-3$

$\Rightarrow m^2(\alpha^2-2)-2\alpha\beta m+\beta^2+3=0$

$m_1m_2=\frac{\beta^2+3}{\alpha^2-2}=2=\tan\theta\tan\phi$

$\therefore \beta^2+3=2\alpha^2-4 \Rightarrow 7=2\alpha^2-\beta^2$