# JEE MAIN 2021

**Held On 25th February, 2021**

**Evening Session**

## PHYSICS

### SECTION-A

**1.** For extrinsic semiconductors; when doping level is increased;

(1) Fermi-level of p-type semiconductor will go upward and Fermi-level of n-type semiconductors will go downward.

(2) Fermi-level of p-type semiconductors will go downward and Fermi-level of n-type semiconductor will go upward.

(3) Fermi-level of both p-type and n-type semiconductros will go upward for $T > T_F$ K and downward for $T < T_F$ K, where $T_F$ is Fermi temperature.

(4) Fermi-level of p and n-type semiconductors will not be affected.

**Official Ans. by NTA (2)**

**Sol.** (2) conceptual

**2.** In a ferromagnetic material, below the curie temperature, a domain is defined as :

(1) a macroscopic region with zero magnetization.

(2) a macroscopic region with consecutive magnetic dipoles oriented in opposite direction.

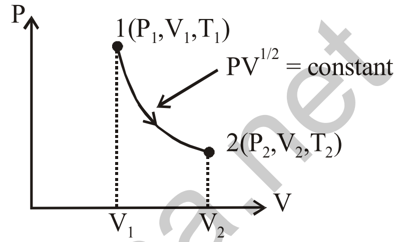
(3) a macroscopic region with randomly oriented magnetic dipoles.

(4) a macroscopic region with saturation magnetization.

**Official Ans. by NTA (4)**

**Sol.** **(4) conceptual**

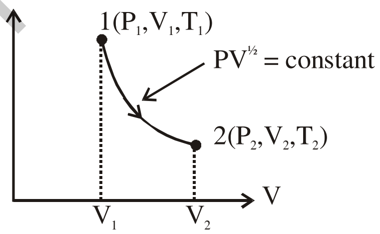
**3.** Thermodynamic process is shown below on a P-V diagram for one mole of an ideal gas. If $V_2 = 2V_1$ then the ratio of temperature $T_2/T_1$ is :




(1) $\frac{1}{2}$ (2) 2 (3) $\sqrt{2}$ (4) $\frac{1}{\sqrt{2}}$

**Official Ans. by NTA (3)**

**Sol.**




$PV^{½} = c$

$\frac{nRT}{V} V^{½} = c$

$T = c^1\ V^{½}$

$\frac{T_2}{T_1} = \left(\frac{V_2}{V_1}\right)^{1/2} = \left(\frac{2V_1}{V_1}\right)^{1/2}$

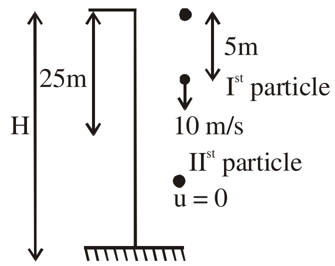
$\frac{T_2}{T_1} = \sqrt{2}$

**4.** A stone is dropped from the top of a building. When it crosses a point 5 m below the top, another stone starts to fall from a point 25 m below the top. Both stones reach the bottom of building simultaneously. The height of the building is :

(1) 35 m (2) 45m (3) 50 m (4) 25m

**Official Ans. by NTA (2)**

**Sol.**




Time for particle to meet = t' = $\frac{S_{rel}}{S_{rel}} = \frac{20}{10}$ = 2sec

Time taken by Ist particle to reach ground = 3sec

$H = \frac{1}{2} g\,(3)^2 = 45$ m

**5.** Given below are two statements :

Statement I : In a diatomic molecule, the rotational energy at a given temperature obeys Maxwell's distribution.

Statement II : In a diatomic molecule, the rotational energy at a given temperature equals the translational kinetic energy for each molecule.

In the light of the above statements, choose the correct answer from the options given below :

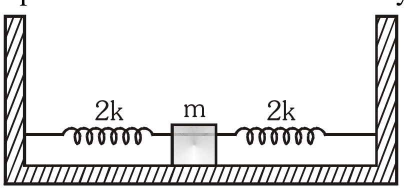
(1) Statement I is false but Statement II is true.
(2) Both Statement I and Statement II are false.
(3) Both Statement I and Statement II are true.
(4) Statement I is true but Statement II is false.

**Official Ans. by NTA (4)**

**Sol.** (4) Translational degree of freedom = 3

Rotational degree of freedom = 2

**6.** Two identical springs of spring constant '2k' are attached to a block of mass m and to fixed support (see figure).When the mass is displaced from equilibrium position on either side, it executes simple harmonic motion. The time period of oscillations of this sytem is :




(1) $2\pi\sqrt{\frac{m}{k}}$ (2) $\pi\sqrt{\frac{m}{2k}}$ (3) $2\pi\sqrt{\frac{m}{2k}}$ (4) $\pi\sqrt{\frac{m}{k}}$

**Official Ans. by NTA (4)**

**Sol.** (4) For parallel combination $k_{eq} = k_1 + k_2$

$k_{eq} = 4k$

$T = 2\pi\sqrt{\frac{m}{k_{eq}}}$

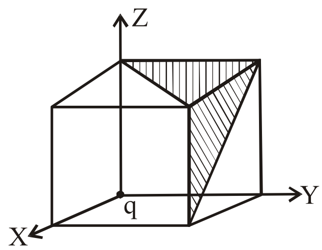
**7.** If a message signal of frequency '$f_m$' is amplitude modulated with a carrier signal of frequency '$f_c$' and radiated through an antenna, the wavelength of the corresponding signal in air is :

(1) $\frac{c}{f_c - f_m}$ (2) $\frac{c}{f_m}$ (3) $\frac{c}{f_c + f_m}$ (4) $\frac{c}{f_c}$

**Official Ans. by NTA (4)**

**Sol.** (4) $\lambda = \frac{v}{f} = \frac{c}{f_c}$

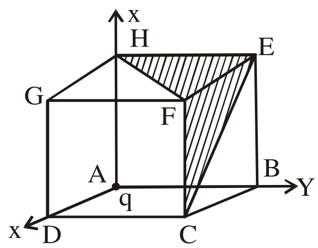
**8.** A charge 'q' is placed at one corner of a cube as shown in figure.The flux of electrostatic field $\vec{E}$ through the shaded area is :




(1) $\frac{q}{4\varepsilon_0}$ (2) $\frac{q}{24\varepsilon_0}$ (3) $\frac{q}{48\varepsilon_0}$ (4) $\frac{q}{8\varepsilon_0}$

**Official Ans. by NTA (2)**

**Sol.**




flux through cube = $\frac{q}{8\epsilon_0}$

flux through surfaces ABEH, ADGH, ABCD will be zero

$\phi\,(EFGH) = \phi\,(DCFG) = \phi\,(EBCF) = \frac{1}{3}\left(\frac{q}{8\epsilon_0}\right)$

$= \frac{q}{24\epsilon_0}$

**9.** The wavelength of the photon emitted by a hydrogen atom when an electron makes a transition from n = 2 to n = 1 state is :

(1) 194.8 nm (2) 913.3 nm

(3) 490.7 nm (4) 121.8 nm

**Official Ans. by NTA (4)**

**Sol.** $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$

$\lambda = 121.8$ nm.

**10.** An LCR circuit contains resistance of 110 Ω and a supply of 220 V at 300 rad/s angular frequency. If only capacitance is removed from the circuit, current lags behind the voltage by 45°. If on the other hand, only inductor is removed the current leads by 45° with the applied voltage. The rms current flowing in the circuit will be :

(1) 1A (2) 2.5 A

(3) 1.5 A (4) 2A

**Official Ans. by NTA (4)**

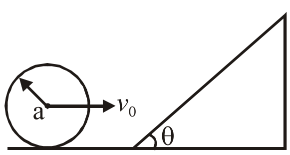
**Sol.** $\tan 45° = \frac{1}{\omega CR} = \frac{\omega L}{R} \Rightarrow X_L = X_C$

$\Rightarrow$ resonance

$i = \frac{V}{R}$

$= \frac{220}{110} = 2A$

**11.** A sphere of radius 'a' and mass 'm' rolls along a horizontal plane with constant speed $v_0$. It encounters an inclined plane at angle θ and climbs upward. Assuming that it rolls without slipping, how far up the sphere will travel ?



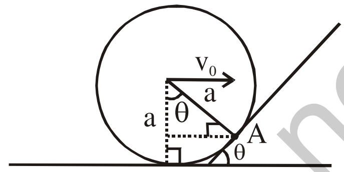

(1) $\frac{10v_0^2}{7g\sin\theta}$ (2) $\frac{v_0^2}{5g\sin\theta}$

(3) $\frac{2}{5}\frac{v_0^2}{g\sin\theta}$ (4) $\frac{v_0^2}{2g\sin\theta}$

**Official Ans. by NTA (1)**

**Official Ans. by ALLEN (BONUS)**

**Sol.**




Angular momentum conservation about A

$mv_0 \, a \cos\theta + \frac{2}{5}ma^2\omega$

$= mva + \frac{2}{5}ma^2\omega^1$

$mv_0 a\left[\frac{2}{5} + \cos\theta\right] = \frac{7}{5}mva$

$v = \frac{5}{7} = v_0\left[\frac{2}{5} + \cos\theta\right]$

$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{7}{10}mv^2 = mgh$

No option Maching

**12.** An electron of mass $m_e$ and a proton of mass $m_p = 1836\ m_e$ are moving with the same speed. The ratio of their de Broglie wavelength $\frac{\lambda_{electron}}{\lambda_{proton}}$ will be :

(1) 1836 (2) 1

(3) 918 (4) $\frac{1}{1836}$

**Official Ans. by NTA (1)**

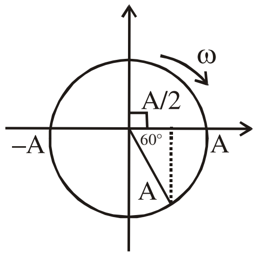
**Sol.** $\frac{\lambda_e}{\lambda_p} = \frac{\frac{h}{m_e v}}{\frac{h}{m_p v}} = 1836$

**13.** Y = A sin(ωt + $\phi_0$) is the time-displacement equation of a SHM. At t = 0 the displacement of the particle is Y = $\frac{A}{2}$ and it is moving along negative $x$-direction. Then the initial phase angle $\phi_0$ will be :

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{3}$ (3) $\frac{5\pi}{6}$ (4) $\frac{2\pi}{3}$

**Official Ans. by NTA (3)**

**Sol.**




initial phase $\frac{\pi}{2}+\frac{\pi}{3}=\frac{5\pi}{6}$

**14.** If e is the electronic charge, c is the speed of light in free space and h is Planck's constant, the quantity $\frac{1}{4\pi\varepsilon_0}\frac{|e|^2}{hc}$ has dimensions of :

(1) $[M^0 L^0 T^0]$ (2) $[L C^{-1}]$

(3) $[M L T^{-1}]$ (4) $[M L T^0]$

**Official Ans. by NTA (1)**

**Sol.** $F = \frac{1}{4\pi \epsilon_0}\frac{e^2}{r^2}$

$E = \frac{hc}{\lambda}$

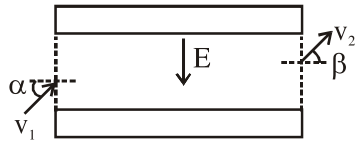
$\left[\frac{e^2}{4\pi\varepsilon_0}\times\frac{1}{hc}\right]=\frac{Fr^2}{E\lambda}=\left(M^0L^0T^0\right)$

**15.** An electron with kinetic energy $K_1$ enters between parallel plates of a capacitor at an angle 'α' with the plates. It leaves the plates at angle 'β' with kinetic energy $K_2$. Then the ratio of kinetic energies $K_1 : K_2$ will be :

(1) $\frac{\sin^2\beta}{\cos^2\alpha}$ (2) $\frac{\cos^2\beta}{\cos^2\alpha}$ (3) $\frac{\cos\beta}{\cos\alpha}$ (4) $\frac{\cos\beta}{\sin\alpha}$

**Official Ans. by NTA (2)**

**Sol.**

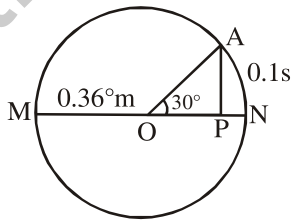



velocity along the plate will not change.

$\therefore v_1 \cos\alpha = v_2 \cos\beta$

$\frac{K_1}{K_2} \Rightarrow \frac{v_1^2}{v_2^2} = \frac{\cos^2\beta}{\cos^2\alpha}$

**16.** The point A moves with a uniform speed along the circumference of a circle of radius 0.36 m and covers 30° in 0.1 s. The perpendicular projection 'P' from 'A' on the diameter MN represents the simple harmonic motion of 'P'. The restoration force per unit mass when P touches M will be :

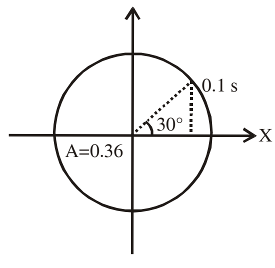



(1) 100 N (2) 0.49 N

(3) 50 N (4) 9.87 N

**Official Ans. by NTA (4)**

**Sol.**



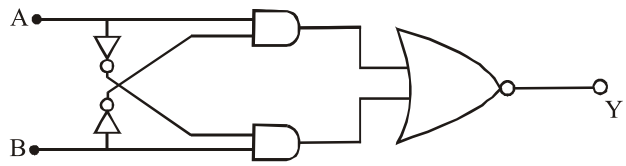

30°→ 0.1 s

360°→1.2 s = T

$\omega = \frac{2\pi}{T} = \frac{5\pi}{3}$

At M, $F = m\omega^2 A \Rightarrow \frac{F}{m} = \omega^2 A$

**17.** The truth table for the followng logic circuit is :




(1)

| A | B | Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

(2)

| A | B | Y |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

(3)

| A | B | Y |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

(4)

| A | B | Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

**Official Ans. by NTA (2)**

**Sol.** $y = \overline{\left(A\bar{B} + \bar{A}B\right)}$

$y = \overline{A\bar{B}} \cdot \overline{\bar{A}B}$

$y = \left(\bar{A} + B\right) . \left(A + \bar{B}\right)$

$y = \bar{A} . A + \bar{A}\bar{B} + A.B + B\bar{B}$

$y = AB + \bar{A}\bar{B}$

| A | B | $Y = AB + \bar{A}\bar{B}$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

**18.** The stopping potential for electrons emitted from a photosensitive surface illuminated by light of wavelength 491 nm is 0.710 V. When the incident wavelength is changed to a new value, the stopping potential is 1.43 V. The new wavelength is :

(1) 329 nm (2) 309 nm

(3) 382 nm (4) 400 nm

**Official Ans. by NTA (3)**

**Sol.** $\frac{hc}{\lambda} = \phi + eV_s$

$\frac{1240}{491} = \phi + 0.71$ .......(1)

$\frac{1240}{\lambda} = \phi + 1.43$ ............(2)

$\therefore \lambda = 382$ nm Ans.

**19.** Match List I with List II.

| List I | List II |
|---|---|
| (a) Rectifier | (i) Used either for stepping up or stepping down the a.c. voltage |
| (b) Stabilizer | (ii) Used to convert a.c. voltage into d.c. voltage |
| (c) Transformer | (iii) Used to remove any ripple in the rectified output voltage |
| (d) Filter | (iv) Used for constant output voltage even when the input voltage or load current change |

Choose the correct answer from the options given below :

(1) (a)–(ii), (b)–(iv), (c)–(i), (d)–(iii)

(2) (a)–(iii), (b)–(iv), (c)–(i), (d)–(ii)

(3) (a)–(ii), (b)–(i), (c)–(iv), (d)–(iii)

(4) (a)–(ii), (b)–(i), (c)–(iii), (d)–(iv)

**Official Ans. by NTA (1)**

**Sol.** (a) Rectifier $\rightarrow$ AC to DC

(b) Stabilizer $\rightarrow$ used for constant output voltage even when input voltage or current change.

(c) Transformer $\rightarrow$ Step - up or step - down ac voltage.

(d) Filter $\rightarrow$ used to remove any ripple in the rectified output voltage.

**20.** Consider the diffraction pattern obtained from the sunlight incident on a pinhole of diameter 0.1$\mu$m. If the diameter of the pinhole is slightly increased, it will affect the diffraction pattern such thtat :

(1) its size decreases, and intensity decreases

(2) its size increases, and intensity increases

(3) its size increases, but intensity decreases

(4) its size decreases, but intensity increases

**Official Ans. by NTA (4)**

**Sol.** $\sin\theta = \dfrac{m\lambda}{a}$

when a increases, $\theta$ decreases,

width decreases

width decreases so intensity will increases

## SECTION-B

**1.** The peak electric field produced by the radiation coming from the 8 W bulb at a distance of 10 m is $\dfrac{x}{10}\sqrt{\dfrac{\mu_0 c}{\pi}}\,\dfrac{V}{m}$. The efficiency of the bulb is 10% and it is a point source. The value of $x$ is____________.

**Official Ans. by NTA (2)**

**Sol.** $I = \dfrac{1}{2}c\,\epsilon_0 E_0^2$

$$\frac{8}{4\pi\times10^2}\times\frac{1}{2}=\frac{1}{4}\times c\times\frac{1}{\mu_0 c^2}\times E_0^2$$

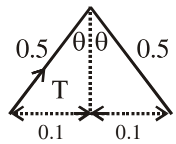
$$E_0=\frac{2}{10}\times\sqrt{\frac{\mu_0 c}{\pi}}\Rightarrow x=2$$

**2.** Two small spheres each of mass 10 mg are suspended from a point by threads 0.5 m long. They are equally charged and repel each other to a distance of 0.20 m. The charge on each of the sphere is $\dfrac{a}{21}\times10^{-8}$ C. The value of 'a' will be ______.

[Given g = 10 ms$^{-2}$]

**Official Ans. by NTA (20)**

**Sol.**




$T\cos\theta = mg = 10\times10^{-6}\times10 = 10^{-4}$

$$T\sin\theta=\frac{9\times10^9\times q^2}{0.04}=F$$

$$\tan\theta = =\frac{0.1}{\sqrt{0.24}}=\frac{F}{mg}$$

$$q=\frac{2\sqrt{10}}{3\sqrt{\sqrt{24}}}\times10^{-8}$$

$$0.95\times10^{-8}=\frac{a}{21}\times10^{-8}$$

a = 20

**3.** The initial velocity $v_i$ required to project a body vertically upward from the surface of the earth to reach a height of 10R, where R is the radius of the earth, may be described in terms of escape velocity $v_e$ such that $v_i=\sqrt{\dfrac{x}{y}}\times v_e$. The value of $x$ will be ________.

**Official Ans. by NTA (10)**

**Sol.** $$\frac{-GMm}{11R}=\frac{-GMm}{R}+\frac{1}{2}mv^2$$

$$v=\sqrt{\frac{20GM}{11R}}$$

**4.** The wavelength of an X-ray beam is 10Å. The mass of a fictitious particle having the same energy as that of the X-ray photons is $\frac{x}{3}$h kg. The value of $x$ is_________.
(h = Planck's constant)
**Official Ans. by NTA (10)**

**Sol.** $\frac{hc}{\lambda} = mc^2$

$m = \frac{h}{c\lambda}$

**5.** A reversible heat engine converts one-fourth of the heat input into work. When the temperature of the sink is reduced by 52 K, its efficiency is doubled. The temperature in Kelvin of the source will be _________.
**Official Ans. by NTA (208)**

**Sol.** $\eta = \frac{1}{4} = 1 - \frac{T_2}{T_1}$

$\frac{T_2}{T_1} = \frac{3}{4}$

$\frac{T_2 - 52}{T_1} = \frac{1}{2}$

**6.** The percentage increase in the speed of transverse waves produced in a stretched string if the tension is increased by 4%, will be __________%.
**Official Ans. by NTA (2)**

**Sol.** $v = \sqrt{\frac{T}{\mu}}$

$\frac{\Delta V}{V} = \frac{1}{2}\frac{\Delta T}{T}$

**7.** If $\vec{P} \times \vec{Q} = \vec{Q} \times \vec{P}$, the angle between $\vec{P}$ and $\vec{Q}$ is θ (0° < θ < 360°). The value of 'θ' will be _________°.
**Official Ans. by NTA (180)**

**Sol.** –PQ sin θ
= PQ sin θ
⇒ θ = 180°

**8.** Two identical conducting spheres with negligible volume have 2.1 nC and –0.1 nC charges, respectively. They are brought into contact and then separated by a distance of 0.5 m. The electrostatic force acting between the spheres is__________× $10^{-9}$ N.

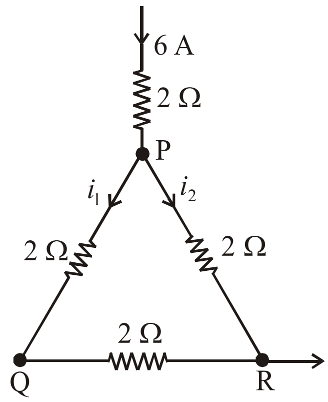
[Given : $4\pi\varepsilon_0 = \frac{1}{9\times 10^9}$ SI unit]

**Official Ans. by NTA (36)**

**Sol.** $q = \frac{(2.1 - 0.1)}{2}$ nC = 1nC

$f = \frac{9\times10^9 \times 10^{-18}}{(0.5)^2} = 36 \times 10^{-9}$

**9.** A current of 6 A enters one corner P of an equilateral triangle PQR having 3 wires of resistance 2Ω each and leaves by the corner R. The currents $i_1$ in ampere is __________.




**Official Ans. by NTA (2)**

**Sol.** For parallel combination current devides in the inverse ratio of resistance.

$i_{PQ} = \frac{2}{6} \times 6$ A

**10.** Two particles having masses 4 g and 16 g respectively are moving with equal kinetic energies. The ratio of the magnitudes of their linear momentum is n : 2. The value of n will be ___________.
**Official Ans. by NTA (1)**

**Sol.** $\frac{p_1^2}{2\times4} = \frac{p_2^2}{2\times16}$

$\frac{p_1}{p_2} = \frac{1}{2}$

## SECTION-A

**1.** Which among the following species has unequal bond lengths ?

(1) $BF_4^-$ (2) $XeF_4$

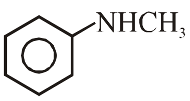
(3) $SF_4$ (4) $SiF_4$

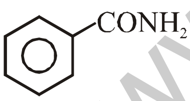
**Official Ans. by NTA (3)**

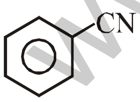
**Sol.**

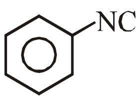
| Species | Hybridisation | Bond length |
|---|---|---|
| $BF_4^{\ominus}$ | $sp^3$(Tetrahedral) | All bond lengths equal |
| $XeF_4$ | $sp^3d^2$(sq. planar) | All bond lengths equal |
| $SF_4$ | $sp^3d$(see – saw) | axial bond length > equitorial bond length |
| $SiF_4$ | $sp^3$(Tetrahedral) | all bond lengths equal |

**2.** Carbylamine test is used to detect the presence of primary amino group in an organic compound. Which of the following compound is formed when this test is performed with aniline?

(1) 

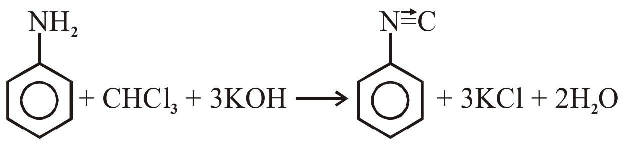

(2) 


(3) 

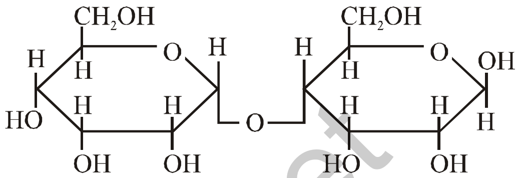

(4) 

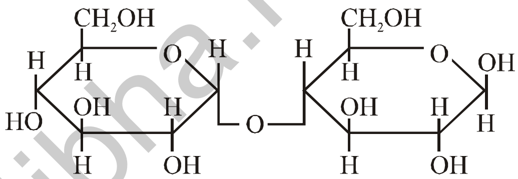

**Official Ans. by NTA (4)**

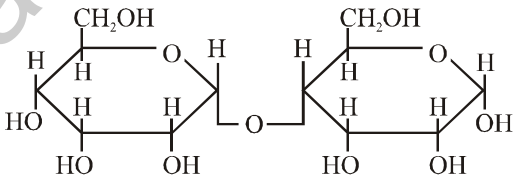
**Sol.** CARBYL amine given by 1° amine

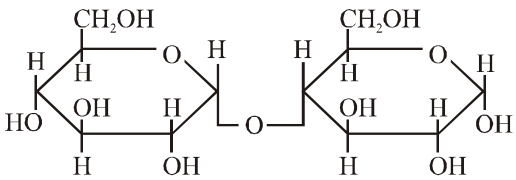



**3.** Which of the following is correct structure of α-anomer of maltose ?

(1) 

(2) 

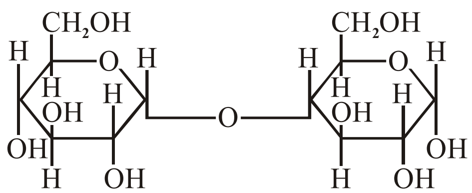
(3) 

(4) 

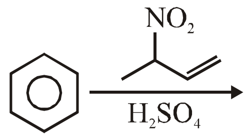
**Official Ans. by NTA (4)**

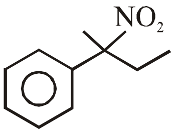
**Sol.** α-ANOMER OF MALTOSE

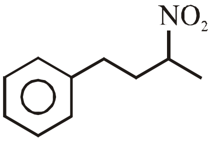
maltose is disaccharides of α-D-glucopyranose by $C_1$–$C_4$ glycosidic linkage

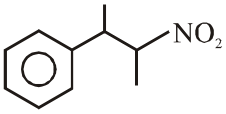


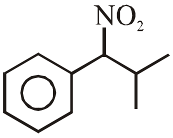
**4.** The major product of the following reaction is:



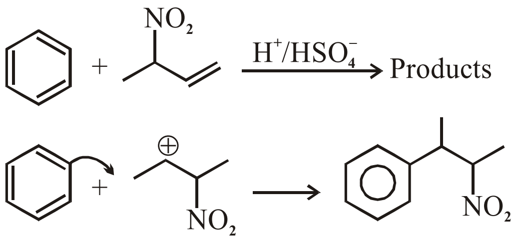

(1) 


(2) 


(3) 


(4) 


**Official Ans. by NTA (3)**

**Sol.** 


**5.** The correct sequence of reagents used in the preparation of 4-bromo-2-nitroethyl benzene from benzene is :

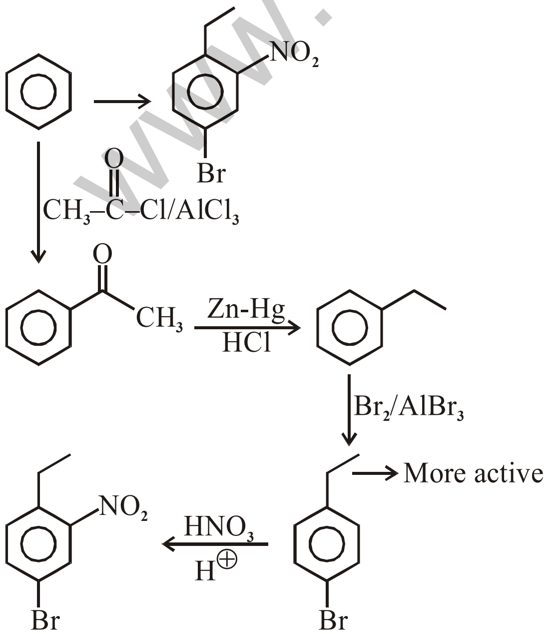
(1) $HNO_3/H_2SO_4$, $Br_2/AlCl_3$, $CH_3COCl/AlCl_3$, Zn-Hg/HCl

(2) $Br_2/AlBr_3$, $CH_3COCl/AlCl_3$, $HNO_3/H_2SO_4$, Zn/HCl

(3) $CH_3COCl/AlCl_3$, $Br_2/AlBr_3$, $HNO_3/H_2SO_4$, Zn/HCl

(4) $CH_3COCl/AlCl_3$, Zn-Hg/HCl, $Br_2/AlBr_3$, $HNO_3/H_2SO_4$

**Official Ans. by NTA (4)**

**Sol.** 


**6.** Water does not produce CO on reacting with:

(1) $CO_2$ (2) C (3) $CH_4$ (4) $C_3H_8$

**Official Ans. by NTA (1)**

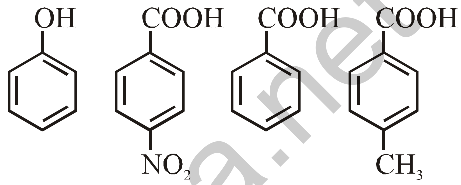
**Sol.** $CO_2 + H_2O \rightarrow H_2CO_3$

$C + H_2O(\text{steam}) \rightarrow CO + H_2$

$$\left.\begin{array}{l} CH_4 + H_2O \rightarrow CO + 3H_2 \\ C_3H_8 + H_2O \rightarrow 3CO + H_2 \end{array}\right] \begin{array}{l}\text{both reactions are carried out at} \\ \text{1270K temp. with Ni catalyst}\end{array}$$

Thus $CO_2$ does not produce CO.

**7.** The correct order of acid character of the following compounds is :



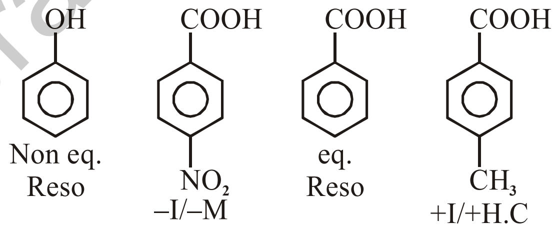

**I** **II** **III** **IV**

**Options:**

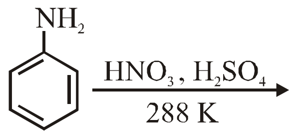
(1) III > II > I > IV (2) IV > III > II > I

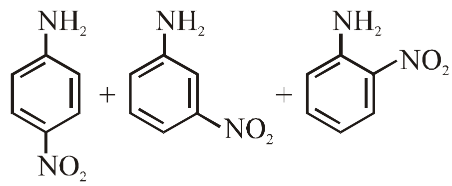
(3) I > II > III > IV (4) II > III > IV > I

**Official Ans. by NTA (4)**

**Sol.** 


2 > 3 > 4 > 1

**8.** 


(A) (B) (C)

Correct statement about the given chemical reaction is :

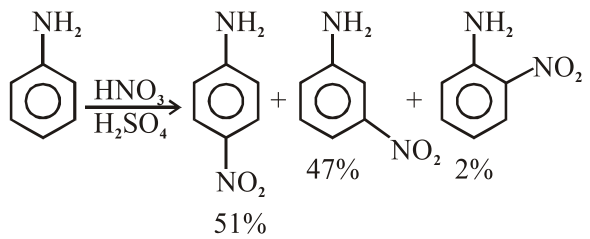
(1) $-\ddot{N}H_2$ group is *ortho* and *para* directive, so product (B) is not possible.

(2) Reaction is possible and compound (B) will be the major product.

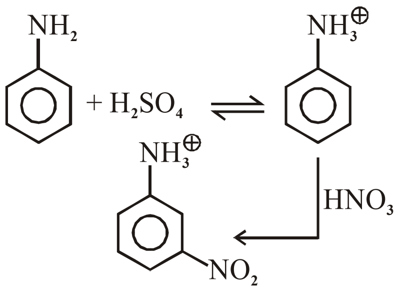
(3) The reaction will form sulphonated product instead of nitration.

(4) Reaction is possible and compound (A) will be major product.

**Official Ans. by NTA (4)**

**Sol.** 


Due to formation of anilinium ion by acid base reaction m-product is form as considerable amount.




**9.** The correct order of bond dissociation enthalpy of halogens is :

(1) $Cl_2 > F_2 > Br_2 > I_2$

(2) $I_2 > Br_2 > Cl_2 > F_2$

(3) $Cl_2 > Br_2 > F_2 > I_2$

(4) $F_2 > Cl_2 > Br_2 > I_2$

**Official Ans. by NTA (3)**

**Sol.** Correct order of bond dissociation enthalpy of halogens is $Cl_2 > Br_2 > F_2 > I_2$.

Due to inter electronic repulsions F–F bond becomes weak and easily broken.

**10.** Given below are two statements :

**Statement I :**

The pH of rain water is normally ~5.6.

**Statement II :**

If the pH of rain water drops below 5.6, it is called acid rain.

In the light of the above statements, choose the correct answer from the options given below:

(1) Statement I is true but Statement II is false.

(2) Both Statement I and Statement II are false.

(3) Statement I is false but Statement II is true.

(4) Both Statement I and Statement II are true.

**Official Ans. by NTA (4)**

**Sol.** Both statements are correct.

Normally rain water has pH of 5.6 due to the presence of $H^+$ ions formed by the reaction of rain water with carbon dioxide present in the atmosphere.

**11.** The major components of German Silver are :

(1) Ge, Cu and Ag (2) Zn, Ni and Ag

(3) Cu, Zn and Ni (4) Cu, Zn and Ag

**Official Ans. by NTA (3)**

**Sol.** Major components of German silver are:

Cu, Zn, Ni

(50%) (30%) (20%)

**12.** In which of the following order the given complex ions are arranged correctly with respect to their decreasing spin only magnetic moment ?

(i) $[FeF_6]^{3-}$ (ii) $[Co(NH_3)_6]^{3+}$

(iii) $[NiCl_4]^{2-}$ (iv) $[Cu(NH_3)_4]^{2+}$

(1) (i) > (iii) > (iv) > (ii)

(2) (ii) > (iii) > (i) > (iv)

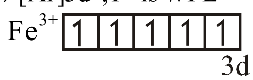
(3) (iii) > (iv) > (ii) > (i)

(4) (ii) > (i) > (iii) > (iv)

**Official Ans. by NTA (1)**

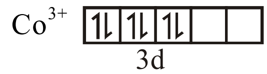
**Sol.** Complex

(i) $[FeF_6]^{3-}$ $Fe^{3+} \Rightarrow [Ar]3d^5, F^-$ is WFL



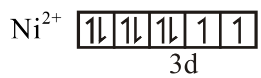

n = 5, $\mu = \sqrt{35}$ B.M.

(ii) $[Co(NH_3)_6]^{3+} \Rightarrow Co^{3+}[Ar]3d^6$, $NH_3$ is SFL



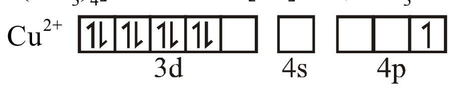

n = 0, $\mu = 0$

(iii) $[NiCl_4]^{2-}$ $Ni^{2+} \Rightarrow [Ar]3d^8$, $Cl^- \rightarrow$ WFL




n = 2, $\mu = \sqrt{8}$ B.M.

(iv) $[Cu(NH_3)_4]^{2+}$ $Cu^{2+} \Rightarrow [Ar]\ 3d^9$, $NH_3 \rightarrow$ SFL




n = 1, $\mu = \sqrt{3}$ B.M.

Thus correct order of spin only magnetic moment is (i) > (iii) > (iv) > (ii)

**13.** Which of the following compound is added to the sodium extract before addition of silver nitrate for testing of halogens?

(1) Nitric acid (2) Ammonia

(3) Hydrochloric acid (4) Sodium hydroxide

**Official Ans. by NTA (1)**

**Sol.** For testing of halogens, Nitric acid is added to the sodium extract because if $CN^-$ or $S^{2-}$ are present then they will be oxidised and removed before the test of halides.

**14.** Which one of the following statements is FALSE for hydrophilic sols ?

(1) Their viscosity is of the order of that of $H_2O$.
(2) The sols cannot be easily coagulated.
(3) They do not require electrolytes for stability.
(4) These sols are reversible in nature.

**Official Ans. by NTA (1)**

**Sol.** $\rightarrow$ Viscosity of hydrophilic sol > viscosity of $H_2O$
$\rightarrow$ Hydrophilic sol is more stable so can't be easily coagulated.
$\rightarrow$ Hydrophilic sols are reversible sols.
$\rightarrow$ No electrolytes are required to stabilise hydrophilic sol.

**15.** The solubility of $Ca(OH)_2$ in water is :
[Given : The solubility product of $Ca(OH)_2$ in water = $5.5 \times 10^{-6}$]

(1) $1.77 \times 10^{-6}$ (2) $1.11 \times 10^{-6}$
(3) $1.11 \times 10^{-2}$ (4) $1.77 \times 10^{-2}$

**Official Ans. by NTA (3)**

**Sol.** $Ca(OH)_2 \rightleftharpoons \underset{s}{Ca^{2+}(aq)} + \underset{2s}{2OH^-(aq)}$

$k_{sp} = s(2s)^2 \Rightarrow 5.5 \times 10^{-6} = 4S^3$

$\Rightarrow s = \left(\frac{5.5}{4}\right)^{\frac{1}{3}} \times 10^{-2} = 1.11 \times 10^{-2}$

**16.** Given below are two statements :

**Statement I :**
The identification of $Ni^{2+}$ is carried out by dimethyl glyoxime in the presence of $NH_4OH$.

**Statement II :**
The dimethyl glyoxime is a bidentate neutral ligand.

In the light of the above statements, choose the correct answer from the options given below:

(1) Statement I is false but Statement II is true.
(2) Both Statement I and Statement II are false.
(3) Statement I is true but Statement II is false.
(4) Both Statement I and Statement II are true.

**Official Ans. by NTA (3)**

**Sol.** Neutral dimethyl glyoxime does not act as ligand.
When $Ni^{2+}$ reacts with dimethyl glyoxime in presence of $NH_4OH$, it produce dimethyl glyoximate then it form rozy red ppt.

$$Ni^{2+}_{(aq)} + 2dmg^- \rightarrow \underset{\text{Rosy red ppt.}}{[Ni(dmg)_2]}$$

**17.** The major product of the following reaction is:

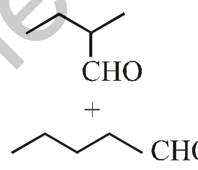
$$CH_3CH_2CH = CH_2 \xrightarrow[\text{Rh catalyst}]{H_2/CO}$$

(1) $CH_3CH_2CH{=}CH{-}CHO$
(2) $CH_3CH_2\underset{\displaystyle CHO}{\underset{|}{C}}{=}CH_2$
(3) $CH_3CH_2CH_2CH_2CHO$
(4) $CH_3CH_2CH_2CHO$

**Official Ans. by NTA (3)**

**Sol.** OXO PROCESS (Hydroformylation) :

$$CH_3 - CH_2 - CH = CH_2 + CO + H_2 \xrightarrow[\text{catalyst}]{Rh}$$



(Major)

**18.** The method used for the purification of Indium is :

(1) van Arkel method
(2) liquation
(3) zone refining
(4) vapour phase refining

**Official Ans. by NTA (3)**

**Sol.** Zone refining is used for the purification of indium.

**19.** What is 'X' in the given reaction?

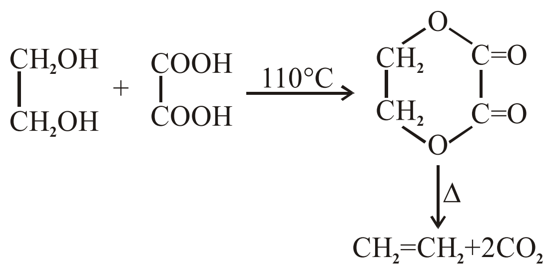
$$\underset{CH_2OH}{\overset{CH_2OH}{|}} + \text{oxalic acid} \xrightarrow{210°C} \underset{\text{(major product)}}{X}$$

(1) $\underset{CH_2}{\overset{CH_2}{\|}}$ (2) $\underset{CH_2}{\overset{CH-OH}{\|}}$

(3) $\underset{CHO}{\overset{CHO}{|}}$ (4) $\underset{CHO}{\overset{CH_2OH}{|}}$

**Official Ans. by NTA (1)**

**Sol.**



**20.** Given below are two statements :

**Statement-I :** $\alpha$ and $\beta$ forms of sulphur can change reversibly between themselves with slow heating or slow cooling.

**Statement-II :** At room temperature the stable crystalline form of sulphur is monoclinic sulphur.

In the light of the above statements, choose the correct answer from the options given below:

(1) Statement I is false but Statement II is true.

(2) Both Statement I and Statement II are true.

(3) Both Statement I is true but Statement II is false.

(4) Both Statement I and Statement II are false.

**Official Ans. by NTA (3)**

**Sol.** $\alpha-\text{sulphur} \underset{<369K}{\overset{>369K}{\rightleftharpoons}} \beta-\text{sulphur}$

at room temperature $\alpha$–sulphur (Rhombic) is most stable form.

### SECTION-B

**1.** If a compound AB dissociates to the extent of 75% in an aqueous solution, the molality of the solution which shows a 2.5 K rise in the boiling point of the solution is ______ molal. (Rounded-off to the nearest integer)

$[K_b = 0.52K\ \text{kg mol}^{-1}]$

**Official Ans. by NTA (3)**

**Sol.** $\alpha = 0.75,\ n = 2$

$i = 1 - \alpha + n\alpha = 1 - 0.75 + 2 \times 0.75 = 1.75$

$\Delta T_b = ik_bm$

or, $2.5 = 1.75 \times 0.52 \times m$

or, $m = \dfrac{2.5}{1.75 \times 0.52} = 2.74$

$\therefore$ nearest integer answer will be 3

**2.** The number of compound/s given below which contain/s –COOH group is _______.

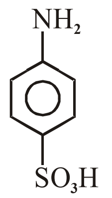
(A) Sulphanilic acid (B) Picric acid

(C) Aspirin (D) Ascorbic acid

**Official Ans. by NTA (1)**

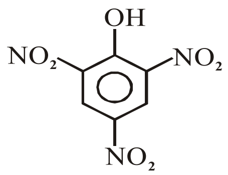
**Sol.** $\longrightarrow$ COOH group present in

(A)



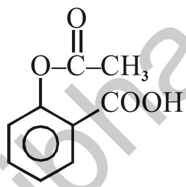
Sulphanilic acid

(B)



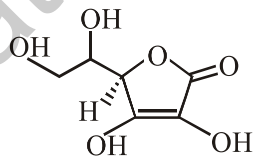
Picric acid

(C)



Asprine

(D)



Ascorbic acid

**3.** The rate constant of a reaction increases by five times on increase in temperature from 27°C to 52°C. The value of activation energy in kJ mol$^{-1}$ is _______ (Rounded-off to the nearest integer)

$[R = 8.314\ \text{J K}^{-1}\ \text{mol}^{-1}]$

**Official Ans. by NTA (52)**

**Sol.** $T_1 = 300K,\ T_2 = 325K,\ K_2 = 5K,$

$\ln\dfrac{K_2}{K_1} = \dfrac{Ea}{R}\left[\dfrac{1}{T_1} - \dfrac{1}{T_2}\right]$

or, $\ln 5 = \dfrac{Ea}{8.314}\left[\dfrac{1}{300} - \dfrac{1}{325}\right]$

or, $Ea = 0.7 \times 2.303 \times 8.314 \times 12 \times 325$

$= 52271\ J = 52.271\ kJ$

Nearest integer answer will be 52 kJ

**4.** Among the following, number of metal/s which can be used as electrodes in the photoelectric cell is _______ (Integer answer)

(A) Li (B) Na

(C) Rb (D) Cs

**Official Ans. by NTA (1)**

**Sol.** Cs is used as electrodes in the photoelectric cell.

**5.** The spin only magnetic moment of a divalent ion in aqueous solution (atomic number 29) is _______ BM.

**Official Ans. by NTA (2)**

**Sol.** Z = 29 (Cu)

$Cu^{2+}$ form $[Cu(H_2O)_4]^{2+}$ complex ion with $H_2O$.

$[Cu(H_2O)_4]^{2+} \Rightarrow Cu^{2+}$ $[Ar]3d^9$, $H_2O \rightarrow$ WFL

$$Cu^{2+} \quad \underbrace{\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow}}_{3d}$$

number of unpaired $e^-$ = 1

$\mu = \sqrt{1(1+2)}$ B.M.

$\mu = \sqrt{3} \Rightarrow 1.73$ B.M. $\Rightarrow$ round off ans. $\Rightarrow 2$

**6.** Electromagnetic radiation of wavelength 663 nm is just sufficient to ionise the atom of metal A. The ionization energy of metal A in kJ $mol^{-1}$ is _______. (Rounded-off to the nearest integer)

$[h = 6.63 \times 10^{-34}$ Js, $c = 3.00 \times 10^8$ $ms^{-1}$, $N_A = 6.02 \times 10^{23}$ $mol^{-1}]$

**Official Ans. by NTA (180)**

**Sol.** $E = \frac{hc}{\lambda} \times \frac{N_A}{1000}$

$$= \frac{6.63\times10^{-34}\times3\times10^{8}\times6.02\times10^{23}}{663\times10^{-9}\times1000}$$

$= 3 \times 6.02 \times 10$ kJ

$= 180.6$ kJ

**7.** Consider titration of NaOH solution versus 1.25M oxalic acid solution. At the end point following burette readings were obtained.

(i) 4.5 mL (ii) 4.5 mL

(iii) 4.4 mL (iv) 4.4 mL

(v) 4.4 mL

If the volume of oxalic acid taken was 10.0 mL then the molarity of the NaOH solution is _______M. (Rounded-off to the nearest integer)

**Official Ans. by NTA (6)**

**Sol.** $V_{NaOH} = 4.4$ ml

eq. of NaOH = eq. of $H_2C_2O_4$

or, $M \times 4.4 \times 1 = 1.25 \times 10 \times 2$

or, M = 5.68 M

$\therefore$ Nearest integer answer is 6

**8.** Five moles of an ideal gas at 293 K is expanded isothermally from an initial pressure of 2.1 MPa to 1.3 MPa against at constant external pressure 4.3 MPa. The heat transferred in this process is _______ kJ $mol^{-1}$. (Rounded-off to the nearest integer) [Use R = 8.314 J $mol^{-1}K^{-1}$]

**Official Ans. by NTA (15)**

**Sol.** n = 5, T = 293K = const, $\Delta U = 0$,

$P_1 = 2.1$ MPa, $P_2 = 1.3$ MPa

$P_{ext} = 4.3$ MPa = const.

$$W = -P_{ext}(V_2 - V_1) = -P_{ext}\left(\frac{nRT}{P_2} - \frac{nRT}{P_1}\right)$$

$$\text{or, } W = -P_{ext}\, nRT\left(\frac{1}{P_2} - \frac{1}{P_1}\right)$$

$$= -4.3 \times 5 \times 8.314 \times 293\left(\frac{1}{1.3} - \frac{1}{2.1}\right)$$

$$= -4.3 \times 5 \times 8.314 \times 293\left(\frac{2.1-1.3}{1.3\times2.1}\right)$$

$= -15347.7$J

or, W = –15.35 kJ

$\Delta U^0 = q + W$

$\therefore$ q = –W

or, q = 15.35 kJ (for 5 moles)

$\therefore$ q/mole = $\frac{15.35}{5} = 3\text{kJ mol}^{-1}$

**9.** Copper reduces $NO_3^-$ into NO and $NO_2$ depending upon the concentration of $HNO_3$ in solution. (Assuming fixed $[Cu^{2+}]$ and $P_{NO} = P_{NO_2}$), the $HNO_3$ concentration at which the thermodynamic tendency for reduction of $NO_3^-$ into NO and $NO_2$ by copper is same is $10^x$ M. The value of 2x is _______. (Rounded-off to the nearest integer)

[Given, $E^o_{Cu^{2+}/Cu} = 0.34$ V, $E^o_{NO_3^-/NO} = 0.96$ V,

$E^o_{NO_3^-/NO_2} = 0.79$ V and at 298 K,

$$\frac{RT}{F}(2.303) = 0.059]$$

**Official Ans. by NTA (1)**

**Sol.** If the partial pressure of NO and $NO_2$ gas is taken as 1 bar, then Answer is 4, else the question is bonus.

$NO_3^- + 4H^+ + 3e^- \longrightarrow NO + 2H_2O$

$$E^o_{NO_3^-/NO} = 0.96V$$

$NO_3^- + 2H^+ + e^- \longrightarrow NO_2 + H_2O$

$$E^o_{NO_3^-/NO_2 = 0.79}$$

Let $[HNO_3] = y \Rightarrow [H^+] = y$ and $[NO_3^-] = y$ for same thermodynamic tendency

$$E_{NO_3^-/NO} = E_{NO_3^-/NO_2}$$

$$\text{or, } E^o_{NO_3^-/NO} - \frac{0.059}{3}\log\frac{P_{NO}}{y \times y^4}$$

$$= E^o_{NO_3^-/NO_2} - \frac{0.059}{1}\log\frac{P_{NO_2}}{y \times y^2}$$

$$\text{or, } 0.96 - \frac{0.059}{3}\log\frac{P_{NO}}{y^5} = 0.79 - \frac{0.059}{1}\log\frac{P_{NO_2}}{y^3}$$

$$\text{or, } 0.17 = -\frac{0.059}{1}\log\frac{P_{NO_2}}{y^3} + \frac{0.059}{3}\log\frac{P_{NO}}{y^5}$$

$$0.17 = -\frac{0.0591}{1}\log\frac{P_{NO_2}}{y^3} + \frac{0.0591}{3}\log\frac{P_{NO}}{y^5}$$

$$0.17 = -\frac{0.0591}{3}\log\frac{P^3_{NO_2}}{y^9} + \frac{0.0591}{3}\log\frac{P_{NO}}{y^5}$$

$$0.17 = \frac{0.0591}{3}\left[\log\frac{P_{NO}}{y^5} - \log\frac{P^3_{NO_2}}{y^9}\right]$$

$$0.17 = \frac{0.0591}{3}\left[\log\frac{P_{NO}}{y^5} \times \frac{y^9}{P^3_{NO_2}}\right]$$

Assume $P_{NO} \simeq P_{NO_2} = 1$ bar

$$\frac{0.17 \times 3}{0.059} = \log y^4 = 8.644$$

$$\log y = \frac{8.644}{4}$$

$\log y = 2.161$

$y = 10^{2.16}$

$\therefore 2x = 2 \times 2.161 = 4.322$

Answer (4)

**10.** The unit cell of copper corresponds to a face centered cube of edge length 3.596 Å with one copper atom at each lattice point. The calculated density of copper in $kg/m^3$ is _______. [Molar mass of Cu : 63.54 g ; Avogadro Number = 6.022 × $10^{23}$]

**Official Ans. by NTA (9077)**

**Sol.** FCC,

$$d = \frac{Z \times M}{N_A \times a^3} = \frac{4 \times 63.54}{1000 \times 6.022 \times 10^{23} \times (3.596 \times 10^{-10})^3}$$

$= 9076\ kg/m^3$

### SECTION-A

**1.** Let A be a 3 × 3 matrix with det(A) = 4. Let $R_i$ denote the $i^{th}$ row of A. If a matrix B is obtained by performing the operation $R_2 \to 2R_2 + 5R_3$ on 2A, then det(B) is equal to :

(1) 16 (2) 80 (3) 128 (4) 64

**Official Ans. by NTA (4)**

**Sol.** $|A| = 4$

$\Rightarrow |2A| = 2^3 \times 4 = 32$

$\because$ B is obtained by $R_2 \to 2R_2 + 5R_3$

$\Rightarrow |B| = 2 \times 32 = 64$

option (4)

**2.** The integral $\int \frac{e^{3\log_e 2x} + 5e^{2\log_e 2x}}{e^{4\log_e x} + 5e^{3\log_e x} - 7e^{2\log_e x}} dx$ , $x > 0$,

is equal to :

(where c is a constant of integration)

(1) $\log_e |x^2 + 5x - 7| + c$

(2) $4\log_e |x^2 + 5x - 7| + c$

(3) $\frac{1}{4}\log_e |x^2 + 5x - 7| + c$

(4) $\log_e \sqrt{x^2 + 5x - 7} + c$

**Official Ans. by NTA (2)**

**Sol.** $\int \frac{e^{3\log_e 2x} + 5e^{2\log_e 2x}}{e^{4\log_e x} + 5e^{3\log_e x} - 7e^{2\log_e x}} dx, x > 0$

$$= \int \frac{(2x)^3 + 5(2x)^2}{x^4 + 5x^3 - 7x^2} dx = \int \frac{4x^2(2x+5)}{x^2(x^2+5x-7)} dx$$

$$= 4\int \frac{d(x^2+5x-7)}{(x^2+5x-7)} = 4\log_e |x^2 + 5x - 7| + c$$

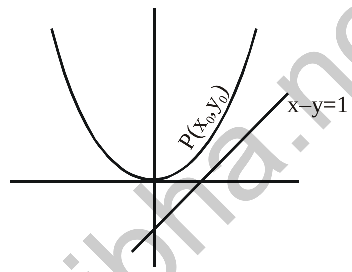
option (2)

**3.** The shortest distance between the line x – y = 1 and the curve $x^2 = 2y$ is :

(1) $\frac{1}{2}$ (2) $\frac{1}{2\sqrt{2}}$ (3) $\frac{1}{\sqrt{2}}$ (4) 0

**Official Ans. by NTA (2)**

**Sol.**




Shortest distance between curves is always along common normal.

$$\left.\frac{dy}{dx}\right|_P = \text{slope of line} = 1$$

$\Rightarrow x_0 = 1$ $\therefore\ y_0 = \frac{1}{2}$

$\Rightarrow P\left(1, \frac{1}{2}\right)$

$$\therefore \text{ Shortest distance } = \left|\frac{1 - \frac{1}{2} - 1}{\sqrt{1^2 + 1^2}}\right| = \frac{1}{2\sqrt{2}}$$

option (2)

**4.** If α, β ∈ R are such that 1 – 2i (here $i^2 = -1$) is a root of $z^2 + \alpha z + \beta = 0$, then (α – β) is equal to :

(1) –3 (2) –7 (3) 7 (4) 3

**Official Ans. by NTA (2)**

**Sol.** $\because$ α,β ∈ R ⇒ other root is 1 + 2i

α = –(sum of roots) = –(1 – 2i + 1 + 2i) = –2

β = product of roots = (1– 2i) (1 + 2i) = 5

$\therefore$ α – β = –7

option (2)

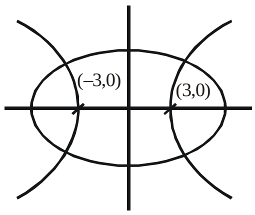
**5.** A hyperbola passes through the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ and its transverse and conjugate axes coincide with major and minor axes of the ellipse, respectively. If the product of their eccentricities in one, then the equation of the hyperbola is :

(1) $\frac{x^2}{9}-\frac{y^2}{25}=1$ (2) $\frac{x^2}{9}-\frac{y^2}{16}=1$

(3) $x^2 - y^2 = 9$ (4) $\frac{x^2}{9}-\frac{y^2}{4}=1$

**Official Ans. by NTA (2)**

**Sol.**




For ellipse $e_1=\sqrt{1-\frac{b^2}{a^2}}=\frac{3}{5}$

for hyperbola $e_2=\frac{5}{3}$

Let hyperbola be

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

$\because$ it passes through (3,0) $\Rightarrow \frac{9}{a_2}=1$

$\Rightarrow a^2 = 9$

$\Rightarrow b^2 = a^2(e^2 - 1)$

$=9\left(\frac{25}{9}-1\right)=16$

$\therefore$ Hyperbola is

$\frac{x^2}{9}-\frac{y^2}{16}=1$ ... option 2.

**6.** If $0<x, y<\pi$ and $\cos x+\cos y-\cos(x+y)=\frac{3}{2}$, then $\sin x + \cos y$ is equal to :

(1) $\frac{1}{2}$ (2) $\frac{1+\sqrt{3}}{2}$

(3) $\frac{\sqrt{3}}{2}$ (4) $\frac{1-\sqrt{3}}{2}$

**Official Ans. by NTA (2)**

**Sol.** $\cos x + \cos y - \cos(x + y) =\frac{3}{2}$

$$\cos^2\left(\frac{x+y}{2}\right)-\cos\left(\frac{x+y}{2}\right).\cos\left(\frac{x-y}{2}\right)$$

$$+\frac{1}{4}.\cos^2\left(\frac{x-y}{2}\right)+\frac{1}{4}\sin^2\left(\frac{x-y}{2}\right)=0$$

$$\Rightarrow\left(\cos\left(\frac{x+y}{2}\right)-\frac{1}{2}\cos\left(\frac{x-y}{2}\right)\right)^2+\frac{1}{4}\sin^2\left(\frac{x-y}{2}\right)=0$$

$\Rightarrow \sin\left(\frac{x-y}{2}\right)=0$ and

$$\cos\left(\frac{x+y}{2}\right)=\frac{1}{2}\cos\left(\frac{x-y}{2}\right)$$

$\Rightarrow x = y$ and $\cos x=\frac{1}{2}=\cos y$

$\therefore \sin x=\frac{\sqrt{3}}{2}$

$\Rightarrow \sin x + \cos y = \frac{1+\sqrt{3}}{2}$

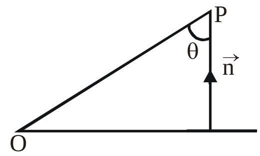
option (2)

**7.** A plane passes through the points A(1, 2, 3), B(2, 3, 1) and C(2, 4, 2). If O is the origin and P is (2, –1, 1), then the projection of $\overline{OP}$ on this plane is of length :

(1) $\sqrt{\frac{2}{7}}$ (2) $\sqrt{\frac{2}{3}}$ (3) $\sqrt{\frac{2}{11}}$ (4) $\sqrt{\frac{2}{5}}$

**Official Ans. by NTA (3)**

**Sol.**




Normal to plane $\vec{n}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & -2\\ 0 & 1 & 1\end{vmatrix}$

$=3\hat{i}-\hat{j}+\hat{k}$

$\overrightarrow{OP}=2\hat{i}-\hat{j}+\hat{k}$

$\cos\theta=\frac{6+1+1}{\sqrt{6}\sqrt{11}}=\frac{8}{\sqrt{66}}\Rightarrow\sin\theta=\sqrt{\frac{2}{66}}$

$\therefore$ Projection of $\overrightarrow{OP}$ on plane $=\left|\overrightarrow{OP}\right|\sin\theta$

$=\sqrt{\frac{2}{11}}$

option (3)

**8.** In a group of 400 people, 160 are smokers and non-vegetarian; 100 are smokers and vegetarian and the remaining 140 are non-smokers and vegetarian. Their chances of getting a particular chest disorder are 35%, 20% and 10% respectively. A person is chosen from the group at random and is found to be suffering from the chest disorder. The probability that the selected person is a smoker and non-vegetarian is :

(1) $\frac{7}{45}$ (2) $\frac{14}{45}$ (3) $\frac{28}{45}$ (4) $\frac{8}{45}$

**Official Ans. by NTA (3)**

**Sol.** Consider following events

A : Person chosen is a smoker and non vegetarian.

B : Person chosen is a smoker and vegetarian.

C : Person chosen is a non-smoker and vegetarian.

E : Person chosen has a chest disorder

Given

$$P(A)=\frac{160}{400} \quad P(B)=\frac{100}{400} \quad P(C)=\frac{140}{400}$$

$$P\left(\frac{E}{A}\right)=\frac{35}{100} \quad P\left(\frac{E}{B}\right)=\frac{20}{100} \quad P\left(\frac{E}{C}\right)=\frac{10}{100}$$

To find

$$P\left(\frac{A}{E}\right)=\frac{P(A)P\left(\frac{E}{A}\right)}{P(A).P\left(\frac{E}{A}\right)+P(B).P\left(\frac{E}{B}\right)+P(C).P\left(\frac{E}{C}\right)}$$

$$=\frac{\frac{160}{400}\times\frac{35}{100}}{\frac{160}{400}\times\frac{35}{100}+\frac{100}{400}\times\frac{20}{100}+\frac{140}{400}\times\frac{10}{100}}$$

$=\frac{28}{45}$ option (3)

**9.** $\operatorname{cosec}\left[2\cot^{-1}(5)+\cos^{-1}\left(\frac{4}{5}\right)\right]$ is equal to :

(1) $\frac{56}{33}$ (2) $\frac{65}{56}$ (3) $\frac{65}{33}$ (4) $\frac{75}{56}$

**Official Ans. by NTA (2)**

**Sol.** $$\operatorname{cosec}\left[2\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{3}{4}\right)\right]$$

$$\operatorname{cosec}\left[\tan^{-1}\left(\frac{5}{12}\right)+\tan^{-1}\left(\frac{3}{4}\right)\right]$$

$$=\operatorname{cosec}\left[\tan^{-1}\left(\frac{56}{33}\right)\right]=\frac{65}{56}$$ option (2)

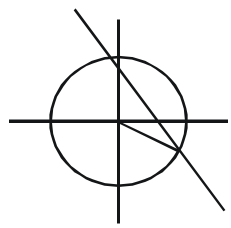
**10.** If the curve $x^2 + 2y^2 = 2$ intersects the line $x + y = 1$ at two points P and Q, then the angle subtended by the line segment PQ at the origin is :

(1) $\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{3}\right)$ (2) $\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{3}\right)$

(3) $\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{4}\right)$ (4) $\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{4}\right)$

**Official Ans. by NTA (4)**

**Sol.**



Homogenising

$$x^2+2y^2-2(x+y)^2=0$$

$$\Rightarrow -x^2-4xy=0 \Rightarrow x^2+4xy=0$$

Lines are $x = 0$ and $y=-\frac{x}{4}$

$\therefore$ Angle between lines $=\frac{\pi}{2}+\tan^{-1}\frac{1}{4}$

option (4)

**11.** The contrapositive of the statement "If you will work, you will earn money" is :

(1) You will earn money, if you will not work

(2) If you will earn money, you will work

(3) If you will not earn money, you will not work

(4) To earn money, you need to work

**Official Ans. by NTA (3)**

**Sol.** Constrapositive of $p \rightarrow q$ is $\sim q \rightarrow \sim p$

$\Rightarrow$ If you will not earn money, you will not work.

option (3)

**12.** A function f(x) is given by $f(x)=\dfrac{5^x}{5^x+5}$,

then the sum of the series

$f\left(\dfrac{1}{20}\right)+f\left(\dfrac{2}{20}\right)+f\left(\dfrac{3}{20}\right)+......+f\left(\dfrac{39}{20}\right)$ is equal to :

(1) $\dfrac{19}{2}$ (2) $\dfrac{49}{2}$ (3) $\dfrac{29}{2}$ (4) $\dfrac{39}{2}$

**Official Ans. by NTA (4)**

**Sol.** $f(x)=\dfrac{5^x}{5^x+5}$ $\qquad f(2-x)=\dfrac{5}{5^x+5}$

$f(x) + f(2 - x) = 1$

$\Rightarrow f\left(\dfrac{1}{20}\right)+f\left(\dfrac{2}{20}\right)+...+f\left(\dfrac{39}{20}\right)$

$=\left(f\left(\dfrac{1}{20}\right)+f\left(\dfrac{39}{20}\right)\right)+...+\left(f\left(\dfrac{19}{20}\right)+f\left(\dfrac{21}{20}\right)+f\left(\dfrac{20}{20}\right)\right)$

$=19+\dfrac{1}{2}=\dfrac{39}{2}$

**13.** If for the matrix, $A=\begin{bmatrix}1 & -\alpha\\ \alpha & \beta\end{bmatrix}$, $AA^T = I_2$, then the value of $\alpha^4 + \beta^4$ is :

(1) 4 (2) 2 (3) 3 (4) 1

**Official Ans. by NTA (4)**

**Sol.** $A=\begin{bmatrix}1 & -\alpha\\ \alpha & \beta\end{bmatrix}$ $\qquad AA^T = I_2$

$\Rightarrow \begin{bmatrix}1 & -\alpha\\ \alpha & \beta\end{bmatrix}\begin{bmatrix}1 & \alpha\\ -\alpha & \beta\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$\Rightarrow \begin{bmatrix}1+\alpha^2 & \alpha-\alpha\beta\\ \alpha-\alpha\beta & \alpha^2+\beta^2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$\Rightarrow \alpha^2 = 0$ & $\beta^2 = 1$

$\therefore \alpha^4 + \beta^4 = 1$

**14.** The minimum value of $f(x)=a^{a^x}+a^{1-a^x}$, where $a, x \in R$ and $a > 0$, is equal to :

(1) 2a (2) $2\sqrt{a}$

(3) $a+\dfrac{1}{a}$ (4) a + 1

**Official Ans. by NTA (2)**

**Sol.** A.M. $\geq$ G.M.

$f(x)=a^{a^x}+a^{1-a^x}=a^{a^x}+\dfrac{a}{a^{a^x}}\geq 2\sqrt{a}$

**15.** If $I_n=\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\cot^n x\,dx$, then :

(1) $\dfrac{1}{I_2+I_4},\dfrac{1}{I_3+I_5},\dfrac{1}{I_4+I_6}$ are in G.P.

(2) $I_2 + I_4$, $I_3 + I_5$, $I_4 + I_6$ are in A.P.

(3) $I_2 + I_4$, $(I_3 + I_5)^2$, $I_4 + I_6$ are in G.P.

(4) $\dfrac{1}{I_2+I_4},\dfrac{1}{I_3+I_5},\dfrac{1}{I_4+I_6}$ are in A.P.

**Official Ans. by NTA (4)**

**Sol.** $I_n=\displaystyle\int_{\pi/4}^{\pi/2}\cot^n x\,dx=\int_{\pi/4}^{\pi/2}\cot^{n-2}x(\mathrm{cosec}^2x-1)\,dx$

$=-\left.\dfrac{\cot^{n-1}x}{n-1}\right]_{\pi/4}^{\pi/2}-I_{n-2}$

$=\dfrac{1}{n-1}-I_{n-2}$

$\Rightarrow I_n + I_{n-2} = \dfrac{1}{n-1}$

$\Rightarrow I_2 + I_4 = \dfrac{1}{3}$

$I_3 + I_5 = \dfrac{1}{4}$

$I_4 + I_6 = \dfrac{1}{5}$

$\therefore \dfrac{1}{I_2+I_4},\dfrac{1}{I_3+I_5},\dfrac{1}{I_4+I_6}$ are in A.P.

**16.** $\lim\limits_{n\to\infty}\left[\frac{1}{n}+\frac{n}{(n+1)^2}+\frac{n}{(n+2)^2}+........+\frac{n}{(2n-1)^2}\right]$

is equal to :

(1) $\frac{1}{2}$ (2) 1

(3) $\frac{1}{3}$ (4) $\frac{1}{4}$

**Official Ans. by NTA (1)**

**Sol.** $\lim\limits_{n\to\infty}\left[\frac{1}{n}+\frac{n}{(n+1)^2}+\frac{n}{(n+2)^2}+...+\frac{n}{(2n-1)^2}\right]$

$$=\lim_{n\to\infty}\sum_{r=0}^{n-1}\frac{n}{(n+r)^2}=\lim_{n\to\infty}\sum_{r=0}^{n-1}\frac{n}{n^2+2nr+r^2}$$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{r=0}^{n-1}\frac{1}{(r/n)^2+2(r/n)+1}$$

$$=\int_0^1\frac{dx}{(x+1)^2}=\left[\frac{-1}{(x+1)}\right]_0^1=\frac{1}{2}$$

**17.** Let A be a set of all 4-digit natural numbers whose exactly one digit is 7. Then the probability that a randomly chosen element of A leaves remainder 2 when divided by 5 is :

(1) $\frac{2}{9}$ (2) $\frac{122}{297}$

(3) $\frac{97}{297}$ (4) $\frac{1}{5}$

**Official Ans. by NTA (3)**

**Sol.** n(s) = n(when 7 appears on thousands place)
+ n(7 does not appear on thousands place)
$= 9 \times 9 \times 9 + 8 \times 9 \times 9 \times 3$
$= 33 \times 9 \times 9$

n(E) = n(last digit 7 & 7 appears once)
+ n(last digit 2 when 7 appears once)
$= 8 \times 9 \times 9 + (9 \times 9 + 8 \times 9 \times 2)$

$$\therefore\ P(E)=\frac{8\times9\times9+9\times25}{33\times9\times9}=\frac{97}{297}$$

**18.** Let $\alpha$ and $\beta$ be the roots of $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10}-2a_8}{3a_9}$ is :

(1) 2 (2) 1 (3) 4 (4) 3

**Official Ans. by NTA (1)**

**Sol.** $\alpha^2 - 6\alpha - 2 = 0$

$\alpha^{10} - 6\alpha^9 - 2\alpha^8 = 0$

Similarly $\beta^{10} - 6\beta^9 - 2\beta^8 = 0$

$(\alpha^{10} - \beta^{10}) - 6(\alpha^9 - \beta^9) - 2(\alpha^8 - \beta^8) = 0$

$\Rightarrow a_{10} - 6a_9 - 2a_8 = 0$

$\Rightarrow \frac{a_{10}-2a_8}{3a_9}=2$

**19.** Let x denote the total number of one-one functions from a set A with 3 elements to a set B with 5 elements and y denote the total number of one-one functions from the set A to the set A × B. Then :

(1) y = 273x (2) 2y = 91x
(3) y = 91x (4) 2y = 273x

**Official Ans. by NTA (2)**

**Sol.** $x = {}^5C_3 \times 3! = 60$

$y = {}^{15}C_3 \times 3! = 15 \times 14 \times 13 = 30 \times 91$

$\therefore 2y = 91x$

**20.** The following system of linear equations

$2x + 3y + 2z = 9$

$3x + 2y + 2z = 9$

$x - y + 4z = 8$

(1) has a solution $(\alpha, \beta, \gamma)$ satisfying $\alpha + \beta^2 + \gamma^3 = 12$

(2) has infinitely many solutions

(3) does not have any solution

(4) has a unique solution

**Official Ans. by NTA (4)**

**Sol.** $2x + 3y + 2z = 9$ ...(1)

$3x + 2y + 2z = 9$ ...(2)

$x - y + 4z = 8$ ...(3)

(1) – (2) $\Rightarrow -x + y = 0 \Rightarrow x - y = 0$

from (3) $4z = 8 \Rightarrow z = 2$

from (1) $2x + 3y = 5$

$\Rightarrow x = y = 1$

$\therefore$ system has unique solution

## SECTION-B

**1.** The total number of two digit numbers 'n', such that $3^n + 7^n$ is a multiple of 10, is ______.

**Official Ans. by NTA (45)**

**Sol.** for $3^n + 7^n$ to be divisible by 10

n can be any odd number

$\therefore$ Number of odd two digit numbers = 45

**2.** A function f is defined on [–3, 3] as

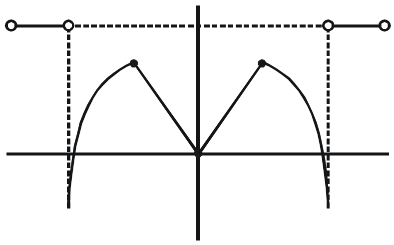
$$f(x)=\begin{cases}\min\{|x|,2-x^2\}, & -2\le x\le 2\\ [|x|] & ,2<|x|\le 3\end{cases}$$

where [x] denotes the greatest integer ≤ x. The number of points, where f is not differentiable in (–3, 3) is ______.

**Official Ans. by NTA (5)**

**Sol.** $$f(x)=\begin{cases}\min\{|x|,\ 2-x^2\} & , \quad -2\le x\le 2\\ [|x|] & , \quad 2<|x|\le 3\end{cases}$$

$\Rightarrow x \in [-3, -2) \cup (2, 3]$



Number of points of non-differentiability in (–3, 3) = 5

**3.** Let $\vec{a}=\hat{i}+\alpha\hat{j}+3\hat{k}$ and $\vec{b}=3\hat{i}-\alpha\hat{j}+\hat{k}$. If the area of the parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$ is $8\sqrt{3}$ square units, then $\vec{a}\cdot\vec{b}$ is equal to ______:

**Official Ans. by NTA (2)**

**Sol.** $\vec{a}=\hat{i}+\alpha\hat{j}+3\hat{k}$

$\vec{b}=3\hat{i}-\alpha\hat{j}+\hat{k}$

area of parallelogram = $|\vec{a}\times\vec{b}|=8\sqrt{3}$.

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & \alpha & 3\\ 3 & -\alpha & 1\end{vmatrix} = \hat{i}(4\alpha)-\hat{j}(-8)+\hat{k}(-4\alpha)$$

$\therefore\ |\vec{a}\times\vec{b}|=\sqrt{64+32\alpha^2}=8\sqrt{3}$

$\Rightarrow 2 + \alpha^2 = 6 \Rightarrow \alpha^2 = 4$

$\therefore\ \vec{a}\cdot\vec{b}=3-\alpha^2+3=2$

**4.** If the remainder when x is divided by 4 is 3, then the remainder when $(2020 + x)^{2022}$ is divided by 8 is _______.

**Official Ans. by NTA (1)**

**Sol.** $x = 4k + 3$

$\therefore\ (2020 + x)^{2022} = (2020 + 4k + 3)^{2022}$

$= (4(505 + k) + 3)^{2022}$

$= (4\lambda + 3)^{2022} = (16\lambda^2 + 24\lambda + 9)^{1011}$

$= (8(2\lambda^2 + 3\lambda + 1) + 1)^{1011}$

$= (8p + 1)^{1011}$

$\therefore$ Remainder when divided by 8 = 1

**5.** If the curves $x = y^4$ and $xy = k$ cut at right angles, then $(4k)^6$ is equal to ______.

**Official Ans. by NTA (4)**

**Sol.** $x = y^4 xy = k$

for intersection $\quad y^5 = k...(1)$

Also $x = y^4$

$\Rightarrow 1=4y^3\dfrac{dy}{dx} \Rightarrow \dfrac{dy}{dx}=\dfrac{1}{4y^3}$

for $xy = k \Rightarrow x=\dfrac{k}{y}$

$\Rightarrow 1=-\dfrac{k}{y^2}.\dfrac{dy}{dx}$

$\Rightarrow \dfrac{dy}{dx}=\dfrac{-y^2}{k}$

$\because$ Curve cut orthogonally

$\Rightarrow \dfrac{1}{4y^3}\times\left(\dfrac{-y^2}{k}\right)=-1$

$\Rightarrow y=\dfrac{1}{4k}$

$\therefore$ from (1) $y^5 = k$

$\Rightarrow \dfrac{1}{(4k)^5}=k$

$\Rightarrow 4 = (4k)^6$

**6.** A line is a common tangent to the circle $(x-3)^2 + y^2 = 9$ and the parabola $y^2 = 4x$. If the two points of contact (a, b) and (c, d) are distinct and lie in the first quadrant, then 2(a + c) is equal to ______.

**Official Ans. by NTA (9)**

**Sol.** Let coordinate of point $A(t^2, 2t)$ $\quad (\because a = 1)$

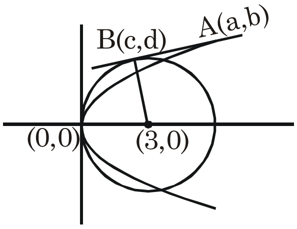
equation of tangent at point A

$yt = x + t^2$

$x - ty + t^2 = 0$

centre of circle (3, 0)

Now PD = radius




$$\left|\frac{3-0+t^2}{\sqrt{1+t^2}}\right| = 3$$

$(3 + t^2)^2 = 9(1 + t^2)$

$9 + t^4 + 6t^2 = 9 + 9t^2$

$t = 0, -\sqrt{3}, \sqrt{3}$

So point $A\left(3, 2\sqrt{3}\right)$

$\Rightarrow a = 3, \ b = 2\sqrt{3}$

(Since it lies in first quadrant)

For point B which is foot of perpendicular from centre (3, 0) to the tangent $x - \sqrt{3}y + 3 = 0$

$$\frac{c-3}{1} = \frac{d-0}{-\sqrt{3}} = \frac{-(3-0+3)}{4}$$

$$\Rightarrow c = \frac{3}{2} \qquad d = \frac{3\sqrt{3}}{2}$$

$$\Rightarrow 2\left(\frac{3}{2}+3\right) = 9$$

**7.** If $\lim\limits_{x\to 0}\dfrac{ax-(e^{4x}-1)}{ax(e^{4x}-1)}$ exists and is equal to b, then the value of a – 2b is ______.

**Official Ans. by NTA (5)**

**Sol.** $$\lim_{x\to 0}\frac{ax-\left(e^{4x}-1\right)}{ax\left(e^{4x}-1\right)} \qquad \left(\frac{0}{0}\right)$$

$$= \lim_{x\to 0}\frac{ax-\left(e^{4x}-1\right)}{ax\,.\,4x} \qquad \boxed{\text{Use } \lim_{x\to 0}\frac{e^{4x}-1}{4x} = 1}$$

Apply L'Hospital Rule

$$= \lim_{x\to 0}\frac{a-4e^{4x}}{8ax} \qquad \left(\frac{a-4}{0}\text{ form}\right)$$

limit exists only when $a - 4 = 0 \Rightarrow a = 4$

$$= \lim_{x\to 0}\frac{4-4e^{4x}}{32x}$$

$$= \lim_{x\to 0}\frac{1-e^{4x}}{8x} \qquad \left(\frac{0}{0}\right)$$

$$= \lim_{x\to 0}\frac{-e^{4x}.4}{8} = -\frac{1}{2} \Rightarrow b = -\frac{1}{2}$$

$$a - 2b = 4 - 2\left(-\frac{1}{2}\right)$$

$= 5$

**8.** If the curve, $y = y(x)$ represented by the solution of the differential equation $(2xy^2 - y)dx + xdy = 0$, passes through the intersection of the lines, $2x - 3y = 1$ and $3x + 2y = 8$, then $|y(1)|$ is equal to ______.

**Official Ans. by NTA (1)**

**Sol.** $(2xy^2 - y)dx + xdy = 0$

$2xy^2\,dx - y\,dx + x\,dy = 0$

$$2x\,dx = \frac{y\,dx - x\,dy}{y^2} = d\left(\frac{x}{y}\right)$$

Now integrate

$$x^2 = \frac{x}{y} + c$$

Now point of intersection of lines are (2, 1)

$$4 = \frac{2}{1} + c \qquad \Rightarrow c = 2$$

$$x^2 = \frac{x}{y} + 2$$

Now $y(1) = -1$

$\Rightarrow |y(1)| = 1$

**9.** The value of $\int_{-2}^{2} |3x^2 - 3x - 6|\,dx$ is ______.

**Official Ans. by NTA (19)**

**Sol.** $\int_{-2}^{2} 3|x^2 - x - 2|\,dx$

$$= 3\int_{-2}^{2} |x^2 - x - 2|\,dx$$

$$= 3\left[\int_{-2}^{-1} (x^2 - x - 2)\,dx + \int_{-1}^{2} -(x^2 - x - 2)\,dx\right]$$

$$= 3\left[\left(\frac{x^3}{3} - \frac{x^2}{2} - 2x\right)\Bigg|_{-2}^{-1} - \left(\frac{x^3}{3} - \frac{x^2}{2} - 2x\right)_{-1}^{2}\right]$$

$$= 3\left[7 - \frac{2}{3}\right]$$

$= 19$

**10.** A line '$l$' passing through origin is perpendicular to the lines

$$l_1 : \vec{r} = (3+t)\hat{i} + (-1+2t)\hat{j} + (4+2t)\hat{k}$$

$$l_2 : \vec{r} = (3+2s)\hat{i} + (3+2s)\hat{j} + (2+s)\hat{k}$$

If the co-ordinates of the point in the first octant on '$l_2$' at a distance of $\sqrt{17}$ from the point of intersection of '$l$' and '$l_1$' are (a, b, c), then 18(a + b + c) is equal to ______.

**Official Ans. by NTA (44)**

**Sol.** $\ell_1 : \vec{r} = (3+t)\hat{i} + (-1+2t)\hat{j} + (4+2t)\hat{k}$

$\ell_2 : \vec{r} = (3+2s)\hat{i} + (3+2s)\hat{j} + (4+s)\hat{k}$

DR of $\ell_1 \equiv (1, 2, 2)$

DR of $\ell_2 \equiv (2, 2, 1)$

DR of $\ell$ (line $\perp$ to $\ell_1$ & $\ell_2$)

$= (-2, 3, -2)$

$\therefore\ \ell :\ \vec{r} = -2\mu\hat{i} + 3\mu\hat{j} - 2\mu\hat{k}$

for intersection of $\ell$ & $\ell_1$

$3 + t = -2\mu$

$-1 + 2t = 3\mu$

$4 + 2t = -2\mu$

$\Rightarrow t = -1$ & $\lambda = -1$

$\therefore$ Point of intersection $P \equiv (2, -3, 2)$

Let point on $\ell_2$ be Q $(3 + 2s, 3 + 2s, 2 + s)$

Given $PQ = \sqrt{17}$ $\qquad \Rightarrow (PQ)^2 = 17$

$\Rightarrow (2s + 1)^2 + (6 + 2s)^2 + (s)^2 = 17$

$\Rightarrow 9s^2 + 28s + 20 = 0$

$$\Rightarrow s = -2,\ -\frac{10}{9}$$

$s \neq -2$ as point lies on $1^{st}$ octant.

$$\therefore\ a = 3 + 2\left(-\frac{10}{9}\right) = \frac{7}{9}$$

$$b = 3 + 2\left(-\frac{10}{9}\right) = \frac{7}{9}$$

$$c = 2 + \left(-\frac{10}{9}\right) = \frac{8}{9}$$

$$\therefore\ 18(a + b + c) = 18\left(\frac{22}{9}\right) = 44$$