JEE MAIN 2021

Held On 25th February, 2021

Morning Session

PHYSICS

SECTION-A

1. Given below are two statement : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : When a rod lying freely is heated, no thermal stress is developed in it.

Reason R : On heating the length of the rod increases.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both A and R are true but R is NOT the correct explanation of A

(2) A is false but R is true

(3) A is true but R is false

(4) Both A and R are true and R is the correct explanation of A

Official Ans. by NTA (1)

Sol. A and R are true but R is not the correct explanation of A.

2. A student is performing the experiment of resonance column. The diameter of the column tube is 6 cm. The frequency of the tuning fork is 504 Hz. Speed of the sound at the given temperature is 336 m/s. The zero of the meter scale coincides with the top end of the resonance column tube. The reading of the water level in the column when the first resonance occurs is:

(1) 13 cm (2) 16.6 cm

(3) 18.4 cm (4) 14.8 cm

Official Ans. by NTA (4)

Sol. d = 6cm, f = 504, v = 336 m/s

e = 0.3d

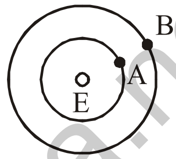
$$l + e = \frac{\lambda}{4} = \frac{V}{4f}$$

$l = 16.66 - 0.3 \times 6$

$l = 14.866$ cm

$l = 14.8$ cm

3. Two satellites A and B of masses 200kg and 400kg are revolving round the earth at height of 600 km and 1600 km respectively. If T_A and T_B are the time periods of A and B respectively then the value of $T_B - T_A$:



[Given : radius of earth = 6400km, mass of earth = 6×10^{24} kg]

(1) 1.33×10^3 s (2) 3.33×10^2 s

(3) 4.24×10^3 s (4) 4.24×10^2 s

Official Ans. by NTA (1)

Sol. $T = 2\pi \sqrt{\frac{r^3}{GM}}$

$$T_A = 2\pi \sqrt{\frac{(6400+600)\times 10^3}{GM}}$$

$$T_A = 2\pi \times 10^9 \sqrt{\frac{7^3}{GM}}$$

$$T_B = 2\pi \times 10^9 \sqrt{\frac{8^3}{GM}}$$

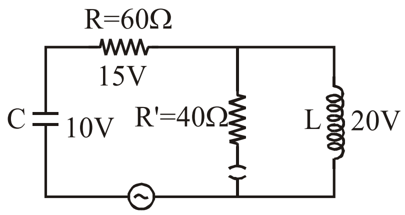
$$T_B - T_A = \frac{2\pi 10^9}{\sqrt{GM}} \left[8\sqrt{8} - 7\sqrt{7}\right]$$

$= 314 \times 4.107$

$= 1289.64$

$= 1.289 \times 10^3$ s

4. The angular frequency of alternating current in a L-C-R circuit is 100 rad/s. The components connected are shown in the figure. Find the value of inductance of the coil and capacity of condenser.



(1) 0.8 H and 150 μF

(2) 0.8 H and 250 μF

(3) 1.33 H and 250 μF

(4) 1.33 H and 150 μF

Official Ans. by NTA (2)

Sol. Current through 60Ω resistance $=\frac{15}{60}=\frac{1}{4}$ A

thus capacitor current = $\frac{1}{4}$ A

$\because$ $V_C = I\,X_C$

$10 = \frac{1}{4} \times \frac{1}{\omega C}$

$\therefore$ $C = \frac{1}{40\omega} = \frac{1}{4000} = 250\ \mu F$

Now,

current through 40Ω resistance = $\frac{20}{40} = \frac{1}{2}$ A

thus current through inductor $=\frac{1}{2}-\frac{1}{4}=\frac{1}{4}$ A

$V_L = IX_L = \frac{1}{4} \times \omega L$

$20 = \frac{1}{4} \times 100 \times L$

$\Rightarrow L = 0.8$ H

5. A proton, a deuteron and an α particle are moving with same momentum in a uniform magnetic field. The ratio of magnetic forces acting on them is ____ and their speed is ____ in the ratio.

(1) 1 : 2 : 4 and 2 : 1 : 1

(2) 2 : 1 : 1 and 4 : 2 : 1

(3) 4 : 2 : 1 and 2 : 1 : 1

(4) 1 : 2 : 4 and 1 : 1 : 2

Official Ans. by NTA (2)

Sol. $F = q\left(\vec{v}\times\vec{B}\right) = \frac{q}{m}\left(\vec{P}\times\vec{B}\right)$

$\Rightarrow F \propto \frac{q}{m}$

thus $F_1 : F_2 : F_3 = \frac{q_1}{m_1} : \frac{q_2}{m_2} : \frac{q_3}{m_3}$

$= \frac{e}{m_p} : \frac{e}{2m_p} : \frac{2e}{4m_p}$

$= \frac{1}{1} : \frac{1}{2} : \frac{2}{4}$

= 2 : 1 : 1

Now for speed calculation

P = constant $\Rightarrow v \propto \frac{1}{m}$

thus $v_1 : v_2 : v_3 = \frac{1}{m_p} : \frac{1}{2m_p} : \frac{1}{4m_p}$

$= \frac{1}{1} : \frac{1}{2} : \frac{1}{4}$

= 4 : 2 : 1

6. Given below are two statement :

Statement-I: A speech signal of 2 kHz is used to modulate a carrier signal of 1 MHz. The band width requirement for the signal is 4 kHz.

Statement-II : The side band frequencies are 1002 kHz. and 998 kHz.

In the light of the above statements, choose the correct answer from the options given below:

(1) Statement I is true but Statement II is false
(2) Statement I is false but Statement II is true
(3) Both Statement I and Statement II are true
(4) Both Statement I and Statement II are false

Official Ans. by NTA (3)

Sol. $f_m = 2\text{kHz}$

$f_c = 1\text{MHz} = 1000\text{ kHz}$

Band width $= 2f_m = 4\text{kHz}$

$\therefore$ Side frequencies will be

$= f_c \pm f_m$

$= (1000 \pm 2)$ kHz

$= 998$ kHz & 1002 kHz

So statement-I & statement-II both are correct.

7. If the time period of a two meter long simple pendulum is 2s, the acceleration due to gravity at the place where pendulum is executing S.H.M. is :

(1) $\pi^2\text{ms}^{-2}$ (2) 9.8 ms^{-2}

(3) $2\pi^2\text{ms}^{-2}$ (4) 16 m/s^2

Official Ans. by NTA (3)

Sol. $T = 2\pi\sqrt{\frac{l}{g}}$

$2 = 2\pi\sqrt{\frac{2}{g}}$

$\Rightarrow g = 2\pi^2$

8. The pitch of the screw gauge is 1mm and there are 100 divisions on the circular scale. When nothing is put in between the jaws, the zero of the circular scale lies 8 divisions below the reference line. When a wire is placed between the jaws, the first linear scale division is clearly visible while 72nd division on circular scale coincides with the reference line. The radius of the wire is

(1) 1.64 mm (2) 0.82 mm

(3) 1.80 mm (4) 0.90 mm

Official Ans. by NTA (2)

Sol. Least count $= \frac{1\text{mm}}{100} = 0.01\text{mm}$

zero error $= +\ 8 \times LC = +\ 0.08$ mm

True reading (Diameter)

$= (1\text{ mm} + 72 \times LC) - (\text{Zero error})$

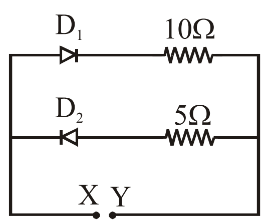
$= (1\text{mm} + 72 \times 0.01\text{mm}) - 0.08\text{ mm}$

$= 1.72\text{ mm} - 0.08\text{ mm}$

$= 1.64$ mm

therefore, radius $= \frac{1.64}{2} = 0.82$ mm.

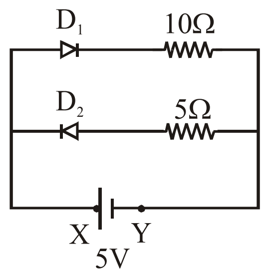
9. A 5V battery is connected across the points X and Y. Assume D_1 and D_2 to be normal silicon diodes. Find the current supplied by the battery if the +ve terminal of the battery is connected to point X.



(1) ~ 0.5 A (2) ~ 1.5 A
(3) ~ 0.86 A (4) ~ 0.43 A

Official Ans. by NTA (4)

Sol.



Here only D_1 will work and we know for silicon diode, potential drop on D_1 will be 0.7V

$$I = \frac{5-0.7}{10} = 0.43 \text{ A}$$

10. An α particle and a proton are accelerated from rest by a potential difference of 200 V. After this, their de Broglie wavelengths are λ_α and λ_p respectively. The ratio $\frac{\lambda_p}{\lambda_\alpha}$ is :

(1) 3.8 (2) 8 (3) 7.8 (4) 2.8

Official Ans. by NTA (4)

Sol. $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}}$

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha}{m_p q_p}} = \sqrt{\frac{4m_p \times 2e}{m_p \times e}} = \sqrt{8} = 2\sqrt{2}$$

$= 2\sqrt{2}$

$$\frac{\lambda_p}{\lambda_\alpha} = 2 \times 1.4 = 2.8$$

11. A diatomic gas, having $C_p = \frac{7}{2}R$ and $C_v = \frac{5}{2}R$, is heated at constant pressure. The ratio dU : dQ : dW :

(1) 5 : 7 : 3 (2) 5 : 7 : 2

(3) 3 : 7 : 2 (4) 3 : 5 : 2

Official Ans. by NTA (2)

Sol. $dU = nC_v dT$

$dQ = nC_p dT$

$dW = PdV = nRdT$ (isobaric process)

$dU : dQ : dW$: $C_v : C_p : R$

$= \frac{5R}{2} : \frac{7R}{2} : R = 5 : 7 : 2$

12. An engine of a train, moving with uniform acceleration, passes the signal-post with velocity u and the last compartment with velocity v. The velocity with which middle point of the train passes the signal post is:

(1) $\sqrt{\frac{v^2+u^2}{2}}$ (2) $\frac{v-u}{2}$

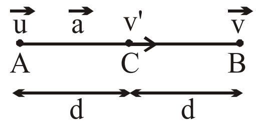
(3) $\frac{u+v}{2}$ (4) $\sqrt{\frac{v^2-u^2}{2}}$

Official Ans. by NTA (1)

Sol. $(v')^2 = u^2 + 2ad$

$v^2 = (v')^2 + 2ad$

solving, we get



$$v' = \sqrt{\frac{v^2+u^2}{2}}$$

13. Match List-I with List-II :

List-I	List-II
(a) h (Planck's constant)	(i) $[M L T^{-1}]$
(b) E (kinetic energy)	(ii) $[M L^2 T^{-1}]$
(c) V (electric potential)	(iii) $[M L^2 T^{-2}]$
(d) P (linear momentum)	(iv) $[M L^2 I^{-1} T^{-3}]$

Choose the correct answer from the options given below :

(1) (a)→(iii), (b)→(iv), (c)→(ii), (d)→(i)

(2) (a)→(ii), (b)→(iii), (c)→(iv), (d)→(i)

(3) (a)→(i), (b)→(ii), (c)→(iv), (d)→(iii)

(4) (a)→(iii), (b)→(ii), (c)→(iv), (d)→(i)

Official Ans. by NTA (2)

Sol. By dimensional analysis.

14. Magnetic fields at two points on the axis of a circular coil at a distance of 0.05m and 0.2 m from the centre are in the ratio 8 : 1. The radius of coil is ____.

(1) 0.2 m (2) 0.1 m

(3) 0.15 m (4) 1.0 m

Official Ans. by NTA (2)

Sol. We know, the magnetic field on the axis of a current carrying circular ring is given by

$$B=\frac{\mu_0}{4\pi}\frac{2NIA}{\left(R^2+x^2\right)^{3/2}}$$

$$\therefore\ \frac{B_1}{B_2}=\frac{8}{1}=\left[\frac{R^2+(0.2)^2}{R^2+(0.05)^2}\right]^{3/2}$$

$4[R^2 + (0.05)^2] = [R^2 + (0.2)^2]$

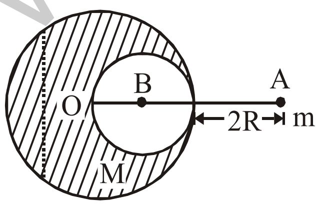
$4R^2 - R^2 = (0.2)^2 - 4 \times (0.05)^2$

$4R^2 - R^2 = (0.2)^2 - (0.1)^2$

$3R^2 = 0.3 \times 0.1$

$R^2 = (0.1)^2 \Rightarrow R = 0.1$

15. A solid sphere of radius R gravitationally attracts a particle placed at 3R form its centre with a force F_1. Now a spherical cavity of radius $\left(\frac{R}{2}\right)$ is made in the sphere (as shown in figure) and the force becomes F_2. The value of $F_1 : F_2$ is :



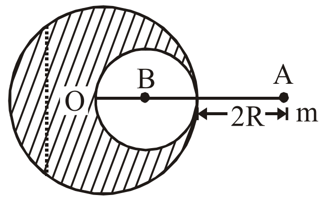
(1) 25 : 36 (2) 36 : 25

(3) 50 : 41 (4) 41 : 50

Official Ans. by NTA (3)

Sol. Let initial mass of sphere is m'. Hence mass of removed portion will be m'/8

$$F_1 = m.E. = \frac{m.Gm'}{9R^2}$$



$$F_2 = m\left[\frac{G.m'}{(3R)^2}-\frac{G.m'/8}{(5R/2)^2}\right]$$

$$=\frac{Gm'}{9R^2}-\frac{Gm'\times 4}{8\times 25}$$

$$=\left(\frac{1}{9}-\frac{1}{50}\right)\frac{Gm'}{R^2}$$

$$F_2=\frac{41}{50\times 9}.\frac{Gm'}{R^2}$$

$$\frac{F_1}{F_2}=\frac{1}{9}\times\frac{50\times 9}{41}=\frac{50}{41}$$

16. Two radioactive substances X and Y originally have N_1 and N_2 nuclei respectively. Half life of X is half of the half life of Y. After three half lives of Y, number of nuclei of both are equal. The ratio $\frac{N_1}{N_2}$ will be equal to :

(1) $\frac{1}{8}$ (2) $\frac{3}{1}$

(3) $\frac{8}{1}$ (4) $\frac{1}{3}$

Official Ans. by NTA (3)

Sol. $T_x = t$; $T_y = 2t$

$3T_y = 6t$,

$N_1' = N_2'$

$$N_1 e^{-\lambda_1 6t} = N_2 e^{-\lambda_2 6t}$$

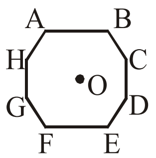
$$\frac{N_1}{N_2} = e^{(\lambda_1-\lambda_2)6t} = e^{ln2\left(\frac{1}{t}-\frac{1}{2t}\right)\times 6t} = e^{(ln2)\times 3} = e^{ln8} = 8$$

$$\frac{N_1}{N_2} = \frac{8}{1}$$

17. In an octagon ABCDEFGH of equal side, what is the sum of

$\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}+\overrightarrow{AG}+\overrightarrow{AH}'$

if, $\overrightarrow{AO}=2\hat{i}+3\hat{j}-4\hat{k}$



(1) $-16\hat{i}-24\hat{j}+32\hat{k}$

(2) $16\hat{i}+24\hat{j}-32\hat{k}$

(3) $16\hat{i}+24\hat{j}+32\hat{k}$

(4) $16\hat{i}-24\hat{j}+32\hat{k}$

Official Ans. by NTA (2)

Sol. We know,

$\because \overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}+\overrightarrow{OE}+\overrightarrow{OF}+\overrightarrow{OG}+\overrightarrow{OH}=\vec{0}$

By triangle law of vector addition, we can write

$\overrightarrow{AB}=\overrightarrow{AO}+\overrightarrow{OB}$; $\overrightarrow{AC}=\overrightarrow{AO}+\overrightarrow{OC}$

$\overrightarrow{AD}=\overrightarrow{AO}+\overrightarrow{OD}$; $\overrightarrow{AE}=\overrightarrow{AO}+\overrightarrow{OE}$

$\overrightarrow{AF}=\overrightarrow{AO}+\overrightarrow{OF}$; $\overrightarrow{AG}=\overrightarrow{AO}+\overrightarrow{OG}$

$\overrightarrow{AH}=\overrightarrow{AO}+\overrightarrow{OH}$

Now

$\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}+\overrightarrow{AG}+\overrightarrow{AH}$

$= (7\overrightarrow{AO})+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}+\overrightarrow{OE}+\overrightarrow{OF}+\overrightarrow{OG}+\overrightarrow{OH}$

$= (7\overrightarrow{AO})+\vec{0}-\overrightarrow{OA}$

$= (7\overrightarrow{AO})+\overrightarrow{AO}$

$= 8\overrightarrow{AO} = 8(2\hat{i}+3\hat{j}-4\hat{k})$

$= 16\hat{i}+24\hat{j}-32\hat{k}$

18. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : The escape velocities of planet A and B are same. But A and B are of unequal mass.

Reason R : The product of their mass and radius must be same, $M_1R_1 = M_2R_2$

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both A and R are correct but R is NOT the correct explanation of A

(2) A is correct but R is not correct

(3) Both A and R are correct and R is the correct explanation of A

(4) A is not correct but R is correct

Official Ans. by NTA (2)

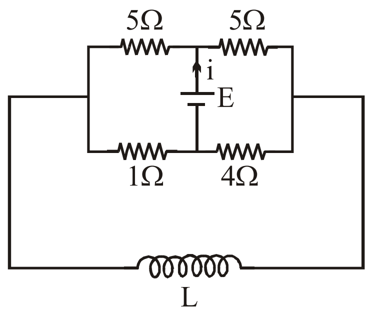
Sol. $V_e = \sqrt{\frac{2GM}{R}}$

$$\frac{M_1}{R_1} = \frac{M_2}{R_2}$$

$M_1R_2 = M_2R_1$

Hence reason R is not correct.

19. The current (i) at time t = 0 and t = ∞ respectively for the given circuit is :



(1) $\frac{18E}{55}, \frac{5E}{18}$ (2) $\frac{10E}{33}, \frac{5E}{18}$

(3) $\frac{5E}{18}, \frac{18E}{55}$ (4) $\frac{5E}{18}, \frac{10E}{33}$

Official Ans. by NTA (4)

Sol. At t = 0, current through inductor is zero,

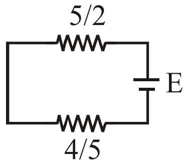
hence $R_{eq} = (5+1) \parallel (5+4) = \dfrac{18}{5}$

$$i_1 = \frac{E}{18/5} = \frac{5E}{18}$$

At t = ∞, inductor becomes a simple wire and now the circuit will be as shown in figure

hence $R_{eq} = (5\parallel 5) + (4\parallel 1) = \dfrac{33}{10}$; ($\parallel \Rightarrow$ parallel)

$$i_2 = \frac{E}{33/10} = \frac{10E}{33}$$



20. Two coherent light sources having intensity in the ratio 2x produce an interference pattern. The ratio $\dfrac{I_{max} - I_{min}}{I_{max} + I_{min}}$ will be :

(1) $\dfrac{2\sqrt{2x}}{x+1}$ (2) $\dfrac{\sqrt{2x}}{2x+1}$

(3) $\dfrac{\sqrt{2x}}{x+1}$ (4) $\dfrac{2\sqrt{2x}}{2x+1}$

Official Ans. by NTA (4)

Sol. Given that, $\dfrac{I_1}{I_2} = 2x$

We know,

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 \ \& \ I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$

$$\therefore \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2} = \frac{2\sqrt{I_1/I_2}}{\frac{I_1}{I_2} + 1} = \frac{2\sqrt{2x}}{2x+1}$$

SECTION-B

1. A transmitting station releases waves of wavelength 960 m. A capacitor of 2.56 μF is used in the resonant circuit. The self inductance of coil necessary for resonance is ____ $\times 10^{-8}$ H.

Official Ans. by NTA (10)

Sol. $\lambda = 960$ m

$C = 2.56\ \mu F = 2.56 \times 10^{-6}$ F

$c = 3 \times 10^8$ m/s

L = ?

Now at resonance, $\omega_0 = \dfrac{1}{\sqrt{LC}}$

[Resoant frequency]

$$2\pi f_0 = \frac{1}{\sqrt{LC}}$$

On substituting $f_0 = \dfrac{c}{\lambda}$, we have $2\pi \dfrac{c}{\lambda} = \dfrac{1}{\sqrt{LC}}$

Squaring both sides : $4\pi^2 \dfrac{c^2}{\lambda^2} = \dfrac{1}{LC}$

$$= \frac{4 \times 10 \times (3\times 10^8)^2}{(960)^2} = \frac{1}{L \times 2.56 \times 10^{-6}}$$

$$\Rightarrow \frac{1}{L} = \frac{4 \times 10 \times 9 \times 10^{16} \times 2.56 \times 10^{-6}}{960 \times 960}$$

$\Rightarrow L = 10 \times 10^{-8}$ H

2. The electric field in a region is given $\vec{E} = \left(\dfrac{3}{5}E_0\hat{i} + \dfrac{4}{5}E_0\hat{j}\right)\dfrac{N}{C}$. The ratio of flux of reported field through the rectangular surface of area 0.2 m² (parallel to y – z plane) to that of the surface of area 0.3 m² (parallel to x – z plane) is a : b, where a = _____.

[Here $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axes respectively]

Official Ans. by NTA (1)

Sol. $\vec{E}=\left(\frac{3E_0}{5}\hat{i}+\frac{4E_0}{5}\hat{j}\right)\frac{N}{C}$

A_1 = 0.2 m² [parallel to y – z plane]

= $\vec{A}_1 = 0.2\ m^2\ \hat{i}$

A_2 = 0.3 m² [parallel to x – z plane]

$\vec{A}_2 = 0.3 m^2\ \hat{j}$

Now $\phi_a = \left[\frac{3E_0}{5}\hat{i}+\frac{4E_0}{5}\hat{j}\right]\cdot\left[0.2\,\hat{i}\right] = \frac{3\times0.2}{5}E_0$

& $\phi_b = \left[\frac{3E_0}{5}\hat{i}+\frac{4E_0}{5}\hat{j}\right]\cdot\left[0.3\,\hat{j}\right] = \frac{4\times0.3}{5}E_0$

Now $\frac{\phi_a}{\phi_b} = \frac{0.6}{1.2} = \frac{1}{2} = \frac{a}{b}$

$\Rightarrow$ a : b = 1 : 2

$\Rightarrow$ a = 1

3. In a certain thermodynamical process, the pressure of a gas depends on its volume as kV^3. The work done when the temperature changes from 100°C to 300°C will be ___ nR, where n denotes number of moles of a gas.

Official Ans. by NTA (50)

Sol. $P = kV^3$

T_i = 100°C & T_f = 300°C

$\Delta T = 300 - 100$

ΔT = 200°C

$P = kV^3$

now PV = nRT

$\therefore kV^4 = nRT$

now $4kV^3\,dV = nRdT$

$\therefore PdV = nRdT/4$

$\therefore$ Work = $\int PdV = \int\frac{nRdT}{4} = \frac{nR}{4}\Delta T$

$= \frac{200}{4}\times nR = 50nR$

4. A small bob tied at one end of a thin string of length 1m is describing a vertical circle so that the maximum and minimum tension in the string are in the ratio 5 : 1. The velocity of the bob at the height position is _____ m/s.

(Take g = 10 m/s²)

Official Ans. by NTA (5)

Sol. Let the speed of bob at lowest position be v_1 and at the highest position be v_2.

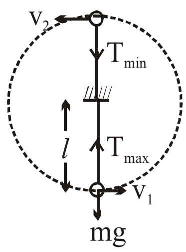
Maximum tension is at lowest position and minimum tension is at the highest position.

Now, using, conservation of mechanical energy,

$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_2^2 + mg\,2l$

$\Rightarrow v_1^2 = v_2^2 + 4gl$ ... (1)

Now $T_{max} - mg = \frac{mv_1^2}{l}$



$\Rightarrow T_{max} = mg + \frac{mv_1^2}{l}$

& $T_{min} + mg = \frac{mv_2^2}{l}$

$\Rightarrow T_{min} = \frac{mv_2^2}{l} - mg$

$\frac{T_{max}}{T_{min}} = \frac{5}{1}$

$\Rightarrow \frac{mg+\frac{mv_1^2}{l}}{\frac{mv_2^2}{l}-mg} = \frac{5}{1}$

$\Rightarrow mg + \frac{mv_1^2}{l} = \left[\frac{mv_2^2}{l} - mg\right]5$

$\Rightarrow mg + \frac{m}{l}\left[v_2^2 + 4gl\right] = \frac{5mv_2^2}{l} - 5mg$

$$\Rightarrow \; mg+\frac{mv_2^2}{l}+4mg=\frac{5mv_2^2}{l}-5mg$$

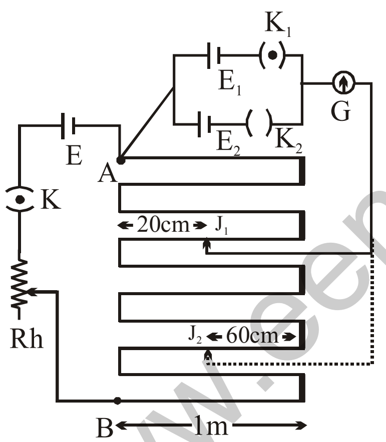
$$\Rightarrow 10mg = \frac{4mv_2^2}{l}$$

$$v_2^2 = \frac{10\times10\times1}{4}$$

$\Rightarrow v_2^2 = 25 \;\; \Rightarrow v_2 = 5$ m/s

Thus, velocity of bob at highest position is 5 m/s.

5. In the given circuit of potentiometer, the potential difference E across AB (10m length) is larger than E_1 and E_2 as well. For key K_1 (closed), the jockey is adjusted to touch the wire at point J_1 so that there is no deflection in the galvanometer. Now the first battery (E_1) is replaced by second battery (E_2) for working by making K_1 open and K_2 closed. The galvanometer gives then null deflection at J_2. The value of $\frac{E_1}{E_2}$ is $\frac{a}{b}$, where a = ___.



Official Ans. by NTA (1)

Sol. Length of AB = 10 m

For battery E_1, balancing length is l_1

l_1 = 380 cm [from end A]

For battery E_2, balancing length is l_2

l_2 = 760 cm [from end A]

Now, we know that $\frac{E_1}{E_2}=\frac{l_1}{l_2}$

$$\Rightarrow \frac{E_1}{E_2}=\frac{380}{760}=\frac{1}{2}=\frac{a}{b}$$

$\therefore$ a = 1 & b = 2.

a = 1

6. The same size images are formed by a convex lens when the object is placed at 20cm or at 10cm from the lens. The focal length of convex lens is _____ cm.

Official Ans. by NTA (15)

Sol. $m=\frac{f}{u+f}$

$$+\,m = \frac{f}{-10+f} \quad \ldots (1)$$

$$-\,m = \frac{f}{-20+f} \quad \ldots (2)$$

(1) / (2)

$$-\,1 = \frac{f-20}{f-10}$$

10 – f = f – 20

30 = 2f

f = 15 cm

7. 512 identical drops of mercury are charged to a potential of 2V each. The drops are joined to form a single drop. The potential of this drop is ____ V.

Official Ans. by NTA (128)

Sol. Q = 512q

$\text{Volume}_i = \text{Volume}_f$

$$512 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

$2^9 r^3 = R^3$

R = 8r

$$2 = \frac{kq}{r}$$

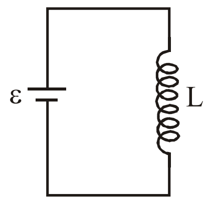
$$V = \frac{kQ}{R} = \frac{k\,512\,q}{8\,r}$$

V = 128.

8. A coil of inductance 2H having negligible resistance is connected to a source of supply whose voltage is given by V = 3t volt. (where t is in second). If the voltage is applied when t = 0, then the energy stored in the coil after 4s is _____ J.

Official Ans. by NTA (144)

Sol. $\varepsilon = \frac{L\,dI}{dt}$



$3\int_0^4 t\,dt = 2\int_0^I dI$

$\frac{3}{2}\times 16 = 2I$

I = 12

$V = \frac{1}{2}LI^2 = \frac{1}{2}\times 2(12)^2 = 144\text{ J}$

9. A monoatomic gas of mass 4.0 u is kept in an insulated container. Container is moving with velocity 30 m/s. If container is suddenly stopped then change in temperature of the gas (R = gas constant) is $\frac{x}{3R}$. Value of x is ___.

Official Ans. by NTA (3600)

Sol. Given that mass of gas is 4u hence its molar mass M is 4g/mol

$\therefore\ \frac{1}{2}mv^2 = nC_v\Delta T$

$\frac{1}{2}m\times(30)^2 = \frac{m}{M}\times\frac{3R}{2}\times\Delta T$

$\therefore\ \Delta T = \frac{3600}{3R}$

10. The potential energy (U) of a diatomic molecule is a function dependent on r (interatomic distance) as

$$U = \frac{\alpha}{r^{10}} - \frac{\beta}{r^5} - 3$$

where, α and β are positive constants. The equilibrium distance between two atoms will be

$\left(\frac{2\alpha}{\beta}\right)^{\frac{a}{b}}$, where a = ____.

Official Ans. by NTA (1)

Sol. For equilibrium

$\frac{dU}{dr} = 0$

$\frac{-10\alpha}{r^{11}} + \frac{5\beta}{r^6} = 0$

$\frac{5\beta}{r^6} = \frac{10\alpha}{r^{11}}$

$r^5 = \frac{2\alpha}{\beta}$

$r = \left(\frac{2\alpha}{\beta}\right)^{\frac{1}{5}}$

a =1

SECTION-A

1. Given below are two statements:

Statement I : CeO_2 can be used for oxidation of aldehydes and ketones.

Statement II : Aqueous solution of $EuSO_4$ is a strong reducing agent.

In the light of the above statements, choose the correct answer from the options given below:

(1) Statement I is false but statement II is true

(2) Statment I is true but statement II is false

(3) Both statement I and statement II are true

(4) Both statement I and statement II are false

Official Ans. by NTA (3)

Sol. The +3 oxidation state of lanthanide is most stable and therefore lanthanide in +4 oxidation state has strong tendence to gain e^- and converted into +3 and therefore act as strong oxidizing agent.

eg Ce^{+4}

And there fore CeO_2 is used to oxidized alcohol aldehyde and ketones.

Lanthanide in +2 oxidation state has strong tendency to loss e^- and converted into +3 oxidation state therefore act as strong reducing agent.

$\therefore$ $EuSO_4$ act as strong reducing agent.

2. According to molecular theory, the species among the following that does not exist is:

(1) He_2^+ (2) He_2^- (3) Be_2 (4) O_2^{2-}

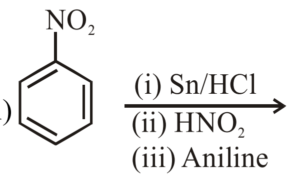
Official Ans. by NTA (3)

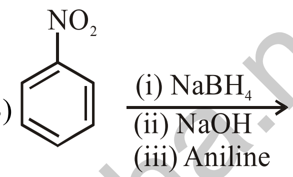
Sol.

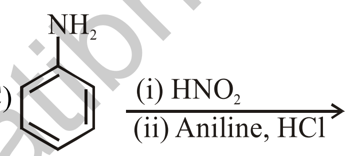
Chemical Species	Bond Order
He_2^+	0.5
He_2^-	0.5
Be_2	0
O_2^{2-}	1

According to M.O.T. If bond order of chemical species is zero then that chemical species does not exist.

3. Which of the following reaction/s will not give p-aminoazobenzene?

(A) 

(B) 

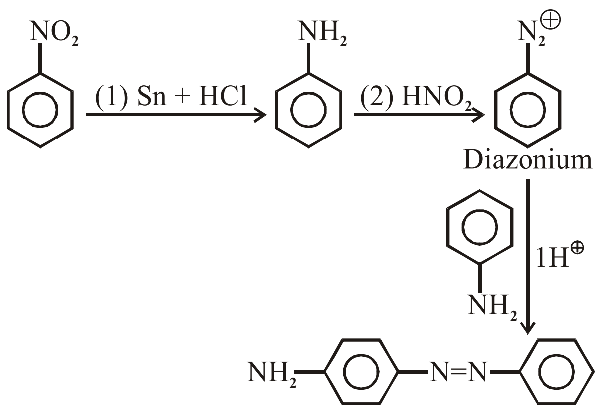
(C) 

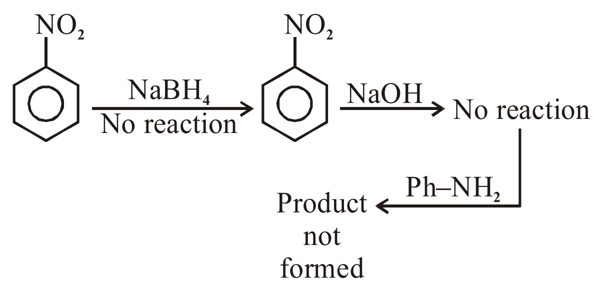
(1) A only (2) B only

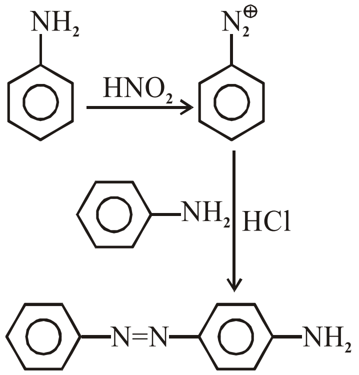
(3) C only (4) A and B

Official Ans. by NTA (2)

Sol. In basic or neutral medium N–N coupling favourable while in slightly acidic medium C–N coupling favourable.

(A) 

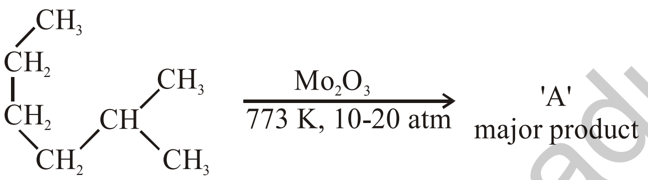
(B) 

(C) 

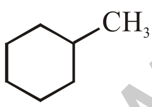
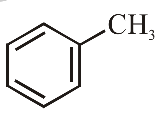
4. Which of the following equation depicts the oxidizing nature of H_2O_2?
(1) $KIO_4 + H_2O_2 \rightarrow KIO_3 + H_2O + O_2$
(2) $2I^- + H_2O_2 + 2H^+ \rightarrow I_2 + 2H_2O$
(3) $I_2 + H_2O_2 + 2OH^- \rightarrow 2I^- + 2H_2O + O_2$
(4) $Cl_2 + H_2O_2 \rightarrow 2HCl + O_2$
Official Ans. by NTA (2)

Sol. I^- is oxidised to I_2 by H_2O_2
Hence answer is (2)

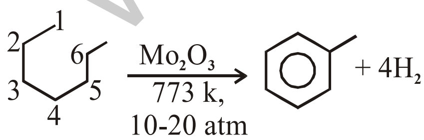
5. Identify A in the given chemical reaction.



(1)  (2) 

(3)  (4) 

Official Ans. by NTA (4)

Sol. 

Mo_2O_3 at 773 K temperature and 10-20-atm pressure is aromatising agent.

6. Complete combustion of 1.80 g of an oxygen containing compound ($C_xH_yO_z$) gave 2.64 g of CO_2 and 1.08 g of H_2O. The percentage of oxygen in the organic compound is:
(1) 51.63 (2) 63.53 (3) 53.33 (4) 50.33
Official Ans. by NTA (3)

Sol. $n_c = n_{co_2} = \frac{2.64}{44} = 0.06$

$n_H = 2 \times n_{H_2O} = \frac{1.08}{18} \times 2 = 0.12$

$m_0 = 1.80 - 12 \times \frac{2.64}{44} - \frac{1.08}{18} \times 2$

$= 1.80 - 0.72 - 0.12 = 0.96$ gm

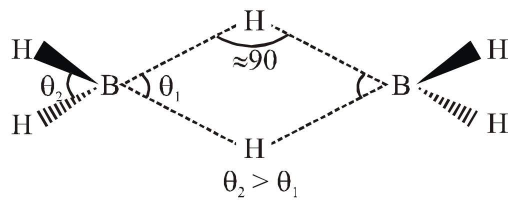
$\%0 = \frac{0.96}{1.80} \times 100 = 53.33\%$

Hence answer is (3)

7. Which one of the following reactions will not form acetaldehyde?
(1) $CH_3CH_2OH \xrightarrow[573K]{Cu}$
(2) $CH_3CN \xrightarrow[(ii) H_2O]{(i) DIBAL-H}$
(3) $CH_2{=}CH_2 + O_2 \xrightarrow[H_2O]{Pd(II)/Cu(II)}$
(4) $CH_3CH_2OH \xrightarrow{CrO_3-H_2SO_4}$
Official Ans. by NTA (4)

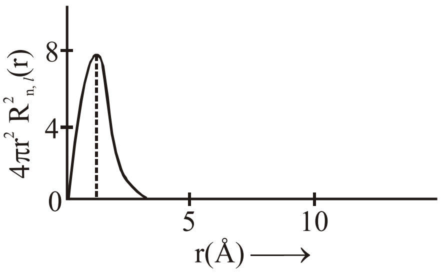
Sol. $CH_3–CH_2–OH \xrightarrow[\text{Strong oxidising agent}]{CrO_3\cdot H_2SO_4} CH_3–\overset{\overset{O}{||}}{C}–OH$ (Carboxylic acid is formed by complete oxidation)

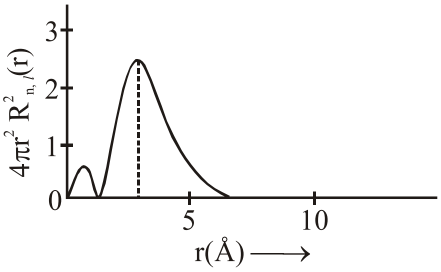
8. The correct statement about B_2H_6 is:
(1) Terminal B–H bonds have less p-character when compared to bridging bonds.
(2) The two B–H–B bonds are not of same length
(3) All B–H–B angles are of 120°
(4) Its fragment, BH_3, behaves as a Lewis base
Official Ans. by NTA (1)

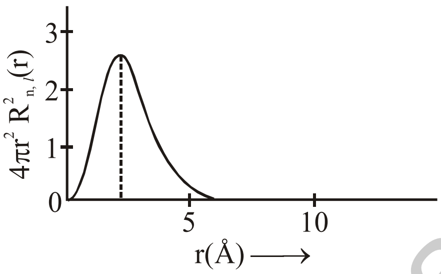
Sol. 

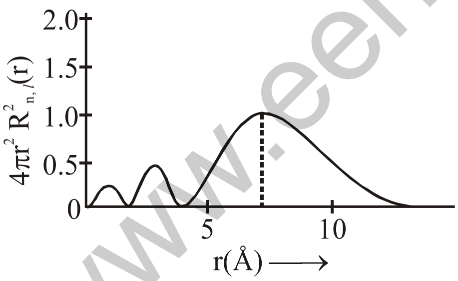
- $\theta_2 > \theta_1$, $\therefore$ B–H (terminal) having less p-character as compare to bridge bond.
- Both B–H–B bridge bond having same bond length.
- B–H–B bond angle is ≈90°
- BH_3 is e^- deficient species and therefore act as lewis acid

9. The plots of radial distribution functions for various orbitals of hydrogen atom against 'r' are given below:

(A) 

(B) 

(C) 

(D) 

The correct plot for 3s orbital is:

(1) (B) (2) (A)

(3) (D) (4) (C)

Official Ans. by NTA (3)

Sol. Number of radial nodes $= n - \ell - 1$

$= 3 - 0 - 1 = 2$

Therefor corresponding graph is (D)

Hence answer is (3)

10. Given below are two statements:

Statement I : An allotrope of oxygen is an important intermediate in the formation of reducing smog.

Statement II : Gases such as oxides of nitrogen and sulphur present in troposphere contribute to the formation of photochemical smog.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both statement I and Statement II are false

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are true

(4) Statement I is false but Statement II is true

Official Ans. by NTA (1)

Sol. Reducing smog is a mixture of smoke, fog and sulphur dioxide.

Tropospheric pollutants such as hydrocarbon and nitrogen oxide contirbute to the formation of photochemical smog.

11. In which of the following pairs, the outer most electronic configuration will be the same?

(1) Cr^+ and Mn^{2+} (2) Ni^{2+} and Cu^+

(3) Fe^{2+} and Co^+ (4) V^{2+} and Cr^+

Official Ans. by NTA (1)

Sol.

Option – 1	$Mn^{+2}[Ar]3d^5, Cr^+[Ar]3d^5$
Option – 2	$Ni^{+2}[Ar]3d^8, Cu^+[Ar]3d^{10}$
Option – 3	$Fe^{+2}[Ar]3d^6, Co^+[Ar]3d^7 4s^1$
Option – 4	$V^{+2}[Ar]3d^3, Cr^+[Ar]3d^5$

12. Which of the glycosidic linkage between galactose and glucose is present in lactose?

(1) C-1 of galactose and C-4 of glucose

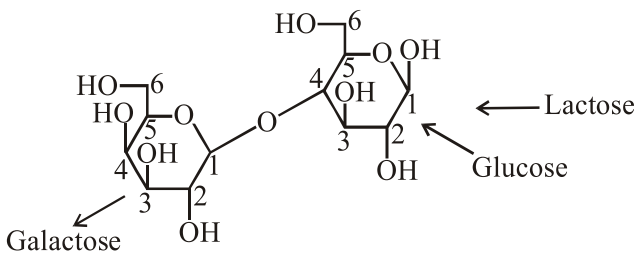
(2) C-1 of glucose and C-6 of galactose

(3) C-1 of glucose and C-4 of galactose

(4) C-1 of galactose and C-6 of glucose

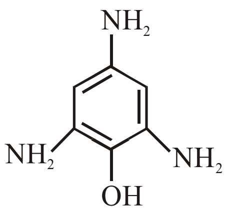
Official Ans. by NTA (1)

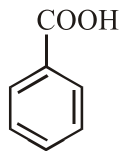
Sol.

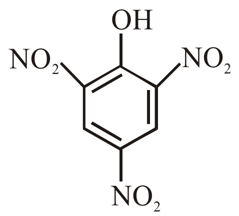


In lactose linkage is formed between C_1 of galactose and C_4 of gluocse.

13. Compound(s) which will liberate carbon dioxide with sodium bicarbonate solution is/are:

A = 

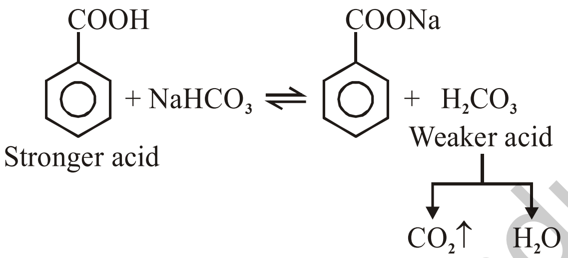
B = 

C = 

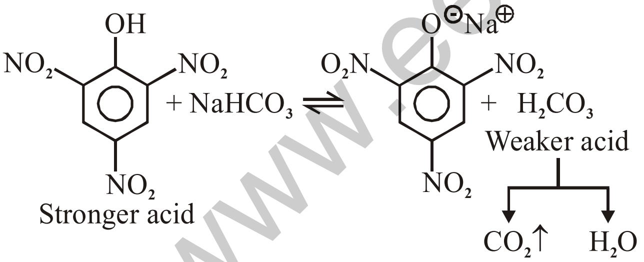
(1) B only (2) C only
(3) B and C only (4) A and B only

Official Ans. by NTA (3)

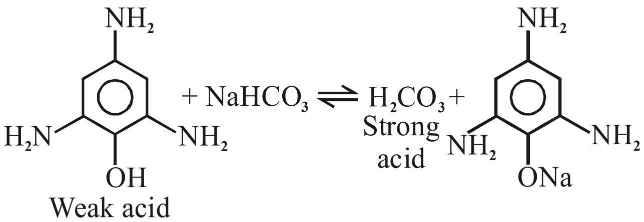
Sol.



equilibrium favours forward direction and $CO_2\uparrow$ is librated.



Equilibrium favours forward direction and $CO_2\uparrow$ is librated.



Equilibrium favours back word direction and $CO_2\uparrow$ is not librated.

14. The hybridization and magnetic nature of $[Mn(CN)_6]^{4-}$ and $[Fe(CN)_6]^{3-}$, respectively are:

(1) d^2sp^3 and diamagnetic

(2) sp^3d^2 and diamagnetic

(3) d^2sp^3 and paramagnetic

(4) sp^3d^2 and paramagnetic

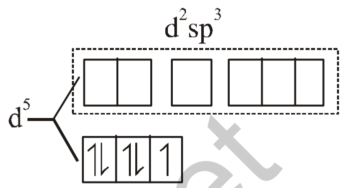
Official Ans. by NTA (3)

Sol. $[Mn(CN)_6]^{4-}$

Mn^{+2} d^5

CN^- – SFL

$\Delta_0 > P$



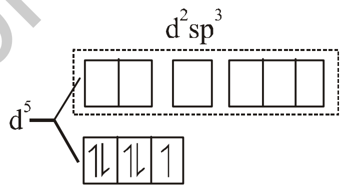
$\therefore$ hybridisation is d^2sp^3 and due to presence of unpaired e^- complex is paramagnetic in nature

$[Fe(CN)_6]^{3-}$

Fe^{+3} d^5

CN^- – SFL

$\Delta_0 > P$



$\therefore$ hybridisation is d^2sp^3 and due to presence of unpaired e^- complex paramagnetic in nature

15. Ellingham diagram is a graphical representation of:

(1) ΔH vs T (2) ΔG vs T

(3) ΔG vs P (4) $(\Delta G - T\Delta S)$ vs T

Official Ans. by NTA (2)

Sol. Ellingham diagram is a graphical representation of ΔG vs T when metal heated with oxygen to form metal oxide

16. The solubility of AgCN in a buffer solution of pH = 3 is x. The value of x is:

[Assume : No cyano complex is formed; $K_{sp}(AgCN) = 2.2 \times 10^{-16}$ and $K_a(HCN) = 6.2 \times 10^{-10}$]

(1) 0.625×10^{-6} (2) 1.9×10^{-5}

(3) 2.2×10^{-16} (4) 1.6×10^{-6}

Official Ans. by NTA (2)

Sol. $\frac{K_{sp}}{Ka} = \frac{s^2}{(H^+)}$; $s = \sqrt{\frac{K_{sp}}{K_a}(H^+)}$

$$s = \sqrt{\frac{2.2\times10^{-16}}{6.2\times10^{-10}}\times10^{-3}}$$

$s = 1.9 \times 10^{-5}$

Hence answer is (2)

17. In Freundlich adsorption isotherm at moderate pressure, the extent of adsorption $\left(\frac{x}{m}\right)$ is directly proportional to P^x. The value of x is

(1) zero (2) $\frac{1}{n}$

(3) 1 (4) ∞

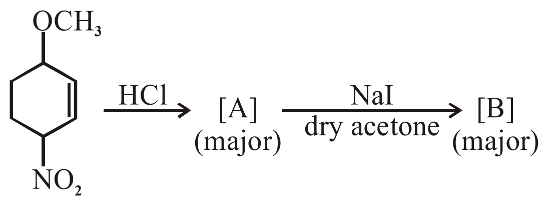
Official Ans. by NTA (2)

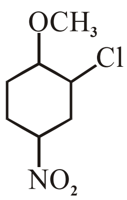
Sol. As per Freundlich adsorption isotherm

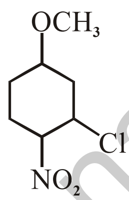
$$\left(\frac{x}{m}\right) = KP^{\frac{1}{n}} \rightarrow x = \frac{1}{n}$$

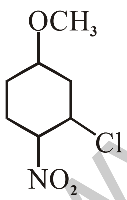
Hence answer is (2)

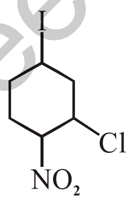
18. Identify A and B in the chemical reaction.

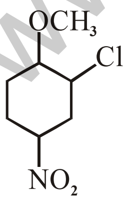


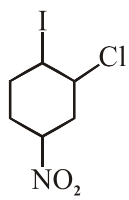
(1) A = 

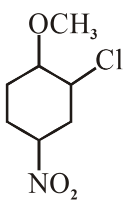
B = 

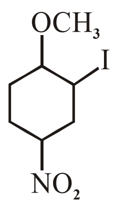
(2) A = 

B = 

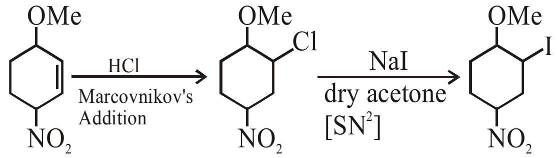
(3) A = 

B = 

(4) A = 

B = 

Official Ans. by NTA (4)

Sol. 

$\Rightarrow$ Ist reaction marcovnikov's addition of HCl on double bond while 2nd reaction is halide substitution by finkelstein reaction.

19. Which statement is correct ?

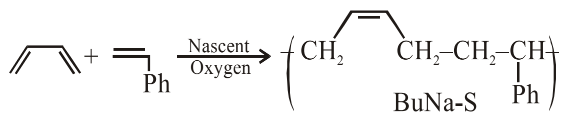
(1) Synthesis of Buna-S needs nascent oxygen.

(2) Neoprene is an addition copolymer used in plastic bucket manufacturing.

(3) Buna-S is a synthetic and linear thermosetting polymer.

(4) Buna-N is a natural polymer.

Official Ans. by NTA (1)

Sol. 

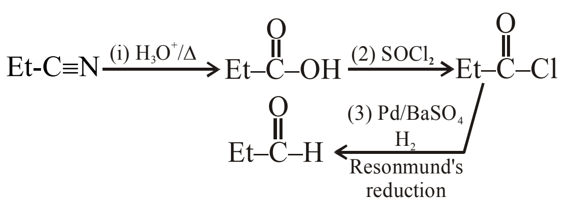
20. The major product of the following chemical reaction is :

$$CH_3CH_2CN \xrightarrow[(3)\ Pd/BaSO_4,H_2]{(1)\ H_3O^+,\ \Delta \quad (2)\ SOCl_2} ?$$

(1) $CH_3CH_2CH_3$ (2) $CH_3CH_2CH_2OH$

(3) $(CH_3CH_2CO)_2O$ (4) CH_3CH_2CHO

Official Ans. by NTA (4)

Sol. 

Final product of reaction is propanaldehyde.

SECTION-B

1. Among the following, the number of halide(s) which is/are inert to hydrolysis is _______.

(A) BF_3 (B) $SiCl_4$

(C) PCl_5 (D) SF_6

Official Ans. by NTA (1)

Sol. SF_6 is inert towards hydrolysis

$\therefore$ answere is (1)

2. 1 molal aqueous solution of an electrolyte A_2B_3 is 60% ionised. The boiling point of the solution at 1 atm is _______ K. (Rounded-off to the nearest integer)

[Given K_b for (H_2O) = 0.52 K kg mol^{-1}]

Official Ans. by NTA (375)

Sol. $\Delta T_b = iK_bm$

$= (1 + 4\alpha) \times 0.52 \times 1$

$= 3.4 \times 0.52 \times 1 = 1.768$

$T_b = 1.768 + 313.15 = 374.918$ K

$= 375$K

Hence answer is (375)

3. In basic medium CrO_4^{2-} oxidises $S_2O_3^{2-}$ to form SO_4^{2-} and itself changes into $Cr(OH)_4^-$. The volume of 0.154 M CrO_4^{2-} required to react with 40 mL of 0.25 M $S_2O_3^{2-}$ is _______ mL. (Rounded-off to the nearest integer)

Official Ans. by NTA (173)

Sol. $\overset{+6}{Cr}O_4^{2-} + \overset{+2}{S_2}O_3^{2-} \rightarrow \overset{+6}{S}O_4^{2-} + \overset{+3}{Cr}(OH)_4^-$

gm equi. of $CrO_4^{2-} = S_2O_3^{2-}$

$0.154 \times 3 \times v = 0.25 \times 40 \times 8$

$v = 173.16 = 173$ ml

Hence answer is (173)

4. A car tyre is filled with nitrogen gas at 35 psi at 27°C. It will burst if pressure exceeds 40 psi. The temperature in °C at which the car tyre will burst is _______. (Rounded-off to the nearest integer)

Official Ans. by NTA (70)

Sol. $P \propto T$

$$\frac{P_2}{P_1} = \frac{T_2}{T_1} \Rightarrow \frac{40}{35} = \frac{T_2}{300}$$

$T_2 = 342.854$ K

$= 69.70°C \simeq 70°C$

Hence answer is (70)

5. The reaction of cyanamide, $NH_2CN_{(s)}$ with oxygen was run in a bomb calorimeter and ΔU was found to be -742.24 kJ mol^{-1}. The magnitude of ΔH_{298} for the reaction

$$NH_2CN_{(s)} + \frac{3}{2}O_2(g) \rightarrow N_{2(g)} + O_2(g) + H_2O_{(l)}$$

is _______ kJ. (Rounded off to the nearest integer)

[Assume ideal gases and R = 8.314 J mol^{-1} K^{-1}]

Official Ans. by NTA (741)

Sol. $\Delta H = \Delta U + \Delta n_g RT$

$$= -742.24 + \frac{1}{2} \times \frac{8.314}{1000} \times 298$$

$= -741$ kJ/mol

Hence answer is (741)

6. Using the provided information in the following paper chromatogram :

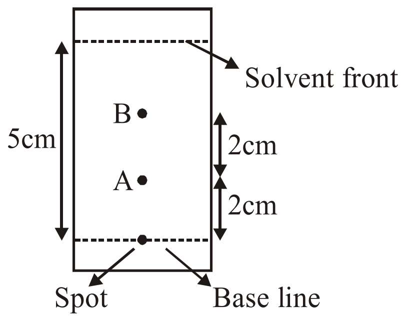


Figure : Paper chromatography for compounds A and B.

the calculate R_f value of A _______ $\times 10^{-1}$.

Official Ans. by NTA (4)

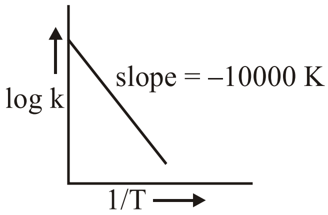
Sol. $$R_f = \frac{\text{Distance travelled by compound}}{\text{Distance travelled by solvent}}$$

on chromatogram distance travelled by cmopound is $\rightarrow$ 2 cm

Distance travelled by solvent = 5 cm

So $R_f = \frac{2}{5} = 4 \times 10^{-1} = 0.4$

7. For the reaction, $aA + bB \rightarrow cC + dD$, the plot of log k vs $\frac{1}{T}$ is given below :



The temperature at which the rate constant of the reaction is $10^{-4}\ s^{-1}$ is _______ K.

(Rounded-off to the nearest integer)

[Given : The rate constant of the reaction is $10^{-5}\ s^{-1}$ at 500 K.]

Official Ans. by NTA (526)

Sol. $\log K = \log A - \dfrac{Ea}{2.303RT}$

$$|\text{Slope}| = \frac{Ea}{2.303R} = 10{,}000$$

$$\log\left(\frac{K_2}{K_1}\right) = \frac{Ea}{2.303R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

$$\log\left(\frac{10^{-4}}{10^{-5}}\right) = 10{,}000\left[\frac{1}{500} - \frac{1}{T_2}\right]$$

$T_2 = 526.31 \simeq 526K$

Hence answer is (526)

8. 0.4 g mixture of NaOH, Na_2CO_3 and some inert impurities was first titrated with $\frac{N}{10}$ HCl using phenolphthalein as an indicator, 17.5 mL of HCl was required at the end point. After this methyl orange was added and titrated. 1.5 mL of same HCl was required for the next end point. The weight percentage of Na_2CO_3 in the mixture is _______. (Rounded-off to the nearest integer)

Official Ans. by NTA (4)

Sol. Upto first end point

gm equi. of $(NaOH + Na_2CO_3) = HCl$

$$x + y \times 1 = \frac{1}{10} \times 17.5$$

$x + y = 1.75$ …(1)

Upto second end point

$NaOH + Na_2CO_3 \equiv HCl$

$$x + y \times 2 = \frac{1}{10} \times 19$$

$x + 2y = 1.9$ …(2)

$y = 0.15$

$$\%Na_2CO_3 = \frac{0.15 \times 10^{-3} \times 106}{0.4} \times 100$$

$= 3.975\%$

$= 4\%$

Hence answer is (4)

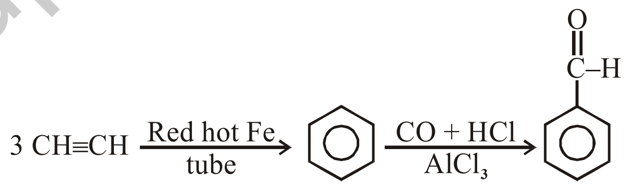
9. Consider the following chemical reaction.

$$CH \equiv CH \xrightarrow[\text{(2) CO, HCl, AlCl}_3]{\text{(1) Red hot Fe tube, 873 K}} \text{Product}$$

The number of sp^2 hybridized carbon atom(s) present in the product is _______.

Official Ans. by NTA (7)

Sol.

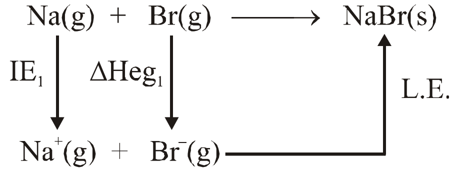


In benzaldehyde total number of sp^2 'C' are 7.

10. The ionization enthalpy of Na^+ formation from $Na_{(g)}$ is 495.8 kJ mol^{-1}, while the electron gain enthalpy of Br is –325.0 kJ mol^{-1}. Given the lattice enthalpy of NaBr is –728.4 kJ mol^{-1}. The energy for the formation of NaBr ionic solid is (–) _______ $\times 10^{-1}$ kJ mol^{-1}.

Official Ans. by NTA (5576)

Sol.



$\Delta H_{formation} = IE_1 + \Delta Heg_1 + LE$

$= 495.8 + (-325) + (-728.4)$

$= -557.6$

$= -5576 \times 10^{-1}$ KJ/mol.

Note: The above calculation is not for $\Delta H_{formation}$ but for $\Delta H_{Reaction}$.

But on the basis of given data it is the best ans.

SECTION-A

1. When a missile is fired from a ship, the probability that it is intercepted is $\frac{1}{3}$ and the probability that the missile hits the target, given that it is not intercepted, is $\frac{3}{4}$. If three missiles are fired independently from the ship, then the probability that all three hit the target, is :

(1) $\frac{1}{27}$ (2) $\frac{3}{4}$ (3) $\frac{1}{8}$ (4) $\frac{3}{8}$

Official Ans. by NTA (3)

Sol. Required probability = $\left(\frac{2}{3}\times\frac{3}{4}\right)^3 = \frac{1}{8}$

2. If $0<\theta,\phi<\frac{\pi}{2}, x=\sum_{n=0}^{\infty}\cos^{2n}\theta,\ y=\sum_{n=0}^{\infty}\sin^{2n}\phi$

and $z=\sum_{n=0}^{\infty}\cos^{2n}\theta\cdot\sin^{2n}\phi$ then :

(1) $xy - z = (x + y) z$ (2) $xy + yz + zx = z$
(3) $xyz = 4$ (4) $xy + z = (x + y)z$

Official Ans. by NTA (4)

Sol. $x = \frac{1}{1-\cos^2\theta} \quad \Rightarrow \sin^2\theta = \frac{1}{x}$

Also, $\cos^2\theta = \frac{1}{y}$ & $1-\sin^2\theta\cos^2\theta = \frac{1}{z}$

So, $1-\frac{1}{x}\times\frac{1}{y}=\frac{1}{z}\Rightarrow z(xy-1)=xy$ ....(1)

Also, $\frac{1}{x}+\frac{1}{y}=1 \quad \Rightarrow x+y=xy$ .....(2)

From (i) and (ii)

$xy + z = xyz = (x + y) z$

3. Let f, g : N $\to$ N such that $f(n + 1) = f(n) + f(1)$ $\forall$ n $\in$ N and g be any arbitrary function. Which of the following statements is NOT true?

(1) If fog is one-one, then g is one-one
(2) If f is onto, then $f(n) = n \ \forall n\in N$
(3) f is one-one
(4) If g is onto, then fog is one-one

Official Ans. by NTA (4)

Sol. $f(n + 1) - f(n) = f(1)$

$\Rightarrow f(n) = nf(1)$

$\Rightarrow$ f is one-one

Now, Let $f(g(x_2)) = f(g(x_1))$

$\Rightarrow g(x_2) = g(x_1)$ (as f is one-one)

$\Rightarrow x_1 = x_2$ (as fog is one-one)

$\Rightarrow$ g is one-one

Now, $f(g(n)) = g(n)\ f(1)$

may be many-one if

g(n) is many-one

4. The equation of the line through the point (0,1,2) and perpendicular to the line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{-2}$ is :

(1) $\frac{x}{3}=\frac{y-1}{4}=\frac{z-2}{3}$

(2) $\frac{x}{3}=\frac{y-1}{-4}=\frac{z-2}{3}$

(3) $\frac{x}{3}=\frac{y-1}{4}=\frac{z-2}{-3}$

(4) $\frac{x}{-3}=\frac{y-1}{4}=\frac{z-2}{3}$

Official Ans. by NTA (4)

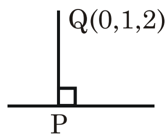
Sol. $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{-2}=r$

$\Rightarrow P(x, y, z) = (2r + 1, 3r - 1, -2r + 1)$

Since, $\overline{QP}\perp(2\hat{i}+3\hat{j}-2\hat{k})$

$\Rightarrow 4r + 2 + 9r - 6 + 4r + 2 = 0$

$\Rightarrow r=\frac{2}{17}$



$\Rightarrow P\left(\frac{21}{17},\frac{-11}{17},\frac{13}{17}\right)$

$\Rightarrow \overline{PQ}=\frac{21\hat{i}-28\hat{j}-21\hat{k}}{17}$

So, $\overleftrightarrow{QP}: \frac{x}{-3}=\frac{y-1}{4}=\frac{z-2}{3}$

5. Let α be the angle between the lines whose direction cosines satisfy the equations $l + m - n = 0$ and $l^2 + m^2 - n^2 = 0$. Then the value of $\sin^4\alpha + \cos^4\alpha$ is :

(1) $\frac{3}{4}$ (2) $\frac{3}{8}$ (3) $\frac{5}{8}$ (4) $\frac{1}{2}$

Official Ans. by NTA (3)

Sol. $n = \ell + m$

Now, $\ell^2 + m^2 = n^2 = (\ell + m)^2$

$\Rightarrow 2\ell m = 0$

If $\ell = 0 \Rightarrow m = n = \pm\frac{1}{\sqrt{2}}$

And, If $m = 0 \Rightarrow n = \ell = \pm\frac{1}{\sqrt{2}}$

So, direction cosines of two lines are

$\left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $\left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$

Thus, $\cos\alpha = \frac{1}{2} \Rightarrow \alpha = \frac{\pi}{3}$

6. The value of the integral

$$\int \frac{\sin\theta.\sin 2\theta(\sin^6\theta + \sin^4\theta + \sin^2\theta)\sqrt{2\sin^4\theta + 3\sin^2\theta + 6}}{1 - \cos 2\theta} d\theta$$

is :

(where c is a constant of integration)

(1) $\frac{1}{18}\left[11 - 18\sin^2\theta + 9\sin^4\theta - 2\sin^6\theta\right]^{\frac{3}{2}} + c$

(2) $\frac{1}{18}\left[9 - 2\cos^6\theta - 3\cos^4\theta - 6\cos^2\theta\right]^{\frac{3}{2}} + c$

(3) $\frac{1}{18}\left[9 - 2\sin^6\theta - 3\sin^4\theta - 6\sin^2\theta\right]^{\frac{3}{2}} + c$

(4) $\frac{1}{18}\left[11 - 18\cos^2\theta + 9\cos^4\theta - 2\cos^6\theta\right]^{\frac{3}{2}} + c$

Official Ans. by NTA (4)

Sol. $I = \int \frac{\sin\theta.\sin 2\theta(\sin^6\theta + \sin^4\theta + \sin^2\theta)\sqrt{2\sin^4\theta + 3\sin^2\theta + 6}}{1 - \cos 2\theta} d\theta$

$\Rightarrow I = \int \frac{\sin\theta.2\sin\theta\cos\theta.\sin^2\theta(\sin^4\theta + \sin^2\theta + 1)(2\sin^4\theta + 3\sin^2\theta + 6)^{1/2}}{2\sin^2\theta} d\theta$

$= \int \sin^2\theta.\cos\theta(\sin^4\theta + \sin^2\theta + 1)(2\sin^4\theta + 3\sin^2\theta + 6)^{1/2} d\theta$

Let $\sin\theta = t \Rightarrow \cos\theta\, d\theta = dt$

$\therefore\ I = \int t^2(t^4 + t^2 + 1)(2t^4 + 3t^2 + 6)^{1/2} dt$

$= \int (t^5 + t^3 + t)t(2t^4 + 3t^2 + 6)^{1/2} dt$

$= \int (t^5 + t^3 + t)(t^2)^{1/2}(2t^4 + 3t^2 + 6)^{1/2} dt$

$= \int (t^5 + t^3 + t)(2t^6 + 3t^4 + 6t^2)^{1/2} dt$

Let $2t^6 + 3t^4 + 6t^2 = u^2$

$\Rightarrow 12(t^5 + t^3 + t)\, dt = 2u\,du$

$\therefore\ I = \int (u^2)^{1/2} \cdot \frac{2u\,du}{12}$

$= \int \frac{u^2}{6} du = \frac{u^3}{18} + C$

$= \frac{(2t^6 + 3t^4 + 6t^2)^{3/2}}{18} + C$

when $t = \sin\theta$

and $t^2 = 1 - \cos^2\theta$ will give option (4)

7. The value of $\int_{-1}^{1} x^2 e^{[x^3]} dx$, where [t] denotes the greatest integer $\leq$ t, is :

(1) $\frac{e-1}{3e}$ (2) $\frac{e+1}{3}$ (3) $\frac{e+1}{3e}$ (4) $\frac{1}{3e}$

Official Ans. by NTA (3)

Sol. $I = \int_{-1}^{1} x^2 e^{[x^3]} dx$

$= \int_{-1}^{0} x^2 e^{[x^3]} dx + \int_{0}^{1} x^2 e^{[x^3]} dx$

$= \int_{-1}^{0} x^2 e^{-1} dx + \int_{0}^{1} x^2 e^{0} dx$

$= \frac{1}{e} \times \left.\frac{x^3}{3}\right|_{-1}^{0} + \left.\frac{x^3}{3}\right|_{0}^{1}$

$= \frac{1}{e} \times \left(0 - \left(\frac{-1}{3}\right)\right) + \frac{1}{3}$

$= \frac{1}{3e} + \frac{1}{3} = \frac{1+e}{3e}$

8. A man is observing, from the top of a tower, a boat speeding towards the tower from a certain point A, with uniform speed. At that point, angle of depression of the boat with the man's eye is

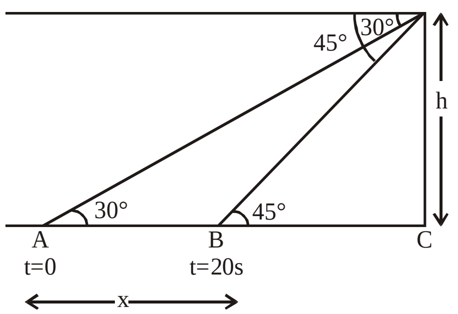
30° (Ignore man's height). After sailing for 20 seconds, towards the base of the tower (which is at the level of water), the boat has reached a point B, where the angle of depression is 45°. Then the time taken (in seconds) by the boat from B to reach the base of the tower is:

(1) 10 (2) $10\sqrt{3}$

(3) $10(\sqrt{3}+1)$ (4) $10(\sqrt{3}-1)$

Official Ans. by NTA (3)

Sol.



Let speed of boat is u m/s and height of tower is h meter & distance AB = x metre

$\therefore$ x = h cot 30° – h cot 45°

$\Rightarrow x = h(\sqrt{3}-1)$

$$\therefore \ u = \frac{x}{20} = \frac{h(\sqrt{3}-1)}{20} \text{m/s}$$

$\therefore$ Time taken to travel from B to C (Distance = h meter)

$$= \frac{h}{u} = \frac{h}{h\frac{(\sqrt{3}-1)}{20}} = \frac{20}{\sqrt{3}-1} = 10(\sqrt{3}+1)\text{sec}.$$

9. A tangent is drawn to the parabola $y^2 = 6x$ which is perpendicular to the line $2x + y = 1$. Which of the following points does NOT lie on it?

(1) (–6, 0) (2) (4, 5) (3) (5, 4) (4) (0, 3)

Official Ans. by NTA (3)

Sol. Slope of tangent = m_T = m

So, m (–2) = –1 $\Rightarrow m = \frac{1}{2}$

Equation : $y = mx + \frac{a}{m}$

$$\Rightarrow y = \frac{1}{2}x + \frac{3}{2\times\frac{1}{2}}\left(a = \frac{6}{4} = \frac{3}{2}\right)$$

$$\Rightarrow y = \frac{x}{2} + 3$$

$\Rightarrow$ 2y = x + 6

Point (5, 4) will not lie on it

10. All possible values of $\theta \in [0, 2\pi]$ for which $\sin 2\theta + \tan 2\theta > 0$ lie in :

(1) $\left(0,\frac{\pi}{2}\right)\cup\left(\pi,\frac{3\pi}{2}\right)$

(2) $\left(0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\frac{3\pi}{4}\right)\cup\left(\pi,\frac{7\pi}{6}\right)$

(3) $\left(0,\frac{\pi}{4}\right)\cup\left(\frac{\pi}{2},\frac{3\pi}{4}\right)\cup\left(\frac{3\pi}{2},\frac{11\pi}{6}\right)$

(4) $\left(0,\frac{\pi}{4}\right)\cup\left(\frac{\pi}{2},\frac{3\pi}{4}\right)\cup\left(\pi,\frac{5\pi}{4}\right)\cup\left(\frac{3\pi}{2},\frac{7\pi}{4}\right)$

Official Ans. by NTA (4)

Sol. $\sin 2\theta + \tan 2\theta > 0$

$$\Rightarrow \sin 2\theta + \frac{\sin 2\theta}{\cos 2\theta} > 0$$

$$\Rightarrow \sin 2\theta\frac{(\cos 2\theta + 1)}{\cos 2\theta} > 0 \Rightarrow \tan 2\theta(2\cos^2\theta) > 0$$

Note : $\cos 2\theta \neq 0$

$$\Rightarrow 1-2\sin^2\theta \neq 0 \Rightarrow \sin\theta \neq \pm\frac{1}{\sqrt{2}}$$

Now, $\tan 2\theta\,(1 + \cos 2\theta) > 0$

$\Rightarrow \tan 2\theta > 0$ (as $\cos 2\theta + 1 > 0$)

$$\Rightarrow 2\theta \in \left(0,\frac{\pi}{2}\right)\cup\left(\pi,\frac{3\pi}{2}\right)\cup\left(2\pi,\frac{5\pi}{2}\right)\cup\left(3\pi,\frac{7\pi}{2}\right)$$

$$\Rightarrow \theta \in \left(0,\frac{\pi}{4}\right)\cup\left(\frac{\pi}{2},\frac{3\pi}{4}\right)\cup\left(\pi,\frac{5\pi}{4}\right)\cup\left(\frac{3\pi}{2},\frac{7\pi}{4}\right)$$

As $\sin\theta \neq \pm\frac{1}{\sqrt{2}}$; which has been already considered

11. Let the lines $(2 - i)z = (2 + i)\bar{z}$ and $(2 + i)z + (i - 2)\bar{z} - 4i = 0$, (here $i^2 = -1$) be normal to a circle C. If the line $iz + \bar{z} + 1 + i = 0$ is tangent to this circle C, then its radius is:

(1) $\frac{3}{\sqrt{2}}$ (2) $\frac{1}{2\sqrt{2}}$ (3) $3\sqrt{2}$ (4) $\frac{3}{2\sqrt{2}}$

Official Ans. by NTA (4)

Sol. (i) $(2 - i)z = (2 + i)\bar{z}$

$\boxed{y = \frac{x}{2}}$

(ii) $(2 + i)z + (i - 2)\bar{z} - 4i = 0$

$\boxed{x + 2y = 2}$

(iii) $iz + \bar{z} + 1 + i = 0$

Eq^n of tangent $\boxed{x - y + 1 = 0}$

Solving (i) and (ii)

$x = 1,\ y = \frac{1}{2}$

Now, $p = r \Rightarrow \left|\frac{1 - \frac{1}{2} + 1}{\sqrt{2}}\right| = r$

$\Rightarrow r = \frac{3}{2\sqrt{2}}$

12. The image of the point (3, 5) in the line $x - y + 1 = 0$, lies on :

(1) $(x - 2)^2 + (y - 2)^2 = 12$

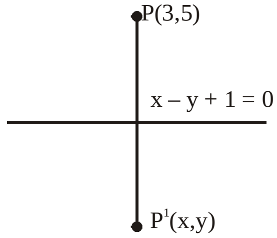
(2) $(x - 4)^2 + (y + 2)^2 = 16$

(3) $(x - 4)^2 + (y - 4)^2 = 8$

(4) $(x - 2)^2 + (y - 4)^2 = 4$

Official Ans. by NTA (4)

Sol.



$$\frac{x-3}{1} = \frac{y-5}{-1} = -2\left(\frac{3-5+1}{1+1}\right)$$

So, $x = 4, y = 4$

Hence, $(x - 2)^2 + (y - 4)^2 = 4$

13. If the curves, $\frac{x^2}{a} + \frac{y^2}{b} = 1$ and $\frac{x^2}{c} + \frac{y^2}{d} = 1$ intersect each other at an angle of 90°, then which of the following relations is TRUE?

(1) $a + b = c + d$

(2) $a - b = c - d$

(3) $a - c = b + d$

(4) $ab = \frac{c+d}{a+b}$

Official Ans. by NTA (2)

Sol. For orthogonal curves $a - c = b - d$

$\Rightarrow a - b = c - d$

14. $\lim\limits_{n\to\infty}\left(1 + \frac{1 + \frac{1}{2} + \dots\dots + \frac{1}{n}}{n^2}\right)^n$ is equal to :

(1) $\frac{1}{2}$ (2) 0 (3) $\frac{1}{e}$ (4) 1

Official Ans. by NTA (4)

Sol. Given limit is of 1^∞ form

So, $l = \exp\left(\lim\limits_{n\to\infty}\frac{1 + \frac{1}{2} + \frac{1}{3} + \dots\dots + \frac{1}{n}}{n}\right)$

Now,

$$0 \le 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} \le 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots + \frac{1}{\sqrt{n}}$$

$$\le 2\sqrt{n} - 1$$

So, l = exp (0) (from sandwich theorem)

= 1

15. The coefficients a, b and c of the quadratic equation, $ax^2 + bx + c = 0$ are obtained by throwing a dice three times. The probability that this equation has equal roots is:

(1) $\frac{1}{72}$ (2) $\frac{5}{216}$ (3) $\frac{1}{36}$ (4) $\frac{1}{54}$

Official Ans. by NTA (2)

Sol. $ax^2 + bx + c = 0$

For equal roots D = 0

$\Rightarrow b^2 = 4ac$

Case I : ac = 1

(a, b, c) = (1, 2, 1)

Case II : ac = 4

(a, b, c) = (1, 4, 4)

or (4, 4, 1)

or (2, 4, 2)

Case III : ac = 9

(a, b, c) = (3, 6, 3)

Required probability = $\frac{5}{216}$

16. The total number of positive integral solutions (x, y, z) such that xyz = 24 is :

(1) 36 (2) 24 (3) 45 (4) 30

Official Ans. by NTA (4)

Sol. $xyz = 2^3 \times 3^1$

Let $x = 2^{\alpha_1} \times 3^{\beta_1}$

$y = 2^{\alpha_2} \times 3^{\beta_2}$

$z = 2^{\alpha_3} \times 3^{\beta_2}$

Now $\alpha_1 + \alpha_2 + \alpha_3 = 3$.

No. of non-negative intergal sol = ${}^5C_2 = 10$

& $\beta_1 + \beta_2 + \beta_3 = 1$

No. of non-negative intergal sol^n = ${}^3C_2 = 3$

Total ways = 10 × 3 = 30.

17. The integer 'k', for which the inequality $x^2 - 2(3k - 1)x + 8k^2 - 7 > 0$ is valid for every x in R, is :

(1) 3 (2) 2 (3) 0 (4) 4

Official Ans. by NTA (1)

Sol. $x^2 - 2(3K - 1)x + 8K^2 - 7 > 0$

Now, D < 0

$\Rightarrow 4(3K - 1)^2 - 4 \times 1 \times (8K^2 - 7) < 0$

$\Rightarrow 9K^2 - 6K + 1 - 8K^2 + 7 < 0$

$\Rightarrow K^2 - 6K + 8 < 0$

$\Rightarrow (K - 4)(K - 2) < 0$

$\Rightarrow \boxed{K \in (2,4)}$

18. If a curve passes through the origin and the slope of the tangent to it at any point (x, y) is $\frac{x^2 - 4x + y + 8}{x - 2}$, then this curve also passes through the point:

(1) (5, 4) (2) (4, 5)

(3) (4, 4) (4) (5, 5)

Official Ans. by NTA (4)

Sol. Given

y (0) = 0

$\& \ \frac{dy}{dx} = \frac{(x-2)^2 + y + 4}{x - 2}$

$\Rightarrow \frac{dy}{dx} - \frac{y}{x-2} = (x-2) + \frac{4}{x-2}$

$\Rightarrow$ I.F. $= e^{-\int \frac{1}{x-2}dx} = \frac{1}{x-2}$

Solution of L.D.E.

$\Rightarrow y\frac{1}{x-2} = \int \frac{1}{x-2}\left((x-2) + \frac{4}{x-2}\right).dx$

$\Rightarrow \frac{y}{x-2} = x - \frac{4}{x-2} + C$

Now, at x = 0, y = 0 $\Rightarrow$ C = –2

y = x (x – 2) – 4 – 2 (x – 2)

$\Rightarrow y = x^2 - 4x$

This curve passes through (5, 5)

19. The statement $A \to (B \to A)$ is equivalent to :

(1) $A \to (A \wedge B)$ (2) $A \to (A \to B)$

(3) $A \to (A \leftrightarrow B)$ (4) $A \to (A \vee B)$

Official Ans. by NTA (4)

Sol. $A \to (B \to A)$

$\equiv A \to (\sim B \vee A)$

$\equiv \sim A \vee (\sim B \vee A)$

$\equiv (\sim A \vee A) \vee \sim B$

$\equiv T \vee \sim B \equiv T$

$\therefore T \vee B = T$

$\equiv (\sim A \vee A) \vee B$

$\equiv \sim A \vee (A \vee B)$

$\equiv A \to (A \vee B)$

20. If Rolle's theorem holds for the function $f(x) = x^3 - ax^2 + bx - 4$, $x \in [1, 2]$ with $f'\left(\frac{4}{3}\right) = 0$, then ordered pair (a, b) is equal to :

(1) (5, 8) (2) (–5, 8)

(3) (5, –8) (4) (–5, –8)

Official Ans. by NTA (1)

Sol. $f(1) = f(2)$

$\Rightarrow 1 - a + b - 4 = 8 - 4a + 2b - 4$

$\Rightarrow 3a - b = 7$ ......(1)

Also $f^1\left(\frac{4}{3}\right) = 0$ (given)

$\Rightarrow \left(3x^2 - 2ax + b\right)_{x=\frac{4}{3}} = 0$

$\Rightarrow \frac{16}{3} - \frac{8a}{3} + b = 0$

$\Rightarrow 8a - 3b - 16 = 0$ ....(2)

Solving (1) and (2)

a = 5, b = 8

SECTION-B

1. Let f(x) be a polynomial of degree 6 in x, in which the coefficient of x^6 is unity and it has extrema at x = –1 and x = 1. If $\lim\limits_{x\to 0}\frac{f(x)}{x^3} = 1$, then $5\cdot f(2)$ is equal to _______.

Official Ans. by NTA (144)

Sol. Let $f(x) = x^6 + ax^5 + bx^4 + cx^3 + dx^2 + ex + f$

as $\lim\limits_{x\to 0}\frac{f(x)}{x^3} = 1$ non-zero finite

So, d = e = f = 0

and $f(x) = x^3(x^3 + ax^2 + bx + c)$

Hence, $\lim\limits_{x\to 0}\frac{f(x)}{x^3} = c = 1$

Now, as $f(x) = x^6 + ax^5 + bx^4 + x^3$

and $f'(x) = 0$ at x = 1 and x = –1

i.e., $f'(x) = 6x^5 + 5ax^4 + 4bx^3 + 3x^2$

$f'(1) = 0$

$\Rightarrow 6 + 5a + 4b + 3 = 0$

$\Rightarrow 5a + 4b = -9$

& $f'(-1) = 0$

$\Rightarrow -6 + 5a - 4b + 3 = 0$

$\Rightarrow 5a - 4b = 3$

Solving both we get,

$a = \frac{-6}{10} = \frac{-3}{5}; \quad b = \frac{-3}{2}$

$\therefore \; f(x) = x^6 - \frac{3}{5}x^5 - \frac{3}{2}x^4 + x^3$

$\therefore \; 5f(2) = 5\left[64 - \frac{3}{5}\cdot 32 - \frac{3}{2}\cdot 16 + 8\right]$

$= 320 - 96 - 120 + 40$

$= 144$

2. The number of points, at which the function f(x) $= |2x + 1| - 3|x + 2| + |x^2 + x - 2|$, $x \in R$ is not differentiable, is _______.

Official Ans. by NTA (2)

Sol. $f(x) = |2x+1| - 3|x+2| + |x^2 + x - 2|$

$= |2x+1| - 3|x+2| + |x+2||x-1|$

$= |2x+1| + |x+2|\left(|x-1| - 3\right)$

Critical points are $x = \frac{-1}{2}$, –2, –1

but x = –2 is making a zero.

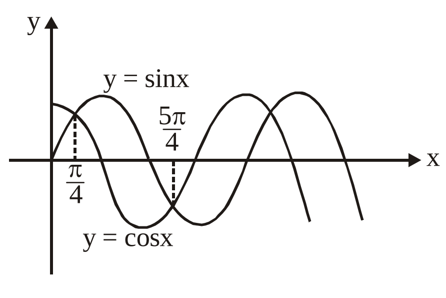
twice in product so, points of non differentability are $\boxed{x = \frac{-1}{2} \text{ and } x = -1}$

$\therefore$ Number of points of non-differentiability $= \boxed{2}$

3. The graphs of sine and cosine functions, intersect each other at a number of points and between two consecutive points of intersection, the two graphs enclose the same area A. Then A^4 is equal to _______.

Official Ans. by NTA (64)

Sol.



$$A = \int_{\pi/4}^{5\pi/4} (\sin x - \cos x)\, dx$$

$$= (-\cos x - \sin x)\Big|_{\pi/4}^{5\pi/4}$$

$$= \left(-\left(\frac{-1}{\sqrt{2}}\right) - \left(\frac{-1}{\sqrt{2}}\right)\right) - \left(-\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}}\right)\right)$$

$$\Rightarrow A = \frac{2}{\sqrt{2}} + \frac{2}{\sqrt{2}} = 2\sqrt{2}$$

$$\Rightarrow A^4 = (2\sqrt{2})^4 = 16 \times 4 = 64$$

4. Let A_1, A_2, A_3, ... be squares such that for each $n \geq 1$, the length of the side of A_n equals the length of diagonal of A_{n+1}. If the length of A_1 is 12 cm, then the smallest value of n for which area of A_n is less than one, is ______.

Official Ans. by NTA (9)

Sol. Let a_n be the side length of A_n.

So, $a_n = \sqrt{2}a_{n+1}$, $a_1 = 12$

$$\Rightarrow a_n = 12 \times \left(\frac{1}{\sqrt{2}}\right)^{n-1}$$

Now, $(a_n)^2 < 1 \Rightarrow \frac{144}{2^{(n-1)}} < 1$

$\Rightarrow 2^{(n-1)} > 144$

$\Rightarrow n - 1 \geq 8$

$\Rightarrow n \geq 9$

5. Let $A = \begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$, where x, y and z are real numbers such that $x + y + z > 0$ and $xyz = 2$. If $A^2 = I_3$, then the value of $x^3 + y^3 + z^3$ is_____.

Official Ans. by NTA (7)

Sol. $A^2 = I$

$\Rightarrow AA' = I$ $(\text{as } A' = A)$

$\Rightarrow$ A is orthogonal

So, $x^2 + y^2 + z^2 = 1$ and $xy + yz + zx = 0$

$\Rightarrow (x + y + z)^2 = 1 + 2 \times 0$

$\Rightarrow x + y + z = 1$

Thus,

$x^3 + y^3 + z^3 = 3 \times 2 + 1 \times (1 - 0)$

$= 7$

6. If $A = \begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix}$ and

$(I_2 + A)(I_2 - A)^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then $13(a^2 + b^2)$ is equal to ________ .

Official Ans. by NTA (13)

Sol. $a^2 + b^2 = |I_2 + A||I_2 - A|^{-1}$

$$= \sec^2\frac{\theta}{2} \times \cos^2\frac{\theta}{2} = 1$$

7. The total number of numbers, lying between 100 and 1000 that can be formed with the digits 1, 2, 3, 4, 5, if the repetition of digits is not allowed and numbers are divisible by either 3 or 5, is ______.

Official Ans. by NTA (32)

Sol. We need three digits numbers.

Since $1 + 2 + 3 + 4 + 5 = 15$

So, number of possible triplets for multiple of 15 is $1 \times 2 \times 2$

so Ans. = $4 \times \underline{|3} + 4 \times 3 - 1 \times 2 \times \underline{|2}$ = 32

8. Let $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$, $\vec{b} = \hat{i} - \hat{j}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$ and $\vec{r} \cdot \vec{b} = 0$, then $\vec{r} \cdot \vec{a}$ is equal to _________.

Official Ans. by NTA (12)

Sol. $(\vec{r} - \vec{c}) \times \vec{a} = 0$

$\Rightarrow \vec{r} = \vec{c} + \lambda\vec{a}$

Now, $0 = \vec{b}.\vec{c} + \lambda\vec{a}.\vec{b}$

$$\Rightarrow \lambda = \frac{-\vec{b} \cdot \vec{c}}{\vec{a} \cdot \vec{b}} = -\frac{2}{-1} = 2$$

So, $\vec{r} \cdot \vec{a} = \vec{a} \cdot \vec{c} + 2a^2 = 12$

9. If the system of equations

$kx + y + 2z = 1$

$3x - y - 2z = 2$

$-2x - 2y - 4z = 3$

has infinitely many solutions, then k is equal to ________.

Official Ans. by NTA (21)

Sol. We observe $5P_2 - P_1 = 3P_3$

So, $15 - K = -6$

$\Rightarrow K = 21$

10. The locus of the point of intersection of the lines

$(\sqrt{3})kx + ky - 4\sqrt{3} = 0$ and

$\sqrt{3}x - y - 4(\sqrt{3})k = 0$ is a conic, whose eccentricity is ___________.

Official Ans. by NTA (2)

Sol. $K = \dfrac{4\sqrt{3}}{\sqrt{3}x + y} = \dfrac{\sqrt{3}x - y}{4\sqrt{3}}$

$\Rightarrow 3x^2 - y^2 = 48$

$\Rightarrow \dfrac{x^2}{16} - \dfrac{y^2}{48} = 1$

Now, $48 = 16(e^2 - 1)$

$\Rightarrow \quad e = \sqrt{4} = 2$