PHYSICS

SECTION-A

1. If 'C' and 'V' represent capacity and voltage respectively then what are the dimensions of λ, where $\frac{C}{V} = \lambda$?

(1) $[M^{-2}L^{-3}I^{2}T^{6}]$ (2) $[M^{-3}L^{-4}I^{3}T^{7}]$

(3) $[M^{-1}L^{-3}I^{-2}T^{-7}]$ (4) $[M^{-2}L^{-4}I^{3}T^{7}]$

Official Ans. by NTA (4)

Sol. $\lambda = \frac{C}{V} = \frac{Q/V}{V} = \frac{Q}{V^2}$

$$V = \frac{\text{work}}{Q}$$

$$\lambda = \frac{Q^3}{(\text{work})^2} = \frac{(It)^3}{(F.s)^2}$$

$$= \frac{[I^3T^3]}{[ML^2T^{-2}]^2} = [M^{-2}L^{-4}I^3T^7]$$

2. The length of metallic wire is ℓ_1 when tension in it is T_1. It is ℓ_2 when the tension is T_2. The original length of the wire will be -

(1) $\frac{\ell_1 + \ell_2}{2}$ (2) $\frac{T_2\ell_1 + T_1\ell_2}{T_1 + T_2}$

(3) $\frac{T_2\ell_1 - T_1\ell_2}{T_2 - T_1}$ (4) $\frac{T_1\ell_1 - T_2\ell_2}{T_2 - T_1}$

Official Ans. by NTA (3)

Sol. Assuming Hooke's law to be valid.

$T \propto (\Delta\ell)$

$T = k(\Delta\ell)$

Let, ℓ_0 = natural length (original length)

$\Rightarrow T = k(\ell - \ell_0)$

so, $T_1 = k(\ell_1 - \ell_0)$ & $T_2 = k(\ell_2 - \ell_0)$

$$\Rightarrow \frac{T_1}{T_2} = \frac{\ell_1 - \ell_0}{\ell_2 - \ell_0}$$

$$\Rightarrow \ell_0 = \frac{T_2\ell_1 - T_1\ell_2}{T_2 - T_1}$$

3. An aeroplane, with its wings spread 10 m, is flying at a speed of 180 km/h in a horizontal direction. The total intensity of earth's field at that part is 2.5×10^{-4} Wb/m^2 and the angle of dip is 60°. The emf induced between the tips of the plane wings will be :-

(1) 108.25 mV (2) 54.125 mV

(3) 88.37 mV (4) 62.50 mV

Official Ans. by NTA (1)

Sol. $\in = [\vec{B}\vec{V}\vec{L}] = BVL\sin\theta$

$$= (2.5 \times 10^{-4}T)\left(180 \times \frac{5}{18} m/s\right)(10m)\sin 60^\circ$$

$= 108.25 \times 10^{-3}V$

4. A tuning fork A of unknown frequency produces 5 beats/s with a fork of known frequency 340 Hz. When fork A is filed, the beat frequency decreases to 2 beats/s. What is the frequency of fork A ?

(1) 342 Hz (2) 345 Hz

(3) 335 Hz (4) 338 Hz

Official Ans. by NTA (3)

Sol. Initially beat frequency = 5 Hz

so, $\rho_A = 340 \pm 5 = 345$ Hz, or 335 Hz

after filing frequency increases slightly

so, new value of frequency of A > ρ_A

Now, beat frequency = 2Hz

$\Rightarrow$ new $\rho_A = 340 \pm 2 = 342$ Hz, or 338 Hz

hence, original frequency of A is $\rho_A = 335$ Hz

5. A particle executes S.H.M., the graph of velocity as a function of displacement is :-

(1) A circle (2) A parabola

(3) An ellipse (4) A helix

Official Ans. by NTA (3)

Sol. $v^2 = \omega^2(A^2 - x^2)$

$$\frac{v^2}{\omega^2} + x^2 = A^2$$

$$\frac{v^2}{(\omega A)^2} + \frac{x^2}{A^2} = 1$$

This is an equation of an ellipse.

6. The trajectory of a projectile in a vertical plane is $y = \alpha x - \beta x^2$, where α and β are constants and x & y are respectively the horizontal and vertical distances of the projectile from the point of projection. The angle of projection θ and the maximum height attained H are respectively given by :-

(1) $\tan^{-1}\alpha, \dfrac{\alpha^2}{4\beta}$ (2) $\tan^{-1}\beta, \dfrac{\alpha^2}{2\beta}$

(3) $\tan^{-1}\alpha, \dfrac{4\alpha^2}{\beta}$ (4) $\tan^{-1}\left(\dfrac{\beta}{\alpha}\right), \dfrac{\alpha^2}{\beta}$

Official Ans. by NTA (1)

Sol. $y = \alpha x - \beta x^2$

comparing with trajectory equation

$$y = x \tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}$$

$\tan\theta = \alpha \Rightarrow \theta = \tan^{-1}\alpha$

$$\beta = \frac{1}{2}\frac{g}{u^2\cos^2\theta}$$

$$u^2 = \frac{g}{2\beta\cos^2\theta}$$

Maximum height : H

$$H = \frac{u^2\sin^2\theta}{2g} = \frac{g}{2\beta\cos^2\theta}\frac{\sin^2\theta}{2g}$$

$$H = \frac{\tan^2\theta}{4\beta} = \frac{\alpha^2}{4\beta}$$

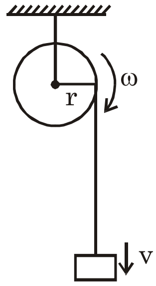
7. A cord is wound round the circumference of wheel of radius r. The axis of the wheel is horizontal and the moment of inertia about it is I. A weight mg is attached to the cord at the end. The weight falls from rest. After falling through a distance 'h', the square of angular velocity of wheel will be :-

(1) $\dfrac{2mgh}{I+2mr^2}$ (2) $\dfrac{2mgh}{I+mr^2}$

(3) $2gh$ (4) $\dfrac{2gh}{I+mr^2}$

Official Ans. by NTA (2)

Sol. $mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$



$v = \omega r$

$$mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}m\omega^2 r^2$$

$$\frac{2mgh}{\left(I+mr^2\right)} = \omega^2$$

8. The internal energy (U), pressure (P) and volume (V) of an ideal gas are related as U = 3PV + 4. The gas is :-

(1) Diatomic only

(2) Polyatomic only

(3) Either monoatomic or diatomic

(4) Monoatomic only

Official Ans. by NTA (2)

Sol. U = 3PV + 4

$$\frac{nf}{2}RT = 3PV + 4$$

$$\frac{f}{2}PV = 3PV + 4$$

$$f = 6 + \frac{8}{PV}$$

Since degree of freedom is more than 6 therefore gas is polyatomic

9. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : For a simple microscope, the angular size of the object equals the angular size of the image.

Reason R : Magnification is achieved as the small object can be kept much closer to the eye than 25 cm and hence it subtends a large angle.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) A is true but R is false

(2) Both A and R are true but R is NOT the correct explanation of A.

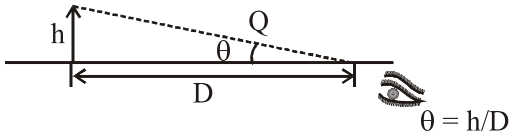
(3) Both A and R are true and R is the correct explanation of A

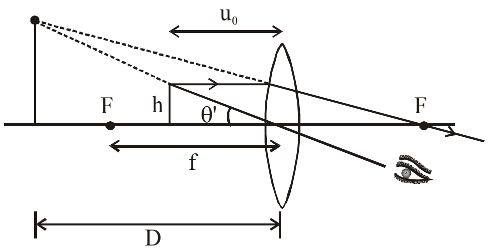
(4) A is false but R is true

Official Ans. by NTA (3)

Allen Ans. (2)

Sol.





$\theta' = \dfrac{h}{u_0}$; θ' is same for both object and image

$m = \dfrac{\theta'}{\theta} = \dfrac{D}{\mu_0}$

$u_0 < D$

Hence m > 1

10. Given below are two statements :

Statement I : An electric dipole is placed at the centre of a hollow sphere. The flux of electric field through the sphere is zero but the electric field is not zero anywhere in the sphere.

Statement II : If R is the radius of a solid metallic sphere and Q be the total charge on it. The electric field at any point on the spherical surface of radius r (< R) is zero but the electric flux passing through this closed spherical surface of radius r is not zero.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both Statement I and Statement II are true

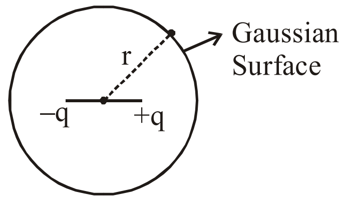
(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are false

(4) Statement I is false but Statement II is true.

Official Ans. by NTA (2)

Sol.

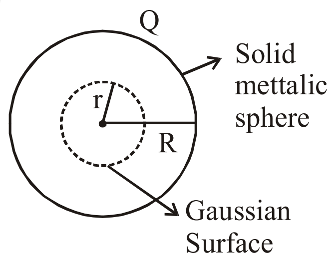


$$\oint \vec{E}.\vec{ds} = \frac{q_{in}}{\varepsilon_0} = 0 = \phi$$

Flux of $\vec{E}$ through sphere is zero.

But $\oint \vec{E}.\vec{ds} = 0 \Rightarrow \{\vec{E}.\vec{ds} \neq 0\}$ for small section ds only

Statement-2



As change encloses within gaussian surface is equal to zero.

$$\phi = \oint \vec{E}.\vec{ds} = 0$$

Option(2) statement-1 correct statement-2 false.

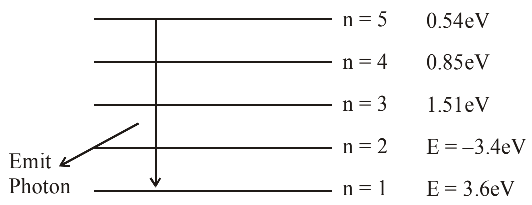
11. The recoil speed of a hydrogen atom after it emits a photon in going from n = 5 state to n = 1 state will be :-

(1) 4.17 m/s (2) 2.19 m/s

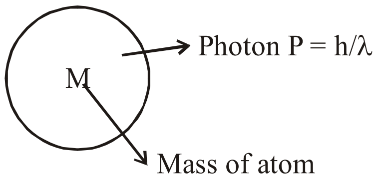
(3) 3.25 m/s (4) 4.34 m/s

Official Ans. by NTA (1)

Sol.



(ΔE) Releases when photon going from n = 5 to n = ΔE = (13.6 – 0.54) eV = 13.06 eV.



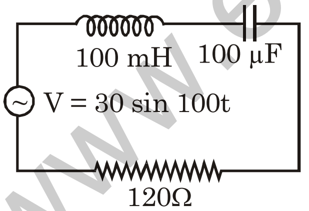
$P_i = P_f$ (By linear momentum conservation)

$$0 = \frac{h}{\lambda} - Mv = V_{Recoil} = \frac{h}{\lambda M} \quad ....(i)$$

$$\& \quad \Delta E = \frac{hc}{\lambda} = \frac{hc}{\lambda M} \times M \Rightarrow McV_{Recoil}$$

$$V_{Recoil} = \frac{\Delta E}{Mc} = \frac{13.06 \times 1.6 \times 10^{-19}}{1.67 \times 10^{-27} \times 3 \times 10^{8}} = 4.17\ m/sec$$

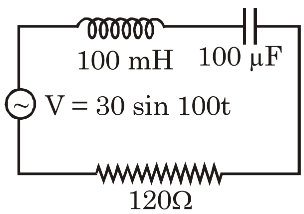
12. Find the peak current and resonant frequency of the following circuit (as shown in figure).



(1) 0.2 A and 50 Hz (2) 0.2 A and 100 Hz
(3) 2 A and 100 Hz (4) 2A and 50 Hz

Official Ans. by NTA (1)

Sol.



as given $z = \sqrt{(x_L - x_C)^2 + R^2}$

$x_L = \omega_L = 100 \times 100 \times 10^{-3} = 10\Omega$

$$x_C = \frac{1}{\omega_C} = \frac{1}{100 \times 100 \times 10^{-6}} = 10\Omega$$

$$z = \sqrt{(10-100)^2 + R^2} = \sqrt{90^2 + 120^2}$$

= 30 × 5= 150Ω

$$i_{peak} = \frac{\Delta v}{z} = \frac{30}{150} = \frac{1}{5} amp = 0.2 amp$$

& For resonant frequency

$$\Rightarrow \omega L = \frac{1}{\omega C} \Rightarrow \omega^2 = \frac{1}{LC} \Rightarrow \omega = \frac{1}{\sqrt{LC}}$$

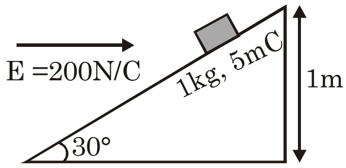
$$\& \quad f = \frac{1}{2\pi\sqrt{LC}} \Rightarrow \frac{1}{2\pi\sqrt{100 \times 10^{-3} \times 100 \times 10^{-6}}}$$

$$= \frac{100\sqrt{10}}{2\pi} = \frac{100\pi}{2\pi} = 50 Hz$$

as $\sqrt{10} \approx \pi$

Answer (1)

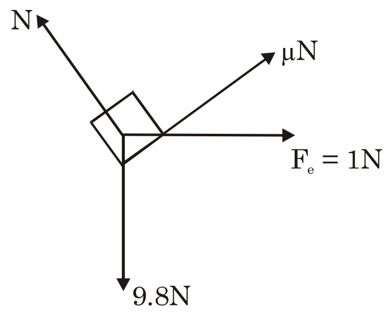
13. An inclined plane making an angle of 30° with the horizontal is placed in a uniform horizontal electric field $200\frac{N}{C}$ as shown in the figure. A body of mass 1kg and charge 5 mC is allowed to slide down from rest at a height of 1m. If the coefficient of friction is 0.2, find the time taken by the body to reach the bottom.[g = 9.8 m/s^2, sin 30° = $\frac{1}{2}$; cos 30° = $\frac{\sqrt{3}}{2}$]



(1) 0.92 s (2) 0.46 s
(3) 2.3 s (4) 1.3 s

Official Ans. by NTA (4)

Sol. FBD



here N = 9.8 cos 30 + 1 sin 30

$\approx$ 9N

so a = $\dfrac{9.8\sin 30 - 1\cos 30 - \mu N}{1}$

a = 2.233 m/s^2

By S = ut + $\dfrac{1}{2}at^2$

= $\dfrac{1}{2}(2.233)t^2$

sin30°

t $\approx$ 1.3 sec

14. Two masses A and B, each of mass M are fixed together by a massless spring. A force acts on the mass B as shown in figure. If the mass A starts moving away from mass B with acceleration 'a', then the acceleration of mass B wil be :-

(1) $\dfrac{Ma - F}{M}$ (2) $\dfrac{MF}{F + Ma}$

(3) $\dfrac{F + Ma}{M}$ (4) $\dfrac{F - Ma}{M}$

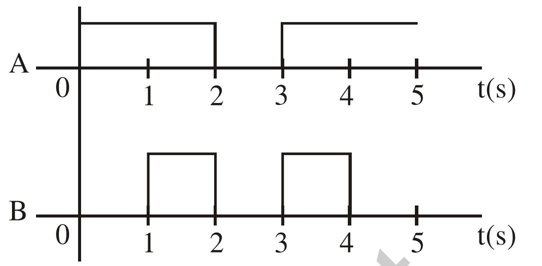
Official Ans. by NTA (4)

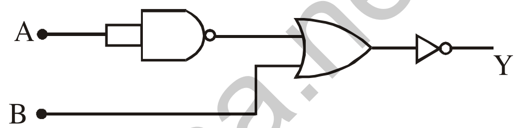
Sol. $a_{cm} = \dfrac{m_1a_1 + m_2a_2}{m_1 + m_2}$

$\dfrac{F}{2M} = \dfrac{Ma + Ma_B}{2M}$

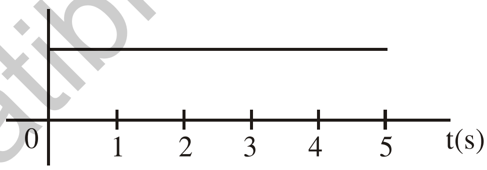
$a_B = \dfrac{F - Ma}{M}$

15. Draw the output signal Y in the given combination of gates :-

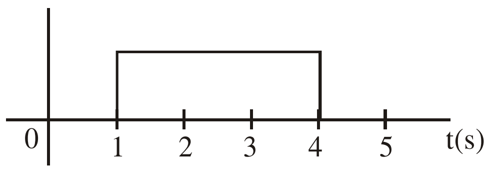




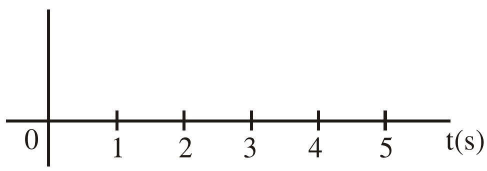
(1)



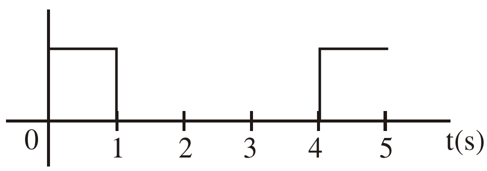
(2)



(3)



(4)



Official Ans. by NTA (4)

Sol. According to gates

by Demorgan's law

$\overline{\overline{A} + B} = A \,.\, \overline{B}$

By observation.

16. A radioactive sample is undergoing α decay. At any time t_1, its activity is A and another time t_2, the activity is $\frac{A}{5}$. What is the average life time for the sample ?

(1) $\frac{\ell n5}{t_2 - t_1}$ (2) $\frac{t_1 - t_2}{\ell n5}$

(3) $\frac{t_2 - t_1}{\ell n5}$ (4) $\frac{\ell n(t_2 + t_1)}{2}$

Official Ans. by NTA (3)

Sol. Let initial activity be A_0

$A = A_0 e^{-\lambda t_1}$ .....(i)

$\frac{A}{5} = A_0 e^{-\lambda t_2}$ .....(ii)

(i) $\div$ (ii)

$5 = e^{\lambda(t_2 - t_1)}$

$$\lambda = \frac{\ell n5}{t_2 - t_1} = \frac{1}{\tau}$$

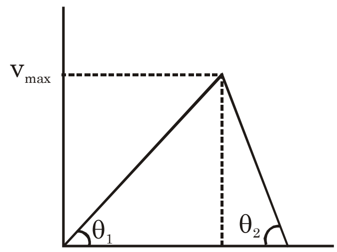
$$\tau = \frac{t_2 - t_1}{\ell n5}$$

17. A scooter accelerates from rest for time t_1 at constant rate a_1 and then retards at constant rate a_2 for time t_2 and comes to rest. The correct value of $\frac{t_1}{t_2}$ will be :-

(1) $\frac{a_1 + a_2}{a_2}$ (2) $\frac{a_2}{a_1}$ (3) $\frac{a_1}{a_2}$ (4) $\frac{a_1 + a_2}{a_1}$

Official Ans. by NTA (2)

Sol. Draw vt curve



$\tan\theta_1 = a_1 = \frac{v_{max}}{t_1}$

& $\tan\theta_2 = a_2 = \frac{v_{max}}{t_2}$

$\div$ above

$$\frac{t_1}{t_2} = \frac{a_2}{a_1}$$

18. Given below are two statements :

Statement I : A second's pendulum has a time period of 1 second.

Statement II : It takes precisely one second to move between the two extreme positions.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both Statement I and Statement II are false.

(2) Statement I is false but Statement II is true

(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are true.

Official Ans. by NTA (2)

Sol. Second pendulum has a time period of 2 sec so statement 1 is false but from one extreme to other it takes only half the time period so statement 2 is true.

19. A wire of 1Ω has a length of 1m. It is stetched till its length increases by 25%. The percentage change in resistance to the neartest integer is :-

(1) 56% (2) 25% (3) 12.5% (4) 76%

Official Ans. by NTA (1)

Sol. $R_0 = 1\Omega$ $R_1 = ?$

$\ell_0 = 1m$ $\ell_1 = 1.25m$

$A_0 = A$

As volume of wire remains constant so

$A_0\ell_0 = A_1\ell_1 \Rightarrow A_1 = \frac{\ell_0 A_0}{\ell_1}$

Now

Resistance (R) = $\frac{\rho\ell}{A}$

$$\frac{R_0}{R_1} = \frac{\ell_0 / A_0}{\rho\ell_1 / A_1}$$

$$\frac{1}{R_1} = \frac{\ell_0}{A_0}\left(\frac{\ell_0 A_0}{\ell_1 \times \ell_1}\right) \quad R_1 = \frac{\ell_1^2}{\ell_0^2} = 1.5625\Omega$$

So % change in resistance

$$= \frac{R_1 - R_0}{R_0} \times 100\%$$

$$= \frac{1.5625 - 1}{1} \times 100\%$$

= 56.25%

20. The incident ray, reflected ray and the outward drawn normal are denoted by the unit vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively. Then choose the correct relation for these vectors.

(1) $\vec{b} = \vec{a} + 2\vec{c}$ (2) $\vec{b} = 2\vec{a} + \vec{c}$

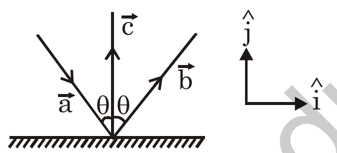
(3) $\vec{b} = \vec{a} - 2(\vec{a}.\vec{c})\vec{c}$ (4) $\vec{b} = \vec{a} - \vec{c}$

Official Ans. by NTA (3)

Sol. $\vec{a} = \sin\theta\hat{i} - \cos\theta\hat{j}$

$\vec{b} = \sin\theta\hat{i} + \cos\theta\hat{j}$

$\vec{c} = \hat{j}$



$\vec{a} - 2(\vec{a}.\vec{c})\vec{c} = \sin\theta\hat{i} + \cos\theta\hat{j}$

SECTION-B

1. The volume V of a given mass of monoatomic gas changes with temperature T according to the relation $V = KT^{2/3}$. The workdone when temperature changes by 90 K will be xR. The value of x is [R = universal gas constant]

Official Ans. by NTA (60)

Sol. We know that work done is

$$W = \int PdV \quad \text{... (1)}$$

$$\Rightarrow P = \frac{nRT}{V} \quad \text{... (2)}$$

$$\Rightarrow W = \int \frac{nRT}{V} dv \quad \text{... (3)}$$

and $V = KT^{2/3}$ ... (4)

$$\Rightarrow W = \int \frac{nRT}{KT^{2/3}} \cdot dv \quad \text{... (5)}$$

$\Rightarrow$ from (4) : $dv = \frac{2}{3}KT^{-1/3}dT$

$$\Rightarrow W = \int_{T_1}^{T_2} \frac{nRT}{KT^{2/3}} \frac{2}{3} K \frac{1}{T^{1/3}} dT$$

$$\Rightarrow W = \frac{2}{3} nR \times (T_2 - T_1) \quad \text{... (6)}$$

$\Rightarrow T_2 - T_1 = 90$ K ... (7)

$$\Rightarrow W = \frac{2}{3} nR \times 90$$

$\Rightarrow$ W = 60 nR

Assuming 1 mole of gas

n = 1

So W = 60R

2. If the highest frequency modulating a carrier is 5 kHz, then the number of AM broadcast stations accommodated in a 90 kHz bandwidth are ....

Official Ans. by NTA (9)

Sol. B. W. (Bandwidth) = 2 × maximum frequency at modulating signal

= 2 × 5kHz

= 10 kHz

∴ No of stations accommodate

$$= \frac{90}{10} = 9$$

3. Two stream of photons, possessing energies equal to twice and ten times the work function of metal are incident on the metal surface successively. The value of ratio of maximum velocities of the photoelectrons emitted in the two respective cases is x : y. The value of x is ....

Official Ans. by NTA (1)

Sol. $KE_{max} = h\nu - \phi$

$$\frac{1}{2}mv^2 = h\nu - \phi$$

$$v = \sqrt{\frac{2(h\nu - \phi)}{m}}$$

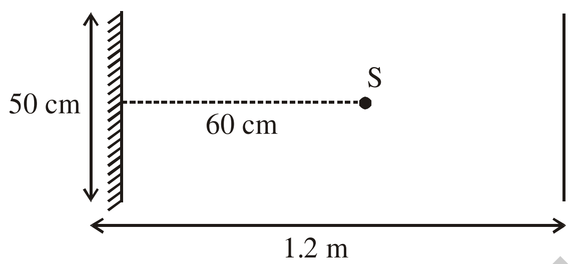
Given $h\nu_1 = 2\phi$

$h\nu_2 = 10\ \phi$

$$\therefore \frac{v_1}{v_2} = \sqrt{\frac{h\nu_1 - \phi}{h\nu_2 - \phi}}$$

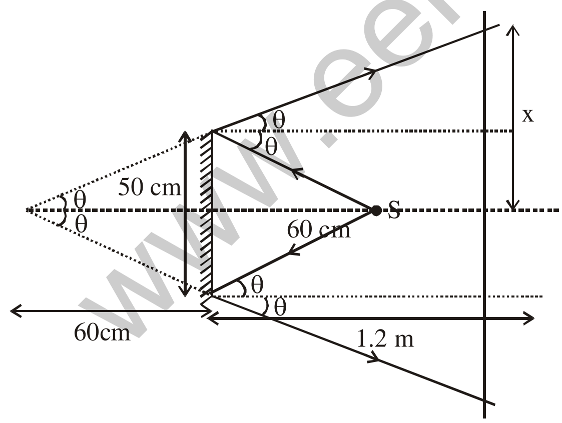
$$\frac{v_1}{v_2} = \sqrt{\frac{2\phi - \phi}{10\phi - \phi}} = \frac{1}{3}$$

4. A point source of light S, placed at a distance 60 cm infront of the centre of a plane mirror of width 50 cm, hangs vertically on a wall. A man walks infront of the mirror along a line parallel to the mirror at a distance 1.2 m from it (see in the figure). The distance between the extreme points where he can see the image of the light source in the mirror is .... cm.



Official Ans. by NTA (150)

Sol.



$$\tan\theta = \frac{25}{60} = \frac{x}{180}$$

x = 75 cm

so distance between extreme point = 2x = 2 × 75 = 150 cm

5. A particle executes S.H.M. with amplitude 'a' and time period V. The displacement of the particle when its speed is half of maximum speed is $\frac{\sqrt{xa}}{2}$. The value of x is ....

Official Ans. by NTA (3)

Sol. $V = \omega\sqrt{A^2 - x^2}$ $\qquad V_{max} = A\omega$

$$\frac{A\omega}{2} = \omega\sqrt{A^2 - x^2}$$

$$\frac{A^2}{4} = A^2 - x^2$$

$$x^2 = \frac{3A^2}{4}$$

$$x = \frac{\sqrt{3}}{2}A$$

6. 27 similar drops of mercury are maintained at 10 V each. All these spherical drops combine into a single big drop. The potential energy of the bigger drop is .... times that of a smaller drop.

Official Ans. by NTA (243)

Sol. $(27)\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3$

R = 3r

Potential energy of smaller drop :

$$U_1 = \frac{3}{5}\frac{kq^2}{r}$$

Potential energy of bigger drop :

$$U = \frac{3}{5}\frac{kQ^2}{R}$$

$$U = \frac{3}{5}\frac{k(27q)^2}{R}$$

$$U = \frac{3}{5}k\frac{(27)(27)q^2}{3r}$$

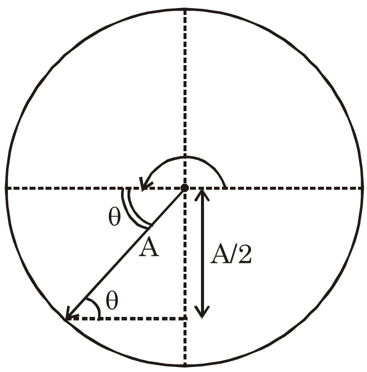
$$U = \frac{(27)(27)}{3}\left(\frac{3}{5}\frac{kq^2}{r}\right)$$

$U = 243\ U_1$

7. Time period of a simple pendulum is T. The time taken to complete 5/8 oscillations starting from mean position is $\frac{\alpha}{\beta}T$. The value of α is ....

Official Ans. by NTA (7)

Sol. $\frac{5}{8}$th of oscillation = $\left(\frac{1}{2}+\frac{1}{8}\right)^{th}$ of oscillation



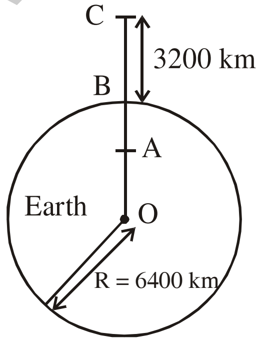
$\pi + \theta = \omega t$

$\pi + \frac{\pi}{6} = \left(\frac{2\pi}{T}\right)t$

$\frac{7\pi}{6} = \left(\frac{2\pi}{T}\right)t$

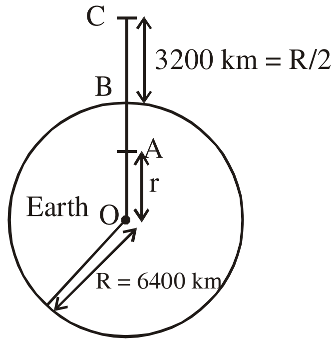
$t = \frac{7T}{12}$

8. In the reported figure of earth, the value of acceleration due to gravity is same at point A and C but it is smaller than that of its value at point B (surface of the earth). The value of OA : AB will be x : y. The value of x is ....



Official Ans. by NTA (4)

Sol. $g_A = \frac{GM(r)}{R^3}$



$g_C = \frac{GM}{\left(R+\frac{R}{2}\right)^2}$

$g_A = g_C$

$\frac{r}{R^3} = \frac{1}{\frac{9}{4}R^2} \Rightarrow r = \frac{4R}{9}$

so OA = $\frac{4R}{9}$; AB = R – r = $\frac{5R}{9}$

OA : AB = 4 : 5

9. 1 mole of rigid diatomic gas performs a work of Q/5 when heat Q is supplied to it. The molar heat capacity of the gas during this transformation is $\frac{xR}{8}$, The value of x is ....

[K = universal gas constant]

Official Ans. by NTA (25)

Sol. $Q = \Delta U + W$

$Q = \Delta U + \frac{Q}{5}$

$\Delta U = \frac{4Q}{5}$

$nC_v\Delta T = \frac{4}{5}nC\Delta T$

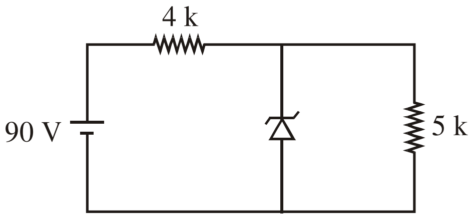
$\frac{5}{4}C_V = C$

$C = \frac{5}{4}\left(\frac{f}{2}\right)R = \frac{5}{4}\left(\frac{5}{2}\right)R$

$C = \frac{25}{8}R$

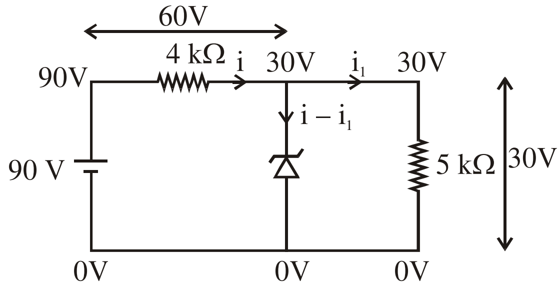
x = 25

10. The zener diode has a V_z = 30 V. The current passing through the diode for the following circuit is .... mA.



Official Ans. by NTA (9)

Sol.



$$i = \frac{60}{4000} A$$

$$i_1 = \frac{30}{5000} A$$

$$i - i_1 = \frac{60}{4000} - \frac{30}{5000} = \frac{9}{1000} A$$

current from zener diode

$i_z = i - i_1$ = 9mA

SECTION-A

1. Which of the following forms of hydrogen emits low energy β^- particles?

(1) Deuterium $^{2}_{1}H$ (2) Tritium $^{3}_{1}H$

(3) Protium $^{1}_{1}H$ (4) Proton H^+

Official Ans. by NTA (2)

Sol. For tritium $(^{3}_{1}H)$

No. of neutron (n) = 2

No. of proton (p) = 1

$\frac{n}{p} = 2$

$\because \frac{n}{p}$ is high ,

tritium wil emit β particle.

2. Given below are two statements :one is labelled as Assertion A and the other is labelled as Reason R

Assertion A : In $T\ell I_3$, isomorphous to CsI_3, the metal is present in +1 oxidation state.

Reason R : $T\ell$ metal has fourteen f electrons in the electronic configuration.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) **A** is correct but **R** is not correct

(2) Both **A** and **R** are correct and **R** is the correct explanation of **A**.

(3) **A** is not correct but **R** is correct

(4) Both **A** and **R** are correct but **R** is NOT the correct explanation of **A**.

Official Ans. by NTA (4)

Sol. $T\ell I_3 \Rightarrow \left(T\ell^{\oplus} \,\&\, I_3^{\ominus}\right)$

$CsI_3 \Rightarrow \left(Cs^{\oplus} \,\&\, I_3^{\ominus}\right)$

[Both have same crystalline structure is called isomorphous]

$T\ell^{\oplus}_{(81)} = [Xe_{54}]4f^{14}, 5d^{10}, 6s^2$

(It is correct due to present 14 f electrons in $T\ell^{\oplus}$ ion)

3. Match List-I with List-II

List-I	List-II
(a) Sucrose	(i) β-D-Galactose and β-D-Glucose
(b) Lactose	(ii) α-D-Glucose and β-D-Fructose
(c) Maltose	(iii) α-D-Glucose and α-D-Glucose

Choose the correct answer from the options given below :

Options :

(1) (a) $\rightarrow$ (i), (b) $\rightarrow$ (iii), (c) $\rightarrow$ (ii)

(2) (a) $\rightarrow$ (iii), (b) $\rightarrow$ (i), (c) $\rightarrow$ (iii)

(3) (a) $\rightarrow$ (ii), (b) $\rightarrow$ (i), (c) $\rightarrow$ (iii)

(4) (a) $\rightarrow$ (iii), (b) $\rightarrow$ (ii), (c) $\rightarrow$ (i)

Official Ans. by NTA (3)

Sol.

(1) Sucrose $\rightarrow$ α-D-Glucose and β-D-Fructose

(2) Lactose $\rightarrow$ β-D-Galactose and β-D-Glucose

(3) Maltose $\rightarrow$ α-D-Glucose and α-D-Glucose

a $\rightarrow$ II

b $\rightarrow$ I

c $\rightarrow$ III

4. A. Phenyl methanamine

B. N,N-Dimethylaniline

C. N-Methyl aniline

D. Benzenamine

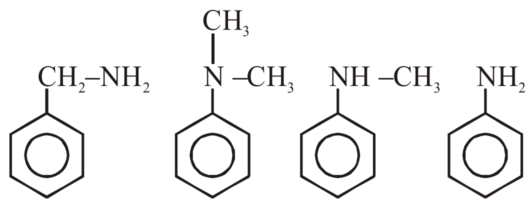
Choose the correct order of basic nature of the above amines.

(1) A > C > B > D (2) D > C > B > A

(3) D > B > C > A (4) A > B > C > D

Official Ans. by NTA (4)

Sol.



(A) (B) (C) (D)

B.S. order (A) > (B) > (C) > (D)

5. The correct order of electron gain enthalpy is

(1) S > Se > Te > O (2) Te > Se > S > O

(3) O > S > Se > Te (4) S > O > Se > Te

Official Ans. by NTA (1)

Sol. correct order of electron gain enthalpy is :-

O < S > Se > Te

$\Rightarrow$ S > Se > Te > O

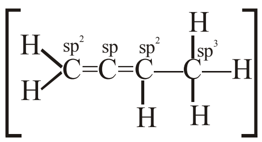
(Oxygen shows least electron gain enthalpy due to small size of atom)

6. In $\overset{1}{C}H_2=\overset{2}{C}=\overset{3}{C}H-\overset{4}{C}H_3$ molecule, the hybridization of carbon 1,2,3 and 4 respectively are :

(1) sp^3, sp, sp^3, sp^3 (2) sp^2, sp^2, sp^2, sp^3
(3) sp^2, sp, sp^2, sp^3 (4) sp^2, sp^3, sp^2, sp^3

Official Ans. by NTA (3)

Sol. $[\overset{1}{C}H_2=\overset{2}{C}=\overset{3}{C}H—\overset{4}{C}H_3]$



7. Seliwanoff test and Xanthoproteic test are used for the identification of _______and _____ respectively

(1) Aldoses, ketoses (2) Proteins, ketoses
(3) Ketoses, proteins (4) Ketoses, aldoses

Official Ans. by NTA (3)

Sol. Seliwanoff test for ketose and Xenthoprotic test for proteins.

8. 2,4-DNP test can be used to identify :

(1) Amine (2) Aldehyde
(3) Ether (4) Halogens

Official Ans. by NTA (2)

Sol. 2,4-DNP test is useful for the identification of carbonyl compounds.

9. Ceric ammonium nitrate and $CHCl_3$ / alc. KOH are used for the identification of functional groups present in ______ and ___ respectively.

(1) Alcohol, phenol (2) Amine, alcohol
(3) Alcohol, amine (4) Amine, phenol

Official Ans. by NTA (3)

Sol. Ceric ammonium nitrate for alcohol and $CHCl_3$/ KOH is carbyl amine test for primary amines

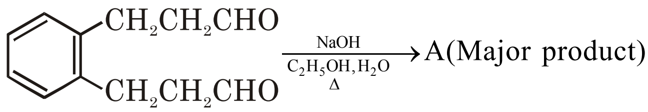
10. Which pair of oxides is acidic in nature?

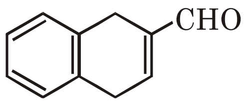
(1) B_2O_3, CaO (2) B_2O_3, SiO_2
(3) N_2O, BaO (4) CaO, SiO_2

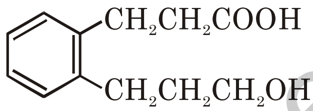
Official Ans. by NTA (2)

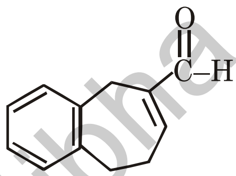
Sol. $\left\{\begin{array}{l} CaO, BaO = \text{Basic Nature} \\ B_2O_3, SiO_2 = \text{Acidic Nature} \\ N_2O = \text{Neutral oxide} \end{array}\right.$

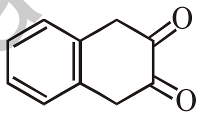
11. Identify A in the given chemical reaction,



(1) 

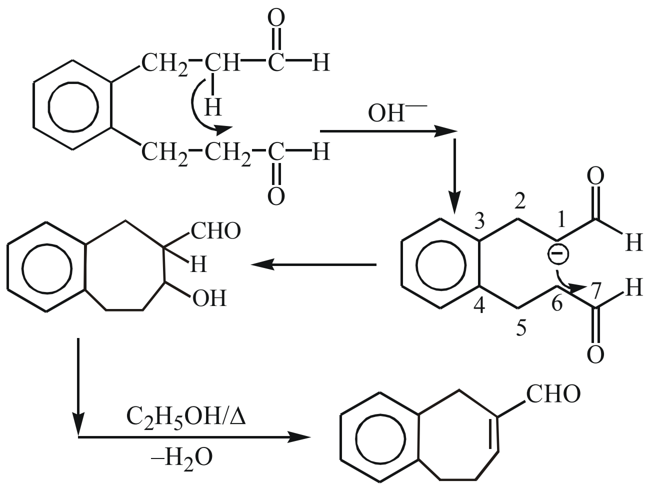
(2) 

(3) 

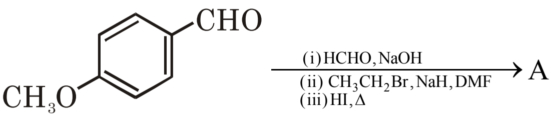
(4) 

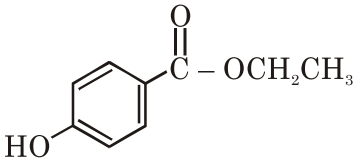
Official Ans. by NTA (3)

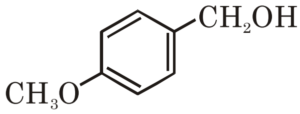
Sol.

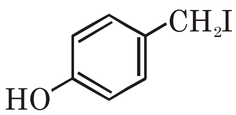


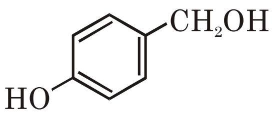
12. Identify A in the following chemical reaction



(1) 

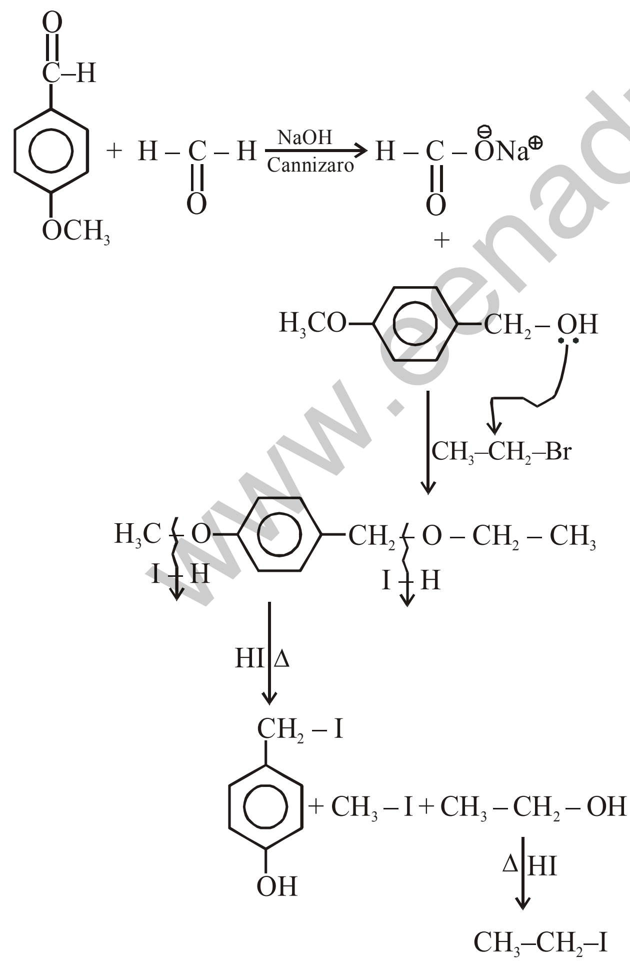
(2) 

(3) 

(4) 

Official Ans. by NTA (3)

Sol.



13. Calgon is used for water treatment. Which of the following statement is NOT true about Calgon?

(1) Calgon contains the 2nd most abundant element by weight in the Earth's crust.

(2) It is polymeric compound and is water soluble.

(3) It is also known as Graham's salt

(4) It does not remove Ca^{2+} ion by precipitation.

Official Ans. by NTA (1)

Sol. → 2nd most abundant element is "Si" and it is not present in calgon

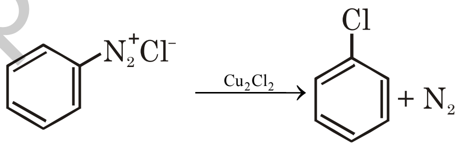
$Na_6P_6O_{18}$ = (Graham's salt) (Sodium hexametaphosphate)

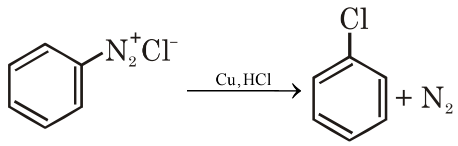
→ It exist in polymeric form as $(NaPO_3)_6$ and water soluble compound

→ It removes Ca^{2+} in soluble ion but not by precipitation

14. Match List-I with List-II

List-I

(a) 

(b) 

(c) $2CH_3CH_2Cl + 2Na \xrightarrow{\text{Ether}} C_2H_5–C_2H_5 + 2NaCl$

(d) $2C_6H_5Cl + 2Na \xrightarrow{\text{Ether}} C_6H_5–C_6H_5 + 2NaCl$

List-II

(i) Wurtz reaction

(ii) Sandmeyer reaction

(iii) Fittig reaction

(iv) Gatterman reaction

Choose the correct answer from the options given below :

(1) (a) → (iii), (b) → (i), (c) → (iv), (d) → (ii)

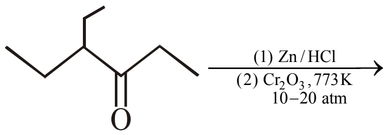
(2) (a) → (ii), (b) → (i), (c) → (iv), (d) → (iii)

(3) (a) → (ii), (b) → (iv), (c) → (i), (d) → (iii)

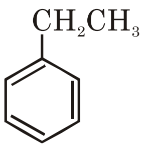
(4) (a) → (iii), (b) → (iv), (c) → (i), (d) → (ii)

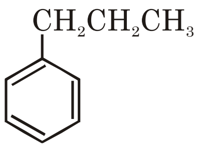
Official Ans. by NTA (3)

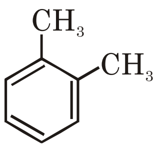
Sol. (a) $\rightarrow$ (ii) Sand Meyer reaction
(b) $\rightarrow$ (iv) Gatterman reaction
(c) $\rightarrow$ (i) Wurtz reaction
(d) $\rightarrow$ (iii) Fittig reaction
(a) $\rightarrow$ (ii),
(b) $\rightarrow$ (iv),
(c) $\rightarrow$ (i),
(d) $\rightarrow$ (iii)

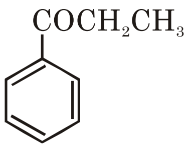
15. 

considering the above reaction, the major product among the following is :

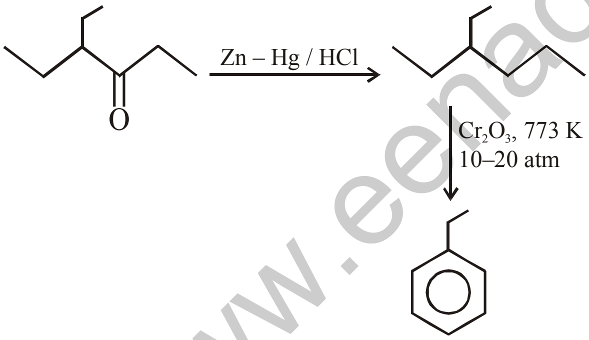
(1) 

(2) 

(3) 

(4) 

Official Ans. by NTA (1)

Sol. 

16. Match List-I with List-II.

	List-I (Molecule)		List-II (Bond order)
(a)	Ne_2	(i)	1
(b)	N_2	(ii)	2
(c)	F_2	(iii)	0
(d)	O_2	(iv)	3

Choose the correct answer from the options given below :
(1) (a)$\rightarrow$(iii), (b)$\rightarrow$(iv), (c)$\rightarrow$(i), (d)$\rightarrow$(ii)
(2) (a)$\rightarrow$(i), (b)$\rightarrow$(ii), (c)$\rightarrow$(iii), (d)$\rightarrow$(iv)
(3) (a)$\rightarrow$(ii), (b)$\rightarrow$(i), (c)$\rightarrow$(iv), (d)$\rightarrow$(iii)
(4) (a)$\rightarrow$(iv), (b)$\rightarrow$(iii), (c)$\rightarrow$(ii), (d)$\rightarrow$(i)

Official Ans by NTA (1)

Sol. (a) Ne_2 = Total $e^{\ominus}$ = 20

$$B.O. = \frac{10-10}{2} = 0$$

(b) N_2 = Total $e^{\ominus}$ = 14

$$B.O. = \frac{10-4}{2} = 3$$

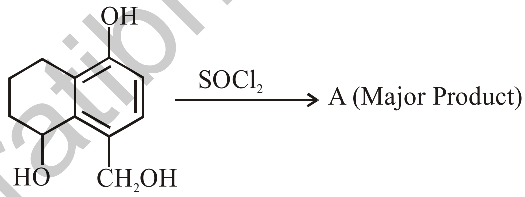
(c) F_2 = Total $e^{\ominus}$ = 18

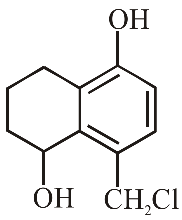
$$B.O. = \frac{10-8}{2} = 1$$

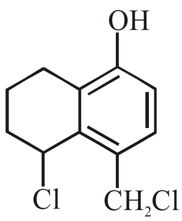
(d) O_2 = Total $e^{\ominus}$ = 16

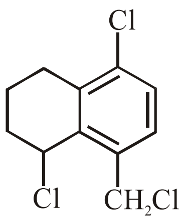
$$B.O. = \frac{10-6}{2} = 2$$

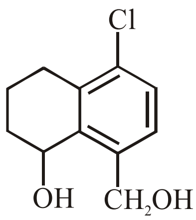
17. Identify A in the given reaction.



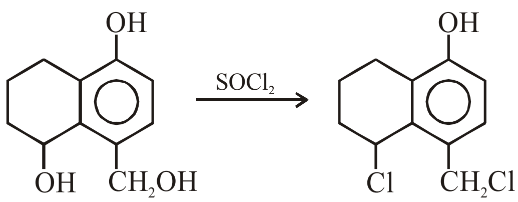
(1) 

(2) 

(3) 

(4) 

Official Ans by NTA (2)

Sol. 

18. Match List-I with List-II.

	List-I		List-II
(a)	Siderite	(i)	Cu
(b)	Calamine	(ii)	Ca
(c)	Malachite	(iii)	Fe
(d)	Cryolite	(iv)	Al
		(v)	Zn

Choose the correct answer from the options given below :

(1) (a)→(iii), (b)→(i), (c)→(v), (d)→(ii)
(2) (a)→(i), (b)→(ii), (c)→(v), (d)→(iii)
(3) (a)→(iii), (b)→(v), (c)→(i), (d)→(iv)
(4) (a)→(i), (b)→(ii), (c)→(iii), (d)→(iv)

Official Ans by NTA (3)

Sol. (a) Siderite = $FeCO_3$ = Fe-metal
(b) Calamine = $ZnCO_3$ = Zn-metal
(c) Malachite = $Cu(OH)_2.CuCO_3$ = Cu-metal
(d) Cryolite = $Na_3A\ell F_6$ = $A\ell$-metal

19. The nature of charge on resulting colloidal particles when $FeCl_3$ is added to excess of hot water is :

(1) Positive
(2) Sometimes positive and sometimes negative
(3) Neutral
(4) Negative

Official Ans by NTA (1)

Sol. If $FeCl_3$ is added to hot water, a positively charged sol, hydrated ferric oxide is formed due to adsorption of Fe^{3+} ions.

$Fe_2O_3.\ xH_2O/Fe^{3+}$

Positively charged.

20. Match List-I with List-II.

	List-I		List-II
(a)	Sodium Carbonate	(i)	Deacon
(b)	Titanium	(ii)	Castner-Kellner
(c)	Chlorine	(iii)	Van-Arkel
(d)	Sodium hydroxide	(iv)	Solvay

Choose the correct answer from the options given below :

(1) (a)→(iv), (b)→(iii), (c)→(i), (d)→(ii)
(2) (a)→(i), (b)→(iii), (c)→(iv), (d)→(ii)
(3) (a)→(iv), (b)→(i), (c)→(ii), (d)→(iii)
(4) (a)→(iii), (b)→(ii), (c)→(i), (d)→(iv)

Official Ans by NTA (1)

Sol. (a) Sodium carbonate is prepared by Solvay process
(b) Titanium is refined by Van-Arkel process
(c) Chlorine is prepared by Deacon process
(d) Sodium hydroxide is prepared by Castner-Kellner process

SECTION-II

1. The $NaNO_3$ weighed out to make 50 mL of an aqueous solution containing 70.0 mg Na^+ per mL is ______g. (Rounded off to the nearest integer)

[Given : Atomic weight in g mol^{-1} – Na : 23 ; N : 14 ; O : 16]

Official Ans by NTA (13)

Sol. Na^+ present in 50 ml

$$= \frac{70mg}{1ml} \times 50ml = 3500 \text{ mg} = 3.5 \text{ gm}$$

$$\text{moles of } Na^+ = \frac{3.5}{23} = \text{moles of } NaNO_3$$

$$\text{weight of } NaNO_3 = \frac{3.5}{23} \times 85 = 12.993gm$$

2. Emf of the following cell at 298 K in V is $x \times 10^{-2}$. $Zn|Zn^{2+}$ (0.1 M)||Ag^+ (0.01 M)| Ag

The value of x is ______. (Rounded off to the nearest integer)

[Given : $E^0_{Zn^{2+}/Zn} = -0.76V; E^0_{Ag^+/Ag} = +0.80V; \frac{2.303RT}{F} = 0.059$]

Official Ans by NTA (147)

Sol. $Zn_{(s)} \rightarrow Zn^{2+}_{(aq.)} + 2e^-$

$2Ag^+_{(aq.)} + 2e^- \rightarrow 2Ag_{(s)}$

$Zn_{(s)} + 2Ag^+_{(aq.)} \rightarrow Zn^{2+}_{(aq.)} + 2Ag_{(s)}$

$$E^0_{cell} = E^0_{Ag^+/Ag} - E^0_{Zn^{2+}/Zn}$$
$= 0.80 - (-0.76)$
$= 1.56$ V

$$E_{cell} = 1.56 \frac{-0.059}{2} \log\frac{[Zn^{2+}]}{[Ag^+]^2}$$

$$= 1.56 - \frac{0.059}{2} \log\frac{0.1}{(0.01)^2}$$

$$= 1.56 - \frac{0.059}{2} \times 3$$

$= 1.56 - 0.0885$
$= 1.4715$
$= 147.15 \times 10^{-2}$

3. When 12.2 g of benzoic acid is dissolved in 100 g of water, the freezing point of solution was found to be –0.93°C ($K_f(H_2O)$ = 1.86K kg mol^{-1}). The number (n) of benzoic acid molecules associated (assuming 100% association) is ____.

Official Ans by NTA (2)

Sol. $\Delta T_f = i \times k_f \times m$

$$0-(-0.93) = i \times 1.86 \times \frac{12.2}{122 \times 100} \times 1000$$

$$i = \frac{0.93}{1.86} = 0.5$$

$$i = 1 + \left(\frac{1}{n} - 1\right)\alpha$$

$$\frac{1}{2} = 1 + \left(\frac{1}{n} - 1\right) \times 1$$

n = 2

4. The average S–F bond energy in kJ mol^{-1} of SF_6 is ______.(Rounded off to the nearest integer)

[Given : The values of standard enthalpy of formation of SF_6(g), S(g) and F(g) are –1100, 275 and 80 kJ mol^{-1} respectively.]

Official Ans by NTA (309)

Sol. $SF_6(g) \rightarrow S(g) + 6F(g)$

If $\in$ - bond enthalpy

$$\Delta_r H = 6 \times \in_{S-F}$$

$$= \Delta_f H(S,g) + 6 \times \Delta_f H(F,g) - \Delta_f H(SF_6,g)$$

= 275 + 6 × 80 – (–1100)

= 1855 kJ

$$\in_{S-F} = \frac{1855}{6} = 309.16 \text{ kJ/mol.}$$

5. A ball weighing 10 g is moving with a velocity of 90 ms^{-1}. If the uncertainty in its velocity is 5%, then the uncertainty in its position is_______$\times 10^{-33}$ m. (Rounded off to the nearest integer)

[Given : h = 6.63 × 10^{-34} Js]

Official Ans by NTA (1)

$$\Delta v = 90 \times \frac{5}{100}$$

= 4.5 m/s

$$\Delta v . \Delta x = \frac{h}{4\pi m}$$

$$\Delta x = \frac{h}{4\pi m . \Delta v}$$

$$= \frac{6.63 \times 10^{-34}}{4 \times 3.14 \times 0.01 \times 4.5}$$

= 1.17 × 10^{-33}

6. The number of octahedral voids per lattice site in a lattice is _____.(Rounded off to the nearest integer)

Official Ans by NTA (1)

Sol. If number of lattice points are N.
then effective octahedral voids = N
So, octahedral voids / lattice site = 1

7. In mildly alkaline medium, thiosulphate ion is oxidized by MnO_4^- to "A". The oxidation state of sulphur in "A" is ______.

Official Ans by NTA (6)

Sol. $$MnO_4^- + S_2O_3^{2-} \rightarrow \overset{+4}{M}nO_2 + \underset{(A)}{SO_4^{2-}}$$

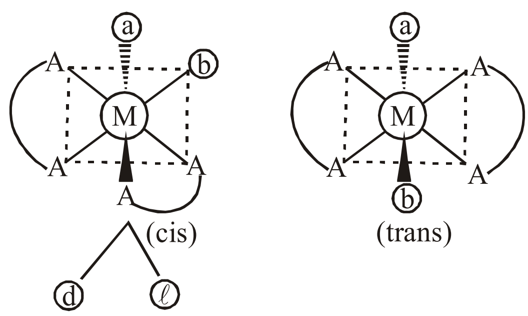
Oxidation state of 'S' in SO_4^{2-}

= + 6

8. The number of stereoisomers possible for $[Co(ox)_2(Br)(NH_3)]^{2-}$ is _______.[ox = oxalate]

Official Ans by NTA (3)

Sol. Total number of stereoisomers in $[Co(ox)_2Br(NH_3)]^{2\ominus}$ i.e. $\simeq [M(AA)_2ab]^{2-}$



→ cis is optically active isomers and trans is optically inactive isomer

→ Hence total isomers is = 3

9. If the activation energy of a reaction is 80.9 kJ mol^{-1}, the fraction of molecules at 700 K, having enough energy to react to form products is e^{-x}. The value of x is _______.

(Rounded off to the nearest integer)

[Use R = 8.31 J K^{-1} mol^{-1}]

Official Ans by NTA (14)

Sol. Fraction of molecules to have enough energy to react = $e^{-Ea/RT}$

So, $x = \frac{E_a}{RT}$

$= \frac{80.9 \times 10^3}{8.31 \times 700}$

= 13.9

10. The pH of ammonium phosphate solution, if pK_a of phosphoric acid and pk_b of ammonium hydroxide are 5.23 and 4.75 respectively, is ______.

Official Ans by NTA (7)

Sol. Since $(NH_4)_3PO_4$ is salt of weak acid (H_3PO_4) & weak base (NH_4OH).

$pH = 7 + \frac{1}{2}(pka - pkb)$

$= 7 + \frac{1}{2}(5.23 - 4.75)$

$= 7.24 \approx 7.$

SECTION-A

1. If vectors $\vec{a}_1 = x\hat{i} - \hat{j} + \hat{k}$ and $\vec{a}_2 = \hat{i} + y\hat{j} + z\hat{k}$ are collinear, then a possible unit vector parallel to the vector $x\hat{i} + y\hat{j} + z\hat{k}$ is

(1) $\frac{1}{\sqrt{2}}\left(-\hat{j} + \hat{k}\right)$ (2) $\frac{1}{\sqrt{2}}\left(\hat{i} - \hat{j}\right)$

(3) $\frac{1}{\sqrt{3}}\left(\hat{i} + \hat{j} - \hat{k}\right)$ (4) $\frac{1}{\sqrt{3}}\left(\hat{i} - \hat{j} + \hat{k}\right)$

Official Ans. by NTA (4)

Sol. $\vec{a}_1$ and $\vec{a}_2$ are collinear

so $\frac{x}{1} = \frac{-1}{y} = \frac{1}{z}$

unit vector in direction of

$$x\hat{i} + y\hat{j} + z\hat{k} = \pm\frac{1}{\sqrt{3}}\left(\hat{i} - \hat{j} + \hat{k}\right)$$

2. Let A = {1, 2, 3, ..., 10} and $f : A \to A$ be defined as $f(k) = \begin{cases} k+1 & \text{if } k \text{ is odd} \\ k & \text{if } k \text{ is even} \end{cases}$

Then the number of possible functions $g : A \to A$ such that $gof = f$ is

(1) 10^5 (2) ${}^{10}C_5$ (3) 5^5 (4) 5!

Official Ans. by NTA (1)

Sol. $f(x) = \begin{cases} x+1, & \text{if } x \text{ is odd} \\ x, & \text{if } x \text{ is even} \end{cases}$

$\because$ $g : A \to A$ such that $g(f(x)) = f(x)$

$\Rightarrow$ If x is even then $g(x) = x$ ...(1)

If x is odd then $g(x + 1) = x + 1$ ...(2)

from (1) and (2) we can say that

$g(x) = x$ if x is even

$\Rightarrow$ If x is odd then g(x) can take any value in set A

so number of $g(x) = 10^5 \times 1$

3. Let $f : R \to R$ be defined as

$$f(x) = \begin{cases} 2\sin\left(-\frac{\pi x}{2}\right), & \text{if } x < -1 \\ |ax^2 + x + b|, & \text{if } -1 \le x \le 1 \\ \sin(\pi x), & \text{if } x > 1 \end{cases}$$

If $f(x)$ is continuous on R, then a + b equals:

(1) –3 (2) –1 (3) 3 (4) 1

Official Ans. by NTA (2)

Sol. $f(x)$ is continuous on R

$\Rightarrow f(1^-) = f(1) = f(1^+)$

$|a + 1 + b| = \lim_{x\to 1} \sin(\pi x)$

$|a + 1 + b| = 0 \Rightarrow a + b = -1$ ...(1)

$\Rightarrow$ Also $f(-1^-) = f(-1) = f(-1^+)$

$$\lim_{x\to -1} 2\sin\left(\frac{-\pi x}{2}\right) = |a - 1 + b|$$

$|a - 1 + b| = 2$

Either $a - 1 + b = 2$ or $a - 1 + b = -2$

$a + b = 3$ ...(2) or $a + b = -1$ ...(3)

from (1) and (2) $\Rightarrow a + b = 3 = -1$(reject)

from (1) and (3) $\Rightarrow a + b = -1$

4. For $x > 0$, if $f(x) = \int_1^x \frac{\log_e t}{(1+t)} dt$, then $f(e) + f\left(\frac{1}{e}\right)$ is equal to

(1) 1 (2) –1 (3) $\frac{1}{2}$ (4) 0

Official Ans. by NTA (3)

Sol. $f(x) = \int_1^x \frac{\log_e t}{(1+t)} dt$

$f\left(\frac{1}{x}\right) = \int_1^{1/x} \frac{\ell n t}{1+t} dt$, let $t = \frac{1}{y}$

$$=+\int_1^x \frac{\ell ny}{1+y}\cdot\frac{y}{y^2}dy$$

$$=\int_1^x \frac{\ell ny}{y(1+y)}dy$$

hence

$$f(x)+f\left(\frac{1}{x}\right)=\int_1^x \frac{(1+t)\ell nt}{t(1+t)}dt=\int_1^x \frac{\ell nt}{t}dt$$

$$=\frac{1}{2}\ell n^2(x)$$

so $f(e)+f\left(\frac{1}{e}\right)=\frac{1}{2}$ ..(3)

5. A natural number has prime factorization given by $n = 2^x3^y5^z$, where y and z are such that $y + z = 5$ and $y^{-1} + z^{-1} = \frac{5}{6}$, $y > z$. Then the number of odd divisors of n, including 1, is :
(1) 11 (2) 6 (3) 6x (4) 12

Official Ans. by NTA (4)

Sol. Ans. (4)

Sol. $y + z = 5$

$\frac{1}{y}+\frac{1}{z}=\frac{5}{6}$ $\quad y > z$

$\Rightarrow y = 3, z = 2$

$\Rightarrow n = 2^x.3^3.5^2 = (2.2.2 \ldots)(3.3.3)(5.5)$

Number of odd divisors $= 4 \times 3 = 12$

6. Let $f(x) = \sin^{-1}x$ and $g(x)=\frac{x^2-x-2}{2x^2-x-6}$. If $g(2)=\lim_{x\to 2} g(x)$, then the domain of the function fog is :

(1) $(-\infty,-2]\cup\left[-\frac{3}{2},\infty\right)$

(2) $(-\infty,-2]\cup[-1,\infty)$

(3) $(-\infty,-2]\cup\left[-\frac{4}{3},\infty\right)$

(4) $(-\infty,-1]\cup[2,\infty)$

Official Ans. by NTA (3)

Sol. Domain of $fog(x) = \sin^{-1}(g(x))$

$\Rightarrow |g(x)| \le 1$, $g(2) = \frac{3}{7}$

$$\left|\frac{x^2-x-2}{2x^2-x-6}\right|\le 1$$

$$\left|\frac{(x+1)(x-2)}{(2x+3)(x-2)}\right|\le 1$$

$$\frac{x+1}{2x+3}\le 1 \text{ and } \frac{x+1}{2x+3}\ge -1$$

$$\frac{x+1-2x-3}{2x+3}\le 0 \text{ and } \frac{x+1+2x+3}{2x+3}\ge 0$$

$$\frac{x+2}{2x+3}\ge 0 \text{ and } \frac{3x+4}{2x+3}\ge 0$$

$$x\in(-\infty,-2]\cup\left[-\frac{4}{3},\infty\right)$$

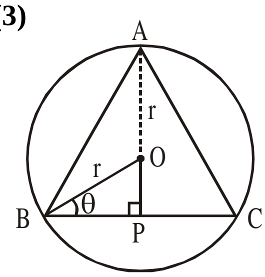
7. The triangle of maximum area that can be inscribed in a given circle of radius 'r' is :
(1) An isosceles triangle with base equal to 2r.
(2) An equilateral triangle of height $\frac{2r}{3}$.
(3) An equilateral triangle having each of its side of length $\sqrt{3}r$.
(4) A right angle triangle having two of its sides of length 2r and r.

Official Ans. by NTA (3)

Sol. $h = r\sin\theta + r$

base $= BC = 2r\cos\theta$

$\theta\in\left[0,\frac{\pi}{2}\right)$



Area of $\Delta ABC = \frac{1}{2}(BC).h$

$$\Delta=\frac{1}{2}(2r\cos\theta).(r\sin\theta+r)$$

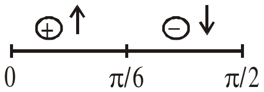
$= r^2(\cos\theta).(1+\sin\theta)$

$$\frac{d\Delta}{d\theta}=r^2\left[\cos^2\theta-\sin\theta-\sin^2\theta\right]$$

$= r^2[1-\sin\theta-2\sin^2\theta]$

$= \underbrace{r^2\left[1+\sin\theta\right]}_{\text{positive}}\left[1-2\sin\theta\right]=0$

$\Rightarrow \theta = \frac{\pi}{6}$



$\Rightarrow$ Δ is maximum where $\theta = \frac{\pi}{6}$

$\Delta_{max.} = \frac{3\sqrt{3}}{4}r^2$ = area of equilateral Δ with $BC = \sqrt{3}r$.

8. Let L be a line obtained from the intersection of two planes $x + 2y + z = 6$ and $y + 2z = 4$. If point $P(\alpha, \beta, \gamma)$ is the foot of perpendicular from (3, 2, 1) on L, then the value of $21(\alpha + \beta + \gamma)$ equals :

(1) 142 (2) 68 (3) 136 (4) 102

Official Ans. by NTA (4)

Sol. $x + 2y + z = 6$

$(y + 2z = 4) \times 2$

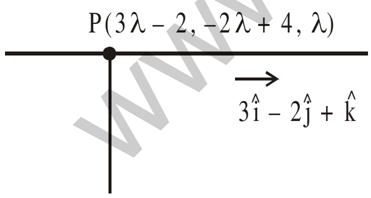
$x - 3z = -2 \Rightarrow x = 3z - 2 \Rightarrow y = 4 - 2z$

$\frac{x+2}{3} = z \qquad \frac{y-4}{-2} = z$

$\Rightarrow$ line of intersection of two planes is

$\frac{x+2}{3} = \frac{y-4}{-2} = z = \lambda$ (Let)

$\because$ AP $\perp^{ar}$ to line



$\therefore \overline{AP}.\left(3\hat{i}-2\hat{j}+\hat{k}\right)=0$

$(3\lambda - 5) . 3 + (-2\lambda + 2)(-2) + (\lambda - 1) . 1 = 0$

$9\lambda - 15 + 4\lambda - 4 + \lambda - 1 = 0$

$14\lambda = 20$

$\lambda = \frac{10}{7} \Rightarrow P\left(\frac{16}{7}, \frac{8}{7}, \frac{10}{7}\right)$

$\Rightarrow \alpha + \beta + \gamma = \frac{16+8+10}{7} = \frac{34}{7}$

$21(\alpha + \beta + \gamma) = 102$

9. Let $F_1(A,B,C) = (A \wedge \sim B) \vee [\sim C \wedge (A \vee B)] \vee \sim A$ and $F_2(A, B) = (A \vee B) \vee (B \rightarrow \sim A)$ be two logical expressions. Then :

(1) F_1 and F_2 both are tautologies

(2) F_1 is a tautology but F_2 is not a tautology

(3) F_1 is not tautology but F_2 is a tautology

(4) Both F_1 and F_2 are not tautologies

Official Ans. by NTA (3)

Sol. $F_1 : (A \wedge \sim B) \vee [\sim C \wedge (A \vee B)] \vee \sim A$

$F_2 : (A \vee B) \vee (B \rightarrow \sim A)$

$F_1 : \{(A \wedge \sim B) \vee \sim A\} \vee [(A \vee B) \wedge \sim C]$

$: \{(A \vee \sim A) \wedge (\sim A \vee \sim B)\} \vee [(A \vee B) \wedge \sim C]$

$: \{t \wedge (\sim A \vee \sim B)\} \vee [(A \vee B) \wedge \sim C]$

$: (\sim A \vee \sim B) \vee [(A \vee B) \wedge \sim C]$

$: \underbrace{\left[(\sim A \vee \sim B) \vee (A \vee B)\right]}_{t} \wedge \left[(\sim A \vee \sim B) \wedge \sim C\right]$

$F_1 : (\sim A \vee \sim B) \wedge \sim C \neq t$ (tautology)

$F_2 : (A \vee B) \vee (\sim B \vee \sim A) = t$ (tautology)

10. Let slope of the tangent line to a curve at any point P(x, y) be given by $\frac{xy^2+y}{x}$. If the curve intersects the line $x + 2y = 4$ at $x = -2$, then the value of y, for which the point (3, y) lies on the curve, is :

(1) $\frac{18}{35}$ (2) $-\frac{4}{3}$ (3) $-\frac{18}{19}$ (4) $-\frac{18}{11}$

Official Ans. by NTA (3)

Sol. $\frac{dy}{dx} = \frac{xy^2+y}{x}$

$\frac{xdy - ydx}{y^2} = xdx$

$-d\left(\frac{x}{y}\right) = xdx$

$-\frac{x}{y} = \frac{x^2}{2} + c$

$\because$ curve intersects the line x + 2y = 4 at x = –2 $\Rightarrow$ point of intersection is (–2, 3)

$\therefore$ curve passes through (–2, 3)

$$\Rightarrow \frac{2}{3} = 2 + c \Rightarrow c = -\frac{4}{3}$$

$$\Rightarrow \frac{-x}{y} = \frac{x^2}{2} - \frac{4}{3}$$

Now put (3, y)

$$\Rightarrow \frac{-3}{y} = \frac{19}{6}$$

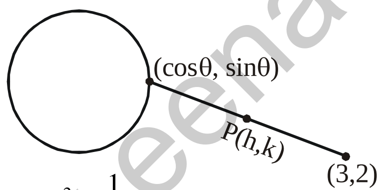
$$\Rightarrow y = \frac{-18}{19}$$

11. If the locus of the mid-point of the line segment from the point (3, 2) to a point on the circle, $x^2 + y^2 = 1$ is a circle of radius r, then r is equal to :

(1) 1 (2) $\frac{1}{2}$ (3) $\frac{1}{3}$ (4) $\frac{1}{4}$

Official Ans. by NTA (2)

Sol. $h = \frac{\cos\theta + 3}{2}$



$k = \frac{\sin\theta + 2}{2}$

$$\Rightarrow \left(h - \frac{3}{2}\right)^2 + (k-1)^2 = \frac{1}{4}$$

$$\Rightarrow r = \frac{1}{2}$$

12. Consider the following system of equations :

x + 2y – 3z = a

2x + 6y – 11z = b

x – 2y + 7z = c,

where a, b and c are real constants. Then the system of equations :

(1) has a unique solution when 5a = 2b + c

(2) has infinite number of solutions when 5a = 2b + c

(3) has no solution for all a, b and c

(4) has a unique solution for all a, b and c

Official Ans. by NTA (2)

Sol. P_1 : x + 2y – 3z = a

P_2 : 2x + 6y – 11z = b

P_3 : x – 2y + 7z = c

Clearly

$5P_1 = 2P_2 + P_3$ if 5a = 2b + c

$\Rightarrow$ All the planes sharing a line of intersection

$\Rightarrow$ infinite solutions

13. If 0 < a, b < 1, and $\tan^{-1}a + \tan^{-1}b = \frac{\pi}{4}$, then the value of

$$(a+b) - \left(\frac{a^2+b^2}{2}\right) + \left(\frac{a^3+b^3}{3}\right) - \left(\frac{a^4+b^4}{4}\right) + \ldots$$

is :

(1) $\log_e 2$ (2) $e^2 - 1$

(3) e (4) $\log_e\left(\frac{e}{2}\right)$

Official Ans. by NTA (1)

Sol. $\tan^{-1}a + \tan^{-1}b = \frac{\pi}{4}$ 0 < a, b < 1

$$\Rightarrow \frac{a+b}{1-ab} = 1$$

a + b = 1 – ab

(a + 1)(b + 1) = 2

Now $\left[a - \frac{a^2}{2} + \frac{a^3}{3} + \ldots\right] + \left[b - \frac{b^2}{2} + \frac{b^3}{3} + \ldots\right]$

$= \log_e(1 + a) + \log_e(1 + b)$

($\because$ expansion of $\log_e(1 + x)$)

$= \log_e[(1 + a)(1 + b)]$

$= \log_e 2$

14. The sum of the series $\sum_{n=1}^{\infty} \frac{n^2+6n+10}{(2n+1)!}$ is equal to :

(1) $\frac{41}{8}e + \frac{19}{8}e^{-1} - 10$

(2) $\frac{41}{8}e - \frac{19}{8}e^{-1} - 10$

(3) $\frac{41}{8}e + \frac{19}{8}e^{-1} + 10$

(4) $-\frac{41}{8}e + \frac{19}{8}e^{-1} - 10$

Official Ans. by NTA (2)

Sol. $T_n = \frac{n^2+6n+10}{(2n+1)!} = \frac{4n^2+24n+40}{4.(2n+1)!}$

$$=\frac{(2n+1)^2+20n+39}{4.(2n+1)!}$$

$$=\frac{(2n+1)^2+(2n+1).10+29}{4(2n+1)!}$$

$$=\frac{1}{4}\left[\frac{(2n+1)^2}{(2n+1)(2n)!}+\frac{(2n+1)10}{(2n+1)(2n)!}+\frac{29}{(2n+1)!}\right]$$

$$=\frac{1}{4}\left[\frac{2n+1}{(2n)!}+\frac{10}{(2n)!}+\frac{29}{(2n+1)!}\right]$$

$$=\frac{1}{4}\left[\frac{1}{(2n-1)!}+\frac{11}{(2n)!}+\frac{29}{(2n+1)!}\right]$$

$$S_1=\frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+...=\frac{e-\frac{1}{e}}{2}$$

$$S_2=11\left[\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+...\right]=11\left[\frac{e+\frac{1}{e}-2}{2}\right]$$

$$S_3=29\left[\frac{1}{3!}+\frac{1}{5!}+\frac{1}{7!}+...\right]=29\left[\frac{e-\frac{1}{e}-2}{2}\right]$$

Now, $S=\frac{1}{4}[S_1+S_2+S_3]$

$$=\frac{1}{4}\left[\frac{e}{2}-\frac{1}{2e}+\frac{11e}{2}+\frac{11}{2e}+\frac{29e}{2}-\frac{29}{2e}-4\right]$$

$$=\frac{41e}{8}-\frac{19}{8e}-10$$

15. Let $f(x)$ be a differentiable function at x = a with $f'(a)$ = 2 and $f(a)$ = 4. Then $\lim\limits_{x\to a}\frac{xf(a)-af(x)}{x-a}$ equals :

(1) 2a + 4 (2) 4 – 2a

(3) 2a – 4 (4) a + 4

Official Ans. by NTA (2)

Sol. $f'(a)=2,\ f(a)=4$

$$\lim_{x\to a}\frac{xf(a)-af(x)}{x-a}$$

$$\Rightarrow \lim_{x\to a}\frac{f(a)-af'(x)}{1} \qquad \text{(Lopitals rule)}$$

$= f(a) - af'(a)$

$= 4 - 2a$

16. Let A(1, 4) and B(1, –5) be two points. Let P be a point on the circle $(x-1)^2+(y-1)^2=1$ such that $(PA)^2+(PB)^2$ have maximum value, then the points P, A and B lie on :

(1) a straight line (2) a hyperbola

(3) an ellipse (4) a parabola

Official Ans. by NTA (1)

Sol. P be a point on $(x-1)^2+(y-1)^2=1$

so $P(1+\cos\theta,\ 1+\sin\theta)$

A(1,4) B(1,–5)

$(PA)^2+(PB)^2$

$=(\cos\theta)^2+(\sin\theta-3)^2+(\text{cso}\theta)^2+(\sin\theta+6)^2$

$=47+6\sin\theta$

is maximum if $\sin\theta=1$

$\Rightarrow \sin\theta=1,\ \cos\theta=0$

P(1,1) A(1,4) B(1,–5)

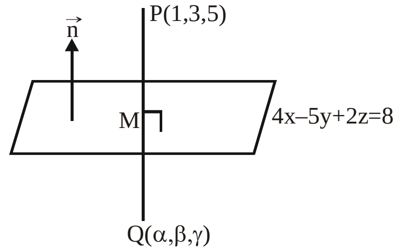
P,A,B are collinear points.

17. If the mirror image of the point (1, 3, 5) with respect to the plane $4x-5y+2z=8$ is (α,β,γ), then $5(\alpha+\beta+\gamma)$ equals :

(1) 47 (2) 43 (3) 39 (4) 41

Official Ans. by NTA (1)

Sol.



Point Q is image of point P w.r.to plane, M is mid point of P and Q, lies in plane

$$M\left(\frac{1+\alpha}{2},\frac{3+\beta}{2},\frac{5+\gamma}{2}\right)$$

$4x - 5y + 2z = 8$

$$4\left(\frac{1+\alpha}{2}\right)-5\left(\frac{3+\beta}{2}\right)+2\left(\frac{5+\gamma}{2}\right)=8 \quad ..(1)$$

Also PQ perpendicualr to the plane

$\Rightarrow \overrightarrow{PQ} \parallel \vec{n}$

$$\frac{\alpha-1}{4}=\frac{\beta-3}{-5}=\frac{\gamma-5}{2}=k \text{ (let)}$$

$$\left.\begin{array}{l}\alpha=1+4k\\ \beta=3-5k\\ \gamma=5+2k\end{array}\right\} \quad ...(2)$$

use (2) in (1)

$$2(1+4k)-5\left(\frac{6-5k}{2}\right)+(10+2k)=8$$

$$k=\frac{2}{5}$$

from (2) $\alpha=\frac{13}{5},\beta=1,\gamma=\frac{29}{5}$

$5(\alpha + \beta + \gamma) = 13 + 5 + 29 = 47$

18. Let $f(x)=\int_0^x e^t f(t)\,dt+e^x$ be a differentiable function for all $x \in R$. Then $f(x)$ equals :

(1) $2e^{(e^x-1)}-1$ (2) $e^{e^x}-1$

(3) $2e^{e^x}-1$ (4) $e^{(e^x-1)}$

Official Ans. by NTA (1)

Sol. $f(x)=\int_0^x e^t f(t)\,dt+e^x \Rightarrow f(0) = 1$

differentiating with respect to x

$f'(x) = e^x f(x) + e^x$

$f'(x) = e^x(f(x) + 1)$

$$\int_0^x \frac{f'(x)}{f(x)+1}\,dx=\int_0^x e^x\,dx$$

$$\ell n\left(f(x)+1\right)\Big|_0^x=e^x\Big|_0^x$$

$\ell n(f(x) + 1) - \ell n(f(0) + 1) = e^x - 1$

$$\ell n\left(\frac{f(x)+1}{2}\right)=e^x-1 \qquad \{\text{as } f(0) = 1\}$$

$$f(x)=2e^{(e^x-1)}-1$$

19. Let A_1 be the area of the region bounded by the curves y = sinx, y = cosx and y-axis in the first quadrant. Also, let A_2 be the area of the region bounded by the curves y = sinx, y = cosx, x-axis and $x=\frac{\pi}{2}$ in the first quadrant. Then,

(1) $A_1 : A_2 = 1:\sqrt{2}$ and $A_1 + A_2 = 1$

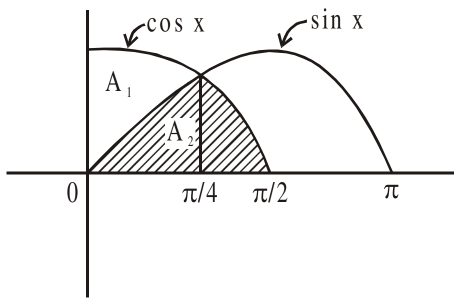
(2) $A_1 = A_2$ and $A_1 + A_2 = \sqrt{2}$

(3) $2A_1 = A_2$ and $A_1 + A_2 = 1+\sqrt{2}$

(4) $A_1 : A_2 = 1:2$ and $A_1 + A_2 = 1$

Official Ans. by NTA (1)

Sol.



$$A_1=\int_0^{\pi/4}(\cos x-\sin x)\,dx$$

$$A_1=(\sin x+\cos x)_0^{\pi/4}=\sqrt{2}-1$$

$$A_2=\int_0^{\pi/4}\sin x\,dx+\int_{\pi/4}^{\pi/2}\cos x\,dx$$

$$= \left(-\cos x\right)_0^{\pi/4} + \left(\sin x\right)_{\pi/4}^{\pi/2}$$

$$A_2 = \sqrt{2}\left(\sqrt{2}-1\right)$$

$A_1 : A_2 = 1 : \sqrt{2}$, $A_1 + A_2 = 1$

20. A seven digit number is formed using digits 3, 3, 4, 4, 4, 5, 5. The probability, that number so formed is divisible by 2, is :

(1) $\frac{6}{7}$ (2) $\frac{1}{7}$ (3) $\frac{3}{7}$ (4) $\frac{4}{7}$

Official Ans. by NTA (3)

Sol. Digits = 3, 3, 4, 4, 4, 5, 5

Total 7 digit numbers = $\frac{7!}{2!2!3!}$

Number of 7 digit number divisible by 2

$\Rightarrow$ last digit = 4

						4

3 , 3 , 4 , 4 , 5 , 5

Now 7 digit numbers which are divisible by 2

$$= \frac{6!}{2!2!2!}$$

Required probability = $\dfrac{\frac{6!}{2!2!2!}}{\frac{7!}{3!2!2!}} = \frac{3}{7}$

SECTION B

1. Let z be those complex numbers which satisfy $|z + 5| \le 4$ and $z(1+i) + \bar{z}(1-i) \ge -10, i = \sqrt{-1}$.

If the maximum value of $|z + 1|^2$ is $\alpha + \beta\sqrt{2}$, then the value of $(\alpha + \beta)$ is _______.

Official Ans. by NTA (48)

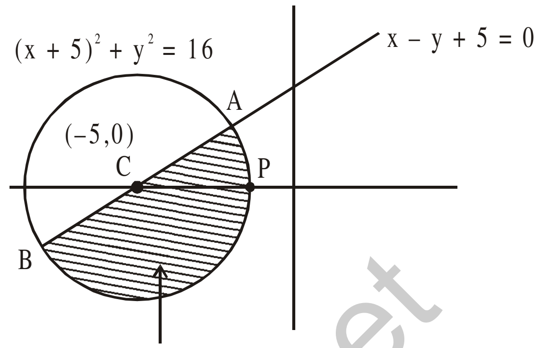
Sol. $|z + 5| \le 4$

$(x + 5)^2 + y^2 \le 16$ .... (1)

$z(1+i) + \bar{z}(1-i) \ge -10$

$(z + \bar{z}) + i(z - \bar{z}) \ge -10$

$x - y + 5 \ge 0$ .... (2)



Region bounded by inequalities (1) & (2)

$|z + 1|^2 = |z - (-1)|^2$

Let P(–1, 0)

$\boxed{|z+1|^2_{\text{Max.}} = PB^2}$ (where B is in 3[rd] quadrant)

for point of intersection

$$\left.\begin{array}{l}(x+5)^2 + y^2 = 16 \\ x - y + 5 = 0\end{array}\right\rangle y = \pm 2\sqrt{2}$$

$A\left(2\sqrt{2}-5, 2\sqrt{2}\right)$ $B\left(-2\sqrt{2}-5, -2\sqrt{2}\right)$

$$PB^2 = \left(+2\sqrt{2}+4\right)^2 + \left(2\sqrt{2}\right)^2$$

$$|z+1|^2 = 8 + 16 + 16\sqrt{2} + 8$$

$$\alpha + \beta\sqrt{2} = 32 + 16\sqrt{2}$$

$\alpha = 32$, $\beta = 16 \Rightarrow \alpha + \beta = 48$

2. Let the normals at all the points on a given curve pass through a fixed point (a, b). If the curve passes through (3, –3) and $\left(4, -2\sqrt{2}\right)$, and given that $a - 2\sqrt{2}\,b = 3$, then $(a^2 + b^2 + ab)$ is equal to ______.

Official Ans. by NTA (9)

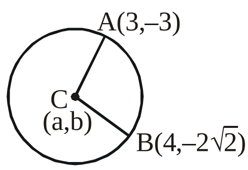
Sol. All normals of circle passes through centre

Radius = CA = CB

$CA^2 = CB^2$

$(a - 3)^2 + (b + 3)^2$

$= (a - 4)^2 + \left(b - 2\sqrt{2}\right)^2$



$$a+(3-2\sqrt{2})b=3$$

$$a-2\sqrt{2}b+3b=3 \qquad ...(1)$$

given that $a-2\sqrt{2}b=3$ ...(2)

from (1) & (2) $\Rightarrow$ a = 3, b = 0

$a^2 + b^2 + ab = 9$

3. Let α and β be two real numbers such that $\alpha + \beta = 1$ and $\alpha\beta = -1$. Let $p_n = (\alpha)^n + (\beta)^n$, $p_{n-1} = 11$ and $p_{n+1} = 29$ for some integer $n \geq 1$. Then, the value of p_n^2 is ______.

Official Ans. by NTA (324)

Sol. $x^2 - x - 1 = 0$ roots = α, β

$\alpha^2 - \alpha - 1 = 0 \Rightarrow \alpha^{n+1} = \alpha^n + \alpha^{n-1}$

$\beta^2 - \beta - 1 = 0 \Rightarrow \beta^{n+1} = \beta^n + \beta^{n-1}$

+

$P_{n+1} = P_n + P_{n-1}$

$29 = P_n + 11$

$P_n = 18$

$P_n^2 = 324$

4. If $I_{m,n} = \int_0^1 x^{m-1}(1-x)^{n-1}dx$, for m, n ≥ 1 and

$\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx = \alpha I_{m,n}$, $\alpha \in R$, then α equals ____.

Official Ans. by NTA (1)

Sol. $I_{m,n} = \int_0^1 x^{m-1}(1-x)^{n-1}dx = I_{n,m}$

Now Let $x = \frac{1}{y+1} \Rightarrow dx = -\frac{1}{(y+1)^2}dy$

so

$$I_{m,n} = -\int_\infty^0 \frac{1}{(y+1)^{m-1}}\frac{y^{n-1}}{(y+1)^{n-1}}\frac{dy}{(y+1)^2} = \int_0^\infty \frac{y^{n-1}}{(1+y)^{m+n}}dy$$

similarly $I_{m,n} = \int_0^\infty \frac{y^{m-1}}{(1+y)^{m+n}}dy$

Now $2I_{m,n} = \int_0^\infty \frac{y^{m-1}+y^{n-1}}{(1+y)^{m+n}}dy$

$$= \int_0^\infty \frac{y^{m-1}+y^{n-1}}{(1+y)^{m+n}}dy$$

$$= \int_0^1 \frac{y^{m-1}+y^{n-1}}{(1+y)^{m+n}}dy + \underbrace{\int_1^\infty \frac{y^{m-1}+y^{n-1}}{(1+y)^{m+n}}dy}_{\text{substitute } y=\frac{1}{t}}$$

$$\Rightarrow 2I_{m,n} = \int_0^1 \frac{y^{m-1}+y^{n-1}}{(1+y)^{m+n}}dy - \int_1^0 \frac{t^{n-1}+t^{m-1}}{t^{m+n-2}}\frac{t^{m+n}}{(1+t)^{m+n}}\frac{dt}{t^2}$$

$$\Rightarrow \text{Hence } 2I_{m,n} = 2\int_0^1 \frac{y^{m-1}+y^{n-1}}{(1+y)^{m+n}}dy \Rightarrow \alpha = 1$$

5. If the arithmetic mean and geometric mean of the p^{th} and q^{th} terms of the sequence –16, 8, –4, 2, ... satisfy the equation $4x^2 - 9x + 5 = 0$, then p + q is equal to ______.

Official Ans. by NTA (10)

Sol. $4x^2 - 9x + 5 = 0 \Rightarrow x = 1, \frac{5}{4}$

Now given $\frac{5}{4} = \frac{t_p+t_q}{2}, t = t_pt_q$ where

$$t_r = -16\left(-\frac{1}{2}\right)^{r-1}$$

so $\frac{5}{4} = -8\left[\left(-\frac{1}{2}\right)^{p-1} + \left(-\frac{1}{2}\right)^{q-1}\right]$

$$1 = 256\left(-\frac{1}{2}\right)^{p+q-2} \Rightarrow 2^{p+q-2} = (-1)^{p+q-2}2^8$$

hence p + q = 10

6. The total number of 4-digit numbers whose greatest common divisor with 18 is 3, is ______.

Official Ans. by NTA (1000)

Sol. Let N be the four digit number

gcd(N,18) = 3

Hence N is an odd integer which is divisible by 3 but not by 9.

4 digit odd multiples of 3

1005, 1011,......., 9999 $\rightarrow$ 1500

4 digit odd multiples of 9

1017, 1035,......., 9999 $\rightarrow$ 500

Hence number of such N = 1000

7. Let L be a common tangent line to the curves $4x^2 + 9y^2 = 36$ and $(2x)^2 + (2y)^2 = 31$. Then the square of the slope of the line L is ______.

Official Ans. by NTA (3)

Sol. Given curves are $\frac{x^2}{9}+\frac{y^2}{4}=1$

$$x^2+y^2=\frac{31}{4}$$

let slope of common tangent be m

so tangents are $y=mx\pm\sqrt{9m^2+4}$

$$y=mx\pm\frac{\sqrt{31}}{2}\sqrt{1+m^2}$$

hence $9m^2+4=\frac{31}{4}\left(1+m^2\right)$

$$\Rightarrow 36m^2+16=31+31m^2 \Rightarrow m^2=3$$

8. Let a be an integer such that all the real roots of the polynomial $2x^5+5x^4 + 10x^3 + 10x^2 + 10x + 10$ lie in the interval $(a, a + 1)$. Then, $|a|$ is equal to ______.

Official Ans. by NTA (2)

Sol. Let $2x^5 + 5x^4 + 10x^3 + 10x^2 + 10x + 10 = f(x)$

Now $f(-2) = -34$ and $f(-1) = 3$

Hence $f(x)$ has a root in $(-2,-1)$

Further $f'(x) = 10x^4 + 20x^3 + 20x^2 + 20x + 10$

$$=10x^2\left[\left(x^2+\frac{1}{x^2}\right)+2\left(x+\frac{1}{x}\right)+20\right]$$

$$=10x^2\left[\left(x+\frac{1}{x}+1\right)^2+17\right]>0$$

Hence $f(x)$ has only one real root, so $|a| = 2$

9. Let $X_1, X_2, ..., X_{18}$ be eighteen observations such that $\sum_{i=1}^{18}(X_i-\alpha)=36$ and $\sum_{i=1}^{18}(X_i-\beta)^2=90$, where α and β are distinct real numbers. If the standard deviation of these observations is 1, then the value of $|\alpha - \beta|$ is ______.

Official Ans. by NTA (4)

Sol. $\sum_{i=1}^{18}(x_i-\alpha)=36, \sum_{i=1}^{18}(x_i-\beta)^2=90$

$$\Rightarrow \sum_{i=1}^{18}x_i=18(\alpha+2), \sum_{i=1}^{18}x_i^2-2\beta\sum_{i=1}^{18}x_i+18\beta^2=90$$

Hence $\sum x_i^2=90-18\beta^2+36\beta(\alpha+2)$

Given $\frac{\sum x_i^2}{18}-\left(\frac{\sum x_i}{18}\right)^2=1$

$\Rightarrow 90 - 18\beta^2 + 36\beta(\alpha + 2) - 18(\alpha + 2)^2 = 18$

$\Rightarrow 5 - \beta^2 + 2\alpha\beta + 4\beta - \alpha^2 - 4\alpha - 4 = 1$

$\Rightarrow (\alpha - \beta)^2 + 4(\alpha - \beta) = 0 \Rightarrow |\alpha - \beta| = 0$ or 4

As α and β are distinct $|\alpha - \beta| = 4$

10. If the matrix $A=\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 3 & 0 & -1\end{bmatrix}$ satisfies the equation $A^{20}+\alpha A^{19}+\beta A=\begin{bmatrix}1 & 0 & 0\\ 0 & 4 & 0\\ 0 & 0 & 1\end{bmatrix}$ for some real numbers α and β, then $\beta-\alpha$ is equal to ______.

Official Ans. by NTA (4)

Sol. $A=\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 3 & 0 & -1\end{bmatrix}$

$$A^2=\begin{bmatrix}1 & 0 & 0\\ 0 & 4 & 0\\ 0 & 0 & 1\end{bmatrix}, A^3=\begin{bmatrix}1 & 0 & 0\\ 0 & 8 & 0\\ 3 & 0 & -1\end{bmatrix}$$

$$A^4=\begin{bmatrix}1 & 0 & 0\\ 0 & 16 & 0\\ 0 & 0 & 1\end{bmatrix}$$

Hence

$$A^{20}=\begin{bmatrix}1 & 0 & 0\\ 0 & 2^{20} & 0\\ 0 & 0 & 1\end{bmatrix}, A^{19}=\begin{bmatrix}1 & 0 & 0\\ 0 & 2^{19} & 0\\ 3 & 0 & -1\end{bmatrix}$$

So $A^{20}+\alpha A^{19}+\beta A=\begin{bmatrix}1+\alpha+\beta & 0 & 0\\ 0 & 2^{20}+\alpha.2^{19}+2\beta & 0\\ 3\alpha+3\beta & 0 & 1-\alpha-\beta\end{bmatrix}$

$$=\begin{bmatrix}1 & 0 & 0\\ 0 & 4 & 0\\ 0 & 0 & 1\end{bmatrix}$$

Therefore $\alpha+\beta=0$ and $2^{20}+2^{19}\alpha-2\alpha=4$

$$\Rightarrow \alpha=\frac{4\left(1-2^{18}\right)}{2\left(2^{18}-1\right)}=-2$$

hence $\beta=2$

so $(\beta-\alpha)=4$