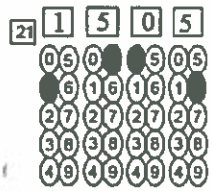
Model Grand Test

2023

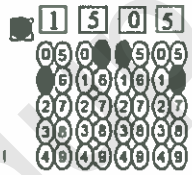
IMPORTANT INSTRUCTION:

1. Immediately fill in the Admission number on this page of the Test Booklet with **Blue/Black** Ball Point **Pen** only.
2. The candidates should not write their Admission Number anywhere (except in the specified space) on the Test Booklet/ Answer Sheet.
3. The test is of **3 hours** duration.
4. The Test Booklet consists of 90 questions. The maximum marks are **300.**
5. There are **three** parts in the question paper 1,2,3 consisting of **Physics, Chemistry** and **Mathematics** having **30 questions** in each subject and subject having **two sections.**
   (I) Section –I contains 20 **multiple choice** questions with only one correct option.
   Marking scheme: +4 for correct answer, 0 if not attempt and -1 in all other cases.
   (II) Section-II contains 10 **Numerical Value Type** questions. Attempt any 5 questions only,if more than 5 questions attempted, First 5 attempted questions will be considered.
   - The Answer should be within **0 to 9999.** If the Answer is in **Decimal** then round off to the **nearest Integer** value (Example i,e. If answer is above **10** and less than **10.5** round off is **10** andIf answer is from **10.5** and less than **11** round off is **11).**

   To cancel any attempted question bubble on the question number box.
   For example: To cancel attempted question 21. Bubble on 21 as shown below



Question Answered for Marking



Question Cancelled for Marking

Marking scheme: +4 for correct answer, 0 if **not attempt** and **-1** in all other cases.

6. Use **Blue / Black Point Pen only** for writing particulars / marking responses on the Answer Sheet. **Use of pencil is strictly prohibited.**
7. No candidate is allowed to carry any textual material, printed or written, bits of papers, mobile phone any electron device etc, except the Identity Card inside the examination hall.
8. Rough work is to be done on the space provided for this purpose in the Test Booklet only.
9. On completion of the test, the candidate must hand over the Answer Sheet to the invigilator on duty in the Hall. **However, the candidate are allowed to take away this Test Booklet with them.**
10. **Do not fold of make any stray marks on the Answer Sheet**

PHYSICS

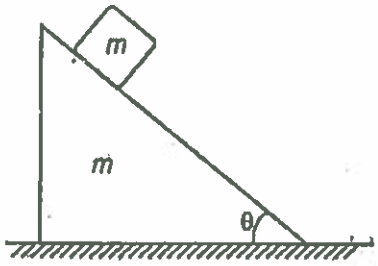
Max Marks: 100

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

1. A block of mass m slides down on an inclined plane of a wedge of same mass m as shown in figure. Friction is absent everywhere. Which of the following statements is/are correct?



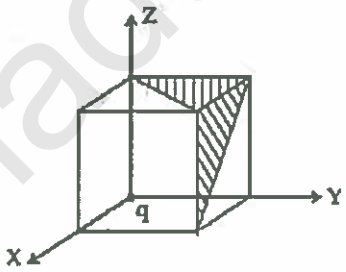
1) Vertical component of acceleration of block is $\frac{g\cos^2\theta}{1+\sin^2\theta}$ downwards

2) Acceleration of the COM of is $\frac{g\sin^2\theta}{1+\sin^2\theta}$ downwards

3) Acceleration of the block is $\frac{g\cos\theta}{1+\sin^2\theta}$ downwards.

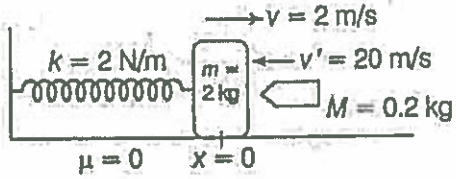
4) Vertical component of acceleration of block is $\frac{2g}{1+\sin^2\theta}$ downwards.

2. A change 'q' is placed at one corner of a cube as shown in figure. The flux of electrostatic field $\vec{E}$ through the shaded area is:



1) $\frac{q}{24\varepsilon_0}$ 2) $\frac{q}{8\varepsilon_0}$ 3) $\frac{q}{48\varepsilon_0}$ 4) $\frac{q}{4\varepsilon_0}$

3. A block is executing SHM on a horizontal floor. At an instant when it was at its equilibrium position and moving towards right a bullet hits it and embedded in it. The new amplitude of oscillation A will be (all surfaces are friction less)



1) A > 2 m 2) A = 4m 3) A = 0 4) 0 < A < 2m

4. Match list I with list II.

List I	List II
(a) Rectifier	(i) Used either for stepping up or stepping down the a.c voltage
(b) Stabilizer	(ii) Used to convert a.c voltage into d.c voltage
(c) Transformer	(iii) Used to remove any ripple in the rectified output voltage
(d) Filter	(iv) Used to constant output voltage even when the input voltage or load current change

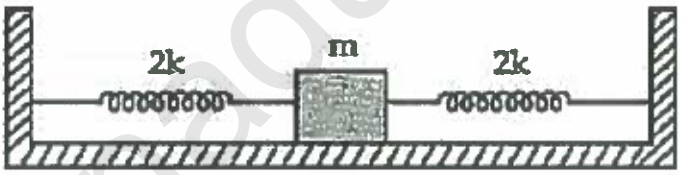
Choose the correct answer from the options given below.

1) (a) –(ii), (b)-(i), (c)-(iv), (d)-(iii)
2) (a) –(iii), (b)-(iv), (c)-(i), (d)-(ii)
3) (a)–(ii), (b)-(iv), (c)-(i), (d)-(iii)
4) (a)–(ii), (b)-(i), (c)-(iii), (d)-(iv)

5. When an AC voltage of variable frequency is applied across the L-C-R series circuit, the current in the circuit was found to be same at 8 MHz and 18 MHz. Then, the frequency at which current will be maximum is

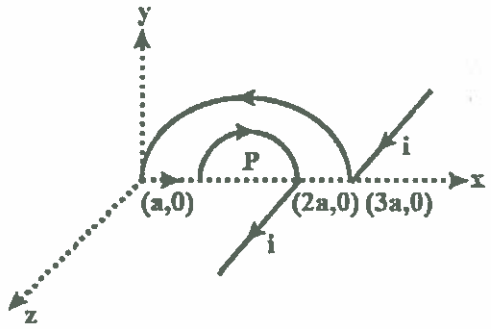
1) 24 MHz
2) 13 MHz
3) 10 MHz
4) 12 MHz

6. Two identical springs of spring constant '2K' are attached to a block of mass m and to fixed support (See figure). When the mass is displaced from equilibrium position on either side, it execute simple harmonic motion. The time period of oscillations of this system is:



1) $2\pi\sqrt{\frac{m}{2K}}$
2) $2\pi\sqrt{\frac{m}{K}}$
3) $\pi\sqrt{\frac{m}{K}}$
4) $\pi\sqrt{\frac{m}{2K}}$

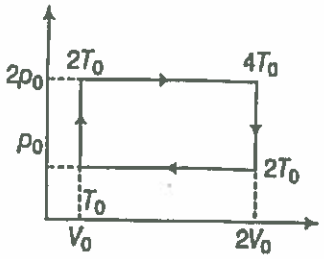
7. In the figure shown, the magnitude of magnetic field at the point P (centre of circular arc) is



1) $\frac{\mu_0 i}{3\pi a}\sqrt{4-\pi^2}$
2) $\frac{2\mu_0 i}{3\pi a}\sqrt{4+\pi^2}$
3) $\frac{\mu_0 i}{3\pi a}\sqrt{\left(4+\pi^2\right)}$
4) $\frac{2\mu_0 i}{3\pi a}\left(4-\pi^2\right)$

8. In a ferromagnetic material, below the Curie temperature, a domain is defined as:
   1) a macroscopic region with randomly oriented magnetic dipoles
   2) a macroscopic region with zero magnetization
   3) a macroscopic region with consecutive magnetic dipoles oriented in opposite direction
   4) a macroscopic region with saturation magnetization.

9. The p-V diagram represents the thermodynamic cycle of an engine, operating with an ideal monoatomic gas. The amount of heat, extracted from the source in a single cycle is



1) p_0V_0 2) $\left(\frac{13}{2}\right)p_0V_0$ 3) $\left(\frac{11}{2}\right)p_0V_0$ 4) $4p_0V_0$

10. $Y = A\sin(\omega t + \phi_0)$ is the time –displacement equation of a SHM. At t=0 the displacement of the particle is $Y = \frac{A}{2}$ and it is moving along negative x-direction. Then the initial phase angle ϕ_0 will be:

1) $\frac{\pi}{3}$ 2) $\frac{2\pi}{3}$ 3) $\frac{\pi}{6}$ 4) $\frac{5\pi}{6}$

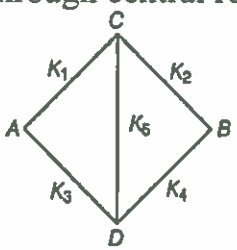
11. A Carnot engine takes 3×10^6 cal of heat from a reservoir at 627°C and gives it to a sink at 27°C. The work done by the engine is

1) $4.2\times10^6 J$ 2) $8.4\times10^6 J$ 3) $16.8\times10^6 J$ 4) Zero

12. The wavelength of the photon emitted by a hydrogen atom when an electron makes a transition from n = 2, to n = 1 state is:

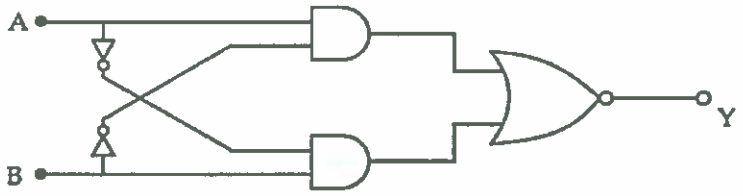
1) 490.7 nm 2) 913.3 nm 3) 121.8 nm 4) 194.8 nm

13. Five rods of same dimensions are arranged as shown in figure. They have thermal conductivities K_1, K_2, K_3, K_4 and K_5. When points A and B are maintained at different temperatures, no heat would flow through central rod, if



1) $K_1K_4 = K_2K_3$ 2) $K_1 = K_4$ *and* $K_2 = K_3$ 3) $\frac{K_1}{K_4} = \frac{K_2}{K_3}$ 4) $K_1K_2 = K_3K_4$

14. The truth table for the following logic circuit is:



1)

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

2)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

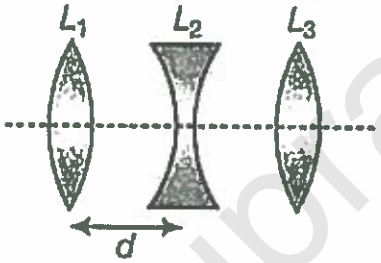
3)

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

4)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

15. Three lenses L_1, L_2, L_3 are placed co-axially as shown in figure. Focal length of lenses are given 30 cm, 10 cm and 5 cm, respectively. If a parallel beam of light falling on lens L_1, emerges from L_3 as a convergent beam such that it converges at the focus of L_3. The distance between L_1 and L_2 will be



1) 40 cm 2) 30 cm 3) 20 cm 4) 10 cm

16. An LCR circuit contains resistance of 110 Ω and a supply 220 V at 300 rad/s angular frequency. If only capacitance is removed from the circuit, current lags behind the voltage by 45^o If on the other hand, only inductor is removed the current leads by 45^o with the applied voltage. The rms current flowing in the circuit will be:

1) 1.5A 2) 1A 3) 2A 4) 2.5A

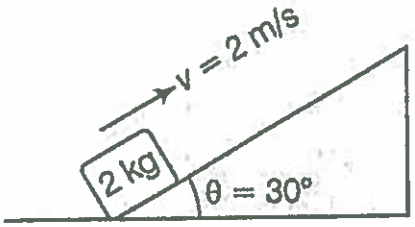
17. Consider a transistor with $\alpha = 0.98$. This transistor is 1st used as common base amplifier and then as common emitter amplifier. Then, the ratio of power gain in two arrangements shall be (given $\frac{R_{out}}{R_{in}}$ is same in both cases)

1) 1 : 2500 2) 1 : 500 3) 500 : 1 4) 2500 : 1

18. An electron of mass m_e and a proton of mass $m_p = 1836\ m_e$ are moving with the same speed. The ratio of their de Broglie wavelength $\frac{\lambda_{electron}}{\lambda_{proton}}$ will be:

1) 1836 2) 918 3) $\frac{1}{1836}$ 4) 1

19. A block is projected along in incline with a speed of 2 m/s. It covers 0.2 m along the incline and stops. Then,



1) $W_{resistive\ force} = -2J$

2) $W_{gravity} = -1J$

3) Total mechanical energy of the system is conserved

4) None of the above

20. Consider the diffraction pattern obtained from the sunlight incident on a pinhole of diameter 0.1μm. if the diameter of the pinhole is slightly increased, it will affect the diffraction pattern

1) Its size decreases, but intensity increases

2) Its size decreases, but intensity decreases

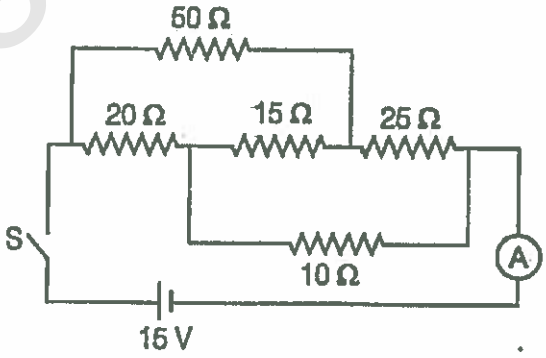
3) Its size increases, but intensity decreases

4) Its size increases, but intensity increases

(NUMERICAL VALUE TYPE)

Section-II contains 10 Numerical Value Type questions. Attempt any 5 questions only. First 5 attempted questions will be considered if more than 5 questions attempted.The Answer should be within 0 to 9999. If the Answer is in Decimal then round off to the nearest Integer value (Example I,e. If answer is above 10 and less than 10.5 round off is 10 andIf answer is from 10.5 and less than 11 round off is 11).
Marking scheme: +4 for correct answer, 0 if not attempt and -1 in all other cases.

21. When switch S is closed, then the reading of ammeter is ____ A.

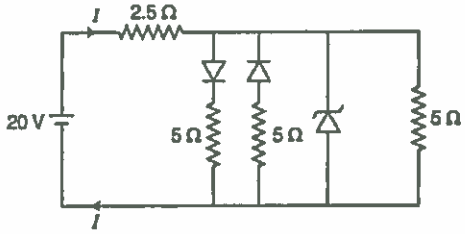


22. A reversible heat engine converts one – fourth of the heat input into work. When the temperature of the sink is reduced by 52K, its efficiency is doubled. The temperature in Kelvin of the source will be ___

23. A transparent cube contains a small air bubble. Its apparent distance is 2 cm when seen through one face and 5 cm when seen through opposite face. If the refractive index of the material of the cube is 1.5, real length of the edge of the cube must be ___ cm

24. The initial velocity v_i required to project a body vertically upward from the surface of the earth to each a height of 10R, where R is the radius of the earth, may be described in terms of escape velocity v_e such that $v_i = \sqrt{\frac{x}{y}} \times v_e$. The value of x will be ___

25. Consider the diagram given below.



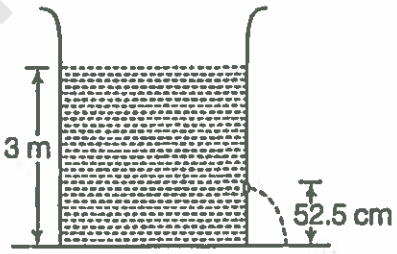
Breakdown voltage for Zener diode is 10V. Then, the value of current (I) will be ___ A. (Consider the diodes as ideal diodes)

26. If $\vec{P} \times \vec{Q} = \vec{Q} \times \vec{P}$, the angle between $\vec{P}$ and $\vec{Q}$ is $\theta (0^0 < \theta < 360^\circ)$. The value of 'θ' will be ——°

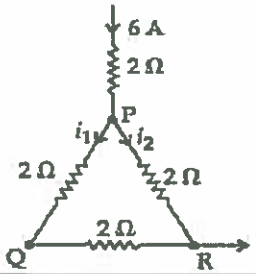
27. A parallel plate capacitor is made of two circular plates separated by a distance of 5 mm and with a dielectric of dielectric constant 2.2 between them. When the electric field in the dielectric is 3×10^4 V/m, the charge density of the positive plate will be close to ___ 10^{-7} C/m^2.

28. The percentage increase in the speed of transverse waves produced in a stretched string if the tension is increased by 4%, will be ___ %

29. Water is filled in a cylindrical container to a height of 3m. The ratio of the cross-sectional area of the orifice and the beaker is 0.1. The square of the speed of the liquid coming out from the orifice is (g = 10 m/s^2) ____ m^2/s^2.



30. A current of 6A enters one corner P of an equilateral triangle PQR having 3 wires of resistance 2Ω each and leaves by the corner R. The current i_1 in ampere is ___



(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.
Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

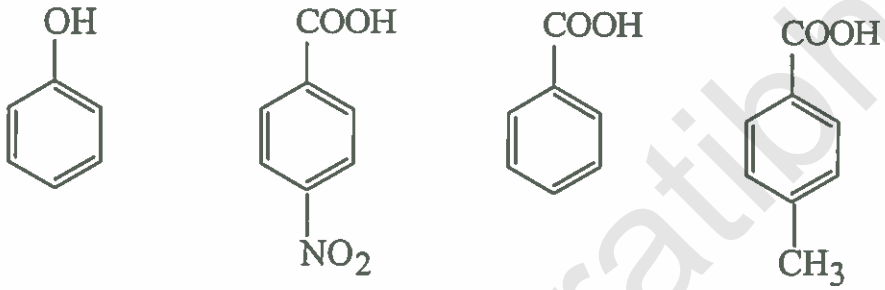
31. At 25°C, the solubility product of $Pb(OH)_2$ (s) is

[Given: $E^0_{Pb^{2+}|Pb} = -0.13V; E^0_{OH|Pb(OH)_2|Pb} = -0.55V$ and $\frac{2.303RT}{F} = 0.06$]

1) 1.36×10^{-5} 2) 1.0×10^{-7}

3) 1.0×10^{-14} 4) 1.25×10^{-15}

32. The correct order of acid character of the following compounds is



I II III IV

1) IV > III > II > I 2) III > II > I > IV

3) II > III > IV> I 4) I > II > III > IV

33. On adding $AgNO_3$ solution into KI solution, a negatively charged colloidal sol is obtained when they are in

1) 100 ml of 0.1 M $AgNO_3$ + 100 ml of 0.1 M KI

2) 100 ml of 0.1 M $AgNO_3$ + 50 ml of 0.2 M KI

3) 100 ml of 0.1 M $AgNO_3$ + 50 ml of 0.1 M KI

4) 100 ml of 0.1 M $AgNO_3$ + 100 ml of 0.15 M KI

34. In which of the following order the given complex ions are arranged correctly with respect to their decreasing spin only magnetic moment?

i. $[FeF_6]^{3-}$ ii. $[Co(NH_3)_6]^{3+}$

iii. $[NiCl_4]^{2-}$ iv. $[Cu(NH_3)_4]^{2+}$

1) ii > iii > i > iv 2) i > iii > iv > ii

3) iii > iv > ii > i 4) ii > i > iii > iv

35. The following reaction is performed at 298 K.

$2NO(g)+O_2(g) \rightleftharpoons 2NO_2(g)\left(K_p = 1.6\times10^{12}\right)$

The standard free energy of formation of NO(g) is 86.6 kJ/mol at 298 K. What is the standard free energy of formation of $NO_2(g)$ at 298 K?

1) $86600 - \dfrac{\ln\left(1.6\times10^{12}\right)}{R(298)}$

2) $0.5\left[2\times86,600 - R(298)\ln\left(1.6\times10^{12}\right)\right]$

3) $R(298)\ln\left(1.6\times10^{12}\right) - 86600$

4) $86600 + R(298)\ln\left(1.6\times10^{12}\right)$

36. The correct sequence of reagents used in the preparation of 4-bromo-2-nitroethyl benzene from benzene is?

1) $CH_3COCl / AlCl_3, Zn-Hg / HCl, Br_2 / AlBr_3, HNO_3 / H_2SO_4$

2) $HNO_3 / H_2SO_4, Br_2 / AlCl_3, CH_3COCl / AlCl_3, Zn-Hg / HCl$

3) $CH_3COCl / AlCl_3, Br_2 / AlBr_3, HNO_3 / H_2SO_4, Zn / HCl$

4) $Br_2 / AlBr_3, CH_3COCl / AlCl_3, HNO_3 / H_2SO_4, Zn / HCl$

37. Which of the following oxide is acidic in nature?

1) B_2O_3 2) Al_2O_3 3) Ga_2O_3 4) In_2O_3

38. Which among the following species has unequal bond lengths?

1) SF_4 2) BF_4^- 3) SiF_4 4) XeF_4

39. Ozonisation of water is carried to remove

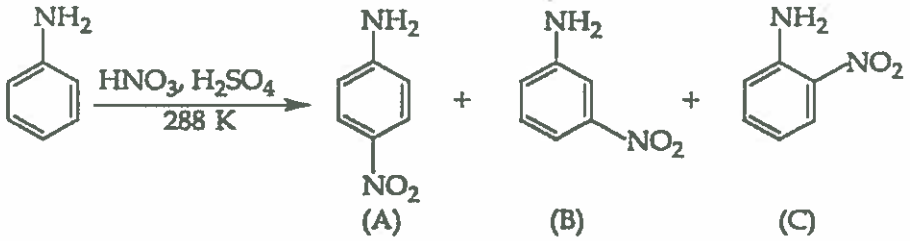
1) Bacterial impurities present
2) Bad taste
3) Excess of chlorine present
4) Calcium and magnesium salts present in it.

40. The method used for the purification of Indium is:

1) vapour phase refining 2) Van Arkel method
3) liquation 4) Zone refining

41. A considerable part of the harmful ultraviolet radiation of the sun does not reach the earth's surface due to

1) CO_2 2) H_2 3) O_3 4) NH_3

42. 

(A) (B) (C)

Correct statement about the given chemical reaction is

1) $-\ddot{N}H_2$ group is ortho and para directive, so product (B) is not possible

2) The reaction will form sulphonated product instead of nitration

3) Reaction is possible and compound (B) will be the major product

4) Reaction is possible and compound (A) will be major product

43. During initial treatment, preferential wetting of ore by oil and gangue by water takes place in

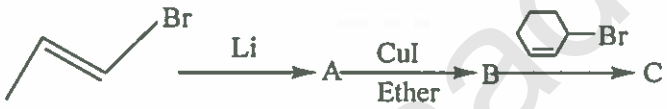
1) Levigation 2) Froth floatation

3) Leaching 4) Bessemerisation

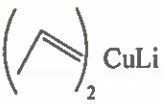
44. Water does not produce CO on reacting with:

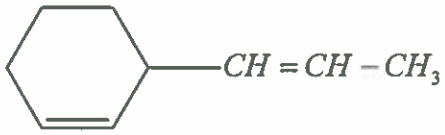
1) C_3H_8 2) CO_2 3) C 4) CH_4

45. Find the product of the following reaction.

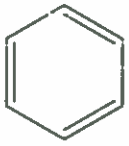


Find out the final product (C):

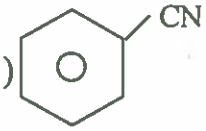
1) 

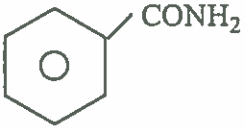
2) 

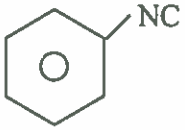
3) 

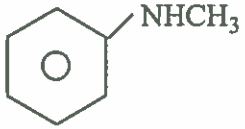
4) 

46. Carbylamine test is used to detect the presence of primary amino group in an organic compound. Which of the following compound is formed when this test is performed with aniline

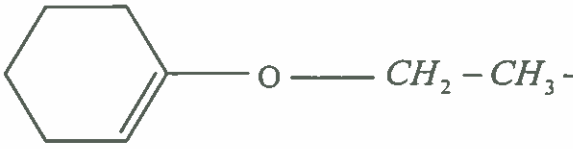
1) 

2) 

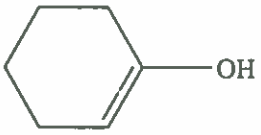
3) 

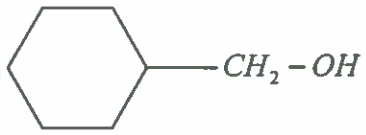
4) 

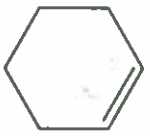
47. In the given reaction

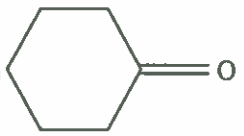


. Here P will be:

1) 

2) 

3) 

4) 

48. Given below are two statements.

Statement I : The pH of rain water is normally ~ 5.6

Statement II : If the pH of rain water drops below 5.6, it is called acid rain.

In the light of the above statements, Choose the correct answer from the options given below

1) Both statement I and statement II are true

2) Statement I is true but statement II is false

3) Statement I is false but statement II is true

4) Both Statement I and statement II are false

49. The magnetic property and the shape of $\left[Cr(NH_3)_6\right]^{+3}$ complex ions are

1) Paramagnetic, octahedral
2) Diamagnetic, square planar
3) Paramagnetic, tetrahedral
4) Diamagnetic, octahedral

50. Given below are two statements:

Statement I : The identification of Ni^{2+} is carried out by dimethyl glyoxime in the presence of NH_4OH

Statement II : The dimethyl glyoxime is a bidentate neutral ligand.

In the light of the above statements, choose the correct answer from the options given below.

1) Both Statement I and statement II are false

2) Both statement 1 and statement II are true

3) Statement I is false but statement II is true

4) Statement I is true but statement II is false

(NUMERICAL VALUE TYPE)

Section-II contains 10 Numerical Value Type questions. Attempt any 5 questions only. First 5 attempted questions will be considered if more than 5 questions attempted.The Answer should be within 0 to 9999. If the Answer is in Decimal then round off to the nearest Integer value (Example i,e. If answer is above 10 and less than 10.5 round off is 10 andIf answer is from 10.5 and less than 11 round off is 11).
Marking scheme: +4 for correct answer, 0 if not attempt and -1 in all other cases.

51. A gas expands from 3 dm^3 to 5 dm^3 against a constant pressure of 3 atm. The work done during expansion is used to heat 10 mole of water of temperature 290 K. Calculate $(\Delta T \times 10)$ of water. Specific heat of water =4.184J/g/K.

(Report your answer in nearest integer)

52. The number of compound/s given below which contain/s –COOH group is ____ [integer answer]

1. Sulphanilic acid 2. Picric acid 3. Aspirin 4. Ascorbic acid

53. Solutions A and B have osmotic pressures of 2.4 atm and 4.2 atm, respectively, at a certain temperature. The osmotic pressure (in atm) of a solution prepared by mixing the solutions in 2 : 1 volume ratio, respectively, at the same temperature is

54. Electromagnetic radiation of wavelength 663 nm is just sufficient to ionise the atom of metal A. The ionization energy of metal A in kJ mol^{-1} is ___ (Rounded off to the nearest integer) [$h = 6.63\times10^{-34}$ Js, $c = 3.00\times10^{8}$ ms^{-1}, $N_A = 6.02\times10^{23}$ mol^{-1}]

55. In carius method of estimation of halogen. 0.15 g of an organic compound gave 0.12g of AgBr. Find out the percentage of bromine in the compound.
(Report your answer in nearest integer)

56. Among the following number of metal/s which can be used as electrodes in the photoelectric cell is ___ (integer answer)

1. Li 2. Na 3. Rb 4. Cs

57. If pH of resultant solution obtained on mixing 100 ml of 0.1 M ammonium hydroxide and 50 ml of 0.1 M ammonium sulphate at 25°C is $x \times 10^{-1}$, then x = ____

(pK_b of $NH_4OH = 4.7$) (log2 = 0.3)

58. Five moles of an ideal gas at 293K is expanded isothermally from an initial pressure of 2.1MPa to 1.3 MPa against at constant external pressure 4.3 MPa. The heat transferred in this process is ___ kJ mol^{-1} (Rounded-off to the nearest integer) [Use $R = 8.314 J mol^{-1}K^{-1}$]

59. Number of C – O – C bonds present in sucralose is __

60. Consider titration of NaOH solution versus 1.25 M oxalic acid solution. At the end point following burette readings were obtained.

1. 4.5ml 2. 4.5 ml 3. 4.4 ml 4. 4.4ml. 5. 4.4ml

If the volume of oxalic acid taken was 10.0ml then the molarity of the NaOH solution is ____ M. (Rounded-off to the nearest integer)

MATHEMATICS

Max Marks: 100

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

61. If the complex number z satisfies $z+|z|=2+8i$ then the value of $|z|=$

1) 8 2) 17 3) 15 4) 24

62. Let α and β be the roots of $x^2-6x-2=0$. If $a_n=\alpha^n-\beta^n$ for $n\geq 1$, then the value of $\frac{a_{10}-2a_8}{3a_9}$ is

1) 3 2) 1 3) 4 4) 2

63. Let set $S_1=\{1\}$, set $S_2=\{4,7\}$, set $S_3=\{10,13,16\}$ and so on. If 355 lies in set S_n, then the value of n is

1) 14 2) 15 3) 16 4) 17

64. The shortest distance between the line $x-y=1$ and the curve $x^2=2y$ is

1) $\frac{1}{2}$ 2) $\frac{1}{\sqrt{2}}$ 3) 0 4) $\frac{1}{2\sqrt{2}}$

65. C_1 and C_2 are circles of unit radius with centres at (0, 0) and (1, 0) respectively. C_3 is a circle of unit radius, passes through the centres of the circles C_1 and C_2 and have its centre above x-axis. Equation of the common tangent to C_1 and C_3 which does not pass through C_2 is

1) $x-\sqrt{3}y+2=0$ 2) $\sqrt{3}x-y+2=0$ 3) $\sqrt{3}x-y-2=0$ 4) $x+\sqrt{3}y+2=0$

66. Let x denote the total number of one –one functions from a set A with 3 elements to a set B with 5 elements and y denote the total number of one – one functions from the set A to the set $A\times B$ then:

1) $y=91x$ 2) $2y=273x$ 3) $y=273x$ 4) $2y=91x$

67. If a variable tangent of the circle $x^2+y^2=1$ intersect the ellipse $x^2+2y^2=4$ at P and Q then the locus of the points of intersection of the tangents at P and Q is

1) a circle of radius 2 units

2) a parabola with focus as (2, 3)

3) an ellipse with eccentricity $\frac{\sqrt{3}}{4}$

4) an ellipse with length of latus rectum is 2 units

68. The integral $\int \frac{e^{3\log_e 2x} + 5e^{2\log_e 2x}}{e^{4\log_e x} + 5e^{3\log_e x} - 7e^{2\log_e x}} dx, x > 0$ is equal to (where c is a constant of integration)

1) $4\log_e\left|x^2+5x-7\right|+c$ 2) $\log_e\left|x^2+5x-7\right|+c$

3) $\frac{1}{4}\log_e\left|x^2+5x-7\right|+c$ 4) $\log_e\sqrt{x^2+5x-7}+c$

69. If maximum and minimum values of the determinant $\begin{vmatrix} 1+\sin^2 x & \cos^2 x & \sin 2x \\ \sin^2 x & 1+\cos^2 x & \sin 2x \\ \sin^2 x & \cos^2 x & 1+\sin 2x \end{vmatrix}$ are α and β respectively, then which of the statements are FALSE?

1) $\alpha+\beta^{99}=4$ 2) $\alpha^3-\beta^{17}=26$

3) $\left(\alpha^{2n}-\beta^{2n}\right)$ is always an even integer for $n \in N$

4) a triangle can be constructed having it's sides as α, β and $\alpha-\beta$

70. If for the matrix $A=\begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix}$, $AA^T = I_2$, then the value of $\alpha^4+\beta^4$ is:

1) 2 2) 1 3) 4 4) 3

71. f is a continuous function on the real numbers. Given that $x^2+\left(f(x)-2\right)x-\sqrt{3}f(x)+2\sqrt{3}-3=0$. Then the value of $f\left(\sqrt{3}\right)$ is

1) can not be determined 2) $2\left(1-\sqrt{3}\right)$

3) zero 4) $\frac{2\left(\sqrt{3}-2\right)}{\sqrt{3}}$

72. A hyperbola passes through the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ and its transverse and conjugate axes coincide with major and minor axes of the ellipse, respectively. If the product of their eccentricities is one, then the equation of the hyperbola is:

1) $\frac{x^2}{9}-\frac{y^2}{16}=1$ 2) $\frac{x^2}{9}-\frac{y^2}{4}=1$ 3) $x^2-y^2=9$ 4) $\frac{x^2}{9}-\frac{y^2}{25}=1$

73. Let $y=f(x)$ be the solution of the differential equation $\frac{dy}{dx}-ky=0$ satisfying the condition $f(0)=1$. If the value of $\lim_{x\to\infty} f(x)=0$ then which of the following is TRUE?

1) $K = 0$ 2) $k > 0$ 3) $k < 0$ 4) no such value of k exists

74. The following system of linear equations

$2x+3y+2z=9$

$3x+2y+2z=9$

$x-y+4z=8$

1) does not have any solution
2) has infinitely many solutions
3) has a unique solution
4) has a solution (α,β,γ) satisfying $\alpha+\beta^2+\gamma^3=12$

75. A tetrahedron has vertices at O(0,0,0), A(1,2,1), B(2,1,3) and C(-1,1,2). Then the angle between faces OAB and ABC will be

1) $\cos^{-1}\left(\frac{19}{35}\right)$ 2) $\cos^{-1}\left(\frac{17}{31}\right)$ 3) 30° 4) 90°

76. If $I_n=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\cot^n x\,dx$, then:

1) $I_2+I_4, I_3+I_5, I_4+I_6$ are in AP
2) $I_2+I_4,(I_3+I_5)^2,I_4+I_6$ are in GP
3) $\frac{1}{I_2+I_4},\frac{1}{I_3+I_5},\frac{1}{I_4+I_6}$ are in GP
4) $\frac{1}{I_2+I_4},\frac{1}{I_3+I_5},\frac{1}{I_4+I_6}$ are in AP

77. $a>0(a\neq 1), b>0(b\neq 1)$ such that $a^{(\log_a b)^x}=b^{(\log_b a)^x}$ then $x=$

1) 1 2) -1 3) $\frac{1}{2}$ 4) 2

78. In a group of 400 people, 160 are smokers and non – vegetarian; 100 are smokers and vegetarian and the remaining 140 are non – smokers and vegetarian. Their chances of getting a particular chest disorder are 35%, 20% and 10% respectively. A person is chosen from the group at random and is found to be suffering from the chest disorder. The probability that the selected person is a smoker and non – vegetarian is:

1) $\frac{28}{45}$ 2) $\frac{7}{45}$ 3) $\frac{8}{45}$ 4) $\frac{14}{45}$

79. The ratio of sum of first three terms of a G.P. to the sum of first six terms is 64 : 91, the common ratio of G.P. is

1) $\frac{1}{4}$ 2) $\frac{3}{4}$ 3) $\frac{5}{4}$ 4) $\frac{7}{4}$

80. $\lim_{n\to\infty}\left[\frac{1}{n}+\frac{n}{(n+1)^2}+\frac{n}{(n+2)^2}+.....+\frac{n}{(2n-1)^2}\right]$ is equal to

1) $\frac{1}{4}$ 2) $\frac{1}{2}$ 3) 1 4) $\frac{1}{3}$

(NUMERICAL VALUE TYPE)

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Marking scheme: +4 for correct answer, 0 if not attempt and -1 in all other cases.

81. Let PN be the ordinate of a point P on the hyperbola $\frac{x^2}{(97)^2}-\frac{y^2}{(79)^2}=1$ and the tangent at P meets the transverse axis in T, O is the origin. Then $\left[\frac{ON.OT}{2011}\right]$ is equal to

(where [.] denotes G.I.F.)

82. The value of $\int_{-2}^{2}\left|3x^2-3x-6\right|dx$ is ___

83. A function f from integers to integers is defined as $f(x)=\begin{cases}n+3, & n\in \text{odd}\\ n/2 & n\in \text{even}\end{cases}$. Suppose $k\in$ odd and f(f(f(k)))=27, then the sum of digits of k is

84. If the curves $x=y^4$ and $xy=k$ cut at right angles, then $(4k)^6$ is equal to

85. Let $f:[0,\infty)\to R$ be a continuous, strictly increasing function such that $(f(x))^3=\int_0^x t(f(t))^2\,dt$, If a normal is drawn to the curve. y = f(x) with gradient $\frac{-1}{2}$, then the intercept made by it on the y-axis is ____.

86. If $\lim\limits_{x\to 0}\frac{ax-(e^{4x}-1)}{ax(e^{4x}-1)}$ exists and is equal to b, then the value of $a-2b$ is ___

87. There are three coins. One is two headed coin (having head on both faces), another is biased coin that comes up heads 75% of the time and third is an unbiased coin. One of the three coins is chosen at random and tossed. If it shows heads, then the probability that it was 2 headed coin is P, then the value of 81P is _____

88. Let $\vec{a}=\hat{i}+\alpha\hat{j}+3\hat{k}$ and $\vec{b}=3\hat{i}-\alpha\hat{j}+\hat{k}$. If the area of the parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$ is $8\sqrt{3}$ square units, then $\vec{a}.\vec{b}$ is equal to ____

89. In triangle ABC the equation of altitudes AM and BN are $x+5y-3=0, x+y-k=0$ .If the altitude CL is given by $3x-y-1=0$, then k = ____

90. A line is a common tangent to the circle $(x-3)^2+y^2=9$ and the parabola $y^2=4x$. If the two points of contact (a, b) and (c,d) are distinct and lie in the first quadrant then 2(a+c) is equal to ____

*****

KEY SHEET

PHYSICS

1	**2**	2	**1**	3	**3**	4	**3**	5	**4**
6	**3**	7	**3**	8	**4**	9	**2**	10	**4**
11	**2**	12	**3**	13	**1**	14	**2**	15	**3**
16	**3**	17	**1**	18	**1**	19	**1**	20	**1**
21	**1**	22	**208**	23	**11**	24	**10**	25	**4**
26	**180**	27	**6**	28	**2**	29	**50**	30	**2**

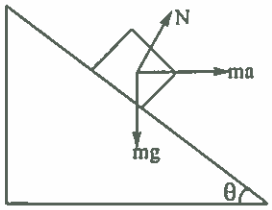
CHEMISTRY

31	**3**	32	**3**	33	**4**	34	**2**	35	**2**
36	**1**	37	**1**	38	**1**	39	**1**	40	**4**
41	**3**	42	**4**	43	**2**	44	**2**	45	**2**
46	**3**	47	**4**	48	**1**	49	**1**	50	**4**
51	**8**	52	**1**	53	**3**	54	**181**	55	**34**
56	**1**	57	**93**	58	**15**	59	**3**	60	**6**

MATHEMATICS

61	**2**	62	**4**	63	**2**	64	**4**	65	**2**
66	**4**	67	**4**	68	**1**	69	**4**	70	**2**
71	**2**	72	**1**	73	**3**	74	**3**	75	**1**
76	**4**	77	**3**	78	**1**	79	**2**	80	**2**
81	**4**	82	**19**	83	**6**	84	**4**	85	**9**
86	**5**	87	**36**	88	**2**	89	**1**	90	**9**

HINTS & SOLUTIONS
PHYSICS

1. 

Let a be the acceleration of the wedge leftwards and a_r be the relative acceleration of block down the plane. $F_{horizontal} = 0$ $a_{horizontal}=0$

$a_r\cos\theta - a = a, a_r\cos\theta = 2a$ ___(1) $N\sin\theta = ma$ ____(2)

And $N + ma\sin\theta = mg\cos\theta$ _____(3) On solving (1),(2)and(3)

we get $a_r = \dfrac{2g\sin\theta}{1+\sin^2\theta}$ For block $a_r = a_r\sin\theta = \dfrac{2g\sin^2\theta}{1+\sin^2\theta}$

$$\therefore a_{COM} = \frac{m(0)+m\left(\dfrac{2g\sin^2\theta}{1+\sin^2\theta}\right)}{m+m} = \frac{g\sin^2\theta}{1+\sin^2\theta}$$

2. Flux through 3 opposite faces $= \dfrac{Q}{8\epsilon_0}$

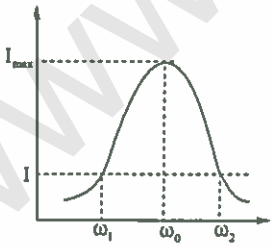
Flux through two half faces = flux through on full face $\phi = \dfrac{Q}{24\epsilon_0}$

3. At the instant of collision we can apply conservation of linear momentum for the system of block and bullet $\Rightarrow mv + Mv' = (2\times 2) - (0.2\times 20) = 0$ (before collision)

So, the system of block and bullet will come to rest just after collision.

So, amplitude, A=0

4. a) Rectifier – converts AC to DC
   b) stabilizer – used for constant output voltage even when the input voltage (or) load current charge
   c) Transformer – used either for stepping up (or) stepping down the ac voltage
   d) Filter - used to remove any ripple in the rectified output voltage

5. 

Z_1 at $\omega_1 = \dfrac{1}{\omega_1 C} - \omega_1 L$ Z_2at $\omega_2 = \omega_2 L - \dfrac{1}{\omega_2 C}$

For current to be same $Z_1 = Z_2 = \frac{1}{\omega_1 C} - \omega_1 L = \omega_2 L - \frac{1}{\omega_2 C}$

$\Rightarrow \left(\frac{1}{\omega_1} + \frac{1}{\omega_2}\right)\frac{1}{C} = (\omega_1 + \omega_2)L \Rightarrow \frac{\omega_1 + \omega_2}{\omega_1 \omega_2} = (\omega_1 + \omega_2)LC$

Now, current is maximum at $\omega_0 = \frac{1}{\sqrt{LC}}$

$\therefore$ Resonant frequency, $\omega_0 = \sqrt{\omega_1 \omega_2}$ For given problem, $\omega_0 = \sqrt{8 \times 18} = 12\text{MHz}$

6. Springs are in parallel, $K_{eff} = 2k + 2k = 4k$, $T = 2\pi\sqrt{\frac{m}{K_{eff}}} = 2\pi\sqrt{\frac{m}{4K}} = \pi\sqrt{\frac{m}{K}}$

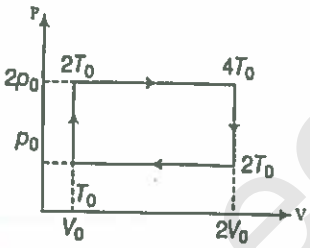
7. $B_P = (B_1)_P + (B_2)_P + (B_3)_P + (B_4)_P + (B_5)_P$

Where, $(B_1)_P = \frac{\mu_0^i}{4\pi\left(\frac{3a}{2}\right)}\left(-\hat{j}\right)(B_2)_P = \frac{\mu_0 i}{4\left(\frac{3a}{2}\right)}\left(-\hat{k}\right), (B_3)_P = 0\,(B_4)_P = \frac{\mu_0 i}{4\left(\frac{a}{2}\right)}\left(-\hat{k}\right)$

$(B_5)_P = \frac{\mu_0 i}{4\pi\left(\frac{a}{2}\right)}\left(-\hat{j}\right) \Rightarrow |B_P| = \frac{\mu_0 i}{3\pi a}\sqrt{1+\pi^2}$

8. 

Below Curie temperature, the domain is a macroscopic region with saturation magnetization

9. 

Heat supplied, $H = nC_v\Delta t + nC_\rho\Delta T$

$= nC_V(2T_0 - T_0) + nC_P(4T_0 - 2T_0)$ For monoatomic gas, $C_V = \frac{3R}{2}$

and $C_P = \frac{5R}{2} \therefore H = n\frac{3RT_0}{2} + n\left(\frac{5R}{2}\right)2T_0 = \frac{3R}{2}nT_0 + 5nRT_0 = \frac{13}{2}nRT_0 = \frac{13}{2}\rho_0 V_0$

10. $y = A\sin(\omega t + \phi_o)$, $at\ t = 0 : y = -A/2$, $-\frac{A}{2} = A\sin\phi_o$, $\phi_o = 5\frac{\pi}{6}$

11. As, $\frac{Q_1}{T_1} = \frac{Q_2}{T_2} \Rightarrow Q_2 = \frac{T_2}{T_1} \times Q_1 = 1 \times 10^6\,\text{cal}$ Workdone $= Q_1 - Q_2 \times 4.2\text{J} = 8.4 \times 10^6\,\text{J}$

12. $E_n = \frac{-13.6}{n^2} eV, E_1 = -13.6 eV, E_2 = -3.4 eV, \Delta E = E_2 - E_1$

$\Delta E = 10.2 eV, \lambda = \frac{1240}{\Delta E(eV)} nm, \lambda = \frac{124}{10.2} nm, \lambda = 121.8\ nm$

13. The condition for balanced bridge is $\frac{P}{Q} = \frac{R}{S}$ (in terms of resistance).

$\frac{1/K_1}{1/K_2} = \frac{1/K_3}{1/K_4} \Rightarrow \frac{K_2}{K_1} = \frac{K_4}{K_3} \Rightarrow K_1K_4 = K_2K_3$

14.

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

$y = \overline{A.\bar{B} + \bar{A}.B}$

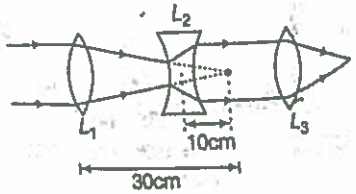
$y = \overline{A.\bar{B}}.\overline{\bar{A}.B}$

$y = (\bar{A} + B).(A + \bar{B})$

$y = A\bar{A} + \overline{AB} + BA + B\bar{B}$

15. L_1 must from image at focus of L_2 foci of L_1 and L_2 must coincide

$\therefore \quad d = 20cm$



16. $R = 110V\ ;\ V_{rms} = 220V$

$\omega = 300\ rad/s, \tan\phi = \frac{X}{R}, \tan 45^\circ = \frac{X_L}{R}, \tan 45^\circ = \frac{X_C}{R}$

$X_L = X_C, \qquad \therefore Z = R\ ,\ I_{rms} = \frac{V_{rms}}{Z} = \frac{220}{110} \qquad I_{rms} = 2A$

17. Since, $\alpha = 0.98$ Then, $\beta = \frac{\alpha}{1-\alpha} = \frac{0.98}{0.02} = 49$ Now, power gain $= \alpha^2 \frac{R_{out}}{R_{in}}$

(for common base) Power gain $= \beta^2 . \frac{R_{out}}{R_{in}}$ (for common emitter)

$\therefore \text{Ratio} = \frac{\alpha^2}{\beta^2} = \frac{0.98 \times 0.98}{49 \times 49} = \frac{1}{2500}$

18. $\lambda = \frac{h}{p} = \frac{h}{mV} \alpha \frac{1}{m}; \ \frac{\lambda_{electron}}{\lambda_{proton}} = \frac{m_{proton}}{m_{electron}} = \frac{1836}{1}$

19. $(KE)_i = \frac{1}{2}mv^2 = \frac{1}{2} \times 2 \times 4 = 4J$ $PE = mgh = 2 \times 10 \times [0.2 \times \sin 30^\circ] = 2 \times 10 \times 0.1 = 2J$

$\Rightarrow \Delta KE = W_{gravity} + W_{resistive\ force} - 4J = -2J + W_{resistive\ force} \Rightarrow W_{resistive\ force} = -2J$

20. $\beta = 1.22\frac{f\lambda}{d}$; If d is increased. Its size (β) decreases And intensity increases as it collects more light.

21. The given problem is application of balanced Wheatstone bridge, i.e., $R_{eq} = \frac{75\times 30}{75+30} = \frac{150}{7}$ So, current, $I = \frac{15}{150}\times 7 = 0.7A$

22. $\eta = 1-\frac{T_2}{T_1}, \frac{1}{4} = 1-\frac{T_2}{T_1}, \frac{2}{4} = 1-\frac{(T_2-52)}{T_1}, \frac{1}{2} = \frac{T_1-T_2}{T_1-T_2+52}, \frac{3}{4} = \frac{T_2}{T_1}, T_2 = \frac{3}{4}T_1$; Solving 1 & 2, $T_1 = 208K$

23. $\text{Now}, \mu = \frac{\text{real depth}}{\text{apparent depth}}$ Real depth $= \mu \times$ apparent depth

Therefore, real length of the edge of the cube $= \mu \times$(apparent distance of the bubble from one face + apparent distance of the bubble from second face) $= 1.5(2+5) = 1.5\times 7 = 10.5cm$

24. If $V_i = KV_e$, $h = \frac{RK^2}{1-K^2}$, $10R = \frac{RK^2}{1-K^2}$, $10 = 11k^2, k = \sqrt{\frac{10}{11}}, V_i = KV_e, V_i = \sqrt{\frac{x}{y}}V_e, x = 10$

25. Since Zener voltage is 10V. So all the components parallel to Zener diode will be at 10V. Hence, drop at 5Ω resistor will be10V. Diode which is reversely biased will not conduct any electricity and thus this component can be removed.

So,current through 5Ω resistor is $\frac{10}{5} = 2A$ and current through 2.5Ω resistor will be 2×2=4A

26. $\overline{P}\times\overline{Q} = \overline{Q}\times\overline{P}$, $\overline{P}\times\overline{Q} = -(\overline{P}\times\overline{Q})$, $PQ\sin\theta = -PQ\sin\theta$, $\therefore\theta = 180^\circ$

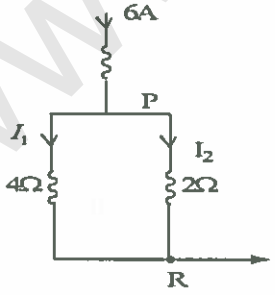
27. When dielectric is introduced between parallel plates of capacitor $E' = \frac{\sigma}{K\varepsilon_0}$

$\Rightarrow \sigma = 2.2\times 8.85\times 10^{-12}\times 3\times 10^4 = 6.6\times 8.85\times 10^{-8} = 5.841\times 10^{-7} C/m^2$

28. $V = \sqrt{\frac{T}{\mu}}$, $\frac{\Delta V}{V}\times 100 = \frac{1}{2}\frac{\Delta T}{T}\times 100 = \frac{1}{2}\times 4\% = 2\%$

29. Appling Bernoulli's theorem, we have $\frac{p}{\rho}+\frac{1}{2}v^2+gh = \frac{p}{\rho}+\frac{1}{2}v^2+0$and equation of continuity

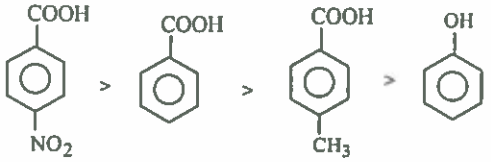
$$Av_1 = av_2 \quad \frac{v_1}{v_2} = \frac{a}{A} = 0.1 \quad v^2 = \frac{2gh}{1-\left(\frac{a}{A}\right)^2} = \frac{2\times 10\times(3-0.525)}{1-(0.1)^2} = 50(m/s)^2 = 50m^2/s^2$$

30. 

From Current dividing rule $I_1 = \frac{6\times 2}{6} = 2A$

31. $E^0_{OH^-/Pb(OH)_2/Pb} = E^0_{Pb^{2P}/Pb} - \frac{0.06}{2}.\log\frac{1}{K_{sp}}$

or, $(-0.55) = (-0.13) - \frac{0.06}{2}.\log\frac{1}{K_{sp}} \therefore K_{sp} = 1.0\times10^{-14}$

32. 

33. For negative charge on colloidal particles $[Ag^+] < [I^-]$

34. $[FeF_6]^{3-}$ $Fe^{+3} = 3d^5 \Rightarrow n = 5$

$[Co(NH_3)_6]^{+3}$ – $Co^{+3} = 3d^6 \Rightarrow n = 0$

$[NiCl_4]^{+2}$ – $Ni = +2$ $3d^8 \Rightarrow n = 2$

$[Cu(NH_3)_4]^{+2}$ – Cu^{+2} $3d^9 \Rightarrow n = 1$

35. $\Delta G^0_{NO(g)} = 86.6\text{kJ/mol} = 86600\text{J/mol}$

$G^0_{NO_2(g)} = x\text{ J/mol}$

$T = 298, K_P = 1.6\times10^{12}$

$\Delta G^0 = -RT\ln K_P$

Given equation, $2NO(g) + O_2(g) \rightleftharpoons 2NO_2(g)$

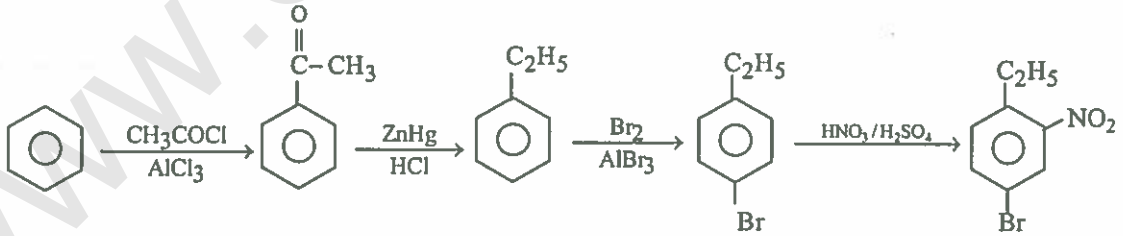
$\therefore 2\Delta G^0_{NO_2} - 2\Delta G^0_{NO} = -R(298)\ln(1.6\times10^{12})$

$2\Delta G^0_{NO_2} - 2\times86600 = -R(298)\ln(1.6\times10^{12})$

$2\Delta G^0_{NO_2} = 2\times86600 - R(298)\ln(1.6\times10^{12})$

$\Delta G^0_{NO_2} = \frac{1}{2}\left[2\times86600 - R(298)\ln(1.6\times10^{12}\right]$

$= 0.5\left[2\times86600 - R(298)\ln(1.6\times10^{12})\right]$

36. 

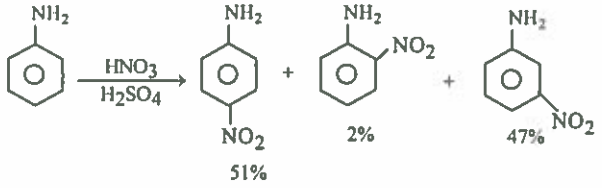
37. Non-metallic oxide is usually acidic

38. SF_4 has see saw structure, have unequal bond lengths. (equilateral and axial bonds are different)

39. O_3 is disinfectant

40. Indium is purified by zone refining

41. O_3 filter U.V. rays from sunlight

42. 

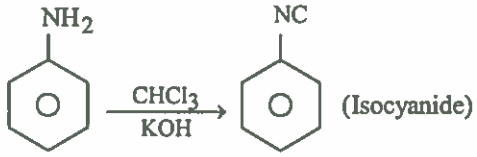
43. Ore dressing techniques in principles of metallurgy.

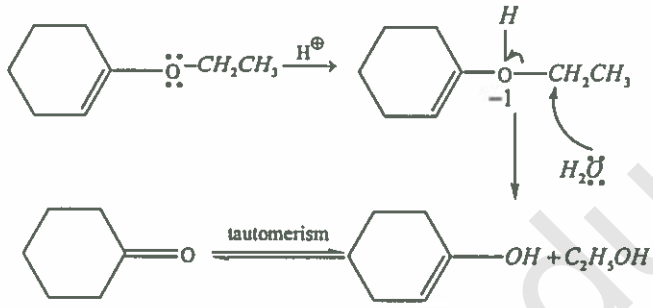
44. $CO_2 + H_2O \rightarrow \xrightarrow{H_2CO_3}$ Not produce CO

45. Corey-House synthesis

$RX + Li \rightarrow RLi \xrightarrow{CuI} R_2CuLi$

$R_2CuLi + R^1X \rightarrow R - R^1$

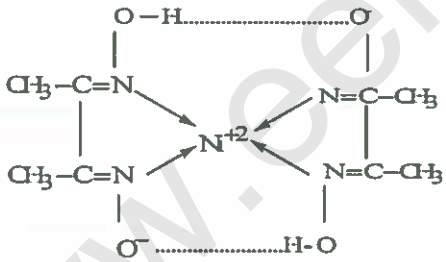
46. 

47. 

48. pH of rain water = 5.6 ; pH of rain water drops below 5.6 is called acid rain

49. Paramagnetic, due to the presence of unpaired electrons.

50. $Ni^{2+} + 2DMg \rightarrow [Ni(DMg)_2]$



DMG is Charged bidentate ligand

51. Work is done against constant P $\therefore$ irreversible

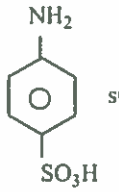
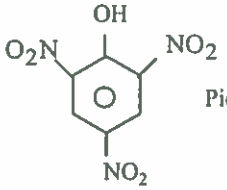
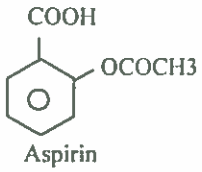
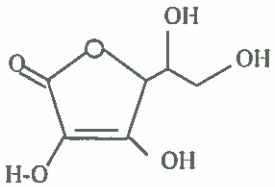
$\Delta V = 5 - 3 = 2dm^3 = 2L; P = 3atm.$

$\therefore W = -P_{ext}.\Delta V = -3 \times 2$ litre atm $= -6 \times 101.3$

Now this work is used up in heating water

$\therefore W = n \times C \times \Delta T = (10 \times 18) \times 4.184 \times \Delta T = 607.8$

$\therefore \Delta T = 0.80 \therefore \Delta T \times 10 = 8$

52.  sulphonilic acid  Picric acid  Aspirin  Ascarbic acid

53. $\pi = \dfrac{\pi_1 V_1 + \pi_2 V_2}{(V_1 + V_2)} = \dfrac{2.4 \times 2V + 4.2 \times V}{2V + V} = 3\text{atm}$

54. $\text{I.E} = \dfrac{hc}{\lambda} = \dfrac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{663 \times 10^{-9}} \times 6.02 \times 10^{23} = 180.6\text{KJ/mole} = 181\text{KJ}$

55. $\%\text{ of bromine} = \dfrac{80 \times 0.12 \times 100}{188 \times 0.15} = 34.04$

56. Cs used as electrode in photoelectric cell

57. $[NH_4OH] = \dfrac{100 \times 0.1}{150}$ $\quad [NH_4^+] = \dfrac{2 \times 50 \times 0.1}{150}$

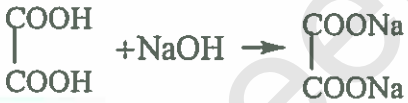
$pOH = pk_b + \log \dfrac{100 \times 0.1}{100 \times 0.1} = 4.7$

$\therefore pH = 14 - 4.7 = 9.3 = 93 \times 10^{-1}$

58. $v_f = \dfrac{5 \times 8.314 \times 293}{1.3 \times 10^{6}} = 9.369 \times 10^{-3}$, $v_i = \dfrac{P_2 V_2}{V_1} = \dfrac{1.3 \times 9.369 \times 10^{-3}}{2.1}$

$w = P_{ext}(\Delta V) = 4.3 \times 5 \times \dfrac{8.314}{10^{6}} \times 293 \left(\dfrac{1}{1.3} - \dfrac{1}{2.1} \right) = 15.3$

59. Structure of sucralose molecule has three C–O–C links

60. 

Mill eq oxalic acid = milli eq NaOH

$v.f \times M_1 \times v_1 = v.f \times M_2 \times v_2$, $2 \times 1.25 \times 10 = 1 \times M_2 \times 4.4$. $M_2 = \dfrac{2.5 \times 10}{4.4} = 5.8\text{M}$

MATHEMATICS

61. Let $z = a + ib$

$a + ib + \sqrt{a^2 + b^2} = 2 + 8i \Rightarrow b = 8$

$a + \sqrt{a^2 + 64} = 2$

$a^2 + 64 = a^2 - 4a + 4 \Rightarrow 4a = -60 \Rightarrow a = -15$

$\therefore |z| = \sqrt{15^2 + 8^2} = \sqrt{289} = 17$

62. $x^2-6x-2=0$; roots α,β

$\left.\begin{aligned}\alpha^2-6\alpha-2=0\\ \beta^2-6\beta-2=0\end{aligned}\right\}$ $\Rightarrow \alpha^{10}=6\alpha^9+2\alpha^8$, $\beta^{10}=6\beta^9+2\beta^8$, $a_8=\alpha^8-\beta^8$

$a_{10}=\alpha^{10}-\beta^{10}=6(\alpha^9-\beta^9)+2(\alpha^8-\beta^8)$

$a_{10}=6a_9+2a_8 \Rightarrow \dfrac{a_{10}-2a_8}{3a_9}=2$

63. 1st term of $S_k=1+3+6+.....+3(k-1)$ $=1+3\dfrac{k(k-1)}{2}$

64. Consider $x^2=2y$, Diff Wrt x, $2x=2\dfrac{dy}{dx}\Rightarrow \dfrac{dy}{dx}=x=1$

Point on the curve $P(1,\dfrac{1}{2})$

Eq of tangent to the curve at P is

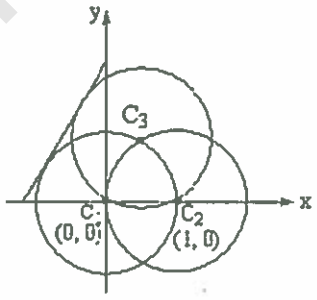
$y-\dfrac{1}{2}=1(x-1)$, $y=x-\dfrac{1}{2}\Rightarrow x-y=\dfrac{1}{2}$

S.D is $=\dfrac{1-\frac{1}{2}}{\sqrt{1+1}}=\dfrac{1}{2\sqrt{2}}$

65. Equation of any circle through (0, 0) and (1, 0)

$$(x-0)(x-1)+(y-0)(y-0)+\lambda\begin{vmatrix}x & y & 1\\ 0 & 0 & 1\\ 1 & 0 & 1\end{vmatrix}=0\Rightarrow x^2+y^2-x+\lambda y=0$$

If it represents C_3, its radius = 1 $\Rightarrow 1=(1/4)+(\lambda^2/4)\Rightarrow \lambda=\pm\sqrt{3}$



As the centre of C_3, lies above the x-axis, we take $\lambda=-\sqrt{3}$ and thus an equation of C_3 is $x^2+y^2-x-\sqrt{3}y=0$. Since C_1 and C_2 interest and are of unit radius, their common tangents are parallel to the joining their centres (0, 0) and $\left(\dfrac{1}{2},\dfrac{\sqrt{3}}{2}\right)$.So, let the equation of a common tangents be $\sqrt{3}x-y+2=0$ It will touch C_1, if $\left|\dfrac{k}{\sqrt{3}+1}\right|=1\Rightarrow k=\pm 2$

From the figure, we observe that the required tangent makes positive intercept on the y-axis and negative on the x-axis and hence its equation to $\sqrt{3}x - y + 2 = 0$

66. x denotes total no of $1-1$ function from A to B

A to B

$n(A) = 3;\ n(B) = 5$

$x = {}^5P_3 = 5 \times 4 \times 3 = 60$

Y denotes total no of $1-1$ function from A to A x B

$n(A) = 3,\ n(A \times B) = 15$

$y = {}^{15}P_3 = 15 \times 14 \times 13 = 2730$

$\frac{Y}{X} = \frac{2730}{60} = \frac{91}{2}, 2Y = 91X$

67. $x^2 + y^2 = 1; x^2 + 2y^2 = 4$

Let $R(x_1, y_1)$ is pt of intersection of tangents drawn at P,Q to ellipse

$\Rightarrow PQ$ is chord of contact of $R(x_1, y_1) \Rightarrow xx_1 + 2yy_1 - 4 = 0$

This touches circle $\Rightarrow r^2(\ell^2 + m^2) = n^2 \Rightarrow 1(x_1^2 + 4y_1^2) = 16$

$\Rightarrow x^2 + 4y^2 = 16$ is ellipse $e = \frac{\sqrt{3}}{2}; LL' = 2$

68. $\int \frac{e^{3\log_e 2x} + 5e^{2\log_e 2x}}{e^{4\log_e x} + 5e^{3\log_e x} - 7e^{2\log_e x}} dx; x > 0$

$= \int \frac{8x^3 + 5.4x^2}{x^4 + 5x^3 - 7x^2} dx = \int \frac{8x + 20}{x^2 + 5x - 7} dx = 4\log\left|x^2 + 5x - 7\right| + C$

69. $\alpha = 3$ and $\beta = 1$

70. $A = \begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix} AA^T = I;\ \alpha^4 + \beta^4 = ...$

$AA^T = \begin{pmatrix} 1 & -\alpha \\ \alpha & \beta \end{pmatrix}\begin{pmatrix} 1 & \alpha \\ -\alpha & \beta \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1+\alpha^2 & \alpha - \alpha\beta \\ \alpha - \alpha\beta & \alpha^2 + \beta^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\Rightarrow \alpha = \alpha\beta \quad 1 + \alpha^2 = 1 \Rightarrow \alpha = 0 \qquad \Rightarrow \alpha(1 - \beta) = 0 \Rightarrow \beta = 1$

$\alpha^4 + \beta^4 = 1$

71. $f(x) = -\frac{\left(x^2 - 2x + 2\sqrt{3} - 3\right)}{x - \sqrt{3}}$

for f(x) to be continuous $f(\sqrt{3}) = f(\sqrt{3}^+) = f(\sqrt{3}^-) = 2(1 - \sqrt{3})$.

72. Equation of ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$, $e=\frac{3}{5}$, s(3,0), s'(−3,0)

Equation of hyperbola $\frac{x^2}{A^2}-\frac{y^2}{B^2}=1$

Satisfies $(\pm 3,0) \Rightarrow A^2=9$

$eE=1 \Rightarrow E=\frac{1}{e}=\frac{5}{3}$, Where "e" is eccentricity of ellipse and "E" is eccentricity of hyperbola

$B^2=A^2(E^2-1) \Rightarrow B^2=16$, Hyperbola $\frac{x^2}{9}-\frac{y^2}{16}=1$

73. $\frac{dy}{dx}-Ky=0, \frac{dy}{y}=Kdx$

$\ln y=Kx+c$

At $x=0, y=1 \therefore C=0$

Now, $\ln y=Kx$; $y=e^{Kx}$

$\lim_{x\to\infty} y=\lim_{x\to\infty} e^{Kx}=0 \quad \therefore K<0$

74. $\left.\begin{matrix}2x+3y+2z=9\\3x+2y+2z=9\\x-y+4z=8\end{matrix}\right\}$; $D=\begin{vmatrix}2 & 3 & 2\\3 & 2 & 2\\1 & -1 & 4\end{vmatrix}=2(10)-3(10)+2(10)=10\neq 0$ Unique solution

75. $\vec{n}_1=OA\times OB=\begin{vmatrix}i & j & k\\1 & 2 & 1\\2 & 1 & 3\end{vmatrix}=5i-j-3k$; $\vec{n}_2=AB\times AC=\begin{vmatrix}i & j & k\\1 & -1 & 2\\-2 & -1 & 1\end{vmatrix}=i-5j-3k$

$\theta=\cos^{-1}\left(\frac{\vec{n}_1.\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}\right)=\cos^{-1}\left(\frac{19}{35}\right)$

76. $I_n=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\cot^n x\,dx$

Consider $I_n+I_{n+2}=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(\cot^n x+\cot^{n+2} x)\,dx=\int_{\frac{\pi}{4}}^{\pi/2}\cot^n x.\operatorname{cosec}^2 x\,dx$

$=-\frac{\cot^{n+1} x}{n+1}\Big]_{\pi/4}^{\pi/2}=\frac{1}{n+1}$; $\frac{1}{I_{2+I_4}}=3;\frac{1}{I_3+I_5}=4;\frac{1}{I_4+I_6}=5$ are in A.P.

77. Taking $\log_b$ both sides we get

$$\left(\log_a^b\right)^x \log_b^a = \left(\log_b^a\right)^x \therefore \left(\log_a^b\right)^x = \left(\log_b^a\right)^{x-1} \quad \therefore 1-x = x \Rightarrow x = \frac{1}{2}$$

78. $$\text{non veg + smokers} \rightarrow \frac{160}{400} \times \frac{35}{100}$$

$$\text{veg + smokers} \rightarrow \frac{100}{400} \times \frac{20}{100}$$

$$\text{Veg + non smokers} \rightarrow \frac{140}{400} \times \frac{10}{100}$$

$$\text{Req proba} = \frac{160 \times 35}{100 \times 20 + 140 \times 10 + 160 \times 35} = \frac{16 \times 35}{200 + 140 + 560} = \frac{560}{900} = \frac{28}{45}$$

79. Given $\frac{S_3}{S_6} = \frac{64}{91} = \frac{a\left(r^3-1\right)}{a\left(r^6-1\right)} \Rightarrow \frac{\left(r^3-1\right)}{\left(r^3+1\right)\left(r^3-1\right)} = \frac{64}{91} \Rightarrow r^3 = \frac{27}{64} \quad \therefore \quad r = \frac{3}{4}$

80. $$\lim_{n\to\infty}\left(\frac{1}{n} + \frac{n}{(n+1)^2} + \frac{n}{(n+2)^2} \ldots\ldots \frac{n}{(2n-1)^2}\right), \quad \lim_{n\to\infty} \frac{n}{n^2}\left\{\sum_{r=1}^{n} \frac{1}{\left(1+\frac{r}{n}\right)^2}\right.$$

$$= \lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{n} \frac{1}{\left(1+\frac{r}{n}\right)^2} = \int_0^1 \frac{1}{(1+x)^2}\,dx \qquad = \frac{-1}{1+x}\Big)_0^1 = \frac{-1}{2} + 1 = \frac{1}{2}$$

81. $$\text{ON.OT} = 97\cos\theta . 97\sec\theta = 97^2 \quad \therefore \left[\frac{\text{ON.OT}}{2011}\right] = \left[\frac{97^2}{2011}\right] = 4$$

82. $$\int_{-2}^{2} \left|3x^2 - 3x - 6\right| dx = 3\int_{-2}^{2} \left|x^2 - x - 2\right| dx$$

$$= 3\int_{-2}^{2} \left|(x-2)(x+1)\right| dx = 3\left\{\int_{-2}^{-1} (x^2 - x - 2)dx + \int_{-1}^{2} (2 + x - x^2)dx\right\} = 19$$

83. Conceptual

84. $x = y^4$, $xy = k$; Diff wrt x

$$1 = 4y^3 . \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{4y^3}; \; y = \frac{k}{x}$$

$$\frac{dy}{dx} = \frac{-k}{x^2}$$ Two curves cut right angle

$$\frac{1}{4y^3}\times\frac{-k}{x^2}=-1\Rightarrow k=4x^2y^3\Rightarrow y^5=4x^2y^3$$

$$\Rightarrow y^2=4x^2\Rightarrow y=\pm 2x$$

$$k=4x^2\times 8x^3\Rightarrow 2x^2=4x^2\times 8x^3\Rightarrow x^3=\frac{1}{16}$$

$$4^6k^6=4^6\times 4^6\times 8^6\times (x^3)^{10}=4^6\times 4^6\times 8^6=\frac{1}{16^{10}}$$

$$4^6\times 4^6\times 8^6\times\frac{1}{4^{10}\times 4^{10}}=4^4\times 4^4=\frac{4\times 2}{4^4\times 4^4}=\frac{64}{16}=4$$

85. Given $f3(x)=\int_0^x tf^2(t)dt$ .... (1)

Differentiating both the sides of equation (1) by using leibnitz, we get

$3f2(x)=xf2(x)\because f(x)=\frac{x^2}{6}, x\geq 0$

Slope of normal $=\frac{-3}{x_1}\Rightarrow x1=6$ and $y1=6$

Hence y – intercepts = 9

86. $\underset{x\to 0}{Lt}\frac{ax-(e^x-1)}{ax.(e^{4x}-1)}=b$

$$\underset{x\to 0}{Lt}\frac{a-4e^{4x}}{ax4e^{4x}+ae^{4x}-a}=b$$

$a-4=0\Rightarrow a=4, b=-\frac{1}{2}$; $a-2b=5$

87. Let E_1,E_2,E_3 be the events that two headed coin, biased coin, unbiased coin is chosen respectively $P(E_1)=P(E_2)=P(E_3)=\frac{1}{3}$ Let E be the even that heads show up

$P(E)=\sum_{i=1}^{3}P(E_i).P\left(\frac{E}{E_i}\right)$ By bayes theorem $P\left(\frac{E}{E_1}\right)=\frac{P(E_1).P(E/E_1)}{P(E)}$

$$=\frac{\left(\frac{1}{3}\right)(1)}{\left(\frac{1}{3}\right)(1)+\frac{1}{3\left(\frac{3}{4}\right)}+\frac{1}{3}\left(-\frac{1}{2}\right)}=\frac{4}{9}$$

88. $\vec{a}=\vec{i}+\alpha\vec{j}+3\vec{k}$ $\vec{b}=3\hat{i}-\alpha\vec{j}+\vec{k}$

$$\left|\vec{a}\times\vec{b}\right|=\begin{vmatrix} i & j & k\\ 1 & \alpha & 3\\ 3 & -\alpha & 1\end{vmatrix}$$

$i(\alpha+3\alpha)-j(-8)+\vec{k}(-\alpha-3\alpha)$

$=\left|4\alpha i+8\vec{j}-4\alpha\vec{k}\right|=\sqrt{16\alpha^3+64+16\alpha^2}=32\alpha^2+64=64\times3$

$\Rightarrow 32\alpha^2=192-64=128\Rightarrow\alpha^2=\frac{128}{32}=4\Rightarrow\alpha=\pm2$

$\vec{a}=i+2j+3k;\ \vec{b}=3i-2j+k\ ;\ \vec{a}.\vec{b}=3-4+3=2$

89. Solving the altitudes AM, CL

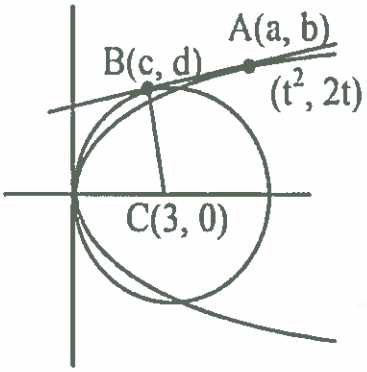
Orthocenter $=\left(\frac{1}{2},\frac{1}{2}\right)$ lies on $x+y-k=0\ ;\ k=1$

90. $(x-b)^2+y^2=9;\ \ y^2=4x$

let $y=mx+\frac{1}{m}$ be a tan gent to $y^2=4x, m^2x-my+1=0$ is also tan gent to circle

C(3,0) r = 3, r = d

$3=\frac{\left|3m^2+1\right|}{\sqrt{m^4+m^2}}\Rightarrow 9(m^4+m^2)=9m^4+1+6m^2$



$\Rightarrow 9m^2+=6m^2+1$

$m^2=\frac{1}{3}$

$y=\frac{1}{\sqrt{3}}x+\sqrt{3}\Rightarrow x-\sqrt{3}y+\sqrt{3}=0$ $\qquad (x-3)^2+\left(\frac{x+3}{\sqrt{3}}\right)^2=9$ $\quad y=\frac{3}{2}.\frac{1}{\sqrt{3}}+\sqrt{3}$

$3(x-3)^2+(x+3)^2=27$ $\qquad y=\frac{\sqrt{3}}{2}+\sqrt{3}=3\frac{\sqrt{3}}{2}$

$\Rightarrow 3x^2+27-18x+x^2+6x+9=27\ \Rightarrow 4x^2-12x+9=0$

$(2x-3)=0$ $\qquad x=\frac{3}{2}$ $\qquad$ and $\qquad \left(\frac{x+3}{\sqrt{3}}\right)^2=4x$

$2(a+c)=2\left(\frac{3}{2}+3\right)=9$ $\qquad x=3$

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