Model Grand Test

2023

IMPORTANT INSTRUCTION:

1. Immediately fill in the Admission number on this page of the Test Booklet with **Blue/Black** Ball Point **Pen** only.
2. The candidates should not write their Admission Number anywhere (except in the specified space) on the Test Booklet/ Answer Sheet.
3. The test is of **3 hours** duration.
4. The Test Booklet consists of 90 questions. The maximum marks are **300.**
5. There are **three** parts in the question paper 1,2,3 consisting of **Physics, Chemistry** and **Mathematics** having **30 questions** in each subject and subject having **two sections.**
   (I) Section –I contains 20 **multiple choice** questions with only one correct option.
   Marking scheme: +4 for correct answer, 0 if not attempt and -1 in all other cases.
   (II) Section-II contains 10 **Numerical Value Type** questions. Attempt any 5 questions only,if more than 5 questions attempted, First 5 attempted questions will be considered.
   - The Answer should be within **0 to 9999.** If the Answer is in **Decimal** then round off to the **nearest Integer** value (Example i,e. If answer is above **10** and less than **10.5** round off is **10** andIf answer is from **10.5** and less than **11** round off is **11).**

   To cancel any attempted question bubble on the question number box.
   For example: To cancel attempted question 21. Bubble on 21 as shown below

Question Answered for Marking **Question Cancelled for Marking**

Marking scheme: +4 for correct answer, 0 if **not attempt** and **-1** in all other cases.

6. Use **Blue / Black Point Pen only** for writing particulars / marking responses on the Answer Sheet. **Use of pencil is strictly prohibited.**
7. No candidate is allowed to carry any textual material, printed or written, bits of papers, mobile phone any electron device etc, except the Identity Card inside the examination hall.
8. Rough work is to be done on the space provided for this purpose in the Test Booklet only.
9. On completion of the test, the candidate must hand over the Answer Sheet to the invigilator on duty in the Hall. **However, the candidate are allowed to take away this Test Booklet with them.**
10. **Do not fold of make any stray marks on the Answer Sheet**

PHYSICS

Max Marks: 100

(SINGLE CORRECT ANSWER TYPE)

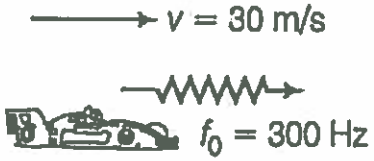
This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

1. If a message signal of frequency 'f_m' is amplitude modulated with a carrier signal of frequency f_c and radiated through an antenna, the wavelength of the corresponding signal in air is:

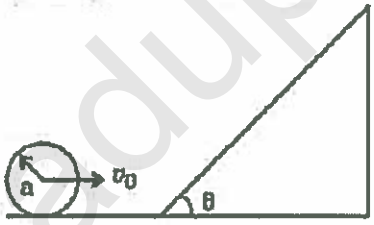
   1) $\frac{c}{f_c}$ 2) $\frac{c}{f_m}$ 3) $\frac{c}{f_c + f_m}$ 4) $\frac{c}{f_c - f_m}$

2. A car is moving towards a fixed wall, the frequency of reflected sound observed by car is $\left(v_{sound} = 330\, m/s\right)$



   1) 330 Hz 2) 360 Hz 3) 300 Hz 4) Zero

3. A sphere of radius 'a' and mass 'm' rolls along a horizontal plane with constant speed v_0. It encounters an inclined plane at angle θ and climbs upward. Assuming that it rolls without slipping, how far up, the sphere will travel ?



   1) $\frac{2}{5}\frac{v_0^2}{g\sin\theta}$ 2) $\frac{v_0^2}{2g\sin\theta}$ 3) $\frac{7v_0^2}{10g\sin\theta}$ 4) $\frac{v_0^2}{5g\sin\theta}$

4. A particle is moving with velocity $v = k\left(y\hat{i} + x\hat{j}\right)$, where k is a constant. The general equation for its path is

   1) $xy = \text{constant}$ 2) $y^2 = x^2 + \text{constant}$ 3) $y^2 = x + \text{constant}$ 4) $y = x^2 + \text{constant}$

5. Given below are two statements:

   Statement 1: In a diatomic molecule, the rotational energy at a given temperature obeys Maxwell's distribution.

   Statement 2: In a diatomic molecule, the rotational energy at a given temperature equals The translational kinetic energy for each molecule.

   In the light of the above statements, choose the correct answers from the options given below.

1) both statement 1 and statement 2 are true

2) both statement 1 and statement 2 are false

3) statement 1 is false but statement 2 is true

4) statement 1 is true but statement 2 is false

6. Poise is the CGS unit of coefficient of viscosity. Suppose we employ a system of units in which unit of mass is α kg, the unit of length is β metre and unit of time is γ second. In this new system, 1 Poise is equal to

1) $1000\,\alpha\beta^{-1}\gamma^{-1}$ 2) $10\,\alpha\beta^{-1}\gamma^{-1}$ 3) $0.1\,\alpha^{-1}\beta\gamma$ 4) $0.01\,\alpha^{-1}\beta\gamma$

7. For extrinsic semiconductors, when doping level is increased;

1) Fermi-level of p-type semiconductors will go downward and Fermi-level of n-type Semiconductor will go upward

2) Fermi-level of both p-type and n-type semiconductors will go upward for $T > T_F K$ and downward $T < T_F K$, where T_F is Fermi temperature.

3) Fermi – level of p and n-type semiconductors will not be affected.

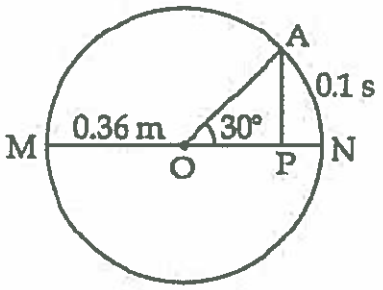
4) Fermi-level of p-type semiconductors will go upward for Fermi-level of n-type Semiconductors will go downward.

8. In a hypothetical Bohr hydrogen, the mass of the electron is doubled. The energy E_0 and radius r_0 of the ground state will be (a_0 is the Bohr's ground state radius of actual H-atom)

1) $E_0 = -27.2eV, r_0 = \frac{a_0}{2}$ 2) $E_0 = -27.2eV, r_0 = 2a_0$

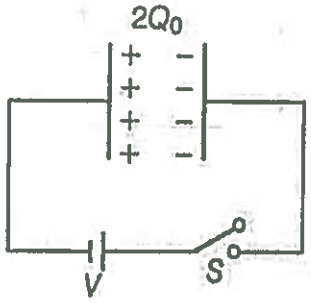
3) $E_0 = -13.6eV, r_0 = \frac{a_0}{2}$ 4) $E_0 = -13.6eV, r_0 = a_0$

9. The point A moves with a uniform speed along the circumference of a circle of radius 0.36 m and covers 30° in 0.1s. The perpendicular projection 'P' from 'A' on the diameter MN represents the simple harmonic motion of 'P'. The restoration force per unit mass when P touches M will be:



1) 0.49 N 2) 9.87 N 3) 50 N 4) 100 N

10. A capacitor of capacitance C having initial charge $2Q_0$, is connected to a battery of potential difference $V = \frac{Q_0}{C}$ as shown, then workdone by the battery is



1) $\frac{Q_0^2}{2C}$ 2) $\frac{3Q_0^2}{2C}$ 3) $\frac{3Q_0^2}{C}$ 4) $\frac{2Q_0^2}{3C}$

11. A stone is dropped from the top of a building. When it crosses a point 5 m below the top, another stone starts to fall from a point 25m below the top. Both stones reach the bottom of building simultaneously. The height of the building is

1) 45 m 2) 50 m 3) 25 m 4) 35 m

12. A TV tower has a height 150 m. What is the total population covered by TV tower, if the population density around the TV tower is 500 person $/km^2$. The radius of the earth is $6.4\times10^6 m$.

1) 30.144 lakh 2) 60.288 lakh 3) 15.07 lakh 4) 120.572 lakh

13. The stopping potential for electrons emitted from a photosensitive surface illuminated by light of wavelength 491 nm is 0.710V. When the incident wavelength is changed to a new value, the stopping potential is 1.43V. The new wavelength is:

1) 309 nm 2) 329 nm 3) 382 nm 4) 400 nm

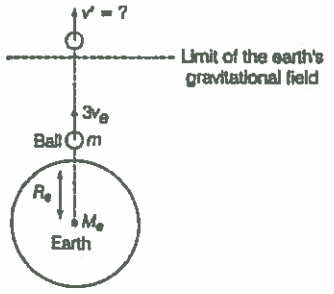
14. Two equal charges of 2C each are fixed at $x=-5m$ and $x=+5m$ on the X-axis. Another point charge Q = 4C is placed at the origin. The final electric potential energy of Q, when charge from origin is displaced by a distance of 2m along X-axis, is (Nearly)

1) $3.43\times10^{10}\,J$ 2) $6.77\times10^{10}\,J$ 3) $2.87\times10^{10}\,J$ 4) $5.86\times10^{10}\,J$

15. An electron with kinetic energy K_1 enters between parallel plates of a capacitor at an angle 'α' with the plates. It leaves the plates at angle 'β' with kinetic energy K_2. Then the ratio of kinetic energies $K_1 : K_2$ will be:

1) $\frac{\sin^2\beta}{\cos^2\alpha}$ 2) $\frac{\cos^2\beta}{\cos^2\alpha}$ 3) $\frac{\cos\beta}{\cos\alpha}$ 4) $\frac{\cos\beta}{\sin\alpha}$

16. A ball is thrown upwards from the earth's surface with a speed $3v_e$, when the ball crosses the earth's gravitational field, then [v_e= escape velocity]



1) $V' = 2V_e$ 2) $V' = \sqrt{7}V_e$ 3) $2\sqrt{2}V_e = V'$ 4) $V' = V_e$

17. If e is the electronic charge, c is the speed of light in free space and h is plank's constant, the quantity $\frac{1}{4\pi\varepsilon_0}\frac{|e|^2}{\hbar c}$ has dimensions of:

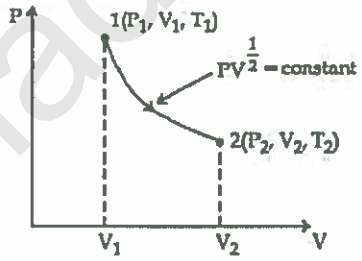
1) $[M\,L\,T^{-1}]$ 2) $[M^0\,L^0\,T^0]$ 3) $[M\,L\,T^0]$ 4) $[L\,C^{-1}]$

18. The average translational energy and the rms speed of molecules in a sample of oxygen at 300 K are $6.21\times10^{-21}J$ and 484 m/s, respectively. The corresponding values at 600 K are nearly (assuming ideal gas behavior).

1) $12.42\times10^{-21}J, 968m/s$ 2) $6.21\times10^{-21}J, 968m/s$

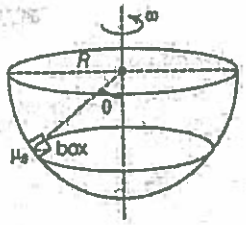
3) $8.78\times10^{-21}J, 684m/s$ 4) $12.42\times10^{-21}J, 684m/s$

19. Thermodynamics process is shown below on a P- V diagram for one mole of an ideal gas. If $V_2 = 2V_1$ then the ratio of temperature T_2/T_1 is:



1) 2 2) $\sqrt{2}$ 3) $\frac{1}{\sqrt{2}}$ 4) $\frac{1}{2}$

20. A hemispherical bowl is rotating with an angular speed ω. If there is no need of friction in revolving a box with bowl, which is placed inside the bowl as shown in the figure, then



1) $\cos\theta = \frac{g}{R\omega^2}$ 2) $\sin\theta = \frac{R\omega^2}{g}$ 3) $\tan\theta = \frac{g}{R\omega^2}$ 4) $\tan\theta = \frac{R\omega}{g}$

(NUMERICAL VALUE TYPE)

Section-II contains 10 Numerical Value Type questions. Attempt any 5 questions only. First 5 attempted questions will be considered if more than 5 questions attempted.The Answer should be within 0 to 9999. If the Answer is in Decimal then round off to the nearest Integer value (Example i,e. If answer is above 10 and less than 10.5 round off is 10 andIf answer is from 10.5 and less than 11 round off is 11).

Marking scheme: +4 for correct answer, 0 if not attempt and -1 in all other cases.

21. The wavelength of an X – ray beam is $10\,\mathring{A}$ the mass of a fictitious particle having the same energy as that of the X- ray photons is $\frac{x}{3}h\,kg$. the value of x is ____ (h=Planck's constant)

22. Bulk modulus of metal is 10^{10} N/m^2 and its density is 11 g/cm^3. The density of this metal under a pressure of 20000 N/cm^2 will be ____ g/cm^3.

23. Two small spheres each of mass 10 mg are suspended from a point by threads 0.5m long. They are equally charged and repel each other to a distance of 0.20m. the charge on each of the sphere is $\frac{a}{21}\times10^{-8}C$. the value of 'a' will be ___ [given g = 10ms^{-2}]

24. In YDSE, the 10th maxima is 3 cm from the central maxima. The YDSE apparatus is immersed in a liquid of refractive index 1.5. Then, the 5th maxima from the central maxima will be (in cm)

25. The peak electric field produced by the radiation coming from the 8W bulb at a distance of 10m is $\frac{x}{10}\sqrt{\frac{\mu_0 C}{\pi}}\frac{V}{m}$.the efficiency of the bulb is 10% and it is a point source. The value of x is (nearest integer)

26. A long solenoid has 200 turns per cm and carries a current *i*. The magnetic field at its centreis $6.28\times10^{-2}Wbm^{-2}$. Another along solenoid has 100 turns per cm and it carries a current *i*/3. The value of the magnetic field at its centre is ___ $10^{-2}Wb/m^2$.

27. Two identical conducting spheres with negligible volume have 2.1 nC and -0.1 nC charges, respectively. They are brought into contact and then separated by a distance of 0.5 m. The electrostatic force acting between the sphere is ____ × 10^{-9} N

[Given: $4\pi\in_0=\frac{1}{9\times10^9}SI\,unit$]

28. A set of 36 tuning forks is arranged in series of decreasing frequencies. Each fork gives 3 beats with succeeding one. The first fork is an octave of the last. The frequency of the 15th tuning fork from start is ____ Hz

29. Two particles having masses 4g and 16g respectively are moving with equal kinetic energies. The ratio of the magnitude of their linear momentum is n : 2. The value of n will be ___

30. In a room, where the temperature is 30°C, a body cools from 61°C to 59°C in 4 min. The time taken by the body to cool from 51°C to 49°C will be (in min)

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

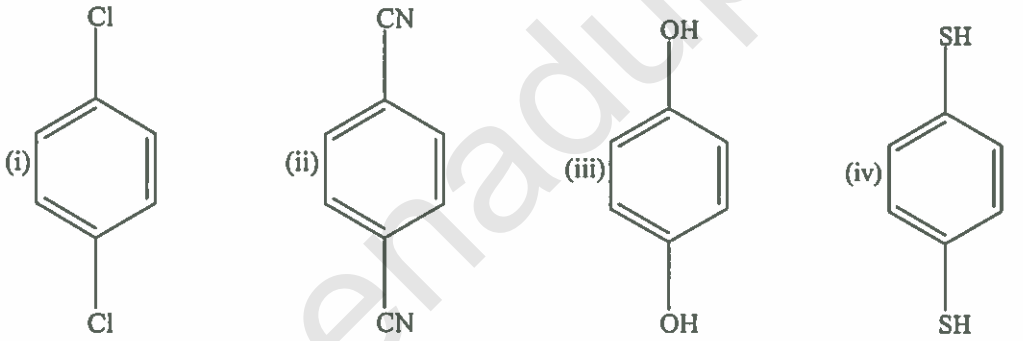
Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

31. Which one of the following statement is FALSE for hydrophilic sols?
    1) The sols cannot be easily coagulated
    2) Their viscosity is of the order of that of H_2O
    3) These sols are reversible in nature
    4) The do not require electrolytes for stability

32. A solid contains A^{n+} and B^{m-} ions. The structure of solid is FCC for B^{m-} ions and A^{n+} ions are present in one-fourth of the tetrahedral voids as well as in one-fourth of octahedral voids. What is the simplest formula of solid?
    1) A_3B_4 2) A_4B_3 3) AB_2 4) A_2B

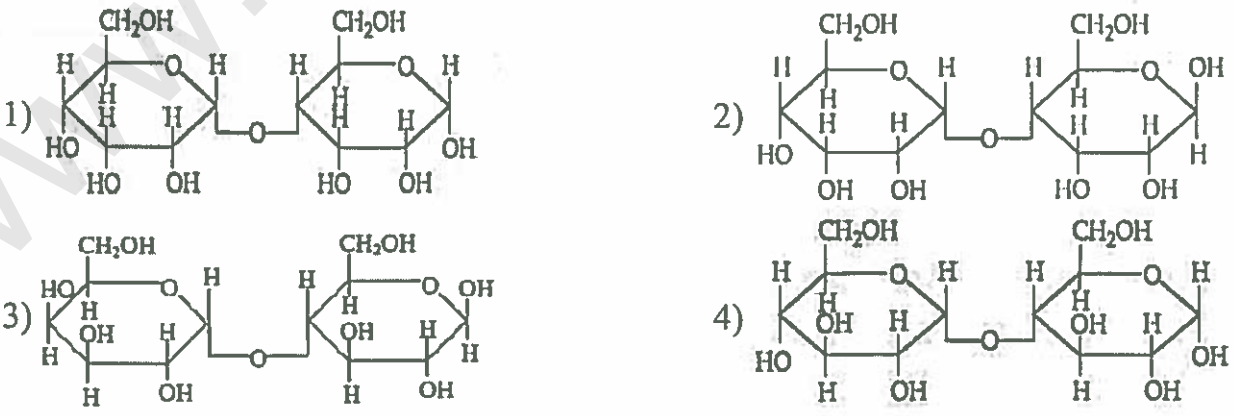
33. Which of the following compound is added to the sodium extract before addition of silver nitrate for testing of halogens?
    1) Ammonia 2) Nitric acid
    3) Hydrochloric acid 4) Sodium hydroxide

34. For which of the following molecule significant $\mu \neq 0$?



    1) Only (i) 2) (i) and (ii) 3) Only (iii) 4) (iii) and (iv)

35. Which of the following is correct structure of α–anomer of maltose?



36. Which statement regarding H_3BO_3 is not correct?

1) It is a strong tribasic acid

2) It is prepared by acidifying an aqueous solution of borax

3) It has a layer structure in which planar BO_3 units are joined by H-bonds

4) It does not act as proton donor but acts as lewis acid by accepting OH^- ions

37. The correct order of bond dissociation enthalpy of halogen is:

1) $I_2 > Br_2 > Cl_2 > F_2$ 2) $Cl_2 > F_2 > Br_2 > I_2$

3) $Cl_2 > Br_2 > F_2 > I_2$ 4) $F_2 > Cl_2 > Br_2 > I_2$

38. Holme's signal can be given by using:

1) $CaC_2 + CaCN_2$ 2) $CaC_2 + Ca_3P_2$

3) $CaC_2 + CaCO_3$ 4) $Ca_3P_2 + CaCN_2$

39. Given below are two statements:

Statement I: α and β forms of sulphur can change reversibly between themselves with slow heating or slow cooling.

Statemetn II: At room temperature the stable crystalline form of slulphur is monoclinic sulphur.

In the light of the above statements, choose the correct answer from the options given below

1) Statement I is false but statement II is true

2) Both Statement I and statement II are true

3) Statement I is true but statement II is false

4) Both Statement I and statement II are false

40. Aluminum (Al) becomes passive by _____ due to formation of_____

1) $dil.HCl, Al_2O_3$ 2) $Conc.HNO_3, Al_2O_3$

3) $conc.H_2SO_4, Al_2S_3$ 4) $conc.HCl, AlCl_3$

41. The major product of the following reaction is

$$CH_3CH_2CH=CH_2 \xrightarrow[\text{Rh catalyst}]{H_2/CO}$$

1) $CH_3CH_2CH_2CHO$

2) $CH_3CH_2CH_2CH_2CHO$

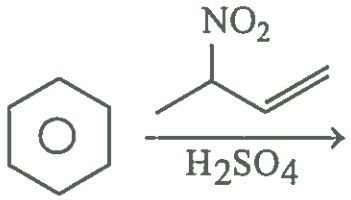
3) $CH_3CH_2\underset{\underset{CHO}{|}}{C}=CH_2$

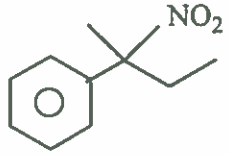
4) $CH_3CH_2CH=CH-CHO$

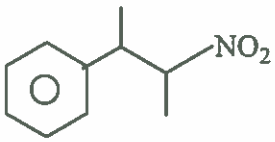
42. If an organic compound contains both nitrogen and sulphur, its fusion with sodium converts these elements into:

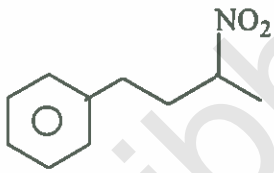
1) Na_2S and Na_4C
2) NaSCN
3) Na_2SO_3 and NaCN
4) Na_2S and NaCNO

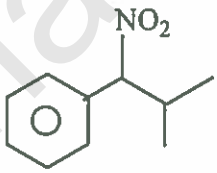
43. The major product of the following reaction is



1) 

2) 

3) 

4) 

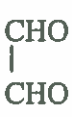
44. Match the following.

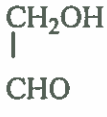
List-I	List-II
A) XeF_4	1) Distorted octahedral
B) XeF_6	2) Tetrahedral
C) XeO_3	3) Squar planar
D) XeO_4	4) Pyramidal

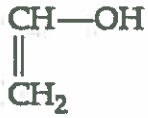
1) A-1, B-2, C-3, 4-D
2) A-3, B-1, C-4, D-2
3) A-1, B-3, C-2, D-4
4) A-2, B-4, C-1, D-3

45. What is X in the given reaction?

1) 

2) 

3) 

4) 

46. An industrial preparation method for the preparation of methanol is:

1) By reacting CH_4 with steam at 900^0C with a nickel catalyst
2) By reduction of HCHO with $LiAlH_4$
3) By catalytical reduction of CO in presence of $ZnO\text{-}Cr_2O_3$.
4) By reaction of HCHO with $NaOH_{(Aq)}$.

47. The major components of German silver are:

1) Cu, Zn and Ni 2) Ge, Cu and Ag 3) Zn, Ni and Ag 4) Cu, Zn and Ag

48. A cyanohydrin of a compound (X) on hydrolysis gives an α-hydroxyacid which show optical activity after resolution. The compound (X) is:

1) Acetone 2) Formaldehyde

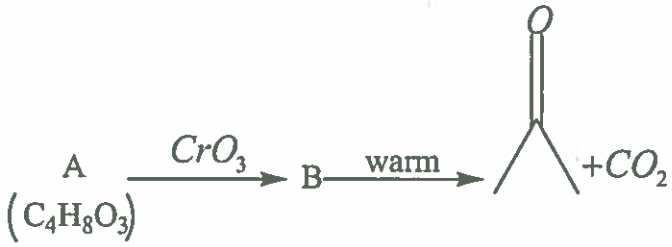
3) Diethyl ketone 4) Acetaldehyde

49. The solubility of $Ca(OH)_2$ in water is:

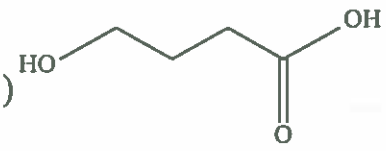
[Given: The solubility product of $Ca(OH)_2$ in water = 5.5×10^{-6}]

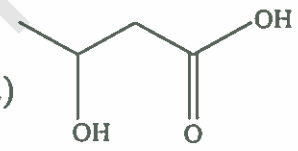
1) 1.11×10^{-2} 2) 1.77×10^{-2} 3) 1.77×10^{-6} 4) 1.11×10^{-6}

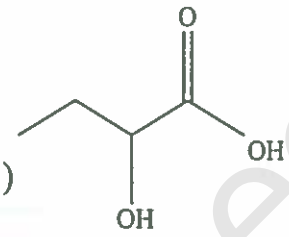
50. Consider the following sequence of reaction?

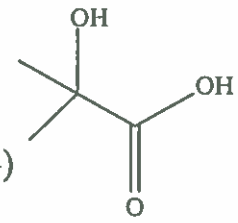


The compound A is

1) 

2) 

3) 

4) 

(NUMERICAL VALUE TYPE)

Section-II contains 10 Numerical Value Type questions. Attempt any 5 questions only. First 5 attempted questions will be considered if more than 5 questions attempted.The Answer should be within 0 to 9999. If the Answer is in Decimal then round off to the nearest Integer value (Example i,e. If answer is above 10 and less than 10.5 round off is 10 andIf answer is from 10.5 and less than 11 round off is 11).
Marking scheme: +4 for correct answer, 0 if not attempt and -1 in all other cases.

51. If a compound AB dissociates to the extent of 75% in an aqueous solution, the molality of the solution which shows a 2.5K rise in the boiling point of the solution is ___ molal. (rounded of to the nearest integer) ($K_b = 0.52\,K\,kg\,mol^{-1}$)

52. How many number of waves are made by an electron in one complete revolution moving in an orbit having maximum magnetic quantum number m = +3?

53. Copper reduces NO_3^- into NO and NO_2 depending upon the concentration of HNO_3 in solution [Assuming fixed $[Cu^{2+}]$ and $P_{NO} = P_{NO_2}$], the HNO_3 concentration at which the thermodynamic tendency for reduction of NO_3^- into NO and NO_2 by copper is same is 10^x M. The value of 2x is ____ (Rounded off to the nearest integer)

[Given $E^\circ_{Cu^{2+}/Cu} = 0.34V$, $E^\circ_{NO_3^-/NO} = 0.96V$, $E^\circ_{NO_3^-/NO_2} = 0.79V$ and

at 298K $\frac{RT}{F}(2.303) = 0.059$]

54. A hydrogenation reaction is carried out at 500 K. If the same reaction is carried out in the presence of a catalyst at the same rate, the temperature required is 400 K. The activation energy of uncatalyzed reaction (in kJ/mol), if the catalyst lowers the activation energy by 20 kJ/mol, is ___

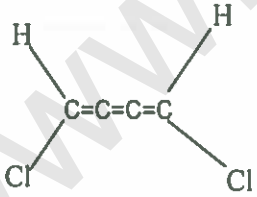
55. The rate constant of a reaction increases by five times on increase in temperature from 27°C to 52°C. The value of activation energy in KJ mol^{-1} is ___ (Rounded off to the nearest integer) [$R = 8.314 Jk^{-1} mol^{-1}$]

56. A sample of butane gas C_4H_{10} of unknown mass is contained in a vessel of unknown volume V at 27^0C and a pressure of 760 mm-Hg. To this vessel, 8.75 g of neon gas is added in such a way that no butane is lost from the vessel. The final pressure in the vessel is 1900 mm-Hg at the same temperature. Calculate the volume of the vessel (in Litre) (Atomic weight of Ne = 20, R = 0.08 Lit.atm/mol-K) (Report your answer in nearest integer)

57. The unit cell of copper corresponds to a face centered cube of edge length 3.596Å with one copper atom at each lattice point. The calculated density of copper in kg/m^3 is ___

[Molar mass of Cu =63.54g: Avogadro number = 6.022×10^{23}]

58. Number of optical isomers possible for



59. The spin only magnetic moment of a divalent ion in aqueous solution (atomic number 29) is ____ BM

60. Double bond equivalence (or) DUS of Guanine?

MATHEMATICS

Max Marks: 100

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

61. If the curve $x^2+2y^2=2$ intersects the line x + y = 1 at two points P and Q, then the angle subtended by the line segment PQ at the origin is

1) $\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{4}\right)$ 2) $\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{3}\right)$

3) $\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{4}\right)$ 4) $\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{3}\right)$

62. The number of rational terms in the expansion of $\left(5^{1/6}+7^{1/9}\right)^{1824}$ is

1) 101 2) 102 3) 103 4) 104

63. Let A be a set of all 4 –digit natural numbers whose exactly one digit is 7. Then the probability that a randomly chosen element of A leaves remainder 2 when divided by 5 is

1) $\frac{97}{297}$ 2) $\frac{1}{5}$ 3) $\frac{2}{9}$ 4) $\frac{122}{297}$

64. If f(x) is a quadratic expression such that $f(-3)=f(3)=0$ and $f(1)=3$ then $\lim\limits_{x\to 0}\frac{\sqrt[3]{f(x)}-\frac{3}{2}}{\ln\cos x}$

1) $\frac{1}{9}$ 2) $\frac{2}{3}$ 3) $\frac{2}{9}$ 4) $\frac{4}{9}$

65. Let A be a 3×3 matrix with det(A) = 4. Let R_i denote the i^{th} row of A. If a matrix B is obtained by performing the operation $R_2\to 2R_2+5R_3$ on 2A, then det(B) is equal to

1) 64 2) 16 3) 128 4) 80

66. The point (1, 2) is one extremity of focal chord of parabola $y^2=4x$. The length of this focal chord is

1) 2 2) 4 3) 6 4) 8

67. A function f(x) is given by $f(x)=\frac{5^x}{5^x+5}$, then the sum of the series $f\left(\frac{1}{20}\right)+f\left(\frac{2}{20}\right)+f\left(\frac{3}{20}\right)+.....+f\left(\frac{39}{20}\right)$ is equal to

1) $\frac{29}{2}$ 2) $\frac{49}{2}$ 3) $\frac{19}{2}$ 4) $\frac{39}{2}$

68. There are 12 pairs of shoes in a box. Then the possible number of ways of picking 7 shoes so that there are exactly two pairs of shoes are

1) 63360 2) 63300 3) 63260 4) 63060

69. A plane passes through the points A (1, 2, 3) B (2, 3, 1) and C(2, 4, 2). If O is the origin and P is (2, -1, 1), then the projection of $\overrightarrow{OP}$ on this plane is of length

1) $\sqrt{\frac{2}{3}}$ 2) $\sqrt{\frac{2}{7}}$ 3) $\sqrt{\frac{2}{11}}$ 4) $\sqrt{\frac{2}{5}}$

70. If $\sum_{i=1}^{18}(x_i-8)=9$ and $\sum_{i=1}^{18}(x_i-8)^2=45$ then the standard deviation of $x_1, x_2 \ldots\ldots x_{18}$ is

1) $\frac{4}{9}$ 2) $\frac{9}{4}$ 3) $\frac{3}{2}$ 4) $\frac{2}{3}$

71. If $\alpha, \beta \in R$ are such that $1-2i$(here $i^2=-1$) is a root of $z^2+\alpha z+\beta=0$, then $(\alpha-\beta)$ is equal to

1) 3 2) 7 3) -3 4) -7

72. If $f(x) = sin^{-1}\left(\frac{\sqrt{x}-\sqrt{x-1}}{\sqrt{x(x+1)}}\right)$ then $f'(x) =$

1) $\frac{1}{2}\left(\frac{1}{\sqrt{x}(x+1)}-\frac{1}{x\sqrt{x-1}}\right)$ 2) $\frac{1}{2}\left(\frac{1}{x\sqrt{x+1}}-\frac{1}{\sqrt{x}(x-1)}\right)$

3) $\frac{1}{2}\left(\frac{1}{x\sqrt{x+1}}+\frac{1}{\sqrt{x}(x-1)}\right)$ 4) $\frac{1}{2}\left(\frac{1}{x\sqrt{x+1}}+\frac{1}{\sqrt{x}(x-1)}\right)$

73. The minimum value of $f(x)=a^{a^x}+a^{1-a^x}$, where $a, x \in R$ and $a>0$, is equal to

1) 2a 2) $a+\frac{1}{a}$ 3) $2\sqrt{a}$ 4) $a+1$

74. Area bounded by the curves $y=e^x, y=\log_e x$ and the lines $x=0, y=0, y=1$ is

1) e^2+2 sq.units 2) $e+1$ sq.units 3) $e+2$ sq.units 4) $e-1$ sq.units

75. If $0<x, y<\pi$ and $\cos x+\cos y-\cos(x+y)=\frac{3}{2}$, then sinx + cos y is equal to

1) $\frac{\sqrt{3}}{2}$ 2) $\frac{1-\sqrt{3}}{2}$ 3) $\frac{1+\sqrt{3}}{2}$ 4) $\frac{1}{2}$

76. Let $f: R \to R$ and $g: R \to R$ be twice differentiable function satisfying $f''(x)=g''(x)$, $2f'(1)=g'(1)=4$ and $3f(2)=g(2)=9$. The value of $f(4)-g(4)$ is equal to

1) -6 2) -16 3) -10 4) -8

77. The contrapositive of the statement 'If you will work, you will earn money' is

1) To earn money, you need to work

2) If you will earn money, you will work

3) You will earn money, if you will not work

4) If you will not earn money, you will not work

78. The equation $x^7+3x^3+4x-9=0$ has

1) No real root
2) 7 real roots
3) a unique rational root
4) a unique irrational root

79. $\operatorname{cosec}\left[2\cot^{-1}(5)+\cos^{-1}\left(\frac{4}{5}\right)\right]$ is equal to

1) $\frac{56}{33}$ 2) $\frac{65}{56}$ 3) $\frac{75}{56}$ 4) $\frac{65}{33}$

80. $\sim(p\vee q)\vee(\sim p\wedge q)$ is logically equivalent to

1) $\sim p$ 2) p 3) q 4) $\sim q$

(NUMERICAL VALUE TYPE)

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Marking scheme: +4 for correct answer, 0 if not attempt and -1 in all other cases.

81. If the remainder when x is divided by 4 is 3, then the remainder when $(2020+x)^{2022}$ is divided by 8 is _____

82. If $\alpha,\beta,\gamma\in(0,\pi/4)$, such that $\frac{1}{2}(1-\tan\alpha)(1-\tan\beta)(1-\tan\gamma)=1-(\tan\alpha+\tan\beta+\tan\gamma)$. Then the value of $\tan(\alpha+\beta+\gamma)$ is

83. A function f is defined on $[-3,3]$ as $f(x)=\begin{cases}\min\{|x|,2-x^2\}, & -2\le x\le 2\\ [|x|], & 2<|x|\le 3\end{cases}$ where $[x]$ denotes the greatest integer $\le x$. The number of points, where f is not differentiable in (-3, 3) is __

84. The number of solutions of $\cos^2\theta+\sin\theta+1=0$, for $\theta\in[0,2\pi]$ is ___

85. If the curve, $y=y(x)$ represented by the solution of the differential equation $(2xy^2-y)dx+xdy=0$, passes through the intersection of the lines $2x-3y=1$ and $3x+2y=8$ then $|y(1)|$ is equal to ___

86. If the shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$ is d. Then the value of $54d^2$ is _____

87. A line 'ℓ' passing through origin is perpendicular to the lines
$\ell_1 : \vec{r} = (3+t)\hat{i} + (-1+2t)\hat{j} + (4+2t)\hat{k}$; $\ell_2 : \vec{r} = (3+2s)\hat{i} + (3+2s)\hat{j} + (2+s)\hat{k}$
If the coordinates of the point in the first octant on 'ℓ_2' at a distance of $\sqrt{17}$ from the point of intersection of 'ℓ' and 'ℓ_1' are (a, b, c) then $18(a+b+c)$ is equal to ___

88. The least integral value of 'a' such that $(a-3)x^2 + 12x + (a+6) > 0 \forall x \in R$ is ___

89. The total number of two digit numbers 'n', such that $3^n + 7^n$ is a multiple of 10, is ___

90. If A is non zero square matrix of order n such that $A^2 = A$ and $(I - 0.4A)^{-1} = I - \alpha A$ where I is the identity matrix of same order as that of A, then value of $\lim_{n\to\infty} \sum_{r=0}^{n} (|\alpha|)^r$ is _____(Here $|x|$ represents modulus function)

*****

KEY SHEET

PHYSICS

1	**1**	2	**2**	3	**3**	4	**2**	5	**4**
6	**3**	7	**1**	8	**1**	9	**2**	10	**3**
11	**1**	12	**1**	13	**3**	14	**1**	15	**2**
16	**3**	17	**2**	18	**4**	19	**2**	20	**1**
21	**10**	22	**11**	23	**20**	24	**1**	25	**2**
26	**1**	27	**36**	28	**168**	29	**1**	30	**6**

CHEMISTRY

31	**2**	32	**1**	33	**2**	34	**4**	35	**4**
36	**1**	37	**3**	38	**2**	39	**3**	40	**2**
41	**2**	42	**2**	43	**2**	44	**2**	45	**1**
46	**3**	47	**1**	48	**4**	49	**1**	50	**2**
51	**3**	52	**4**	53	**4**	54	**100**	55	**52**
56	**7**	57	**9076**	58	**0**	59	**2**	60	**6**

MATHEMATICS

61	**3**	62	**2**	63	**1**	64	**1**	65	**1**
66	**2**	67	**4**	68	**1**	69	**3**	70	**3**
71	**4**	72	**1**	73	**3**	74	**4**	75	**3**
76	**3**	77	**4**	78	**4**	79	**2**	80	**1**
81	**1**	82	**1**	83	**5**	84	**1**	85	**1**
86	**9**	87	**44**	88	**7**	89	**45**	90	**3**

HINTS & SOLUTIONS

PHYSICS

1. Wave transmitted at carrier frequency

$$\lambda = \frac{C}{\text{frequencies of carrier wave}}, \lambda = \frac{C}{fc}$$

2. $$\Rightarrow f = \frac{V}{V - V_{source}} f_0 = \frac{330}{330-30} \times 300 = \frac{330}{300} \times 300$$

$= 330Hz$, Now, wall will act as source with frequency f' and car will act as an observer.

$$\Rightarrow f'' = \frac{V + V_0}{V} f' = \frac{330+30}{330} f' = \frac{360}{330} \times 330 = 360Hz$$

3. $$v^2 - u^2 = 2as, o - u_0^2 = \frac{2g\sin\theta \times s}{1 + \frac{2}{5}}, S = \frac{7v_0^2}{10g\sin\theta}$$

4. The velocity of the particle, $V = ky\hat{i} + kx\hat{j}$

$$\Rightarrow \frac{dx}{dt} = ky \frac{dy}{dt} = kx \; \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{kx}{ky} \; ydy = xdx \Rightarrow y^2 = x^2 + c$$

This is the required general equation

5. In a diatomic molecule even rotational energy at given temperature obeys Maxwell's distribution.

In a diatomic molecule rotational energy $= \frac{2}{2}RT$

Translational energy $= \frac{3}{2}RT$; $E_R \neq E_T$

6. $$[\eta] = \left[M^1L^{-1}T^{-1}\right] \quad n_1u_1 = n_2u_2 \Rightarrow n_2 = \frac{n_1u_1}{u_2} = n_1\left[\frac{M_1}{M_2}\right]^1\left[\frac{L_1}{L_2}\right]^{-1}\left[\frac{T_1}{T_2}\right]^{-1}$$

So, new system unit is $= \left[\frac{1g}{\alpha \times 10^3 g}\right]^1\left[\frac{1cm}{100\beta cm}\right]^{-1}\left[\frac{1s}{\gamma s}\right]^{-1}$ or $= 0.1\alpha^{-1}\beta\gamma$

7. When doping level is increased Fermi level moves down in p- type and moves up in n - type

8. From the Bohr's model, we have the relations for energy and radius as

$E_n = -\frac{me^4}{8\varepsilon_0^2 n^2 h^2}$ and $r_n = \frac{\varepsilon_0 n^2 h^2}{\pi m e^2}$ Hence, $E \propto m$ and $r_n \propto \frac{1}{m}$

So, it mass is doubled then energy will get doubled and r will reduce to half.

$\therefore E_0 = -13.6 \times 2 = -27.2eV$ and $r_0 = \frac{a_0}{2}$

9. When P touches M, $F_{max} = m\omega^2 A = 1 \times \left(\frac{\pi}{0.6}\right)^2 \times 0.36 \Rightarrow F_{max} = 9.87N$

10. Here, initial charge on capacitor is $2Q_0$, battery is attached but with opposite polarity also potential difference is $\frac{Q_0}{C}$. So, total charge through battery $Q_0-(-2Q_0)=3Q_0$ and workdone by battery = (charge supplied) × (potential)

Work done by battery $=\frac{(3Q_0)Q_0}{C}=\frac{3Q_0^2}{C}$

11. Time of release of 2nd stone $t=\sqrt{\frac{2\times5}{10}}=1\therefore\frac{1}{2}gt^2=\frac{1}{2}g(t-1)^2+25,\ 5t^2=5(t^2+1-2t)+25$

$10t=30,\ t=3\text{ sec},\ h=\frac{1}{2}gt^2=\frac{1}{2}\times10\times9=45m$

12. Since, h<<radius of earth

So, the distance upto which transmission could be viewed is $d=\sqrt{2hR}$

$\therefore$Area covered$=\pi d^2=\pi\times2hR=2\pi Rh=500\times10^{-6}\times2\times3.14\times6.4\times10^6\times150$

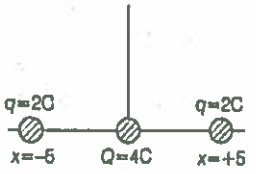
$=30.144$ lakh person

13. Energy of photon $E=\frac{1240}{\lambda(nm)}eV, E=\frac{1240}{491}eV, E=2.53eV, W_0=E-V_0e\ =2.53-0.710\ =1.82eV$

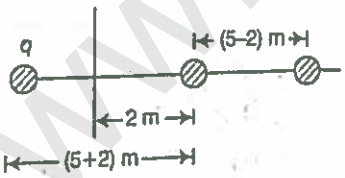
$E=\frac{hc}{\lambda}-\omega_o, V_oe=\frac{hc}{\lambda}-\omega_o, 0.710+\omega_o=\frac{hc}{\lambda}$

$0.710+1.82=\frac{hc}{491}, 1.43+1.82=\frac{hc}{\lambda}, \frac{2.53}{3.25}=\frac{\lambda}{491}, \lambda=382nm$

14. $U_i=\frac{2qQ}{4\pi\varepsilon_0 a}=2.87\times10^{10}\text{J}$



According to given condition when charge Q is displaced by a distance of 2m, then its potential energy will be



$U_f=\frac{1}{4\pi\varepsilon_0}qQ\left[\frac{1}{(5+2)}+\frac{1}{(5-2)}\right]=\frac{2qQ}{4\pi\varepsilon_0}\left[\frac{1}{7}+\frac{1}{3}\right]=6.77\times10^{10}\text{J}$

So, $\Delta U=U_f-U_i=6.77\times10^{10}-2.87\times10^{10}=3.9\times10^{10}\text{J}$

15. $u_1 \cos\alpha = u_2 \cos\beta$

$$\frac{k_1}{k_2} = \frac{\frac{1}{2}mu_1^2}{\frac{1}{2}mu_2^2} = \left(\frac{u_1}{u_2}\right)^2 = \frac{\cos^2\beta}{\cos^2\alpha}$$

16. We can apply mechanical energy conservation

$$(T.E)_i = (T.E)_f \Rightarrow -\frac{GM_e m}{R_e} + \frac{1}{2}m\left(3\times\sqrt{\frac{GM_e}{R_e}}\right)^2 = (PE)_\infty + \frac{1}{2}mv'^2$$

$$\Rightarrow \frac{7}{2}\frac{GM_e}{R_e} = \frac{1}{2}V'^2 \Rightarrow V' = \sqrt{\frac{7GM_e}{R_e}} = \sqrt{7}V_e$$

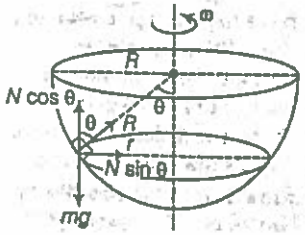
17. $\frac{1}{4\pi\epsilon_o}\frac{e^2}{hc} = \frac{Fr^2}{Er} = M^oL^oT^o$

18. As, $KE = \frac{3}{2}kT$ and $V_{rms} = \sqrt{\frac{3RT}{M}} \therefore KE_2 = 2KE_1 = 2\times 6.21\times 10^{-21} = 12.42\times 10^{-21} J$

$$\frac{V_{rms,2}}{V_{rms,1}} = \sqrt{\frac{T_2}{T_1}} = \sqrt{2} \therefore V_{rms,2} = \sqrt{2}\times V_{rms,1} = 684 m/s$$

19. $PV^{1/2} = const, PV = RT, P = \frac{RT}{V}, \frac{RT}{V}V^{1/2} = const$

$$T \propto V^{1/2}, \frac{T_2}{T_1} = \left(\frac{2V_1}{V}\right)^{1/2} = \sqrt{2}$$

20. 

$mr\omega^2 = N\sin\theta\ N\cos\theta = mg$ _____(ii) Divide Eq.(i) by Eq.(ii), we get

$\tan\theta = \frac{r\omega^2}{g}$ _____(iii) Also, $r = R\sin\theta$ _____(iv). From Eqs. (iii) and (iv), we get

$$\tan\theta = R\sin\theta\frac{\omega^2}{g} \Rightarrow \cos\theta = \frac{g}{R\omega^2}$$

21. Energy of $x-$rays $= \frac{hc}{\lambda} = \frac{h\times 3\times 10^8}{10\times 10^{-10}} = 3h\times 10^{17} J$

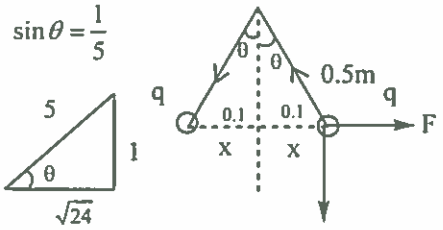
$mc^2 = 3h\times 10^{17}, m\times 9\times 10^{16} = 3h\times 10^{17}, m = \frac{10}{3}h = \frac{xh}{3}, x = 10$

22. Bulk modulus, $K = \frac{-\Delta\rho}{\Delta V/V} \Rightarrow \Delta V = -\frac{\Delta\rho V}{K} = -\frac{2\times 10^8\times V}{10^{10}} = \frac{-V}{50}$

$V' = V - \Delta V = \frac{49V}{50}$. As mass of metal will remains constant, so, $\rho V = \rho' V'$

$$\rho' = \frac{\rho V}{V'} = \frac{11 \times V \times 50}{49 \times V} = 11.22 g/cm^3$$

23. $\sin\theta = \frac{1}{5}$



$$F = mg\tan\theta,\ 9\times10^9 \times \frac{q^2}{0.04} = 10\times10^{-3}\times10^{-3}\times\frac{1}{\sqrt{24}};\ q = \frac{20}{21}\times10^{-8} c\ = 20$$

24. $Y_n = n\beta\, 3 = 10\beta \Rightarrow \beta = \frac{3}{10}\ \beta' = \frac{\beta}{\mu} = \frac{3}{10}\times\frac{2}{3}\ \beta' = \frac{1}{5}$

Now $Y_n^1 = n^1\beta^1$; $Y_n^1 = 5\times\frac{1}{5} = 1$ cm

25. $I = \frac{P}{4\pi r^2}$, $uc = \frac{P}{4\pi r^2}$, $\frac{1}{2}\in_o E_o^2 C = \frac{P}{4\pi r^2}$

$$E_o = \sqrt{\frac{2P}{4\pi r^2 . \in_o \times c}}\ ,\ E_o = \sqrt{\frac{2\times0.8}{4\pi\times100\times\in_o}}$$

26. Magnetic field induction at the centre of a long solenoid is $B = \mu_0 ni$ or $B \propto ni$ (as μ_0 is constant) where, n is the number of turns per unit length of solenoid.

$$\therefore \frac{B_1}{B_2} = \frac{n_1}{n_2}\times\frac{i_1}{i_2} = \frac{200}{100}\frac{i}{(i/3)} = 6\ B_2 = \frac{B_1}{6} = \frac{6.28\times10^{-2}}{6} = 1.05\times10^{-2}\ Wb/m^2$$

27. $q_1 + q_2 = 2nc$, $q_1 = q_2 = 1nc$,

$$F = \frac{1}{4\pi\in_o}\frac{q_1 q_2}{r^2},\ F = 9\times10^9 \times \frac{10^{-18}}{0.25} = 36\times10^{-9} N = 36$$

28. Let the frequencies be $n, n-3, n-6 .....n-35\times3$

Composing give condition, we get, $n - 35\times3 = \frac{n}{2}\ \therefore \frac{n}{2} = 105, n = 210$

$\therefore$ frequency of 15 tuning fork is $(n - 14\times3) = 210 - 42 = 168$

29. $k = \frac{p^2}{2m}, P \propto \sqrt{m}, \frac{P_1}{P_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{4}{16}} = \frac{1}{2},\ n = 1$

30. Newton's law of cooling will be used

Rate of cooling $\frac{d\theta}{dt} = K(\theta_B = \theta_S)\ \frac{61-59}{4} = K\left(\frac{61+59}{2} - 30\right)$ _____(1)

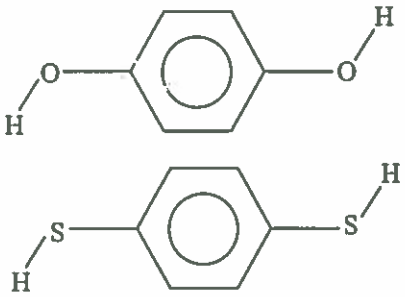
$\frac{51-49}{t} = K\left(\frac{51+49}{2} - 30\right)$ _____(2). Solve (1) & (2), t = 6 min

31. Their viscosity is of the order of that of H_2O is false statement

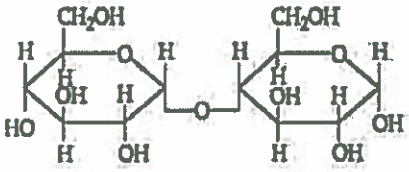
32. $Z_{B^{m-}} = 8\times\frac{1}{8}+6\times\frac{1}{2}=4$

$Z_{A^{n+}} = \frac{1}{4}\times 8+\frac{1}{4}\times 4=3 \therefore$ Formula $= A_3B_4$

33. HNO_3 is added to sodium fusion extraction before addition of $AgNO_3$ for testing halogens becomes HNO_3 oxidize the any impurities like N, S, present, they can be oxidised

34. 

In both the molecules the bond moments are not canceling with each other and hence the molecules has a resultant dipole and hence the molecule is polar.

35. 

36. H_3BO_3 is weak monobasic Lewis acid.

37. Bond dissociation energy of F_2 is less than that of Cl_2 and Br_2

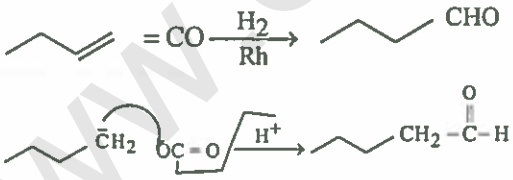
Correct decreasing order is : $Cl_2 > Br_2 > F_2 > I_2$

38. Combination of acetylene and phosphine.

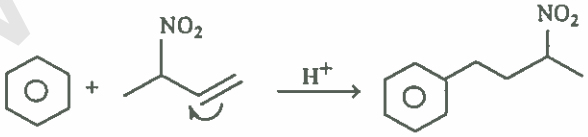
39. α – sulphur $\rightleftharpoons$ β-sulphur (inter converatble)

Ortho rhombic is most stable form

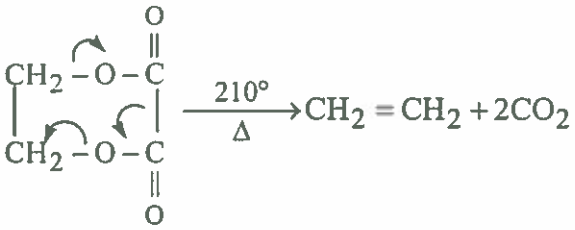
40. Passivity is due to protective oxide film.

41. 

42. $Na + C + S + N \rightarrow NaSCN$

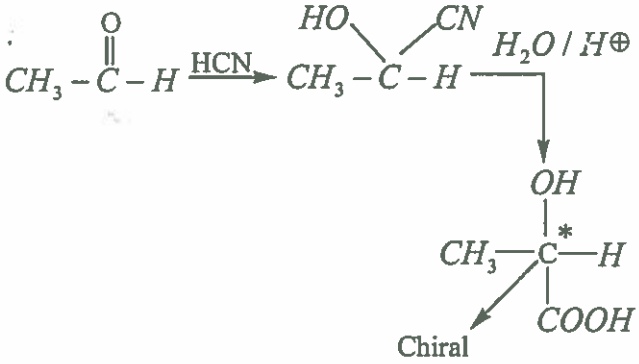
43. 

44. Xenon compounds and hybridization of Xenon.

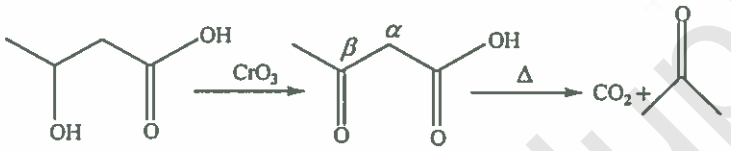
45. 

46. $CO + H_2 \xrightarrow[\Delta]{ZnO-Cr_2O_3} CH_3OH$

47. German silver: Contains Cu, Zn, Ni

48. 

49. $K_{sp} = 4s^3 = 5.5\times10^{-6} \Rightarrow s^3 = \frac{5.5}{4}\times10^{-6} \Rightarrow s = 1.11\times10^{-2}$

50. 

51. $\Delta T_f = i \times k_f \times m$

$\alpha = \frac{i-1}{n-1}, 0.75 = \frac{i-1}{2-1}, i = 1.75,\ 2.5 = 1.75\times0.52\times m,\ m = 2.74$

52. If m = +3 (maximum), then $l = 3$ and $n = 4$

Number of waves in an orbit = n

53. $3Cu + 8H^+ + 2NO_3^- \rightarrow 3Cu^{+2} + 2NO + 4H_2O$

$Cu + 4H^+ + 2NO_3^- \rightarrow Cu^{2+} + 2NO_2 + 2H_2O$

Let con of $HNO_3 = \left[H^+\right] = x\ \left[NO_3^-\right] = x$

$E_{NO_3^-/NO} - E_{Cu^{2+}/Cu} = E_{NO_3^-/NO_2^-} - E_{Cu^{2+}/Cu},\ E_{NO_3^-/NO} = E_{NO_3^-/NO_2^-}$

$0.96 - \frac{0.0591}{3}\log\frac{10^{-3}}{10^5} = 0.79 - \frac{0.059}{3}\log\frac{10^{-3}}{x^3}$

x = 0.66 ; 2x = 1.32 nearst integer is 1

54. $K_{cat} = K_{uncat}$ or, $A.e^{-E_{a_{cat}}/RT_1} = A.e^{-E_{a_{uncat}}/RT_2}$

or, $\frac{E_{a_{cat}}}{T_1} = \frac{E_{uncat}}{T_2} \Rightarrow \frac{E_{a_{uncat}} - 20}{400} = \frac{E_{a_{uncat}}}{500}$

$\therefore E_{a_{uncat}} = 100\,\text{kJ/mol}$

55. $\log\frac{5}{1} = \frac{E_a}{2.303\times8.314}\left[\frac{25}{300\times325}\right]$

$E_a = \frac{0.7\times2.303\times8.314\times300\times325}{25\times100}$; $E_a = 52.27\text{KJ}$

56. $P_{Ne} = 1900 - 760 = 1140\,\text{mm of Hg}$

$PV = nRT$

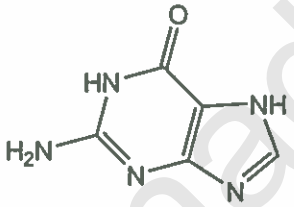
$\frac{1140}{760}\times V = \frac{8.75}{20}\times0.08\times300$; $V = 7\text{ L}$

57. $d = \frac{Z\times M}{a^3\times Na} = \frac{4\times63.54\times10^{-3}}{(3.596)^3\times6.02\times10^{23}\times10^{-30}} = \frac{4\times63.54\times10^{+4}}{(3.596)^3\times6.02} = 9079.28$

58. Odd number of C=C double bonded cumulene is a planar compound. It gives geometrical isomerism.

59. $Cu^{2+} = 3d^9 =$ 

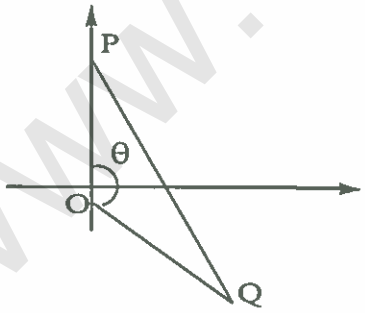
$M.M = \sqrt{n(n+2)} = \sqrt{3} = 1.73$

60. Guanine $\Longrightarrow$ 

MATHEMATICS

61. $x^2 + 2y^2 = 2 \Rightarrow \frac{x^2}{2} + \frac{y^2}{1} = 1$

Line $x + y = 1$; $y = 1\text{-}x$



Solving the equations $x^2 + 2(1-x)^2 = 2$

$x^2 + 2x^2 + 2 - 4x = 2$

$3x^2-4x=0;\ x=0,\ 3x-4=0\ \ x=\frac{4}{3}$

$P(0,1)\ ,Q\left(\frac{4}{3},\frac{-1}{3}\right);\ \ \theta=\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{4}\right)$

62. $T_{r+1}={}^{1824}C_r\ 5^{\frac{1824-r}{6}}\ 7^{\frac{r}{9}}\Rightarrow$ for rational terms $1824-r=6\cdot I$ & $r=9\cdot I$

$r=18\cdot I\Rightarrow$ there are $\left[\frac{1824}{18}\right]+1=102$ rational terms

63. $\boxed{7}\ \_\ \_\ \_ = 1\times9\times9\times9$

$\square\boxed{7}\ \_\ \_ = 8\times{}^{3}C_1\times9\times9$

$n(s)=9\times9\times9+24\times9\times9=81\times(33)=2673$

$7\_\ \_\ \_^{2}=1\times9\times9\times1=81$

$\square\_\_7=8\times9\times9=648$

$\square\_\_\_^{7\ 2}=8\times{}^{2}C_1\times9=144$

n(A)=81+648+144=873

$P(A)=\frac{873}{2673}=\frac{97}{297}$

64. $f(x)=-\frac{3}{8}(x-3)(x+3)$

65. $(A)_{3\times3};\ |A|=4$

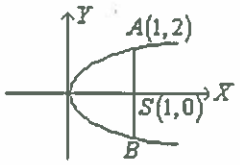
$A=\begin{pmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{pmatrix}\ ;\ 2A=\begin{pmatrix}2a_1 & 2a_2 & 2a_3\\ 2b_1 & 2b_2 & 2b_3\\ 2c_1 & 2c_2 & 2c_3\end{pmatrix}$

$R_2\rightarrow2R_2+5R_3$

$2A=\begin{pmatrix}2a_1 & 2a_2 & 2a_3\\ 4b_1+10c_1 & 4b_2+10c_2 & 4b_3+10c_3\\ 2c_1 & 2c_2 & 2c_3\end{pmatrix}$

$2A=\begin{pmatrix}2a_1 & 2a_2 & 2a_3\\ 4b_1 & 4b_2 & 4b_3\\ 2c_1 & 2c_2 & 2c_3\end{pmatrix}+\begin{pmatrix}2a_1 & 2a_2 & 2a_3\\ 10c_1 & 10c_2 & 10c_3\\ 2c_1 & 2c_2 & 2c_3\end{pmatrix}$

$|B|=|2A|=16|A|\Rightarrow16\times4=64$

66. 

The parabola $y^2 = 4x$, here a = 1 and focus is (1, 0)

The focal chord is ASB. This is clearly latus rectum of parabola, its value = 4

67. $f(x)=\frac{5^x}{5^x+5}; f(\frac{1}{20})+f\left(\frac{2}{20}\right)+f\left(\frac{3}{20}\right)+.....+f\left(\frac{39}{20}\right)=?$

$$f(x)+f(2-x)=\frac{5^x}{5^x+5}+\frac{5^{2-x}}{5^{2-x}+5}=\frac{5^x}{5^x+5}+\frac{\frac{25}{5x}}{\frac{25+5.5x}{5^x}}=\frac{5^x}{5^x+5}+\frac{5}{5^x+5}=1$$

$$f(\frac{1}{20})+f\left(\frac{2}{20}\right)+f\left(\frac{3}{20}\right)+.....+f\left(\frac{39}{20}\right)=19\times1+\frac{1}{2}=\frac{39}{2}$$

68. Total number of ways of picking up 7 shoes with 2 pairs is $12_{C_2}\times10_{C_3}\ 2^3$

69. $A(1,2,3),B(2,3,1),C(2,4,2)$ Eq of plane is $\begin{vmatrix} x-1 & y-2 & z-3 \\ 1 & 1 & -2 \\ 1 & 2 & -1 \end{vmatrix}=0$

$(x-1)(3)-(y-2)(1)+(z-3)(1)=0 \Rightarrow 3x-y+z-4=0$

Projection of O (0,0,0) on the plane is (h,k,l)

$$\frac{h-0}{3}=\frac{k-0}{-1}=\frac{1-0}{1}=\frac{-(-4)}{9+1+1}=\frac{4}{11}\Rightarrow O'(h,k,l)=\left(\frac{12}{11},\frac{-4}{11},\frac{4}{11}\right)$$

Projection of P (2,-1,1) on the plane is (h,k,l)

$$\frac{h-2}{3}=\frac{k+1}{-1}=\frac{1-1}{1}=-\frac{(6+1+1-4)}{9+1+1}=\frac{-4}{11}$$

$$h-2=\frac{-12}{11},k+1=\frac{4}{11},1-1=\frac{-4}{11}$$

$$h=2-\frac{12}{11},k=\frac{4}{11}-1,1=1-\frac{4}{11}$$

$$h=\frac{10}{11},k=\frac{-7}{11},1=\frac{7}{11}=P'=\left(\frac{10}{11},\frac{-7}{11},\frac{7}{11}\right)$$

$$O'P'=\sqrt{\frac{4}{121}+\frac{9}{121}+\frac{9}{121}}=\sqrt{\frac{22}{121}}=\sqrt{\frac{2}{11}}$$

70. Let $d_i=x_i-8$ but $\sigma_x^2=\sigma_d^2=\frac{1}{18}\Sigma d_i^2-\left(\frac{1}{18}\Sigma di\right)^2=\frac{1}{18}\times45-\left(\frac{9}{18}\right)^2=\frac{5}{2}-\frac{1}{4}=\frac{9}{4}.\ \therefore\sigma_x=\frac{3}{2}.$

71. $z^2+\alpha z+\beta=0$ has one root $1-2i$ other root $1+2i$

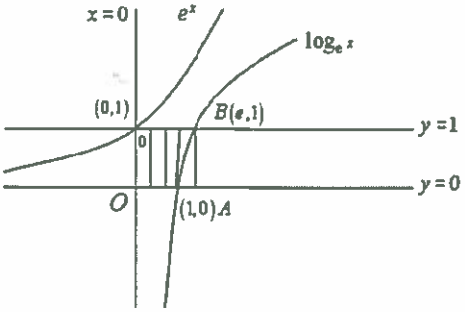
$\alpha=-2;\beta=5\Rightarrow\alpha-\beta=-2-5=-7$

72. $\tan^{-1}\left(\frac{\sqrt{x}-\sqrt{x-1}}{1+\sqrt{x}\sqrt{x-1}}\right)=\tan^{-1}\sqrt{x}-\tan^{-1}\sqrt{x-1}$

73. $f(x)=a^{a^x}+\frac{a}{a^{a^x}}$ is $(a>0)$

$$AM \geq GM \Rightarrow \frac{a^{a^x}+\frac{a}{a^{a^x}}}{2} \geq \sqrt{a^{a^x}.\frac{a}{a^{a^x}}} = 2\sqrt{a}$$

74.



Area = Area of rectangle OABC $-\int_0^1 \log_e x$

75. $0<x,y<\pi, \cos x+\cos y-\cos(x+y)=\frac{3}{2}$; $\sin x+\cos y=?$

$$\cos x+\cos y-\cos(x+y)=\frac{3}{2} \Rightarrow 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}-2\cos^2\frac{x+y}{2}+1=\frac{3}{2}$$

$$\Rightarrow 2\cos\frac{x+y}{2}\left[\cos\frac{x-y}{2}-\cos\frac{x+y}{2}\right]=\frac{1}{2}$$

$$\Rightarrow 2\cos\frac{x+y}{2}2\sin\frac{x}{2}\sin\frac{y}{2}=\frac{1}{2} \Rightarrow \cos\frac{x+y}{2}\sin\frac{x}{2}\sin\frac{y}{2}=\frac{1}{8}$$

$$\Rightarrow \frac{x}{2}=30°, \frac{y}{2}=30°, \frac{x+y}{2}=60° \quad \sin x+\cos y=\frac{\sqrt{3}}{2}+\frac{1}{2}=\frac{\sqrt{3}+1}{2}$$

76. $f'(x)=g(x)-2$; $f(x)=g(x)-2x-2$ $f(u)-g(u)=-10$

77. Contrapositive of "If p then q is $\sim q \rightarrow \sim p$

78. Let $f(x)=x^7+3x^3+4x-9$

$f^1(x)=7x^6+9x^2+4>0 \ \forall x \in R$

$\therefore$ f is strictly increasing

$\therefore f(x)=0$ has unique real root

$f(1).f(2)<0 \Rightarrow$ the real root $\in (1,2)$ if $f(x)=0$ has rational roots, they must be integers. But there are no integers between 1 & 2.

$\therefore$ it has unique irrational root.

79. $\cos ec\left(2\cot^{-1}5+\cos^{-1}\frac{4}{5}\right)=\cos ec(2A+B)$

$\sin(2A+B)=\sin 2A\cos B+\cos 2A\sin B$

$\sin 2A=2\sin A\cos A=\frac{2}{\sqrt{26}}\times\frac{5}{\sqrt{26}}=\frac{10}{26}$

$\cos 2A=1-2\sin^2 A=1-\frac{2}{26}=\frac{24}{26}$

$\text{Sin}(2A+B)=\frac{10}{26}.\frac{4}{15}+\frac{24}{26}.\frac{3}{5}=\frac{112}{130}=\frac{56}{65}$

80.

p	q	$\sim p$	$p\vee q$	$\sim(p\vee q)$	$\sim p\wedge q$	$\sim(p\vee q)\vee(\sim p\wedge q)$
T	T	F	T	F	F	F
T	F	F	T	F	F	F
F	T	T	T	F	T	T
F	F	T	F	T	F	T

$\therefore\sim(p\vee q)\vee(\sim p\wedge q)\equiv\sim p$

81. $x=4k+3, k\in w$ let $x=7,(2020+7)^{2022}=(2027)^{2022}=(2024+3)^{2022}$

$^{2022}C_0.(2024)^{2022}+.....+^{2022}C_{2021}(2024)^1 3^{2021}+^{2022}C_{2022}.3^{2022}$, divided 8 remainder

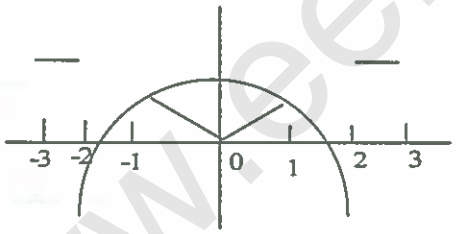
$3^{2022}=(1+8)^{1011}.\text{remainder}=1$

82. Given relation can be written as

$\frac{1-\tan\alpha\tan\beta-\tan\beta\tan\gamma-\tan\gamma\tan\alpha}{\tan\alpha+\tan\beta+\tan\gamma-\tan\alpha\tan\beta\tan\gamma}=1$ therefore, $\cot(\alpha+\beta+\gamma)=1$

$\alpha+\beta+\gamma=\pi/4$

83. $f(x)=\min\begin{cases}|x|, 2-x^2-2\le x\le 2\\ [|x|]; \quad 2<|x|\le 3\end{cases}$



Number of non diff points =5

84. Finding out the number of solutions for a given equation

85. $(2xy^2-y)dx+xdy=0$

$2xy^2dx=ydx-xdy \qquad 2xdx=\frac{ydx-xdy}{y^2}$

$x^2=\frac{x}{y}+C,(2,1),4=2+C\Rightarrow C=2, X^2=\frac{X}{Y}+2\Rightarrow 1=\frac{1}{Y}+2\Rightarrow Y=-1\Rightarrow y=-1, |y|=1$

86. Shortest distance between 2 lines is $\left(\bar{c}-\bar{a}\right).\dfrac{\left(\bar{b}\times\bar{d}\right)}{\left|\bar{b}\times\bar{d}\right|}$

Here $\bar{a}=(1,2,3)\quad \bar{b}=(2,3,4)\ \bar{c}=(2,4,5)\quad \bar{d}=(3,4,5)$

Shortest distance $\dfrac{\begin{vmatrix}2-1 & 4-1 & 5-3\\ 2 & 3 & 4\\ 3 & 4 & 5\end{vmatrix}}{\begin{Vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 4\\ 3 & 4 & 5\end{Vmatrix}}=\dfrac{1}{\sqrt{6}}$

87. $\vec{r}=(3\vec{i}-\vec{j}+4\vec{k})+t(\vec{i}+2\vec{j}+2\vec{k})\ ;\ \vec{r}=(3\vec{i}+3\vec{j}+2\vec{k})+s(\vec{i}+2\vec{j}+\vec{k})$

$\vec{b}=\hat{i}+2\hat{j}+2\hat{k},\quad \vec{d}=2\hat{i}+2\hat{j}+\hat{k}$

$\vec{b}\times\vec{d}=\begin{vmatrix}i & j & k\\ 1 & 2 & 2\\ 2 & 2 & 1\end{vmatrix}=i(-2)-j(-3)+k(-2)=-2i+3j-2k$

$\vec{r}=\lambda(-2i+3j-2k),\ 3+t=-2\lambda,\ -1+2t=3\lambda,\ 4+2t=-2\lambda$

$1+t=0\Rightarrow t=-1,\ \lambda=-1$

Point of l and l_1 = (2, -3, 2)

$17=(2s+1)^2+(6+2s)^2+s^2, 17=4s^2+1+4s+36+4s^2+24s+s^2, 17=9s^2+28s+37$

$9s^2+28s+20=0, 9s^2+18s+10s+20=0, 9s(s+2)+10(s+2)=0, s=-2, s=-\dfrac{10}{9}$

$(3-4,3-4,0)=(a,b,c)\qquad \left(3-\dfrac{20}{9},3-\dfrac{20}{9},2-\dfrac{10}{9}\right)$

Take $\left(\dfrac{7}{9},\dfrac{7}{9},\dfrac{8}{9}\right)=(a,b,c)\quad 18\left(\dfrac{7}{9}+\dfrac{7}{9}+\dfrac{8}{9}\right)=44$

88. $ax^2+bx+c>0\forall x\in R\Rightarrow a>0, D<0\Rightarrow (i)a-3>0(ii)(a+9)(a-6), a>6$

Least integral value of a = 7

89. 3^n+7^n is a multiple of 10

Since n is odd, n={11,13,15,.....99} ; Number of values of n = 45

90. Given $A^2=A(I-0.4A)^{-1}=(I-\alpha A)$

$I-0.4A(I-\alpha A)=I\Rightarrow I-0.4A-\alpha A+0.4\alpha A^2=I$

$\Rightarrow -0.4A-\alpha A+0.4\alpha A=0\ A(-0.6\alpha-0.4)=0\Rightarrow\alpha=\dfrac{-2}{3}$

$\therefore$ required value $=\dfrac{1}{1-\dfrac{2}{3}}=3$

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