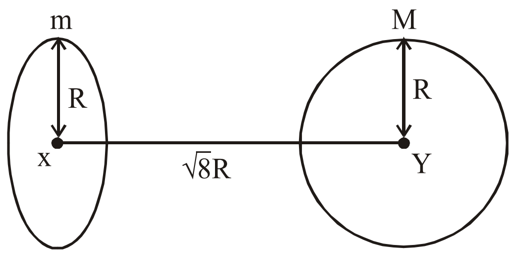
PHYSICS

SECTION-A

1. Find the gravitational force of attraction between the ring and sphere as shown in the diagram, where the plane of the ring is perpendicular to the line joining the centres. If $\sqrt{8}R$ is the distance between the centres of a ring (of mass 'm') and a sphere (mass 'M') where both have equal radius 'R'.



(1) $\frac{\sqrt{8}}{9}\cdot\frac{GmM}{R}$ (2) $\frac{2\sqrt{2}}{3}\cdot\frac{GMm}{R^2}$

(3) $\frac{1}{3\sqrt{8}}\cdot\frac{GMm}{R^2}$ (4) $\frac{\sqrt{8}}{27}\cdot\frac{GmM}{R^2}$

Official Ans. by NTA (4)

Sol. Gravitational field of ring

$$= -\frac{Gmx}{\left(R^2+x^2\right)^{3/2}}$$

Force between sphere & ring

$$= \frac{GmM\left(\sqrt{8}R\right)}{\left(R^2+8R^2\right)^{3/2}}$$

$$= \frac{GmM}{R^2}\times\frac{\sqrt{8}}{27}$$

Ans. (4)

2. Consider the combination of 2 capacitors C_1 and C_2, with $C_2 > C_1$, when connected in parallel, the equivalent capacitance is $\frac{15}{4}$ time the equivalent capacitance of the same connected in series. Calculate the ratio of capacitors, $\frac{C_2}{C_1}$.

(1) $\frac{15}{11}$ (2) $\frac{111}{80}$ (3) $\frac{29}{15}$ (4) $\frac{15}{4}$

Official Ans. by NTA (2)

Allen Ans. (Bonus)

Sol. When connected in parallel

$C_{eq} = C_1 + C_2$

When in series

$$C'_{eq} = \frac{C_1C_2}{C_1+C_2}$$

$$C_1 + C_2 = \frac{15}{4}\left(\frac{C_1C_2}{C_1+C_2}\right)$$

$4\,(C_1 + C_2)^2 = 15\,C_1C_2$

$4\,C_1^2 + 4C_2^2 - 7\,C_1C_2 = 0$

dividing by C_1^2

$$4\left(\frac{C_2}{C_1}\right)^2 - \frac{7C_2}{C_1} + 4 = 0$$

Let $\frac{C_2}{C_1} = x$

$4x^2 - 7x + 4 = 0$

$b^2 - 4ac = 49 - 64 < 0$

No solution exits

Ans. (Bonus)

3. In a typical combustion engine the work done by a gas molecule is given $W = \alpha^2 \beta e^{\frac{-\beta x^2}{kT}}$, where x is the displacement, k is the Boltzmann constant and T is the temperature. If α and β are constants, dimensions of α will be :

(1) $[MLT^{-2}]$ (2) $[M^0LT^0]$
(3) $[M^2LT^{-2}]$ (4) $[MLT^{-1}]$

Official Ans. by NTA (2)

Sol. kT has dimension of energy

$\frac{\beta x^2}{kT}$ is dimensionless

$[\beta]\,[L^2] = [ML^2T^{-2}]$

$[\beta] = [MT^{-2}]$

$\alpha^2\beta$ has dimensions of work

$[\alpha^2]\,[MT^{-2}] = [ML^2\,T^{-2}]$

$[\alpha] = [M^0LT^0]$

Ans. 2

4. If λ_1 and λ_2 are the wavelengths of the third member of Lyman and first member of the Paschen series respectively, then the value of $\lambda_1 : \lambda_2$ is :

(1) 1 : 9 (2) 7 : 108
(3) 7 : 135 (4) 1 : 3

Official Ans. by NTA (3)

Sol. $$\frac{1}{\lambda_1} = R\left[\frac{1}{1^2} - \frac{1}{4^2}\right]$$

$$\frac{1}{\lambda_2} = R\left[\frac{1}{3^2} - \frac{1}{4^2}\right]$$

$$\frac{\lambda_1}{\lambda_2} = \frac{\left[\frac{1}{9} - \frac{1}{16}\right]}{\left[1 - \frac{1}{16}\right]} = \frac{7}{9 \times 15}$$

$$\frac{\lambda_1}{\lambda_2} = \frac{7}{135}$$

Ans. (3)

5. A short straight object of height 100 cm lies before the central axis of a spherical mirror whose focal length has absolute value $|f| = 40$cm. The image of object produced by the mirror is of height 25 cm and has the same orientation of the object. One may conclude from the information :

(1) Image is real, same side of concave mirror.
(2) Image is virtual, opposite side of concave mirror.
(3) Image is real, same side of convex mirror.
(4) Image is virtual, opposite side of convex mirror.

Official Ans. by NTA (4)

Sol. Since orientation is same image is virtual. Since image is smaller the mirror has to be convex

Ans. (4)

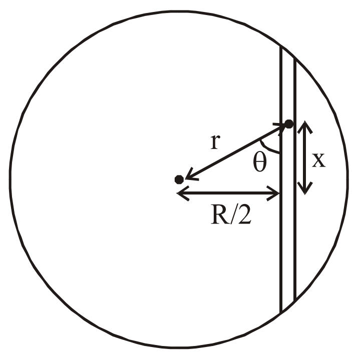
6. Assume that a tunnel is dug along a chord of the earth, at a perpendicular distance (R/2) from the earth's centre, where 'R' is the radius of the Earth. The wall of the tunnel is frictionless. If a particle is released in this tunnel, it will execute a simple harmonic motion with a time period :

(1) $\frac{2\pi R}{g}$ (2) $\frac{g}{2\pi R}$

(3) $\frac{1}{2\pi}\sqrt{\frac{g}{R}}$ (4) $2\pi\sqrt{\frac{R}{g}}$

Official Ans. by NTA (4)

Sol.



Force along the tunnel

$$F=-\left(\frac{GMmr}{R^3}\right)\cos\theta$$

$$F=-\frac{gm}{R}x \quad \left(\frac{GM}{R^2}=g, r\cos\theta=x\right)$$

$$a=-\frac{g}{R}x$$

$$\omega^2=\frac{g}{R} \qquad T=2\pi\sqrt{\frac{R}{g}}$$

Ans. (4)

7. An alternating current is given by the equation $i = i_1 \sin\omega t + i_2 \cos\omega t$. The rms current will be

(1) $\frac{1}{\sqrt{2}}\left(i_1^2+i_2^2\right)^{\frac{1}{2}}$ (2) $\frac{1}{\sqrt{2}}\left(i_1+i_2\right)^2$

(3) $\frac{1}{2}\left(i_1^2+i_2^2\right)^{\frac{1}{2}}$ (4) $\frac{1}{\sqrt{2}}\left(i_1+i_2\right)$

Official Ans. by NTA (1)

Sol. $i = i_1\sin\omega t + i_2\sin(\omega t + 90)$

$$i=\sqrt{i_1^2+i_2^2}\sin(\omega t+\phi)$$

$$i_{rms}=\frac{i_0}{\sqrt{2}}=\frac{\sqrt{i_1^2+i_2^2}}{\sqrt{2}}$$

8. The normal density of a material is ρ and its bulk modulus of elasticity is K. The magnitude of increase in density of material, when a pressure P is applied uniformly on all sides, will be :

(1) $\frac{\rho K}{P}$ (2) $\frac{\rho P}{K}$

(3) $\frac{K}{\rho P}$ (4) $\frac{PK}{\rho}$

Official Ans. by NTA (2)

Sol. $\rho=\frac{M}{V}$

$$\frac{d\rho}{\rho}=-\frac{dV}{V}$$

$$k=-\frac{P}{\frac{dV}{V}}$$

$$-\frac{dV}{V}=\frac{P}{k}$$

$$\frac{d\rho}{\rho}=\frac{P}{k}\Rightarrow d\rho=\frac{\rho P}{k}$$

9. A particle is moving with uniform speed along the circumference of a circle of radius R under the action of a central fictitious force F which is inversely proportional to R^3. Its time period of revolution will be given by :

(1) $T \propto R^2$

(2) $T\propto R^{\frac{3}{2}}$

(3) $T\propto R^{\frac{5}{2}}$

(4) $T\propto R^{\frac{4}{3}}$

Official Ans. by NTA (1)

Sol. $F\propto\frac{1}{R^3}$

$$\frac{K}{R^3}=m\omega^2R$$

$$\omega^2=\frac{K}{m}\times\frac{1}{R^4}$$

$$\left(\frac{2\pi}{T}\right)^2=\frac{K}{m}\times\frac{1}{R^4}$$

$T^2 \propto R^4$

$T \propto R^2$

10. A planet revolving in elliptical orbit has :

(A) a constant velocity of revolution.

(B) has the least velocity when it is nearest to the sun.

(C) its areal velocity is directly proportional to its velocity.

(D) areal velocity is inversely proportional to its velocity.

(E) to follow a trajectory such that the areal velocity is constant.

Choose the correct answer from the options given below :

(1) A only (2) D only

(3) C only (4) E only

Official Ans. by NTA (4)

Sol. As per Keppler's 2nd law, Areal velocity is constant.

11. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Body 'P' having mass M moving with speed 'u' has head-on collision elastically with another body 'Q' having mass 'm' initially at rest. If m << M, body 'Q' will have a maximum speed equal to '2u' after collision.

Reason R : During elastic collision, the momentum and kinetic energy are both conserved.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) A is not correct but R is correct.

(2) Both A and R are correct but R is NOT the correct explanation of A.

(3) Both A and R are correct and R is the correct explanation of A.

(4) A is correct but R is not correct.

Official Ans. by NTA (3)

Sol. For e = 1 & second body at rest

$$V_2 = \frac{2m_1u_1}{m_1+m_2} = \frac{2u(M)}{M+m} \simeq 2u$$

Since M >> m

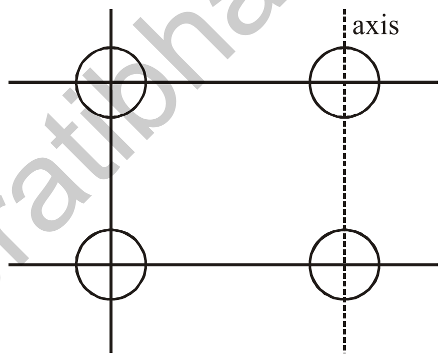
12. Four identical solid spheres each of mass 'm' and radius 'a' are placed with their centres on the four corners of a square of side 'b'. The moment of inertia of the system about one side of square where the axis of rotation is parallel to the plane of the square is :

(1) $\frac{4}{5}ma^2 + 2mb^2$ (2) $\frac{8}{5}ma^2 + mb^2$

(3) $\frac{8}{5}ma^2 + 2mb^2$ (4) $\frac{4}{5}ma^2$

Official Ans. by NTA (3)

Sol.



$$I = 2\times\left(\frac{2}{5}ma^2\right) + 2\times\left(\frac{2}{5}ma^2 + mb^2\right)$$

$$I = \frac{8}{5}ma^2 + 2mb^2$$

13. In a Young's double slit experiment two slits are separated by 2 mm and the screen is placed one meter away. When a light of wavelength 500 nm is used, the fringe separation will be:

(1) 0.25 mm (2) 0.50 mm

(3) 0.75 mm (4) 1 mm

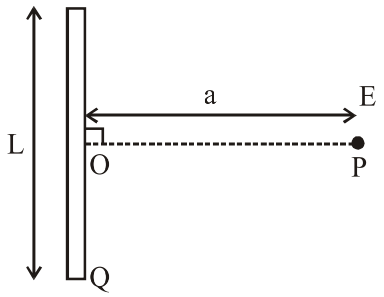
Official Ans. by NTA (1)

Sol. $$\beta = \frac{\lambda D}{d} = \frac{500\times10^{-9}\times1}{2\times10^{-3}}$$

$$\beta = \frac{5}{2}\times10^{-4}m = 2.5\times10^{-1}mm$$

b = 0.25 mm

14. Find the electric field at point P (as shown in figure) on the perpendicular bisector of a uniformly charged thin wire of length L carrying a charge Q. The distance of the point P from the centre of the rod is $a=\frac{\sqrt{3}}{2}L$.



(1) $\frac{\sqrt{3}Q}{4\pi\varepsilon_0 L^2}$ (2) $\frac{Q}{3\pi\varepsilon_0 L^2}$

(3) $\frac{Q}{2\sqrt{3}\pi\varepsilon_0 L^2}$ (4) $\frac{Q}{4\pi\varepsilon_0 L^2}$

Official Ans. by NTA (3)

Sol. $E=\frac{k\lambda}{a}(\sin\theta_1+\sin\theta_2)$

$$E=\frac{1}{4\pi\varepsilon_0}\times\frac{Q}{L}\times\frac{1}{\left(\frac{\sqrt{3}L}{2}\right)}\times(2\sin\theta)$$

$$\tan\theta=\frac{L/2}{\frac{\sqrt{3}L}{2}}=\frac{1}{\sqrt{3}}$$

$\sin\theta = \frac{1}{2}$

$$E=\frac{1}{4\pi\varepsilon_0}\times\frac{2Q}{\sqrt{3}L^2}\times\left(2\times\frac{1}{2}\right)$$

$$E=\frac{Q}{2\sqrt{3}\pi\varepsilon_0 L^2}$$

15. If two similar springs each of spring constant K_1 are joined in series, the new spring constant and time period would be changed by a factor:

(1) $\frac{1}{2},\sqrt{2}$ (2) $\frac{1}{4},\sqrt{2}$

(3) $\frac{1}{4},2\sqrt{2}$ (4) $\frac{1}{2},2\sqrt{2}$

Official Ans. by NTA (1)

Sol. $\frac{1}{k_{eq}}=\frac{1}{k_1}+\frac{1}{k_2}$

$$\frac{1}{k_{eq}}=\frac{1}{k}+\frac{1}{k}\Rightarrow k_{eq}=\frac{k}{2}$$

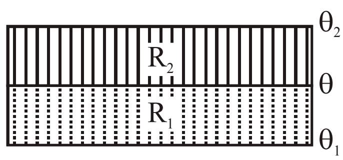
$k'=\frac{k}{2}$

$T=2\pi\sqrt{\frac{M}{k}}$ $\quad$ $T'=2\pi\sqrt{\frac{M}{k'}}$

$$\Rightarrow T'=2\pi\sqrt{\frac{M}{k}}\times\sqrt{2}$$

$T'=\sqrt{2}T$

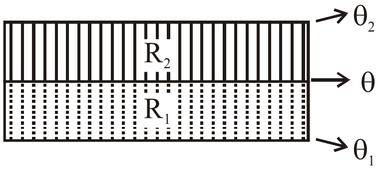
16. The temperature θ at the junction of two insulating sheets, having thermal resistances R_1 and R_2 as well as top and bottom temperatures θ_1 and θ_2 (as shown in figure) is given by :



(1) $\frac{\theta_2R_2-\theta_1R_1}{R_2-R_1}$ (2) $\frac{\theta_1R_2-\theta_2R_1}{R_2-R_1}$

(3) $\frac{\theta_1R_2+\theta_2R_1}{R_1+R_2}$ (4) $\frac{\theta_1R_1+\theta_2R_2}{R_1+R_2}$

Official Ans. by NTA (3)

Sol. 

Heat flow rate will be same through both

$$\therefore \quad \frac{\theta_1-\theta}{R_1}=\frac{\theta-\theta_2}{R_2}$$

$R_2\theta_1 - R_2\theta = R_1\theta - R_1\theta_2$

$$\theta=\frac{R_2\theta_1+R_1\theta_2}{R_1+R_2}$$

Ans. (3)

17. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : An electron microscope can achieve better resolving power than an optical microscope.

Reason R : The de Broglie's wavelength of the electrons emitted from an electron gun is much less than wavelength of visible light.

In the light of the above statements, choose the correct answer from the options given below:

(1) A is true but R is false.

(2) Both A and R are true and R is the correct explanation of A.

(3) Both A and R are true but R is NOT the correct explanation of A.

(4) A is false but R is true.

Official Ans. by NTA (2)

Sol. Resolving power $\propto \frac{1}{\lambda}$

Since wavelength of electron is much less than visible light, its resolving power will be much more.

18. LED is constructed from Ga-As-P semiconducting material. The energy gap of this LED is 1.9 eV. Calculate the wavelength of light emitted and its colour.

[$h = 6.63 \times 10^{-34}$ Js and $c = 3 \times 10^{8}$ ms^{-1}]

(1) 1046 nm and red colour

(2) 654 nm and orange colour

(3) 1046 nm and blue colour

(4) 654 nm and red colour

Official Ans. by NTA (4)

Sol. $$\lambda=\frac{hc}{E}=\frac{6.6\times10^{-34}\times3\times10^{8}}{1.9\times1.6\times10^{-19}}=6.54\times10^{-7}$$

= 654 nm

Red color

19. A large number of water drops, each of radius r, combine to have a drop of radius R. If the surface tension is T and mechanical equivalent of heat is J, the rise in heat energy per unit volume will be:

(1) $\frac{2T}{J}\left(\frac{1}{r}-\frac{1}{R}\right)$ (2) $\frac{2T}{rJ}$

(3) $\frac{3T}{rJ}$ (4) $\frac{3T}{J}\left(\frac{1}{r}-\frac{1}{R}\right)$

Official Ans. by NTA (4)

Sol. $$n\times\frac{4}{3}\pi r^3=\frac{4}{3}\pi R^3$$

$\therefore n^{1/3}r = R$

$\therefore$ Total change in surface energy

$= (n(4\pi r^2) - 4\pi R^2)T$

$\Rightarrow 4\pi T\,(nr^2 - R^2)$

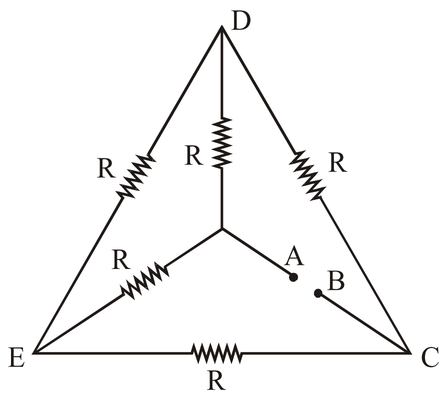
$\therefore$ Heat energy

$$=\frac{4\pi T\left(nr^2-R^2\right)}{J\times\frac{4}{3}\pi R^3}=\frac{3T}{J}\left(\frac{nr^2}{R^3}-\frac{1}{R}\right)$$

Put $nr^3 = R^3$

$$\therefore \frac{3T}{J}\left(\frac{1}{r}-\frac{1}{R}\right)$$

20. Five equal resistances are connected in a network as shown in figure. The net resistance between the points A and B is :

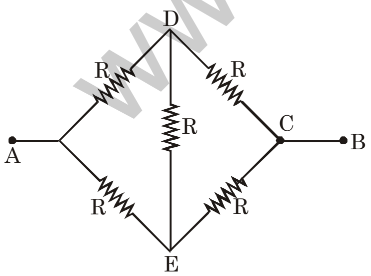


(1) 2R (2) $\frac{R}{2}$

(3) $\frac{3R}{2}$ (4) R

Official Ans. by NTA (4)

Sol. This diagram can be drawn like



It is a wheat stone bridge

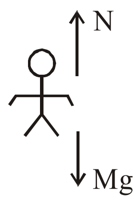
$$\therefore\ R_{eq} = \frac{2R\times 2R}{2R+2R} \Rightarrow R$$

SECTION-B

1. A person standing on a spring balance inside a stationary lift measures 60 kg. The weight of that person if the lift descends with uniform downward acceleration of 1.8 m/s^2 will be_ N. [g = 10 m/s^2]

Official Ans. by NTA (492)

Sol.

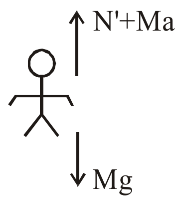


When lift is at rest

N = mg

$\Rightarrow 60 \times 10 = 600$ N

When lift moves with downward acceleration. In frame of lift pseudo force will be in upward direction.



$N' = M(g - a)$

$\Rightarrow 60\,(10 - 1.8)$

$N' \Rightarrow 492$ N

2. In an electrical circuit, a battery is connected to pass 20 C of charge through it in a certain given time. The potential difference between two plates of the battery is maintained at 15 V. The work done by the battery is ______J.

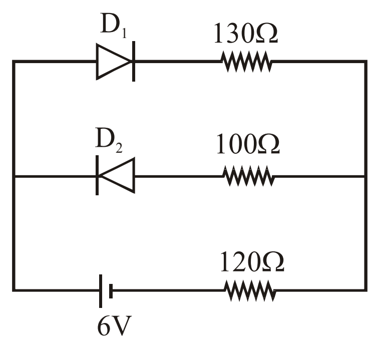
Official Ans. by NTA (300)

Sol. Work done by battery = Q (ΔV)

$\Rightarrow 20 \times 15 = 300$ J

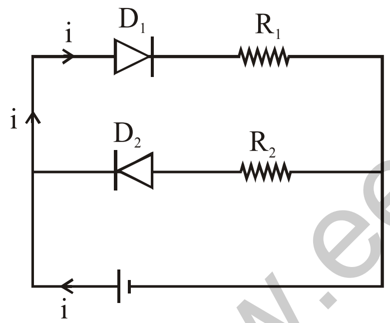
$\therefore$ Ans. 300

3. The circuit contains two diodes each with a forward resistance of 50 Ω and with infinite reverse resistance. If the battery voltage is 6 V, the current through the 120 Ω resistance is_ mA.



Official Ans. by NTA (20)

Sol.



In this circuit D_1 will be forward bias and D_2 will be revers bias.

$\therefore$ There will be no current through D_2 and R_2

Apply KVL in circuit we get

$+ 6 - 50i - 130i - 120i = 0$

$$i = \frac{6}{300} A = \frac{6}{300} \times 1000 mA$$

$\Rightarrow$ 20 mA

4. A radiation is emitted by 1000 W bulb and it generates an electric field and magnetic field at P, placed at a distance of 2 m. The efficiency of the bulb is 1.25%. The value of peak electric field at P is x × 10^{-1} V/m. Value of x is_. (Rounded-off to the nearest integer)

[Take $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2N^{-1}\ m^{-2}$, $c = 3 \times 10^8\ ms^{-1}$]

Official Ans. by NTA (137)

Sol. $I_{avg} = \frac{1}{2}\varepsilon_0 E_0^2 C$

$$\frac{1.25}{100} \times \frac{1000}{4\pi(2)^2} = \frac{1}{2} \times 8.85 \times 10^{-12} \times 3 \times 10^8 \times E_0^2$$

$E_0^2 = 187.4$

$\therefore E_0 = 13.689$ V/m

$= 136.89 \times 10^{-1}$ V/m

$\therefore x = 136.89$

Rounding off to nearest integer

x = 137

5. A boy pushes a box of mass 2 kg with a force $\vec{F} = (20\hat{i} + 10\hat{j})N$ on a frictionless surface. If the box was initially at rest, then ______ m is displacement along the x-axis after 10 s.

Official Ans. by NTA (500)

Sol. $\vec{F} = 20\hat{i} + 10\hat{j}$

$$\vec{a} = \frac{\vec{F}}{m} = \frac{20\hat{i} + 10\hat{j}}{2} \Rightarrow 10\hat{i} + 5\hat{j}$$

$$\therefore \vec{s} = \frac{1}{2}\vec{a}t^2 = \frac{1}{2}(10\hat{i} + 5\hat{j}) \times (10)^2$$

$$\Rightarrow 50(10\hat{i} + 5\hat{j})m$$

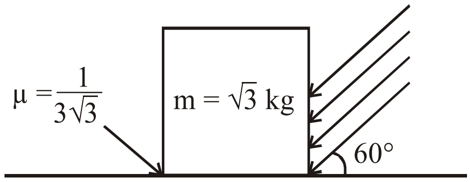
$\therefore$ Displacement along x-axis

$\Rightarrow 50 \times 10 \Rightarrow 500$ m

$\therefore$ Ans. 500

6. As shown in the figure, a block of mass $\sqrt{3}$ kg is kept on a horizontal rough surface of coefficient of friction $\frac{1}{3\sqrt{3}}$. The critical force to be applied on the vertical surface as shown at an angle 60° with horizontal such that it does not move, will be 3x. The value of x will be

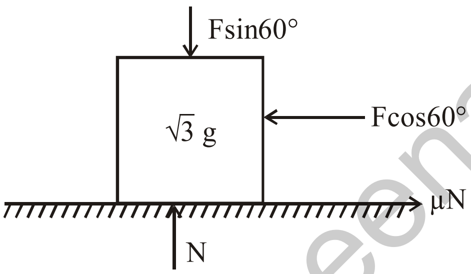
[g = 10 m/s²; sin 60° = $\frac{\sqrt{3}}{2}$; cos 60° = $\frac{1}{2}$]



Official Ans. by NTA (10)

Allen Ans. (3.33)

Sol.



F cos 60° = μN or $\frac{F}{2} = \frac{1}{3\sqrt{3}}N$ ... (1)

& $N = \sin 60° + \sqrt{3}g$ ... (2)

From equation (1) & (2)

$$\frac{F}{2} = \frac{1}{3\sqrt{3}}\left(\frac{F\sqrt{3}}{2} + \sqrt{3}g\right)$$

⇒ F = g = 10 Newton = 3x

So $x = \frac{10}{3}$ = 3.33

7. A container is divided into two chambers by a partition. The volume of first chamber is 4.5 litre and second chamber is 5.5 litre. The first chamber contain 3.0 moles of gas at pressure 2.0 atm and second chamber contain 4.0 moles of gas at pressure 3.0 atm. After the partition is removed and the mixture attains equilibrium, then, the common equilibrium pressure existing in the mixture is x × 10^{-1} atm. Value of x is_.

Official Ans. by NTA (25)

Allen Ans. (25.50)

Sol. Let common equilibrium pressure of mixture is P atmp. then

$U_1 + U_2 = U_{mixutre}$

$$\frac{f}{2}P_1V_1 + \frac{f}{2}P_2V_2 = \frac{f}{2}P(V_1 + V_2)$$

$$\frac{f}{2}(2)(4.5) + \frac{f}{2}(3)(5.5) = \frac{f}{2}P(4.5 + 5.5)$$

⇒ P = 2.55 = x × 10^{-1} atmp

So x = 25.5 ≈ 26 (Nearest integer)

8. The mass per unit length of a uniform wire is 0.135 g/cm. A transverse wave of the form y = –0.21 sin (x + 30t) is produced in it, where x is in meter and t is in second. Then, the expected value of tension in the wire is x × 10^{-2} N. Value of x is . (Round-off to the nearest integer)

Official Ans. by NTA (12)

Allen Ans. (1215)

Sol. μ = 0.135 gm/cm = 0.0135 kg/m

y = –0.21 sin (x + 30t)

(x in meter & t in sec)

$$v = \frac{\omega}{k} = \frac{30}{1} = 30 m/s$$

$v = \sqrt{\frac{T}{\mu}}$ ⇒ T = $v^2\mu$ = $(30)^2$ (0.0135)

= 12.15

= x × 10^{-2} N

⇒ x = 1215

9. In a series LCR resonant circuit, the quality factor is measured as 100. If the inductance is increased by two fold and resistance is decreased by two fold, then the quality factor after this change will be_____.

Official Ans. by NTA (400)

Allen Ans. (282.84)

Sol. $Q = \frac{X_L}{R} = \frac{\omega L}{R} = \frac{1}{\sqrt{LC}}\frac{L}{R} = \frac{\sqrt{L}}{R\sqrt{C}}$

$$Q' = \frac{\sqrt{2L}}{\left(\frac{R}{2}\right)\sqrt{C}} = 2\sqrt{2}Q = 2\sqrt{2}(100)$$

$= 282.84$

10. The maximum and minimum amplitude of an amplitude modulated wave is 16V and 8V respectively. The modulation index for this amplitude modulated wave is $x \times 10^{-2}$. The value of x is______.

Official Ans. by NTA (33)

Sol. Modulation index $= \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$

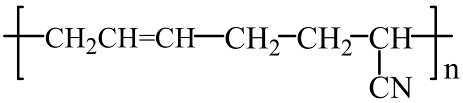
$$= \frac{16-8}{16+8} = \frac{8}{24} = \frac{1}{3} = 0.33$$

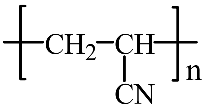
$x \times 10^{-2} = 0.33$

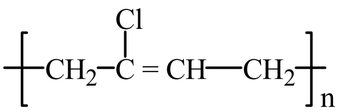
$x = 33$

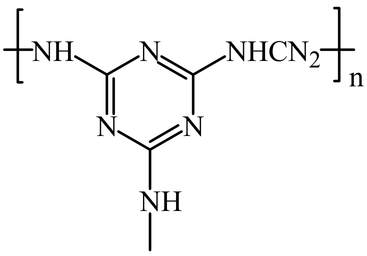
SECTION-A

1. The structure of Neoprene is -

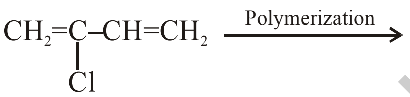
(1) 

(2) 

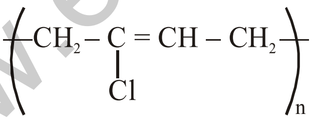
(3) 

(4) 

Official Ans. by NTA (3)

Sol. 

Chloroprene
2–Chloro–1, 3–Butadiene



Neoprene

2. Find A, B and C in the following reactions :

$NH_3 + A + CO_2 \rightarrow (NH_4)_2CO_3$

$(NH_4)_2CO_3 + H_2O + B \rightarrow NH_4HCO_3$

$NH_4HCO_3 + NaCl \rightarrow NH_4Cl + C$

(1) A – O_2 ; B – CO_2 ; C – Na_2CO_3

(2) A – H_2O ; B – O_2 ; C – Na_2CO_3

(3) A – H_2O ; B – O_2 ; C – $NaHCO_3$

(4) A – H_2O ; B – CO_2 ; C – $NaHCO_3$

Official Ans. by NTA (4)

Sol. $2NH_3 + \underset{(A)}{H_2O} + CO_2 \longrightarrow (NH_4)_2CO_3$

$(NH_4)_2CO_3 + H_2O + \underset{(B)}{CO_2} \longrightarrow 2NH_4HCO_3$

$NH_4HCO_3 + NaCl \longrightarrow \underset{(C)}{NaHCO_3} + NH_4Cl$

3. The presence of ozone in troposphere

(1) Protects us from the UV radiation

(2) Protects us from the X-ray radiation

(3) Protects us from greenhouse effect

(4) generates photochemical smog

Official Ans. by NTA (4)

Sol. The presence of ozone in troposphere generates photochemical smog.

4. Match List -I with List - II

List - I	**List - II**
Electronic configuration of elements	**Δ_i in kJ mol^{-1}**
(a) $1s^2 2s^2$	(i) 801
(b) $1s^2 2s^2 2p^4$	(ii) 899
(c) $1s^2 2s^2 2p^3$	(iii) 1314
(d) $1s^2 2s^2 2p^1$	(iv) 1402

Choose the most appropriate answer from the options given below -

(1) (a) → (ii), (b) → (iii), (c) → (iv),(d) → (i)

(2) (a) → (i), (b) → (iv), (c) → (iii),(d) → (ii)

(3) (a) → (i), (b) → (iii), (c) → (iv),(d) → (ii)

(4) (a) → (iv), (b) → (i), (c) → (ii),(d) → (iii)

Official Ans. by NTA (1)

Sol. (a) $1s^2\ 2s^2 \rightarrow Be$

(b) $1s^2\ 2s^2\ 2p^4 \rightarrow O$

(c) $1s^2\ 2s^2\ 2p^3 \rightarrow N$

(d) $1s^2\ 2s^2\ 2p^1 \rightarrow B$

The ionization enthalpy order is

$B < Be < O < N$

Be has more IE compared to B due to extra stability & N has more IE compared to oxygen due to extra stability

Hence, $N \rightarrow 1402$ kJ/mol

$O \rightarrow 1314$ kJ/mol

$B \rightarrow 801$ kJ/mol

$Be \rightarrow 899$ kJ/mol

5. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Dipole-dipole interactions are the only non-covalent interactions, resulting in hydrogen bond formation.

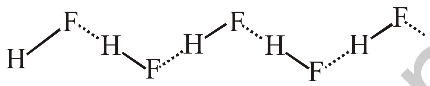
Reason R : Fluorine is the most electronegative element and hydrogen bonds in HF are symmetrical.

In the light of the above statements, choose the most appropriate answer from the options given below.

(1) A is false but R is true
(2) Both A and R are true and R is the correct explanation of A
(3) A is true R is false
(4) Both A and R are true but R is NOT the correct explanation of A

Official Ans. by NTA (1)

Sol. Assertion is incorrect since in hydrogen bonding, Dipole–dipole interactions are non-covalent but ion-dipole interaction can also result in H-bond formation. Reason is correct since F is most electronegative element & structure is



Symmetrical H-bonds are present

6. Statements about heavy water are given below.

A. Heavy water is used in exchange reactions for the study of reaction mechanisms.
B. Heavy water is prepared by exhaustive electrolysis of water
C. Heavy water has higher boiling point than ordinary water.
D. Viscosity of H_2O is greater than D_2O

(1) A, B and C only
(2) A and B only
(3) A and D only
(4) A and C only

Official Ans. by NTA (1)

Sol. Heavy water is used in exchange reactions for study of reaction mechanisms
Heavy water is prepared by exhaustive electrolysis of water.
B.P. of D_2O = 374.4 K
B.P. of H_2O = 373 K
Viscosity of H_2O = 0.89 centipoise
Viscosity of D_2O = 1.107 centipoise

7. The orbital having two radial as well as two angular nodes is -

(1) 3p (2) 4f (3) 4d (4) 5d

7. **Offcial Ans. by NTA (4)**

Sol. $n - l - 1 = 2$
$l = 2$
$n - 2 - 1 = 2$
$n = 5$

8. Match List -I with List - II

List - I (Ore)	**List - II (Element Present)**
(a) Kernite	(i) Tin
(b) Cassiterite	(ii) Boron
(c) Calamine	(iii) Fluorine
(d) Cryolite	(iv) Zinc

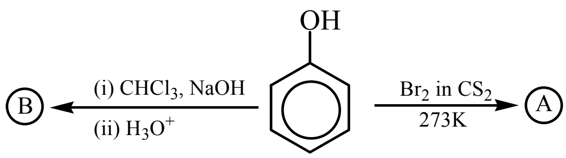
Choose the most appropriate answer from the options given below.

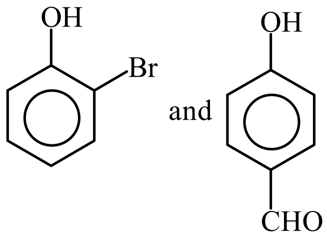
(1) (a) → (i), (b) → (iii), (c) → (iv), (d) → (ii)
(2) (a) → (ii), (b) → (i), (c) → (iv), (d) → (iii)
(3) (a) → (ii), (b) → (iv), (c) → (i), (d) → (iii)
(4) (a) → (iii), (b) → (i), (c) → (ii), (d) → (iv)

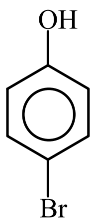
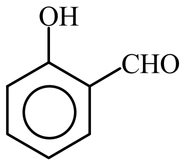
Official Ans. by NTA (2)

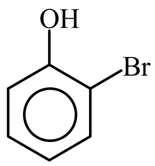
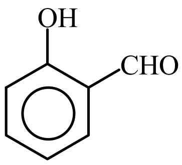
Sol. Kernite = $Na_2B_4O_7.4H_2O$
Cassiterite = SnO_2
Calamine = $ZnCO_3$
Cryolite = $Na_3A\ell F_6$

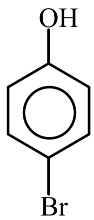
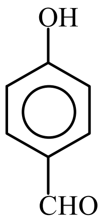
9. Identify the major products A and B respectively in the following reactions of phenol.



(1) 

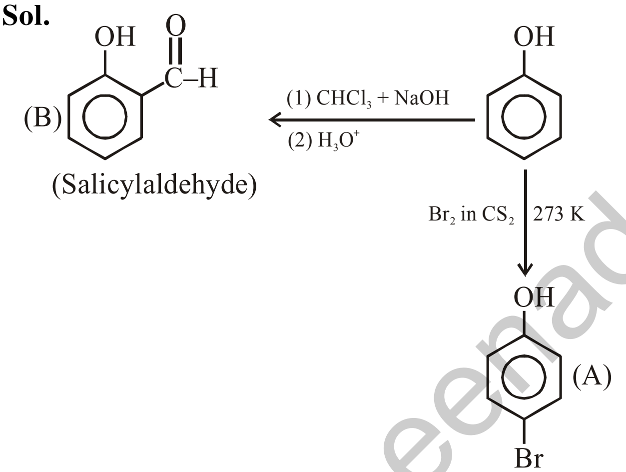
(2)  and 

(3)  and 

(4)  and 

Official Ans. by NTA (2)

Sol.



p–Bromo phenol

10. Given below are two statements :

Statement I : A mixture of chloroform and aniline can be separated by simple distillation.

Statement II : When separating aniline from a mixture of aniline and water by steam distillation aniline boils below its boiling point.

In the light of the above statements, choose the most appropriate answer from the options given below.

(1) **Statement-I** is false but **Statement II** is true

(2) **Both Statement-I** and **Statement II** are false

(3) **Statement-I** is true but **Statement II** is false

(4) **Both Statement-I** and **Statement II** are true

Official Ans. by NTA (4)

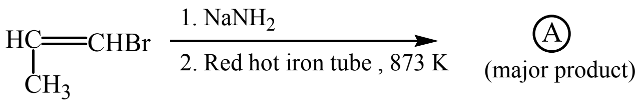
Sol. **Statement 1 :** B.P. of chloroform = 334 K

B.P. of aniline = 457 K

thus can be seprated of simple distillation.

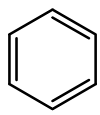
Statement 2 : Mixture of aniline and water seprated by simple distillation.

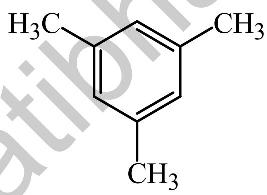
11. For the given reaction :



(1) $CH_3CH_2CH_2NH_2$

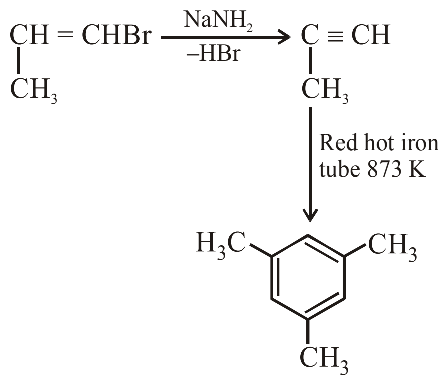
(2) $\underset{\displaystyle CH_3}{\underset{|}{CH}}=CH-NH_2$

(3) 

(4) 

Official Ans. by NTA (4)

Sol.



12. On treating a compound with warm dil. H_2SO_4, gas X is evolved which turns $K_2Cr_2O_7$ paper acidified with dil. H_2SO_4 to a green compound Y. X and Y respectively are -

(1) X = SO_2, Y = Cr_2O_3

(2) X = SO_3, Y = Cr_2O_3

(3) X = SO_2, Y = $Cr_2(SO_4)_3$

(4) X = SO_3, Y = $Cr_2(SO_4)_3$

Official Ans. by NTA (3)

Sol. SO_2 + dil $H_2SO_4 \longrightarrow SO_3(g)$

$SO_3 + K_2Cr_2O_7 \xrightarrow[H_2SO_4]{dil.} Cr_2(SO_4)_3$

13. Which of the following is 'a' FALSE statement?

(1) Carius tube is used in the estimation of sulphur in an organic compound

(2) Carius method is used for the estimation of nitrogen in an organic compound

(3) Phosphoric acid produced on oxidation of phosphorus present in an organic compound is precipitated as $Mg_2P_2O_7$ by adding magnesia mixture.

(4) Kjeldahl's method is used for the estimation of nitrogen in an organic compound

Official Ans. by NTA (2)

Sol. Carius method is used in the estimation of halogen in organic compounds.

14. Which of the following vitamin is helpful in delaying the blood clotting -

(1) Vitamin C (2) Vitamin B

(3) Vitamin E (4) Vitamin K

Official Ans. by NTA (4)

Sol. Vitamin helpful in delaying the blood clotting is Vitamin K

15. $\underset{(C_4H_8Cl_2)}{A} \xrightarrow[373K]{Hydrolysis} \underset{(C_4H_8O)}{B}$

B reacts with Hydroxyl amine but does not give Tollen's test. Identify A and B

(1) 1,1-Dichlorobutane and 2-Butanone

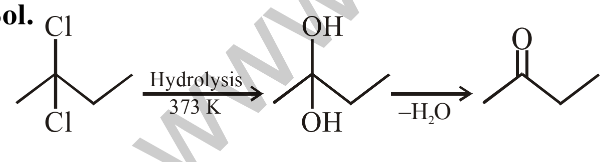
(2) 2,2-Dichlorobutane and Butanal

(3) 1,1-Dichlorobutane and Butanal

(4) 2,2-Dichlorobutane and 2-butan-one

Official Ans. by NTA (4)

Sol.



$(C_4H_8Cl_2)$ (C_4H_8O)

(A) (B)

16. Compound A used as a strong oxidizing agent is amphoteric in nature. It is the part of lead storage batteries. Compound A is :

(1) PbO_2 (2) PbO

(3) $PbSO_4$ (4) Pb_3O_4

Official Ans. by NTA (1)

Sol. PbO_2 is amphoteric and strong oxidizing agent and also a component of lead storage batteries.

17. Which one of the following lanthanoids does not form MO_2? [M is lanthanoid metal]

(1) Pr (2) Dy

(3) Nd (4) Yb

Official Ans. by NTA (4)

Sol. Yb is the only element that do not form MO_2 type oxide

18. Given below are two statements :

Statement I : o-Nitrophenol is steam volatile due to intramolecular hydrogen bonding.

Statement II : o-Nitrophenol has high melting due to hydrogen bonding.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Statement I is false but Statement II is true

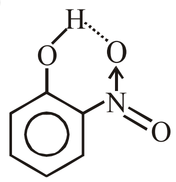
(2) Both statement I and statement II are true

(3) Both statement I and statement II are false

(4) Statement I is true but statement II is false

Official Ans. by NTA (4)

Sol.

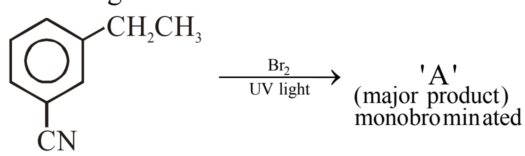


Intramolecular H–bonding

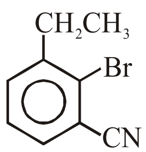
thus it is more volatile due to intramolecular H-bonding.

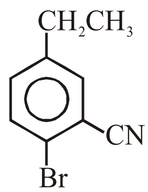
Melting point depends on packing efficiency not on H-bonding thus statement II is false

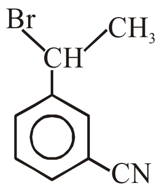
19. For the given reaction :

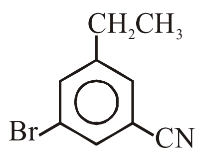


What is 'A'?

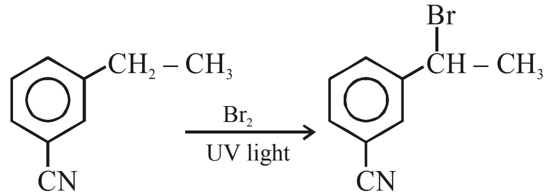
(1)


(2)


(3)


(4)


Official Ans. by NTA (3)

Sol. 

20. An amine on reaction with benzenesulphonyl chloride produces a compound insoluble in alkaline solution. This amine can be prepared by ammonolysis of ethyl chloride. The correct structure of amine is :

(1) 

(2) $CH_3CH_2NH_2$

(3) $CH_3CH_2CH_2NHCH_3$

(4) $CH_3CH_2CH_2\overset{H}{N}-CH_2CH_3$

Official Ans. by NTA (4)

Sol. It has to be 2° amine because on reaction with benzene sulphonylchloride it gives water in soluble product. As it is formed by ammonolysis of ethylchloride, so it has to be R–NH–Et type.

$CH_3-CH_2-CH_2-\ddot{N}H_2 + Et-Cl$

$\downarrow$

$CH_3-CH_2-CH_2-\overset{+}{N}H_2-Et$

$\downarrow -H^+$

$CH_3-CH_2-CH_2-NH-Et$

SECTION-B

1. For a chemical reaction $A + B \rightleftharpoons C + D$ (Δ_rH^0 = 80kJ mol^{-1}) the entropy change Δ_rS^0 depends on the temperature T (in K) as (Δ_rS^0 = 2T (J K^{-1} mol^{-1}).

Minimum temperature at which it will become spontaneous is ______K.(Integer)

1. Official Ans. by NTA (200)

$\Delta G^0 = \Delta H^0 - T \times \Delta S^0$

$\Delta G^0 = \Delta H^0 - T \times (2T)$

T = 200K

2. The number of significant figures in 50000.020×10^{-3} is _____.

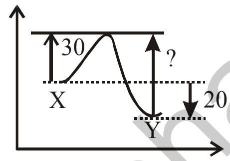
2. Official Ans. by NTA (7)

Sol. $\underline{50000.020} \times 10^{-3}$

3. An exothermic reaction X → Y has an activation energy 30 kJ mol^{-1}. If energy change ΔE during the reaction is –20 kJ, then the activation energy for the reverse reaction in kJ is _______.(Integer answer)

3. Official Ans. by NTA (50)

Sol. $X \longrightarrow Y$



4. Consider the following reaction

$MnO_4^- + 8H^+ + 5e^- \rightarrow Mn^{2+} + 4H_2O, E^o = 1.51$ V.

The quantity of electricity required in Faraday to reduce five moles of MnO_4^- is____.

4. Official Ans. by NTA (25)

5. A certain gas obeys $P(V_m-b) = RT$. The value of $\left(\frac{\partial Z}{\partial P}\right)_T$ is $\frac{xb}{RT}$. The value of x is_____.

(Integer answer) (Z : compressibility factor)

5. Official Ans. by NTA (1)

Sol. $Z = 1 + \frac{Pb}{RT}$

$$\left(\frac{\partial Z}{dP}\right)_T = 0 + \frac{b}{RT} \times 1$$

6. A homogeneous ideal gaseous reaction $AB_{2(g)} \rightleftharpoons A_{(g)} + 2B_{(g)}$ is carried out in a 25 litre flask at 27°C. The initial amount of AB_2 was 1 mole and the equilibrium pressure was 1.9 atm. The value of K_P is $x \times 10^{-2}$. The value of x is______.(Integer answer)

Official Ans. by NTA (74)

Allen Ans.(72 to 75)

Sol. $AB_2 = A + 2B$

$1 \quad - \quad -$

$1-\alpha \quad \alpha \quad 2\alpha$

$= 0.535 \quad 0.465 \quad 0.93$

$1.9 \times 25 = n_T \times 0.08206 \times 300$

$n_T = 1.93 = 1 + 2\alpha$

$\alpha = 0.465$

$$Kp = \frac{\left(\frac{0.465}{1.93}\times 19\right)\left(\frac{0.93}{1.93}\times 1.9\right)^2}{\left(\frac{0.535}{1.93}\times 1.9\right)}$$

$\simeq 73 \times 10^{-2}\ atm^2$

7. Dichromate ion is treated with base, the oxidation number of Cr in the product formed is ______.

Official Ans. by NTA (6)

Sol. $Cr_2O_7^{2-} + OH^- \longrightarrow CrO_4^{2-}$

Oxidation state of Cr in CrO_4^{2-} is +6

8. 224 mL of $SO_{2(g)}$ at 298 K and 1 atm is passed through 100 mL of 0.1 M NaOH solution. The non-volatile solute produced is dissolved in 36 g of water. The lowering of vapour pressure of solution (assuming the solution is dilute) $\left(P_{(H_2O)} = 24 \text{ mm of Hg}\right)$ is $x \times 10^{-2}$ mm of Hg, the value of x is ______. (Integer answer)

8. Official Ans. by NTA (12)

Allen Ans.(18 to 24)

Sol.(1) $SO_2 + 2NaOH \rightarrow Na_2SO_3 + H_2O$

$\frac{224}{0.0821\times 298}$ 10mmol (L.R.) 5mmol (i = 3)

= 9.2 m mol

$P^s = P^0.\ X_{solvent}$

$= 24\times\frac{2}{(2+15\times 10^{-3})}$

$= 23.82$

$\Delta P = 0.18$ torr $= 18 \times 10^{-2}$ torr.

Sol.(2) $SO_2 + NaOH \rightarrow NaHSO_3$

$9.2 \quad 10 \quad -$

$- \quad 0.8 \quad 9.2$

$\Delta P = P^0 .\ X_{solute}$

$= 24 \times \frac{(1.6+18.4)}{2020}$

$= 0.2376 = 23.76 \times 10^{-2}$

9. 3.12 g of oxygen is adsorbed on 1.2 g of platinum metal. The volume of oxygen adsorbed per gram of the adsorbent at 1 atm and 300 K in L is______.

[R = 0.0821 L atm K^{-1} mol^{-1}]

9. Official Ans. by NTA (2)

Sol. $V = \frac{\frac{3.12}{32}\times 0.0821\times 300}{1} = 2.40\,l$

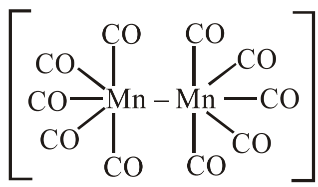
$\because$ 1.2 gm adsorbs 2.40 l

$\therefore$ 1 gm adsorbs 2 l

10. Number of bridging CO ligands in $[Mn_2(CO)_{10}]$ is _______.

Official Ans. by NTA (0)

Sol. $Mn_2(CO)_{10}$ structure is



Zero bridging CO ligands are present

SECTION-A

1. If $\vec{a}$ and $\vec{b}$ are perpendicular, then $\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times\vec{b}\right)\right)\right)$ is equal to

(1) $\vec{0}$ (2) $\frac{1}{2}|\vec{a}|^4\vec{b}$ (3) $\vec{a}\times\vec{b}$ (4) $|\vec{a}|^4\vec{b}$

Official Ans. by NTA (4)

Sol. $\vec{a}.\vec{b}=0$

$$\vec{a}\times\left(\vec{a}\times\vec{b}\right)=\left(\vec{a}.\vec{b}\right)\vec{a}-\left(\vec{a}.\vec{a}\right)\vec{b}=-|\vec{a}|^2\vec{b}$$

Now $\vec{a}\times\left(\vec{a}\times\left(-|\vec{a}|^2\vec{b}\right)\right)$

$$=-|\vec{a}|^2\left(\vec{a}\times\left(\vec{a}\times\vec{b}\right)\right)$$

$$=-|\vec{a}|^2\left(-|\vec{a}|^2\vec{b}\right)=|\vec{a}|^4\vec{b}$$

2. A fair coin is tossed a fixed number of times. If the probability of getting 7 heads is equal to probability of getting 9 heads, then the probability of getting 2 heads is

(1) $\frac{15}{2^{13}}$ (2) $\frac{15}{2^{12}}$ (3) $\frac{15}{2^{8}}$ (4) $\frac{15}{2^{14}}$

Official Ans. by NTA (1)

Sol. Let the coin be tossed n-times

$P(H) = P(T) = \frac{1}{2}$

$$P(7\text{ heads}) = {}^nC_7\left(\frac{1}{2}\right)^{n-7}\left(\frac{1}{2}\right)^7=\frac{{}^nC_7}{2^n}$$

$$P(9\text{ heads}) = {}^nC_9\left(\frac{1}{2}\right)^{n-9}\left(\frac{1}{2}\right)^9=\frac{{}^nC_9}{2^n}$$

P(7 heads) = P(9 heads)

${}^nC_7 = {}^nC_9 \Rightarrow n = 16$

$$P(2\text{ heads}) = {}^{16}C_2\left(\frac{1}{2}\right)^{14}\left(\frac{1}{2}\right)^2=\frac{15\times8}{2^{16}}$$

$$P(2\text{ heads}) = \frac{15}{2^{13}}$$

3. Let A be a symmetric matrix of order 2 with integer entries. If the sum of the diagonal elements of A^2 is 1, then the possible number of such matrices is

(1) 4 (2) 1 (3) 6 (4) 12

Official Ans. by NTA (1)

Sol. $A=\begin{pmatrix}a & b\\ b & c\end{pmatrix}$, a, b, c $\in$ I

$$A^2=\begin{pmatrix}a & b\\ b & c\end{pmatrix}\begin{pmatrix}a & b\\ b & c\end{pmatrix}=\begin{pmatrix}a^2+b^2 & b(a+c)\\ b(a+c) & b^2+c^2\end{pmatrix}$$

Sum of the diagonal entries of

$A^2 = a^2 + 2b^2 + c^2$

Given $a^2 + 2b^2 + c^2 = 1$, a, b, c $\in$ I

$b = 0$ & $a^2 + c^2 = 1$

Case–1 : $a = 0 \Rightarrow c = \pm1$ (2-matrices)

Case-2 : $c = 0 \Rightarrow a = \pm1$ (2-matrices)

Total = 4 matrices

4. In a increasing geometric series, the sum of the second and the sixth term is $\frac{25}{2}$ and the product of the third and fifth term is 25. Then, the sum of 4^{th}, 6^{th} and 8^{th} terms is equal to

(1) 30 (2) 26 (3) 35 (4) 32

Official Ans. by NTA (3)

Sol. a, ar, ar^2, ...

$$T_2+T_6=\frac{25}{2}\Rightarrow ar\left(1+r^4\right)=\frac{25}{2}$$

$$a^2r^2\,(1+r^4)^2=\frac{625}{4} \quad .... (1)$$

$T_3 . T_5 = 25 \Rightarrow (ar^2)(ar^4) = 25$

$a^2r^6 = 25$ .... (2)

On dividing (1) by (2)

$$\frac{\left(1+r^4\right)^2}{r^4}=\frac{25}{4}$$

$4r^8 - 17r^4 + 4 = 0$

$(4r^4 - 1)(r^4 - 4) = 0$

$r^4 = \frac{1}{4}, 4 \Rightarrow r^4 = 4$

(an increasing geometric series)

$a^2r^6 = 25 \Rightarrow (ar^3)^2 = 25$

$T_4 + T_6 + T_8 = ar^3 + ar^5 + ar^7$

$= ar^3(1 + r^2 + r^4)$

$= 5(1 + 2 + 4) = 35$

5. The value of $\sum_{n=1}^{100} \int_{n-1}^{n} e^{x-[x]}dx$, where [x] is the greatest integer $\leq$ x, is

(1) $100(e - 1)$ (2) $100(1 - e)$

(3) $100e$ (4) $100(1 + e)$

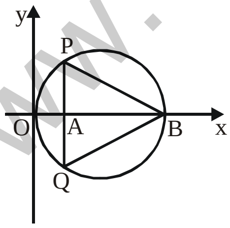
Official Ans. by NTA (1)

Sol. $\sum_{n=1}^{100} \int_{n-1}^{n} e^{\{x\}}dx$, period of $\{x\} = 1$

$$\sum_{n=1}^{100} \int_{0}^{1} e^{\{x\}}\,dx = \sum_{n=1}^{100} \int_{0}^{1} e^{x}dx$$

$$\sum_{n=1}^{100} (e-1) = 100(e-1)$$

6. In the circle given below, let OA = 1 unit, OB = 13 unit and $PQ \perp OB$. Then, the area of the triangle PQB (in square units) is



(1) $24\sqrt{2}$ (2) $24\sqrt{3}$

(3) $26\sqrt{3}$ (4) $26\sqrt{2}$

Official Ans. by NTA (2)

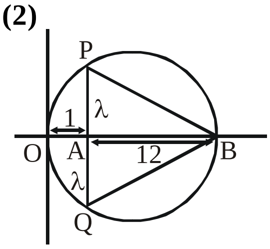
Sol. $PA = AQ = \lambda$

$OA \cdot AB$

$= AP \cdot AQ$

$\Rightarrow 1.12 = \lambda \cdot \lambda$

$\Rightarrow \lambda = 2\sqrt{3}$



Area $\Delta PQB = \frac{1}{2} \times 2\lambda \times AB$

$$\Delta = \frac{1}{2}.4\sqrt{3}\times 12$$

$$= 24\sqrt{3}$$

7. The sum of the infinite series

$1+\frac{2}{3}+\frac{7}{3^2}+\frac{12}{3^3}+\frac{17}{3^4}+\frac{22}{3^5}+\ldots$ is equal to

(1) $\frac{13}{4}$ (2) $\frac{9}{4}$ (3) $\frac{15}{4}$ (4) $\frac{11}{4}$

Official Ans. by NTA (1)

Sol. $S = 1+\frac{2}{3}+\frac{7}{3^2}+\frac{12}{3^3}+\frac{17}{3^4}+\ldots$

$$\frac{S}{3} = \frac{1}{3}+\frac{2}{3^2}+\frac{7}{3^3}+\frac{12}{3^4}+\ldots$$

$$\frac{2S}{3} = 1+\frac{1}{3}+\frac{5}{3^2}+\frac{5}{3^3}+\frac{5}{3^4}+\ldots+\text{up to infinite terms}$$

$$\Rightarrow S = \frac{13}{4}$$

8. The value of

$$\lim_{h\to 0} 2\left\{\frac{\sqrt{3}\sin\left(\frac{\pi}{6}+h\right)-\cos\left(\frac{\pi}{6}+h\right)}{\sqrt{3}h\left(\sqrt{3}\cosh-\sinh\right)}\right\}$$ is

(1) $\frac{4}{3}$ (2) $\frac{2}{\sqrt{3}}$ (3) $\frac{3}{4}$ (4) $\frac{2}{3}$

Official Ans. by NTA (1)

Sol. $$L = \lim_{h\to 0} 2\left(\frac{\sqrt{3}\left(\frac{1}{2}\cosh+\frac{\sqrt{3}}{2}\sinh\right)-\left(\frac{\sqrt{3}}{2}\cosh-\frac{\sinh}{2}\right)}{\left(\sqrt{3}h\right)\left(\sqrt{3}\right)}\right)$$

$$L = \lim_{h\to 0}\frac{4\sinh}{3h}$$

$$\Rightarrow L = \frac{4}{3}$$

9. The maximum value of the term independent of 't' in the expansion of $\left(tx^{\frac{1}{5}}+\frac{(1-x)^{\frac{1}{10}}}{t}\right)^{10}$ where $x \in (0,1)$ is

(1) $\frac{10!}{\sqrt{3}(5!)^2}$ (2) $\frac{2.10!}{3\sqrt{3}(5!)^2}$

(3) $\frac{2.10!}{3(5!)^2}$ (4) $\frac{10!}{3(5!)^2}$

Official Ans. by NTA (2)

Sol. Term independent of t will be the middle term due to exect same magnitude but opposite sign powers of t in the binomial expression given

so $T_6 = {}^{10}C_5\left(tx^{2}5\right)^5\left(\frac{(1-x)^{\frac{1}{10}}}{t}\right)^5$

$T_6 = f(x) = {}^{10}C_5\left(x\sqrt{1-x}\right)$; for maximum

$f'(x) = 0 \Rightarrow x = \frac{2}{3}$ & $f''\left(\frac{2}{3}\right) < 0$

so $f(x)_{max.} = {}^{10}C_5\left(\frac{2}{3}\right).\frac{1}{\sqrt{3}}$

10. The rate of growth of bacteria in a culture is proportional to the number of bacteris present and the bacteria count is 1000 at initial time t = 0. The number of bacteria is increased by 20% in 2 hours. If the population of bacteria is 2000 after $\frac{k}{\log_e\left(\frac{6}{5}\right)}$ hours, then $\left(\frac{k}{\log_e 2}\right)^2$ is equal to

(1) 4 (2) 8 (3) 2 (4) 16

Official Ans. by NTA (1)

Sol. $\frac{dB}{dt} = \lambda B \Rightarrow \int_{1000}^{1200}\frac{dB}{B} = \lambda\int_0^2 dt \Rightarrow \lambda = \frac{1}{2}\ell n\left(\frac{6}{5}\right)$

$\int_{1000}^{2000}\frac{dB}{B} = \frac{1}{2}\ell n\left(\frac{6}{5}\right)\int_0^T dt \Rightarrow T = \frac{2\ell n2}{\ell n\left(\frac{6}{5}\right)}$

$\Rightarrow k = 2\ell n2$

11. If (1,5,35), (7,5,5), (1,λ,7) and (2λ,1,2) are coplanar, then the sum of all possible values of λ is

(1) $\frac{39}{5}$ (2) $-\frac{39}{5}$ (3) $\frac{44}{5}$ (4) $-\frac{44}{5}$

Official Ans. by NTA (3)

Sol. A(1, 5, 35), B(7, 5, 5), C(1, λ, 7), D(2λ, 1, 2)

$\overline{AB} = 6\hat{i} - 30\hat{k}$, $\overline{BC} = -6\hat{i}(\lambda-5)\hat{j} + 2\hat{k}$,

$\overline{CD} = (2\lambda-1)\hat{i} + (1-\lambda)\hat{j} - 5\hat{k}$

Points are coplanar

$$\Rightarrow 0 = \begin{vmatrix} 6 & 0 & -30 \\ -6 & \lambda-5 & 2 \\ 2\lambda-1 & 1-\lambda & -5 \end{vmatrix}$$

$= 6(-5\lambda + 25 - 2 + 2\lambda)$
$-30(-6 + 6\lambda - (2\lambda^2 - \lambda - 10\lambda + 5))$
$= 6(-3\lambda + 23) - 30(-2\lambda^2 + 11\lambda - 5 - 6 + 6\lambda)$
$= 6(-3\lambda + 23) - 30(-2\lambda^2 + 17\lambda - 11)$
$= 6(-3\lambda + 23 + 10\lambda^2 - 85\lambda + 55)$
$= 6(10\lambda^2 - 88\lambda + 78) = 12(5\lambda^2 - 44\lambda + 39)$
$\Rightarrow 0 = 12(5\lambda^2 - 44\lambda + 39)$

$\lambda_1 + \lambda_2 = \frac{44}{5}$

12. If $\frac{\sin^{-1}x}{a} = \frac{\cos^{-1}x}{b} = \frac{\tan^{-1}y}{c}; 0 < x < 1$, then the value of $\cos\left(\frac{\pi c}{a+b}\right)$ is

(1) $\frac{1-y^2}{y\sqrt{y}}$ (2) $1 - y^2$

(3) $\frac{1-y^2}{1+y^2}$ (4) $\frac{1-y^2}{2y}$

Official Ans. by NTA (3)

Sol. $\frac{\sin^{-1}x}{r} = a$, $\frac{\cos^{-1}x}{r} = b$, $\frac{\tan^{-1}y}{r} = c$

So, $a + b = \frac{\pi}{2r}$

$$\cos\left(\frac{\pi c}{a+b}\right)=\cos\left(\frac{\pi\tan^{-1}y}{\frac{\pi}{2r}r}\right)$$

$= \cos(2\tan^{-1}y)$, let $\tan^{-1}y = \theta$

$= \cos(2\theta)$

$$=\frac{1-\tan^2\theta}{1+\tan^2\theta}=\frac{1-y^2}{1+y^2}$$

13. The number of seven digit integers with sum of the digits equal to 10 and formed by using the digits 1,2 and 3 only is

(1) 42 (2) 82 (3) 77 (4) 35

Official Ans. by NTA (3)

Sol. **(I)** First possiblity is 1, 1, 1, 1, 1, 2, 3

required number $= \frac{7!}{5!} = 7 \times 6 = 42$

(II) Second possiblity is 1, 1, 1, 1, 2, 2, 2

required number $= \frac{7!}{4!\ 3!} = \frac{7\times6\times5}{6} = 35$

Total = 42 + 35 = 77

14. Let f be any function defined on R and let it satisfy the condition :

$|f(x) - f(y)| \le |(x - y)^2|,\ \forall\ (x,y) \in R$

If $f(0) = 1$, then :

(1) $f(x)$ can take any value in R

(2) $f(x) < 0,\ \forall\ x \in R$

(3) $f(x) = 0,\ \forall\ x \in R$

(4) $f(x) > 0,\ \forall\ x \in R$

Official Ans. by NTA (4)

Sol. $\left|\frac{f(x)-f(y)}{(x-y)}\right| \le |(x-y)|$

$x - y = h$ let $\Rightarrow x = y + h$

$$\lim_{x\to0}\left|\frac{f(y+h)-f(y)}{h}\right|\le 0$$

$\Rightarrow |f'(y)| \le 0 \Rightarrow f'(y) = 0$

$\Rightarrow f(y) = k$ (constant)

and $f(0) = 1$ given

So, $f(y) = 1 \Rightarrow f(x) = 1$

15. The maximum slope of the curve $y = \frac{1}{2}x^4 - 5x^3 + 18x^2 - 19x$ occurs at the point

(1) (2,2) (2) (0,0)

(3) (2,9) (4) $\left(3, \frac{21}{2}\right)$

Official Ans. by NTA (1)

Sol. $\frac{dy}{dx} = 2x^3 - 15x^2 + 36x - 19$

Since, slope is maximum so,

$\frac{d^2y}{dx^2} = 6x^2 - 30x + 36 = 0$

$\Rightarrow x^2 - 5x + 6 = 0$

$x = 2, 3$

$\frac{d^3y}{dx^3} = 12x - 30$

at $x = 2$, $\frac{d^3y}{dx^3} < 0$

So, maxima

at $x = 2$

$y = \frac{1}{2} \times 16 - 5 \times 8 + 18 \times 4 - 19 \times 2$

$= 8 - 40 + 72 - 38 = 80 - 78 = 2$

point (2, 2)

16. The intersection of three lines $x - y = 0$, $x + 2y = 3$ and $2x + y = 6$ is a

(1) Right angled triangle

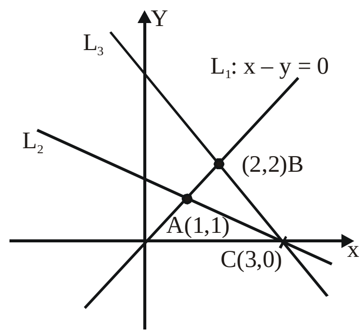
(2) Equilateral triangle

(3) Isosceles triangle

(4) None of the above

Official Ans. by NTA (3)

Sol.



$L_1 : x - y = 0$

$L_2 : x + 2y = 3$

$L_3 : x + y = 6$

on solving L_1 and L_2 :

y = L and x = 1

L_1 and L_3 :

x = 2

y = 2

L_2 and L_3 :

x + y = 3

2x + y = 6

x = 3

y = 0

$$AC = \sqrt{4+1} = \sqrt{5}$$

$$BC = \sqrt{4+1} = \sqrt{5}$$

$$AB = \sqrt{1+1} = \sqrt{2}$$

so its an isosceles triangle

17. Consider the three planes

$P_1 : 3x + 15y + 21z = 9$,

$P_2 : x - 3y - z = 5$, and

$P_3 : 2x + 10y + 14z = 5$

Then, which one of the following is true ?

(1) P_1 and P_2 are parallel

(2) P_1 and P_3 are parallel

(3) P_2 and P_3 are parallel

(4) P_1,P_2 and P_3 all are parallel

Official Ans. by NTA (2)

Sol. $P_1 : x + 5y + 7z = 3$,

$P_2 : x - 3y - z = 5$

$P_3 : x + 5y + 7z = \frac{5}{2}$

so P_1 and P_3 are parallel.

18. The value of $\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$ is

(1) (a + 2) (a + 3) (a + 4)

(2) –2

(3) (a + 1) (a + 2) (a + 3)

(4) 0

Official Ans. by NTA (2)

Sol. $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\Delta = \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3-a-1) & 1 & 0 \\ a^2+7a+12-a^2-3a-2 & 2 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} a^2+3a+2 & a+2 & 1 \\ 2(a+2) & 1 & 0 \\ 4a+10 & 2 & 0 \end{vmatrix}$$

= 4(a + 2) – 4a – 10

= 4a + 8 – 4a – 10 = –2

19. The value of $\int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^x} dx$ is

(1) $\frac{\pi}{4}$ (2) 4π (3) $\frac{\pi}{2}$ (4) 2π

Official Ans. by NTA (1)

Sol. $I = \int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^x} dx$ (using king)

$$I = \int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^{-x}} dx = \int_{-\pi/2}^{\pi/2} \frac{3^x \cos^2 x}{1+3^x} dx$$

$$2I = \int_{-\pi/2}^{\pi/2} \frac{(1+3^x)\cos^2 x}{1+3^x} dx$$

$$= \int_{-\pi/2}^{\pi/2} \cos^2 x dx = 2\int_0^{\pi/2} \cos^2 x\, dx$$

$$\Rightarrow I = \int_0^{\pi/2} \cos^2 x\, dx = \frac{\pi}{4}$$

20. Let R = {(P,Q) | P and Q are at the same distance from the origin} be a relation, then the equivalence class of (1,–1) is the set :

(1) $S = \{(x,y) \mid x^2 + y^2 = 4\}$

(2) $S = \{(x,y) \mid x^2 + y^2 = 1\}$

(3) $S = \{(x,y) \mid x^2 + y^2 = \sqrt{2}\}$

(4) $S = \{(x,y) \mid x^2 + y^2 = 2\}$

Official Ans. by NTA (4)

Sol. Equivalence class of (1, –1) is a circle with centre at (0,0) and radius = $\sqrt{2}$

$\Rightarrow x^2 + y^2 = 2$

$S = \{(x,y) | x^2 + y^2 = 2\}$

SECTION-B

1. The difference between degree and order of a differential equation that represents the family of curves given by $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right), a > 0$ is

Official Ans. by NTA (2)

Sol. $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right) = ax + \frac{a^{3/2}}{2}$ ...(1)

$\Rightarrow 2yy' = a$

put in equation (1)

$$y^2 = (2yy')x + \frac{(2yy')^{3/2}}{2}$$

$$(y^2 - 2xyy') = \frac{(2yy')^{3/2}}{2}$$

squaring

$$(y^2 - 2xyy')^2 = \frac{y^3(y')^3}{2}$$

$\therefore$ order = 1

degree = 3

Degree – order = 3 – 1 = 2

2. The number of integral values of 'k' for which the equation 3sinx + 4 cosx = k + 1 has a solution, k $\in$ R is

Official Ans. by NTA (11)

Sol. 3 sinx + 4 cosx = k + 1

$$\Rightarrow k + 1 \in \left[-\sqrt{3^2 + 4^2}, \sqrt{3^2 + 4^2}\right]$$

$$\Rightarrow k + 1 \in [-5, 5]$$

$$\Rightarrow k \in [-6, 4]$$

No. of integral values of k = 11

3. The number of solutions of the equation $\log_4(x - 1) = \log_2(x - 3)$ is

Official Ans. by NTA (1)

Sol. $\log_4(x - 1) = \log_2(x - 3)$

$$\Rightarrow \frac{1}{2}\log_2(x-1) = \log_2(x-3)$$

$$\Rightarrow \log_2(x-1)^{1/2} = \log_2(x-3)$$

$$\Rightarrow (x-1)^{1/2} = x - 3$$

$\Rightarrow x - 1 = x^2 + 9 - 6x$

$\Rightarrow x^2 - 7x + 10 = 0$

$\Rightarrow (x - 2)(x - 5) = 0$

$\Rightarrow x = 2,5$

But $x \neq 2$ because it is not satisfying the domain of given equation i.e $\log_2(x - 3) \rightarrow$ its domain $x > 3$

finally x is 5

$\therefore$ No. of solutions = 1.

4. The sum of 162[th] power of the roots of the equation $x^3 - 2x^2 + 2x - 1 = 0$ is

Official Ans. by NTA (3)

Sol. $x^3 - 2x^2 + 2x - 1 = 0$

x = 1 satisfying the equation

$\therefore$ x– 1 is factor of

$x^3 - 2x^2 + 2x - 1$

$= (x - 1)(x^2 - x + 1) = 0$

$$x = 1, \frac{1 + i\sqrt{3}}{2}, \frac{1 - i\sqrt{3}}{2}$$

$x = 1, -\omega^2, -\omega$

sum of 162[th] power of roots

$= (1)^{162} + (-\omega^2)^{162} + (-\omega)^{162}$

$= 1 + (\omega)^{324} + (\omega)^{162}$

= 1 + 1 + 1 = 3

5. Let m,n $\in$ N and gcd (2,n) = 1. If

$$30\binom{30}{0}+29\binom{30}{1}+...+2\binom{30}{28}+1\binom{30}{29}=n.2^m\ ,$$

then n + m is equal to

(Here $\binom{n}{k}={}^nC_k$)

Official Ans. by NTA (45)

Sol. $30({}^{30}C_0) + 29({}^{30}C_1) + ... + 2({}^{30}C_{28}) + 1({}^{30}C_{29})$

$= 30({}^{30}C_{30}) + 29({}^{30}C_{29}) + ... + 2({}^{30}C_2) + 1({}^{30}C_1)$

$$=\sum_{r=1}^{30} r\left({}^{30}C_r\right)$$

$$=\sum_{r=1}^{30} r\left(\frac{30}{r}\right)\left({}^{29}C_{r-1}\right)$$

$$=30\sum_{r=1}^{30} {}^{29}C_{r-1}$$

$$=30\left({}^{29}C_0+{}^{29}C_1+{}^{29}C_2+...+{}^{29}C_{29}\right)$$

$$=30\left(2^{29}\right)=15(2)^{30}=n(2)^m$$

$\therefore$ n = 15, m = 30

n + m = 45

6. If y = y(x) is the solution of the equaiton

$$e^{\sin y}\cos y\frac{dy}{dx}+e^{\sin y}\cos x=\cos x,\ y(0)=0;$$

then $1+y\left(\frac{\pi}{6}\right)+\frac{\sqrt{3}}{2}y\left(\frac{\pi}{3}\right)+\frac{1}{\sqrt{2}}y\left(\frac{\pi}{4}\right)$ is equal to

Official Ans. by NTA (1)

Sol. Put $e^{\sin y} = t$

$$\Rightarrow e^{\sin y}\cos y\frac{dy}{dx}=\frac{dt}{dx}$$

$$\Rightarrow \text{D.E is } \frac{dt}{dx}+t\cos x=\cos x$$

I.F. $= e^{\int \cos x\,dx} = e^{\sin x}$

$\Rightarrow$ solution is $t.e^{\sin x} = \int \cos x e^{\sin x}$

$\Rightarrow e^{\sin y}\, e^{\sin x} = e^{\sin x} + c$

$\because$ x = 0, y = 0 $\Rightarrow$ c = 0

$\Rightarrow e^{\sin y} = 1$

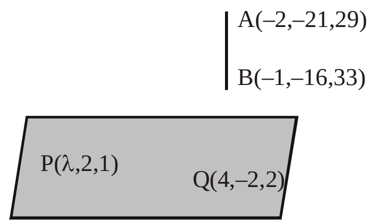
$\Rightarrow$ y = 0

$$\Rightarrow 1+y\left(\frac{\pi}{6}\right)+\frac{\sqrt{3}}{2}y\left(\frac{\pi}{3}\right)+\frac{1}{\sqrt{2}}y\left(\frac{\pi}{4}\right)=1$$

7. Let $(\lambda,2,1)$ be a point on the plane which passes through the ponit (4,–2,2). If the plane is perpendicular to the line joining the points (–2,–21,29) and (–1, –16, 23), then $\left(\frac{\lambda}{11}\right)^2-\frac{4\lambda}{11}-4$ is equal to

Official Ans. by NTA (8)

Sol.



$$\overline{AB}.\ \overrightarrow{PQ}=0$$

$$\Rightarrow\left(\hat{i}+5\hat{j}-6\hat{k}\right).\left((4-\lambda)\hat{i}-4\hat{j}+\hat{k}\right)=0$$

$$\Rightarrow 4-\lambda-20-6=0$$

$$\Rightarrow \lambda=-22$$

$$\Rightarrow\left(\frac{\lambda}{11}\right)^2-\frac{4\lambda}{11}-4=4+8-4=8$$

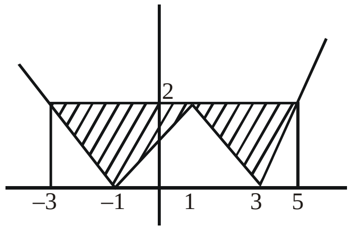
8. The area bounded by the lines y = ||x – 1| –2| is

Official Ans. by NTA (8)

Ans. By ALLEN (BONUS)

Sol. Remark :

Question is incomplete it should be area bounded by y = |x – 1| – 2| and y = 2



Area $= 2\left(\frac{1}{2}.4.2\right)$

9. The value of the integral $\int_0^{\pi} |\sin 2x|\,dx$ is

Official Ans. by NTA (2)

Sol. Put 2x = t ⇒ 2dx = dt

$$\Rightarrow \; I = \frac{1}{2}\int_0^{2\pi} |\sin t|\,dt$$

$$= \int_0^{\pi} |\sin t|\,dt$$

= 2

10. If $\sqrt{3}\left(\cos^2 x\right) = \left(\sqrt{3}-1\right)\cos x + 1$, the number of solutions of the given equation when $x \in \left[0, \frac{\pi}{2}\right]$ is

Official Ans. by NTA (1)

Sol. $\sqrt{3}\left(\cos x\right)^2 - \sqrt{3}\cos x + \cos x - 1 = 0$

$$\Rightarrow \left(\sqrt{3}\cos x + 1\right)\left(\cos x - 1\right) = 0$$

$$\Rightarrow \cos x = 1 \text{ or } \cos x = -\frac{1}{\sqrt{3}} \text{ (reject)}$$

$$\Rightarrow x = 0 \text{ only}$$