JEE MAIN 2021

Held On 24th February, 2021

Evening Session

PHYSICS

SECTION-A

1. When a particle executes SHM, the nature of graphical representation of velocity as a function of displacement is :

(1) circular (2) elliptical

(3) parabolic (4) straight line

Official Ans. by NTA (2)

Sol. For a particle executing SHM,

$x = A\sin(\omega t+\phi)$

$v = \omega A\cos(\omega t+\phi)$

$\Rightarrow \frac{v^2}{\omega^2 A^2}+\frac{x^2}{A^2}=1 \Rightarrow$ equation of ellipse between v and x

Hence option (2)

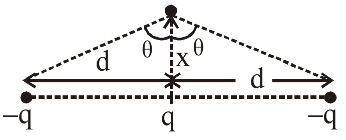
2. Two electrons each are fixed at a distance '2d'. A third charge proton placed at the midpoint is displaced slightly by a distance x (x << d) perpendicular to the line joining the two fixed charges. Proton will execute simple harmonic motion having angular frequency : (m = mass of charged particle)

(1) $\left(\frac{2q^2}{\pi\varepsilon_0 md^3}\right)^{\frac{1}{2}}$ (2) $\left(\frac{\pi\varepsilon_0 md^3}{2q^2}\right)^{\frac{1}{2}}$

(3) $\left(\frac{q^2}{2\pi\varepsilon_0 md^3}\right)^{\frac{1}{2}}$ (4) $\left(\frac{2\pi\varepsilon_0 md^3}{q^2}\right)^{\frac{1}{2}}$

Official Ans. by NTA (3)

Sol. From the given condition, we have



$$F_{netq} = -\left[2F_{q/q}\cos\theta\right]$$

$$F_{netq} = -2.\frac{1}{4\pi\varepsilon_0}.\frac{q^2}{\left(\sqrt{d^2+x^2}\right)^2}.\frac{x}{\sqrt{d^2+x^2}}$$

$$= -\frac{q^2}{2\pi\varepsilon_0}\frac{x}{(d^2+x^2)^{3/2}}$$

For x << d,

$$F_{net\ q} = -\frac{q^2}{2\pi\varepsilon_0 d^3}x$$

$$\therefore\ a = -\frac{q^2}{2\pi\varepsilon_0.md^3}x$$

Comparing with equation of SHM ($a = -\omega^2 x$)

$$\therefore\ \omega = \sqrt{\frac{q^2}{2\pi\varepsilon_0 md^3}}$$

Hence option (3) is correct

3. On the basis of kinetic theory of gases, the gas exerts pressure because its molecules :

(1) continuously lose their energy till it reaches wall.

(2) are attracted by the walls of container.

(3) continuously stick to the walls of container.

(4) suffer change in momentum when impinge on the walls of container.

Official Ans. by NTA (4)

Sol. From the assumption of KTG, the molecules of gas collide with the walls and suffers momentum change which results in force on the wall and hence pressure.

Hence option (4) is correct

4. A soft ferromagnetic material is placed in an external magnetic field. The magnetic domains :

(1) increase in size but no change in orientation.

(2) have no relation with external magnetic field.

(3) decrease in size and changes orientation.

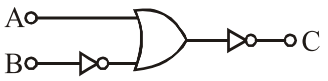
(4) may increase or decrease in size and change its orientation.

Official Ans. by NTA (4)

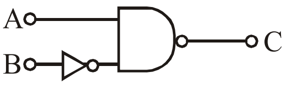
Sol. Soft ferromagnetic materials are materials which can be easily magnetised and demagnetised by external magnetic field. When external field is applied, the domains experiences a net torque hence change its orientation.

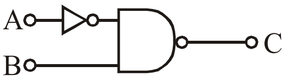
Hence option (4) is correct

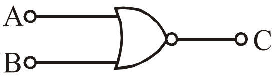
5.

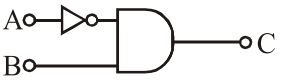


The logic circuit shown above is equivalent to :

(1)


(2)


(3)


(4)


Official Ans. by NTA (4)

Sol. Truth table of the given gate :

A	B	C
0	0	0
0	1	1
1	0	0
1	1	0

Truth table of option (1)

A	B	C
0	0	1
0	1	1
1	0	0
1	1	1

Truth table of option (2)

A	B	C
0	0	1
0	1	0
1	0	1
1	1	1

Truth table of option (3)

A	B	C
0	0	1
0	1	0
1	0	0
1	1	0

Truth table of option (4)

A	B	C
0	0	0
0	1	1
1	0	0
1	1	0

Since option (1) has same truth table, hence answer is option (4) is correct.

Alternative solution :

Given Boolean expression can be written as

$\overline{A+\overline{B}}=C$

$\therefore\ C=\overline{A}\cdot\overline{\overline{B}}=\overline{A}\cdot B$

Hence option (4) is correct

6. The period of oscillation of a simple pendulum is $T=2\pi\sqrt{\frac{L}{g}}$. Measured value of 'L' is 1.0 m from meter scale having a minimum division of 1 mm and time of one complete oscillation is 1.95 s measured from stopwatch of 0.01 s resolution. The percentage error in the determination of 'g' will be :

(1) 1.13% (2) 1.03%

(3) 1.33% (4) 1.30%

Official Ans. by NTA (1)

Sol. $T=2\pi\sqrt{\frac{\ell}{g}}$

$g=\frac{4\pi^2\ell}{T^2}$

$\frac{\Delta g}{g}=\frac{\Delta\ell}{\ell}+\frac{2\Delta T}{T}$

$\frac{\Delta g}{g}=\frac{1\times10^{-3}}{1}+2\times\frac{0.01}{1.95}$

$\frac{\Delta g}{g}=0.0113\ \text{ or }\ 1.13\%$

option (1) is correct

7. Given below are two statements :

Statement I : PN junction diodes can be used to function as transistor, simply by connecting two diodes, back to back, which acts as the base terminal.

Statement II : In the study of transistor, the amplification factor β indicates ratio of the collector current to the base current.

In the light of the above statements, choose the correct answer from the options given below :

(1) Statement I is false but Statement II is true

(2) Both Statement I and Statement II are true

(3) Both Statement I and Statement II are false

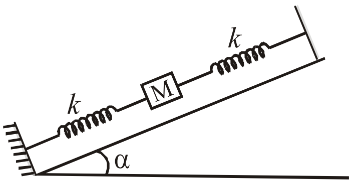
(4) Statement I is true but Statement II is false

Official Ans. by NTA (1)

Sol. Back to back diode will not the make a transistor

$\beta=\frac{i_c}{i_b}$

8. In the given figure, a body of mass M is held between two massless springs, on a smooth inclined plane. The free ends of the springs are attached to firm supports. If each spring has spring constant *k*, the frequency of oscillation of given body is :

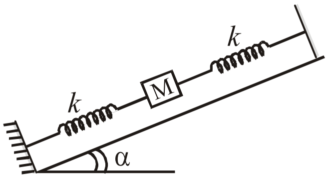


(1) $\frac{1}{2\pi}\sqrt{\frac{k}{2M}}$ (2) $\frac{1}{2\pi}\sqrt{\frac{2k}{Mg\sin\alpha}}$

(3) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ (4) $\frac{1}{2\pi}\sqrt{\frac{k}{Mg\sin\alpha}}$

Official Ans. by NTA (3)

Sol.

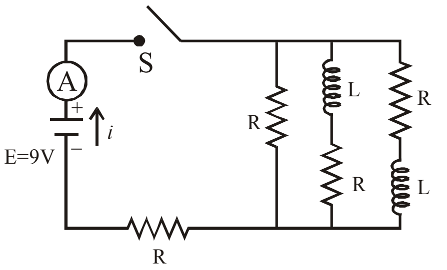


$K_{eq} = K_1 + K_2 = K + K = 2K$

$T = 2\pi\sqrt{\frac{m}{K_{eq}}} = 2\pi\sqrt{\frac{m}{2K}}$

$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{2K}{m}}$ (Option 3) is correct

9. Figure shows a circuit that contains four identical resistors with resistance R = 2.0 Ω, two identical inductors with inductance L = 2.0 mH and an ideal battery with *emf* E = 9 V. The current '*i*' just after the switch 'S' is closed will be :

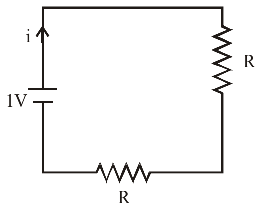


(1) 2.25 A (2) 3.0 A

(3) 3.37 A (4) 9 A

Official Ans. by NTA (1)

Sol. Just after the switch is closed, inductor will behave like infinite resistance (open circuit) so the circuit will look like



$i = \frac{9}{R+R} = \frac{9}{4} = 2.25$

Option (1) is correct.

10. The de Broglie wavelength of a proton and α-particle are equal. The ratio of their velocities is :

(1) 4 : 3 (2) 4 : 1 (3) 4 : 2 (4) 1 : 4

Official Ans. by NTA (2)

Sol. $\lambda = \frac{h}{mv}$

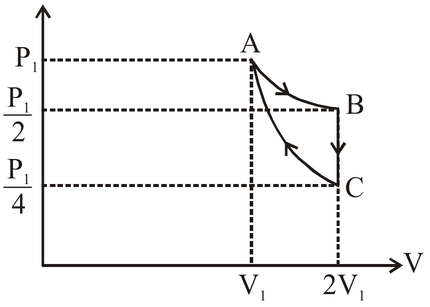
$\lambda_P = \lambda_\alpha$

$m_Pv_P = m_\alpha v_\alpha$

$m_Pv_P = 4m_pv_\alpha$ $(m_\alpha = 4m_P)$

$\frac{v_P}{v_\alpha} = 4$ (Option 2) is correct

11. If one mole of an ideal gas at (P_1, V_1) is allowed to expand reversibly and isothermally (A to B) its pressure is reduced to one-half of the original pressure (see figure). This is followed by a constant volume cooling till its pressure is reduced to one-fourth of the initial value (B→C). Then it is restored to its initial state by a reversible adiabatic compression (C to A). The net workdone by the gas is equal to :



(1) $RT\left(\ln 2 - \frac{1}{2(\gamma-1)}\right)$ (2) $-\frac{RT}{2(\gamma-1)}$

(3) 0 (4) RT ln 2

Official Ans. by NTA (1)

Sol. A – B = isothermal process

$$W_{AB} = P_1V_1 \ln\left[\frac{2V_1}{V_1}\right] = P_1V_1 \ln(2)$$

B – C → Isochoric process

$W_{BC} = 0$

C – A → Adiabatic process

$$W_{CA} = \frac{P_1V_1 - \frac{P_1}{4}\times 2V_1}{1-\gamma} = \frac{P_1V_1\left[1-\frac{1}{2}\right]}{1-\gamma} = \frac{P_1V_1}{2(1-\gamma)}$$

$$W_{net} = W_{AB} + W_{BC} + W_{CA} \qquad \{P_1V_1 = RT\}$$

$$= P_1V_1\ln(2) + 0 + \frac{P_1V_1}{2(1-\gamma)}$$

$$W_{net} = RT\left[\ln(2) - \frac{1}{2(\gamma-1)}\right]$$

Option (1) is correct.

12. An X-ray tube is operated at 1.24 million volt. The shortest wavelength of the produced photon will be :

(1) 10^{-3} nm (2) 10^{-1} nm

(3) 10^{-2} nm (4) 10^{-4} nm

Official Ans. by NTA (1)

Sol. $$\lambda_{min} = \frac{1240}{\Delta V}(nm)$$

$$= \frac{1240}{1.24\times 10^6} = 10^{-3}\,nm$$

Option (1) is correct.

13. Which of the following equations represents a travelling wave ?

(1) y = $A\sin(15x - 2t)$

(2) y = $Ae^{-x^2}(vt+\theta)$

(3) y = $Ae^{x}\cos(\omega t - \theta)$

(4) y = $A\sin x \cos\omega t$

Official Ans. by NTA (1)

Sol. y = F(x, t)

For travelling wave y should be linear function of x and t and they must exist as (x ± vt)

y = $A\sin(15x - 2t)$→linear function in x and t

Option (1) is correct.

14. According to Bohr atom model, in which of the following transitions will the frequency be maximum ?

(1) n = 4 to n = 3 (2) n = 2 to n = 1

(3) n = 5 to n = 4 (4) n = 3 to n = 2

Official Ans. by NTA (2)

Sol. $$\Delta E = 13.6\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] = h\nu$$

It is maximum if $n_1 = 1$ and $n_2 = 2$

n = 5 .... –0.544 eV

n = 4 .... –0.850 eV

n = 3 .... –1.511 eV

n = 2 .... –3.4 eV

n = 1 .... –13.6 eV

Option (2) is correct.

15. If the source of light used in a Young's double slit experiment is changed from red to violet :

(1) consecutive fringe lines will come closer.

(2) the central bright fringe will become a dark fringe.

(3) the fringes will become brighter.

(4) the intensity of minima will increase.

Official Ans. by NTA (1)

Sol. $$\beta = \frac{\lambda.D}{d}$$

$\lambda_R > \lambda_V$

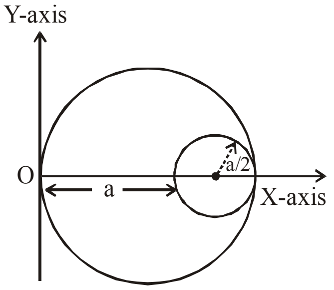
$$\beta_R = \frac{\lambda_R D}{d} \text{ and } \beta_V = \frac{\lambda_V D}{d}$$

$\beta_R > \beta_V$

Fringe pattern will shrink.

Option (1) is correct.

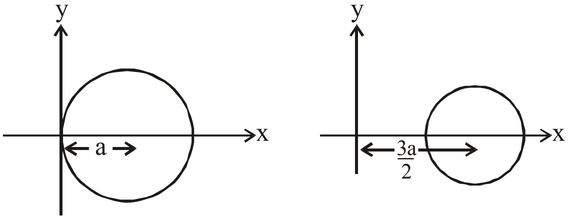
16. A circular hole of radius $\left(\frac{a}{2}\right)$ is cut out of a circular disc of radius 'a' as shown in figure. The centroid of the remaining circular portion with respect to point 'O' will be :



(1) $\frac{1}{6}a$ (2) $\frac{10}{11}a$ (3) $\frac{5}{6}a$ (4) $\frac{2}{3}a$

Official Ans. by NTA (3)

Sol.



Let σ be the uniform mass density of disc then

$$x_{COM} = \frac{(\sigma\pi a^2)a - \sigma\pi\left(\frac{a^2}{4}\right)\times\frac{3a}{2}}{\sigma\pi a^2 - \frac{\sigma\pi a^2}{4}}$$

$$= \frac{a - \frac{3a}{8}}{1 - \frac{1}{4}} = \frac{5a}{6}$$

Option (2) is correct.

17. Zener breakdown occurs in a *p-n* junction having p and n both :

(1) lightly doped and have wide depletion layer.

(2) heavily doped and have narrow depletion layer.

(3) lightly doped and have narrow depletion layer.

(4) heavily doped and have wide depletion layer.

Official Ans. by NTA (2)

Sol. Zener diode is heavily doped and have narrow depletion layer.

Option (2) is correct.

18. Match List - I with List - II.

List - I	**List - II**
(a) Source of microwave frequency	(i) Radioactive decay on nucleus
(b) Source of infrared frequency	(ii) Magnetron
(c) Source of Gamma Rays	(iii) Inner shell electrons
(d) Source of X-rays	(iv) Vibration of atoms and molecules
	(v) LASER
	(vi) RC circuit

Choose the correct answer from the options given below :

(1) (a)-(vi), (b)-(iv), (c)-(i), (d)-(v)

(2) (a)-(vi), (b)-(v), (c)-(i), (d)-(iv)

(3) (a)-(ii), (b)-(iv), (c)-(vi), (d)-(iii)

(4) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

Official Ans. by NTA (4)

Sol. (a) Source of microwave frequency is magnetron.

(b) Source of infrared frequency is vibration of atoms and molecules.

(c) Source of Gamma rays is radioactive decay of nucleus

(d) Source of X-rays inner shell electron transition.

Option (4) is correct.

19. A particle is projected with velocity v_0 along *x*-axis. A damping force is acting on the particle which is proportional to the square of the distance from the origin i.e., $ma = -\alpha x^2$.. The distance at which the particle stops :

(1) $\left(\frac{3v_0^2}{2\alpha}\right)^{\frac{1}{2}}$　(2) $\left(\frac{2v_0}{3\alpha}\right)^{\frac{1}{3}}$

(3) $\left(\frac{2v_0^2}{3\alpha}\right)^{\frac{1}{2}}$　(4) $\left(\frac{3v_0^2}{2\alpha}\right)^{\frac{1}{3}}$

Official Ans. by NTA (4)

Sol. $F = -\alpha x^2$

$ma = -\alpha x^2$

$$a = \frac{-\alpha x^2}{m}$$

$$\frac{vdv}{dx} = -\frac{\alpha}{m}x^2$$

$$\int_{v_0}^{0} v\,dv = \int_{0}^{x} -\frac{\alpha}{m}x^2 dx$$

$$\left(\frac{v^2}{2}\right)_{v_0}^{0} = -\frac{\alpha}{m}\left(\frac{x^3}{3}\right)_0^x$$

$$\frac{-v_0^2}{2} = -\frac{\alpha}{m}\frac{x^3}{3}$$

$$\boxed{x = \left(\frac{3mv_0^2}{2\alpha}\right)^{\frac{1}{3}}}$$

Option(4) is most suitable option as (m) is not given in any option

20. A body weighs 49 N on a spring balance at the north pole. What will be its weight recorded on the same weighing machine, if it is shifted to the equator ?

(Use $g = \frac{GM}{R^2} = 9.8\ ms^{-2}$ and radius of earth, R = 6400 km.]

(1) 49 N
(2) 48.83 N
(3) 49.83 N
(4) 49.17 N

Official Ans. by NTA (2)

Sol. Weight of pole = mg = 49 N

At equator due to rotation = $g_e = g - R\omega^2$

so $W = mg_e = m(g - R\omega^{2)}$

$\therefore W_P > W_e$ $W_P = 49$ N

So, $W_e = 48.83$ N. $W_e < 49$ N

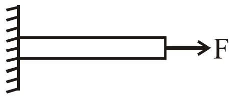
Option (2) is correct.

SECTION-B

1. A uniform metallic wire is elongated by 0.04 m when subjected to a linear force F. The elongation, if its length and diameter is doubled and subjected to the same force will be _____ cm.

Official Ans. by NTA (2)

Sol.



$$F = Y.A.\frac{\Delta\ell}{\ell}$$

$$\Delta\ell = \frac{F}{Y.A.}.\ell$$

$$\Delta\ell = \frac{F.\ell}{Y.\pi r^2}$$

$$\Delta\ell \propto \frac{\ell}{r^2}$$

$$\frac{\Delta\ell_2}{\Delta\ell_1} = \left(\frac{\ell_2}{\ell_1}\right)\left(\frac{r_1}{r_2}\right)^2$$

$$= (2)\left(\frac{1}{2}\right)^2$$

$$\frac{\Delta\ell_2}{\Delta\ell_1} = \frac{1}{2}$$

$$\Delta\ell_2 = \frac{\Delta\ell_1}{2}$$

$$= \frac{0.04}{2}$$

$$= 0.02 \text{ m}$$

$$\boxed{\Delta\ell_2 = 2\text{cm}}$$

$$\boxed{\text{Ans.} = 2}$$

2. A cylindrical wire of radius 0.5 mm and conductivity 5×10^7 S/m is subjected to an electric field of 10 mV/m. The expected value of current in the wire will be $x^3\pi$ mA. The value of x is ____.

Official Ans. by NTA (5)

Sol. Conductivity $\sigma = 5 \times 10^7$ S/m

Radius $r = 0.5$ mm $= 5 \times 10^{-4}$ m

$$E = 10 \times 10^{-3}\frac{V}{m}$$

$$J = \sigma E = 10 \times 10^{-3} \times 5 \times 10^7$$

$$J = 5 \times 10^5$$

$$\frac{i}{A} = 5 \times 10^5$$

$$i = 5 \times 10^5 \times \pi r^2$$

$$= 5 \times 10^5 \times \pi \times (5 \times 10^{-4})^2$$

$$= 125\pi \times 10^{-3} \text{ Amp}$$

$$i = 125\ \pi \text{ mA}$$

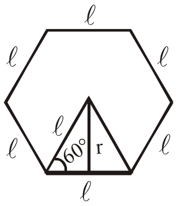
$$\boxed{x = 5}$$

$$\boxed{\text{Ans. } 5}$$

3. A uniform thin bar of mass 6 kg and length 2.4 meter is bent to make an equilateral hexagon. The moment of inertia about an axis passing through the centre of mass and perpendicular to the plane of hexagon is ____ $\times 10^{-1}$ kg m².

Official Ans. by NTA (8)

Sol.



m = mass of one
side of hexagon
= 1 kg

$6l = 2.4$ $\boxed{\ell = 0.4\,\text{m}}$

$\sin 60° = \frac{r}{\ell}$

$r = l\sin 60° = \frac{\ell\sqrt{3}}{2}$

MOI, $I = \left[\frac{m\ell^2}{12} + mr^2\right]6$

$$= \left[\frac{m\ell^2}{12} + m\left(\frac{\ell\sqrt{3}}{2}\right)^2\right]6$$

$= 5\ ml^2$

$= 5 \times 1 \times 0.16$

$= 0.8$

$I = 8 \times 10^{-1}$ kg m^2

$\boxed{\text{Ans. } 8}$

4. Two solids A and B of mass 1 kg and 2 kg respectively are moving with equal linear momentum. The ratio of their kinetic energies $(K.E.)_A : (K.E.)_B$ will be $\frac{A}{1}$, so the value of A will be ___.

Official Ans. by NTA (2)

Sol. Kinetic energy $K = \frac{P^2}{2m}$, $(P_A = P_B)$

$$K \propto \frac{1}{m}$$

$$\frac{K_A}{K_B} = \frac{m_B}{m_A}$$

$$= \frac{2}{1}$$

Ans. (2)

5. The root mean square speed of molecules of a given mass of a gas at 27°C and 1 atmosphere pressure is 200 ms^{-1}. The root mean square speed of molecules of the gas at 127°C and 2 atmosphere pressure is $\frac{x}{\sqrt{3}}\text{ms}^{-1}$. The value of x will be _____.

Official Ans. by NTA (400)

Sol. $v_{rms} = \sqrt{\frac{3RT}{M}}$

$v_{rms} \propto \sqrt{T}$

$\frac{(v_{rms})_2}{(v_{rms})_1} = \sqrt{\frac{T_2}{T_1}}$

$= \sqrt{\frac{400}{300}}$

$= \frac{2}{\sqrt{3}}$

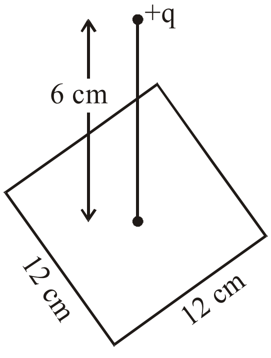
$(v_{rms})_2 = \frac{2}{\sqrt{3}}(v_{rms})_1$

$= \frac{2}{\sqrt{3}} \times 200$

$(v_{rms})_2 = \frac{400}{\sqrt{3}}$ m / s

$\boxed{\text{Ans. } 400}$

6. A point charge of +12 μC is at a distance 6 cm vertically above the centre of a square of side 12 cm as shown in figure. The magnitude of the electric flux through the square will be ______ × 10^3 Nm^2/C.



Official Ans. by NTA (226)

Sol. From symmetry $\phi = \frac{1}{6}\left(\frac{q}{\varepsilon_0}\right)$

$$= \frac{12\times10^{-6}}{6\times8.85\times10^{-12}}$$

$$= 225.98 \times 10^3 \ \frac{\text{Nm}^2}{\text{s}}$$

$$\simeq 226\times10^3 \frac{\text{Nm}^2}{\text{C}}$$

7. A signal of 0.1 kW is transmitted in a cable. The attenuation of cable is –5 dB per km and cable length is 20 km. The power received at receiver is 10^{-x} W. The value of x is ______.

[Gain in dB = 10 $\log_{10}\left(\frac{P_0}{P_i}\right)$]

Official Ans. by NTA (8)

Sol. Sound level decreases by 5dB every km so sound level decreased in 20 km = 100 dB

$$\beta_2 - \beta_1 = 10\log_{10}\frac{I_2}{I_1}$$

$$-100 = 10\log_{10}\frac{I_2}{I_1} \Rightarrow \frac{I_1}{I_2} = 10^{10}$$

$I_2 = 10^{-10}\ I_1 \quad \Rightarrow \quad P_2 = 10^{-10}\ P_1 = 10^{-8}$ W

x = 8 $\boxed{\text{Ans. 8}}$

8. A series LCR circuit is designed to resonate at an angular frequency $\omega_0 = 10^5$ rad/s. The circuit draws 16 W power from 120 V source at resonance. The value of resistance 'R' in the circuit is ___Ω.

Official Ans. by NTA (900)

Sol. At resonance

$$P = \frac{V^2}{R}$$

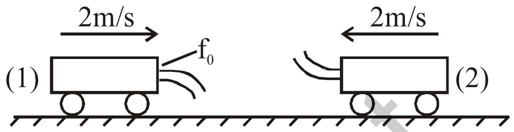
$$R = \frac{V^2}{P} = \frac{(120)^2}{16}$$

$$= 900\Omega$$

9. Two cars are approaching each other at an equal speed of 7.2 km/hr. When they see each other, both blow horns having frequency of 676 Hz. The beat frequency heard by each driver will be _____ Hz. [Velocity of sound in air is 340 m/s.]

Official Ans. by NTA (8)

Sol.



Frequency of sound heard by car-1, which comes by reflection from car-2

$$f_1 = f_0 \left(\frac{340+2}{340-2}\right)\left(\frac{340+2}{340-2}\right)$$

$$= f_0\left(\frac{342}{338}\right)^2$$

Frequency of sound coming directly from car-2

$$f_2 = f_0 \left(\frac{340+2}{340-2}\right)$$

$$\therefore f_1 - f_2 = f_0\left(\frac{342}{338}\right)\left(\frac{342}{338}-1\right) = 8.09 \simeq 8$$

10. An electromagnetic wave of frequency 3 GHz enters a dielectric medium of relative electric permittivity 2.25 from vacuum. The wavelength of this wave in that medium will be ____ × 10^{-2} cm.

Official Ans. by NTA (667)

Sol. λ in vacuum = $\frac{c}{f} = \frac{3\times10^8}{3\times10^9} = 0.1\text{m}$

$\therefore$ λ in medium = $\frac{0.1}{\mu}$

Where refractive index

$$\mu = \sqrt{\mu_r\varepsilon_r}$$

Assuming non-magnetic material μ_r = 1

$$\therefore \quad \mu = \sqrt{2.25} = 1.5$$

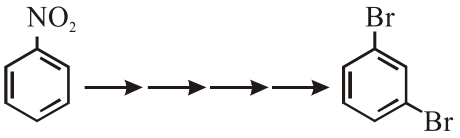
$$\lambda_m = \frac{0.1}{1.5} = \frac{1}{15}\text{m} = 6.67\text{cm}$$

$= 667 \times 10^{-2}$ cm

$\boxed{\text{Ans. 667}}$

SECTION-A

1. What is the correct sequence of reagents used for converting nitrobenzene into *m*-dibromobenzene ?



(1) $\xrightarrow{NaNO_2} / \xrightarrow{HCl} / \xrightarrow{KBr} / \xrightarrow{H^+}$

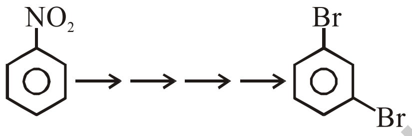
(2) $\xrightarrow{Br_2/Fe} / \xrightarrow{Sn/HCl} / \xrightarrow{NaNO_2/HCl} / \xrightarrow{CuBr/HBr}$

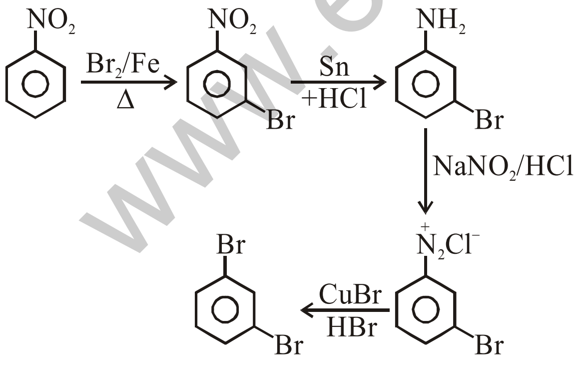
(3) $\xrightarrow{Sn/HCl} / \xrightarrow{KBr} / \xrightarrow{Br_2} / \xrightarrow{H^+}$

(4) $\xrightarrow{Sn/HCl} / \xrightarrow{Br_2} / \xrightarrow{NaNO_2} / \xrightarrow{NaBr}$

Official Ans. by NTA (2)

Sol. Correct sequence of reagents for the following conversion.





2. Most suitable salt which can be used for efficient clotting of blood will be :-

(1) $NaHCO_3$ (2) $FeSO_4$

(3) $Mg(HCO_3)_2$ (4) $FeCl_3$

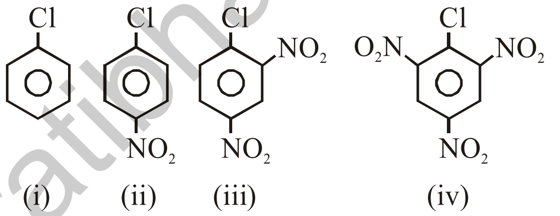
Official Ans. by NTA (4)

Sol. Blood : negatively charged sol

According to Hardly-schulz rule, for the negatively charged sol, most (+) ve ion is needed for its efficient coagulation.

Ans. : $FeCl_3$

3. The correct order of the following compounds showing increasing tendency towards nucleophilic substitution reaction is :-



(1) (iv) < (iii) < (ii) < (i)

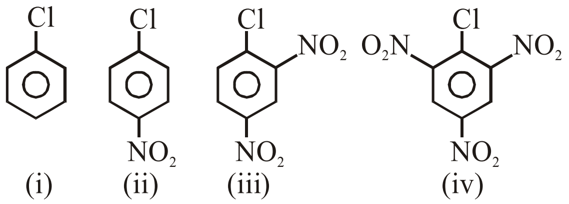
(2) (iv) < (i) < (ii) < (iii)

(3) (iv) < (i) < (iii) < (ii)

(4) (i) < (ii) < (iii) < (iv)

Official Ans. by NTA (4)

Sol. For nucleophile substitution in aromatic halides



Correct order is :

(i) < (ii) < (iii) < (iv)

More No. of NO_2 substituted aromatic halide, increase the rate of nucleophile substitution reaction in aromatic halides.

4. According to Bohr's atomic theory :-

(A) Kinetic energy of electron is $\propto \frac{Z^2}{n^2}$.

(B) The product of velocity (v) of electron and principal quantum number (n), 'vn' $\propto Z^2$.

(C) Frequency of revolution of electron in an orbit is $\propto \frac{Z^3}{n^3}$.

(D) Coulombic force of attraction on the electron is $\propto \frac{Z^3}{n^4}$.

Choose the most appropriate answer from the options given below :

(1) (C) Only

(2) (A) Only

(3) (A), (C) and (D) only

(4) (A) and (D) only

Official Ans. by NTA (3)

Official Ans. by ALLEN (4)

Sol. According to Bohr's theory :

(A) $KE = 13.6\frac{z^2}{n^2}\frac{eV}{atom} \Rightarrow KE\alpha\frac{z^2}{n^2}$

(B) speed of e^- $\alpha\frac{z}{n}$

$\therefore \boxed{v \times n \alpha z}$

(C) Frequency of revolution of $e^- = \frac{v}{2\pi r}$

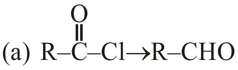
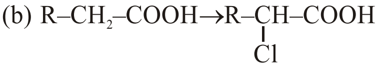
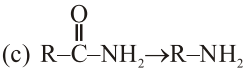
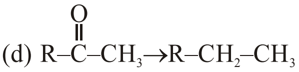
$\therefore \boxed{\text{frequency}\,\alpha\frac{z^2}{n^3}}$

(D) $F = \frac{kq_1q_2}{r^2} = \frac{kze^2}{r^2}$ $\left\{r\alpha\frac{n^2}{z}\right.$

$\Rightarrow F\alpha\frac{z}{\left(\frac{n^2}{z}\right)^2}$

$\Rightarrow \boxed{F\alpha\frac{z^3}{n^4}}$

5. Match list - I and List - II.

List-I	**List-II**
(a)  $R-\overset{O}{\overset{\|}{C}}-Cl \rightarrow R-CHO$	(i) $Br_2/NaOH$
(b)  $R-CH_2-COOH \rightarrow R-\underset{Cl}{\underset{\mid}{C}}H-COOH$	(ii) $H_2/Pd-BaSO_4$
(c)  $R-\overset{O}{\overset{\|}{C}}-NH_2 \rightarrow R-NH_2$	(iii) Zn(Hg)/Conc.HCl
(d)  $R-\overset{O}{\overset{\|}{C}}-CH_3 \rightarrow R-CH_2-CH_3$	(iv) Cl_2/Red P, H_2O

Choose the correct answer from the options given below :

(1) (a)–(ii), (b)–(i), (c)–(iv), (d)–(iii)

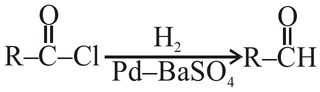
(2) (a)–(iii), (b)–(iv), (c)–(i), (d)–(ii)

(3) (a)–(ii), (b)–(iv), (c)–(i), (d)–(iii)

(4) (a)–(iii), (b)–(i), (c)–(iv), (d)–(ii)

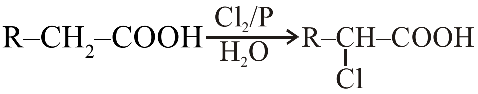
Official Ans. by NTA (3)

Sol. Match list-I & list-II

(a) 

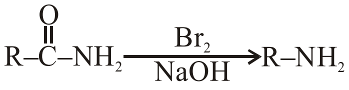
$R-\overset{O}{\overset{\|}{C}}-Cl \xrightarrow[Pd-BaSO_4]{H_2} R-\overset{O}{\overset{\|}{C}}H$ (a) – (ii)

Rosenmund Reduction

(b) 

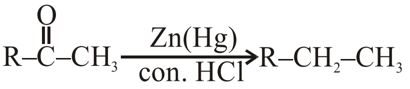
$R-CH_2-COOH \xrightarrow[H_2O]{Cl_2/P} R-\underset{Cl}{\underset{\mid}{C}}H-COOH$

HVZ reaction (b)-(iv)

(c) 

$R-\overset{O}{\overset{\|}{C}}-NH_2 \xrightarrow[NaOH]{Br_2} R-NH_2$ (c) - (i)

Hoffmann Bromamide reaction

(c) 

$R-\overset{O}{\overset{\|}{C}}-CH_3 \xrightarrow[con.\ HCl]{Zn(Hg)} R-CH_2-CH_3$ (d) - (iii)

Clemmenson reduction

6. The calculated magnetic moments (spin only value) for species $[FeCl_4]^{2-}$, $[Co(C_2O_4)_3]^{3-}$ and MnO_4^{2-} respectively are :

(1) 5.82, 0 and 0 BM

(2) 4.90, 0 and 1.73 BM

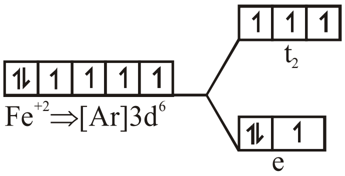
(3) 5.92, 4.90 and 0 BM

(4) 4.90, 0 and 2.83 BM

Official Ans. by NTA (2)

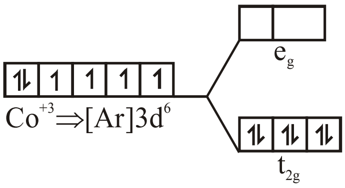
Sol.

(i) $[FeCl_4]^{-2} \Rightarrow$



$$\mu = \sqrt{n(n+2)}\ BM$$
$$= \sqrt{4(4+2)}\ BM$$
$$= \sqrt{24}\ BM \Rightarrow 4.90\ BM$$

(ii) $[Co(C_2O_4)_3]^{-3}$



$\mu = 0$

(iii) MnO_4^{-2}

$Mn^{+6} \Rightarrow [Ar]3d^1$ $\quad \mu = \sqrt{n(n+2)}\ BM$
$$= \sqrt{1(1+2)}\ BM$$
$$= \sqrt{3}\ BM \Rightarrow 1.73\ BM$$

7. Match List-I with List-II :

	List-I (Salt)		**List-II (Flame colour wavelength)**
(a)	LiCl	(i)	455.5 nm
(b)	NaCl	(ii)	670.8 nm
(c)	RbCl	(iii)	780.0 nm
(d)	CsCl	(iv)	589.2 nm

Choose the correct answer from the options given below :

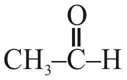
(1) (a)–(iv), (b)–(ii), (c)–(iii), (d)–(i)
(2) (a)–(ii), (b)–(i), (c)–(iv), (d)–(iii)
(3) (a)–(i), (b)–(iv), (c)–(ii), (d)–(iii)
(4) (a)–(ii), (b)–(iv), (c)–(iii), (d)–(i)

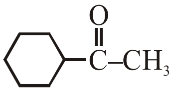
Official Ans. by NTA (4)

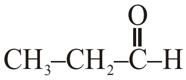
Sol.

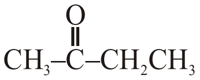
	Colour	**λ/nm**
Li	Crimson red	670.8
Na	Yellow	589.2
Rb	Red violet	780.0
Cs	Blue	455.5

8. Which one of the following carbonyl compounds cannot be prepared by addition of water on an alkyne in the presence of $HgSO_4$ and H_2SO_4 ?

(1) 
$CH_3-\overset{\overset{O}{\|}}{C}-H$

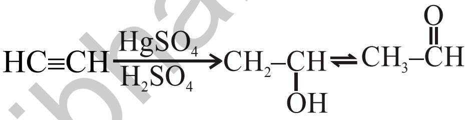
(2) 

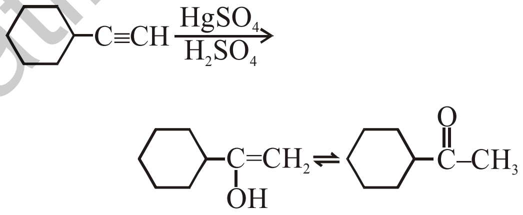
(3) 
$CH_3-CH_2-\overset{\overset{O}{\|}}{C}-H$

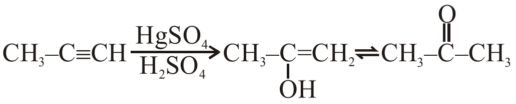
(4) 
$CH_3-\overset{\overset{O}{\|}}{C}-CH_2CH_3$

Official Ans. by NTA (3)

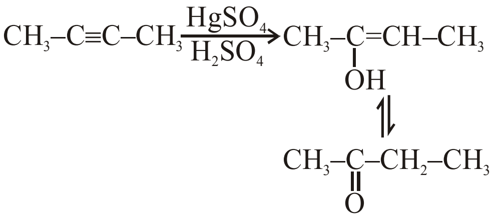
Sol. Reaction of $HgSO_4$/dil.H_2SO_4 with alkyne gives addition of water as per markonikoff's rule.

(1) 
$HC\equiv CH \xrightarrow[H_2SO_4]{HgSO_4} CH_2-\underset{\underset{OH}{|}}{CH} \rightleftharpoons CH_3-\overset{\overset{O}{\|}}{C}H$

(2) 

(3) 
$CH_3-C\equiv CH \xrightarrow[H_2SO_4]{HgSO_4} CH_3-\underset{\underset{OH}{|}}{C}=CH_2 \rightleftharpoons CH_3-\overset{\overset{O}{\|}}{C}-CH_3$

Hence CH_3-CH_2-CHO cannot be form.

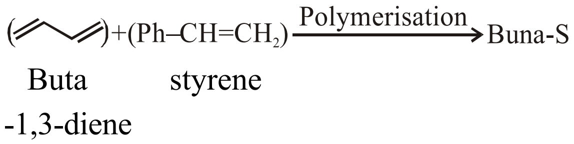
(4) 
$CH_3-C\equiv C-CH_3 \xrightarrow[H_2SO_4]{HgSO_4} CH_3-\underset{\underset{OH}{|}}{C}=CH-CH_3 \rightleftharpoons CH_3-\underset{\underset{O}{\|}}{C}-CH_2-CH_3$

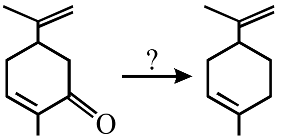
9. In polymer Buna-S: 'S' stands for :-

(1) Sulphonation (2) Strength
(3) Sulphur (4) Styrene

Official Ans. by NTA (4)

Sol. BUN-S, 'S' stand for styrene.



10. 

Which of the following reagent is suitable for the preparation of the product in the above reaction ?

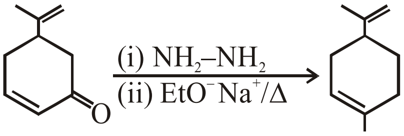
(1) $NaBH_4$

(2) $NH_2–NH_2/ C_2H_5\overset{\ominus}{O}\overset{\oplus}{Na}$

(3) Ni/H_2

(4) Red P + Cl_2

Official Ans. by NTA (2)

Sol. 

To reduce the carbonyl groups into alkane wolf – kischner reduction is used, without affecting the double bond.

11. Match List-I and List-II.

	List-I		**List-II**
(a)	Valium	(i)	Antifertility drug
(b)	Morphine	(ii)	Pernicious anaemia
(c)	Norethindrone	(iii)	Analgesic
(d)	Vitamin B_{12}	(iv)	Tranquilizer

(1) (a)–(iv), (b)–(iii), (c)–(ii), (d)–(i)
(2) (a)–(iv), (b)–(iii), (c)–(i), (d)–(ii)
(3) (a)–(ii), (b)–(iv), (c)–(iii), (d)–(i)
(4) (a)–(i), (b)–(iii), (c)–(iv), (d)–(ii)

Official Ans. by NTA (2)

Sol. (a) Valium – Tranquilizer (a)-(iv)
(b) Morphine – Analgesic (b)-(iii)
(c) Norethindrone – Antifertility Drug (c)-(i)
(d) Vitamin B_{12} – Pernicious anaemia (d)-(ii)

12. Match List-I with List-II.

	List-I (Metal)		**List-II (Ores)**
(a)	Aluminium	(i)	Siderite
(b)	Iron	(ii)	Calamine
(c)	Copper	(iii)	Kaolinite
(d)	Zinc	(iv)	Malachite

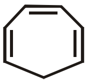
Choose the correct answer from the options given below :

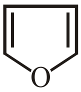
(1) (a)–(iv), (b)–(iii), (c)–(ii), (d)–(i)
(2) (a)–(ii), (b)–(iv), (c)–(i), (d)–(iii)
(3) (a)–(i), (b)–(ii), (c)–(iii), (d)–(iv)
(4) (a)–(iii), (b)–(i), (c)–(iv), (d)–(ii)

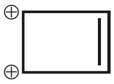
Official Ans. by NTA (4)

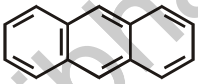
Sol. Siderite – $FeCO_3$
Calamine – $ZnCO_3$
Kaolinite – $Al_2(OH)_4.Si_2O_5$
Malachite – $Cu(OH)_2.CuCO_3$

13. Which one of the following compounds is non-aromatic ?

(1) 

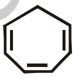
(2) 

(3) 

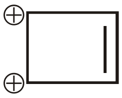
(4) 

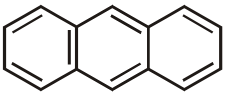
Official Ans. by NTA (1)

Sol. For the following ion/compounds

(1)  – Non aromatic

(2)  – Aromatic

(3)  – Aromatic

(4)  – Aromatic

14. What is the correct order of the following elements with respect to their density ?

(1) Cr < Zn < Co < Cu < Fe
(2) Zn < Cu < Co < Fe < Cr
(3) Zn < Cr < Fe < Co < Cu
(4) Cr < Fe < Co < Cu < Zn

Official Ans. by NTA (3)

Sol. Zn < Cr < Fe < Co < Cu

	Zn	Cr	Fe	Co	Cu
Density/g cm^{-3}	7.1	7.19	7.8	8.7	8.9

15. Given below are two statements :-

Statement I : The value of the parameter "Biochemical Oxygen Demand (BOD)" is important for survival of aquatic life.

Statement II : The optimum value of BOD is 6.5 ppm.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Statement I is false but Statement II is true
(2) Both Statement I and Statement II are true
(3) Statement I is true but Statement II is false
(4) Both Statement I and Statement II are false

Official Ans. by NTA (3)

Sol. Clean water would have BOD value of less than 5 ppm whereas highly polluted water could have a BOD value of 17 ppm or more.

16. The **incorrect** statement among the following is :-

(1) $VOSO_4$ is a reducing agent
(2) Cr_2O_3 is an amphoteric oxide
(3) RuO_4 is an oxidizing agent
(4) Red colour of ruby is due to the presence of Co^{3+}

Official Ans. by NTA (4)

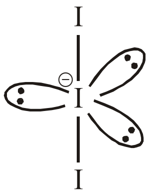
Sol.

(i) In $VOSO_4$, 'V' is in +4 oxidation state. So it act as oxidising agent.
(ii) Cr_2O_3 is an amphoteric oxide.
(iii) In RuO_4, 'Ru' is in +8 oxidation state. So it act as oxidising agent.
(iv) Red colour of ruby is due to the presence of Cr^{+3} ions in Al_2O_3.

17. The correct shape and I–I–I bond angles respectively in I_3^- ion are :-

(1) Distorted trigonal planar; 135° and 90°
(2) T-shaped; 180° and 90°
(3) Trigonal planar; 120°
(4) Linear; 180°

Official Ans. by NTA (4)

Sol. 

Shape : Linear, I–I–I Bond angle ⇒ 180°

18. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : Hydrogen is the most abundant element in the Universe, but it is not the most abundant gas in the troposphere.

Reason R : Hydrogen is the lightest element.

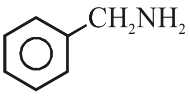
In the light of the above statements, choose the correct answer from the options given below :

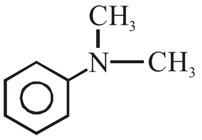
(1) **A** is true but **R** is false
(2) Both **A** and **R** are true and **R** is the correct explanation of **A**
(3) **A** is false but **R** is true
(4) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

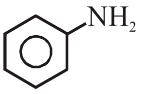
Official Ans. by NTA (2)

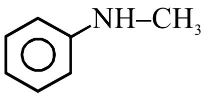
Sol. Most abundant gas in the troposphere is nitrogen.

19. The diazonium salt of which of the following compounds will form a coloured dye on reaction with β-Naphthol in NaOH ?

(1) 

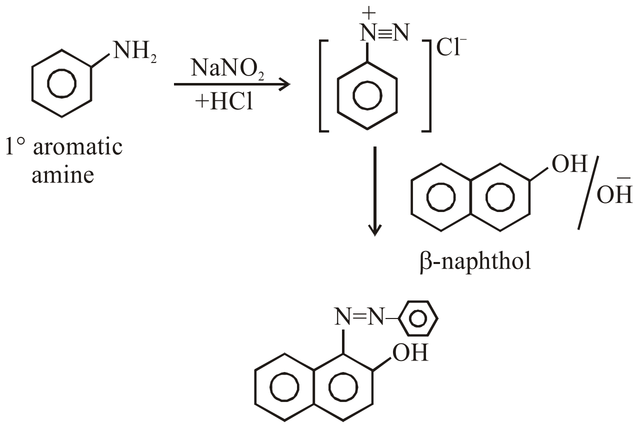
(2) 

(3) 

(4) 

Official Ans. by NTA (3)

Sol.



Orange-Red. dye

20. The correct set from the following in which both pairs are in correct order of melting point is :-

(1) LiF > LiCl ; MgO > NaCl

(2) LiCl > LiF ; NaCl > MgO

(3) LiF > LiCl ; NaCl > MgO

(4) LiCl > LiF ; MgO > NaCl

Official Ans. by NTA (1)

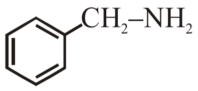
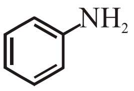
Sol. L.E. $\propto$ M.P.

L.E. : LiF > LiCl, MgO > NaCl

SECTION-B

1. The total number of amines among the following which can be synthesized by Gabriel synthesis is _______.

(A) 
$(CH_3)_2CH{-}CH_2{-}NH_2$ (B) $CH_3CH_2NH_2$

(C) 
(D) 

Official Ans. by NTA (3)

Sol. Gabriel phthalimide synthesis is used to prepare 1° aliphatic/alicyclic amine in common.

Hence amine which can synthesised by Gabriel phthalimide synthesis method is :

(A) $Me_2CH{-}CH_2{-}NH_2$ (B) $CH_3CH_2NH_2$

(C) $Ph{-}CH_2{-}NH_2$

2. Among the following allotropic forms of sulphur, the number of allotropic forms, which will show paramagnetism is ________.

(A) α-sulphur (B) β-sulphur

(C) S_2-form

Official Ans. by NTA (1)

Sol. α-sulphur and β-sulphur are diamagnetic.

S_2-form is paramagnetic.

3. The formula of a gaseous hydrocarbon which requires 6 times of its own volume of O_2 for complete oxidation and produces 4 times its own volume of CO_2 is C_xH_y. The value of y is _______.

Official Ans. by NTA (8)

Sol. Combustion rx^n :

$$C_xH_{y(g)} + \left(x+\frac{y}{4}\right)O_2(g) \rightarrow xCO_2(g) + \frac{y}{2}H_2O(\ell)$$

V 6V –

– – Vx = 4V

$\Rightarrow \boxed{x=4}$

Sinc : (I) $Vo_2 = 6\times V_{C_xH_y}$

$$\Rightarrow V\left(x+\frac{y}{4}\right)=6V$$

$$\Rightarrow \boxed{\left(x+\frac{y}{4}\right)=6} \Rightarrow 4+\frac{y}{4}=6$$

$$\Rightarrow \boxed{y=8}$$

4. The volume occupied by 4.75 g of acetylene gas at 50°C and 740 mmHg pressure is _____ L. (Rounded off to the nearest integer)

[Given R = 0.0826 L atm K^{-1} mol^{-1}]

Official Ans. by NTA (5)

Sol. Given Mass = 4.75 g $\Rightarrow C_2H_2(g)$

$$\Rightarrow \text{Moles} = \frac{4.75}{26} \text{ mol}$$

Temp = 50 + 273 = 323 K

$$P = \frac{740}{760}\text{atm}$$

$$R = 0.0826\ \frac{\ell\,\text{atm}}{\text{mol K}}$$

$$\Rightarrow V = \frac{nRT}{P} = \frac{4.75}{26} \times \frac{0.0826\times 323}{\left(\frac{740}{760}\right)}$$

$$\Rightarrow V = \frac{96314.078}{19240} = 5.0059\,\ell \simeq 5\ell$$

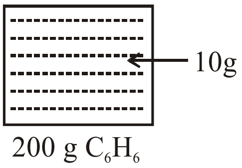
5. C_6H_6 freezes at 5.5˚C. The temperature at which a solution 10 g of C_4H_{10} in 200 g of C_6H_6 freeze is _______ ˚C. (The molal freezing point depression constant of C_6H_6 is 5.12˚C/m.)

Official Ans. by NTA (1)

Sol. Pure Solvent : $C_6H_6\ (\ell)$

Given : $T_f^{\circ} = 5.5°C$

$K_f = 5.12\,°C/m$



: Solute is non dissociative

$\because\ \Delta T_f = k_f \times m$

$$\Rightarrow \left(T_f^0 - T_f^{'}\right) = 5.12 \times \frac{\left(\frac{10}{58}\right)\text{mol}}{\left(\frac{200}{1000}\right)\text{kg}}$$

$$\Rightarrow 5.5 - T_f^{'} = \frac{5.12 \times 5 \times 10}{58}$$

$$\Rightarrow \boxed{T_f^{'} = 1.086\,°C \simeq 1°C}$$

6. The magnitude of the change in oxidising power of the MnO_4^- / Mn^{2+} couple is x × 10^{-4} V, if the H^+ concentration is decreased from 1 M to 10^{-4} M at 25˚C. (Assume concentration of MnO_4^- and Mn^{2+} to be same on change in H^+ concentration). The value of x is _______. (Rounded off to the nearest integer)

$\left[\text{Given} : \frac{2.303\ RT}{F} = 0.059\right]$

Official Ans. by NTA (3776)

Sol. Eqn is-

$$MnO_4^- + H^{\oplus} + 5e^- \rightarrow Mn^{+2} + 4H_2O$$

Nernst equation:

$$E_{cell} = E^0_{Cell} - \frac{0.059}{5}\log\frac{[Mn^{+2}]}{[MnO_4^-]}\left[\frac{1}{H^+}\right]^8$$

(I) Given $[H^{\oplus}] = 1M$

$$E_1 = E^0 - \frac{0.059}{5}\log\frac{[Mn^{+2}]}{[MnO_4^-]}$$

(II) Now : $[H^{\oplus}] = 10^{-4}M$

$$E_2 = E^0 - \frac{0.059}{5}\log\frac{[Mn^{+2}]}{[MnO_4^-]} \times \frac{1}{(10^{-4})^8}$$

$$= E^0 - \frac{0.059}{5}\log\frac{Mn^{+2}}{[MnO_4^-]} + \frac{0.059}{5}\log 10^{-32}$$

therefore : $|E_1 - E_2| = \frac{0.059}{5} \times 32$

= 0.3776 V = 3776 × 10^{-4}

x = 3776

7. The solubility product of PbI_2 is 8.0 × 10^{-9}. The solubility of lead iodide in 0.1 molar solution of lead nitrate is x × 10^{-6} mol/L. The value of x is __________. (Rounded off to the nearest integer)

$\left[\text{Given} : \sqrt{2} = 1.41\right]$

Official Ans. by NTA (141)

Sol. Given : $[K_{sp}]_{PbI_2} = 8 \times 10^{-9}$

To calculate : solubility of PbI_2 in 0.1 M sol of $Pb\ (NO_3)_2$

(I) $Pb\ (NO_3)_2 \rightarrow Pb^{+2}_{(aq)} + 2NO_3^-(aq)$

0.1 M	–	–
–	0.1M	0.2M

(II) $PbI_2(s) \rightleftharpoons Pb^{+2}(aq) + 2I^-(aq)$

s 2s

= s + 0.1

$\simeq 0.1$

Now : K_{sp} = 8 × 10^{-9} = $[Pb^{+2}]\ [I^-]^2$

$\Rightarrow$ 8 × 10^{-9} = 0.1 × $(2s)^2$

$\Rightarrow$ 8 × 10^{-8} = $4s^2$ $\Rightarrow$ s = $\sqrt{2} \times 10^{-4}$

$$\Rightarrow \boxed{S = 141 \times 10^{-6}M}$$

$\Rightarrow$ x = 141

8. Sucrose hydrolyses in acid solution into glucose and fructose following first order rate law with a half-life of 3.33 h at 25°C. After 9 h, the fraction of sucrose remaining is *f*. The value of $\log_{10}\left(\frac{1}{f}\right)$ is _______ × 10^{-2}. (Rounded off to the nearest integer)

[Assume : ln 10 = 2.303, ln 2 = 0.693]

Official Ans. by NTA (81)

Sol. Given :

$$C_{12}H_{22}O_{11} + H_2O \xrightarrow[t_{1/2}=\frac{10}{3}hr]{\text{I order}} \underset{\text{Glucose}}{C_6H_{12}O_6} + \underset{\text{Fructose}}{C_6H_{12}O_6}$$

t = 0 a = $[A]_0$ – –

t = 9hr a –x = $[A]_t$

from I order kinetic : $\frac{k\times t}{2.303} = \log\frac{|A|_0}{|A|_t}$

$$\Rightarrow \frac{\ell n2\times 9}{\frac{10}{3}\times 2.303} = \log\left(\frac{1}{f}\right)$$

$$\Rightarrow \frac{0.693\times 9\times 3}{23.03} = \log\left(\frac{1}{f}\right)$$

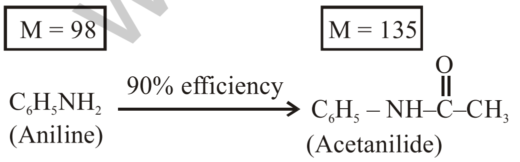
$$\Rightarrow \log\left(\frac{1}{f}\right) = 0.81246 = 81.24\times 10^{-2}$$

⇒ x = 81

9. 1.86 g of aniline completely reacts to form acetanilide. 10% of the product is lost during purification. Amount of acetanilide obtained after purification (in g) is _______ × 10^{-2}.

Official Ans. by NTA (243)

Sol.



$$\underset{\text{(Aniline)}}{\overset{\boxed{M=98}}{C_6H_5NH_2}} \xrightarrow{\text{90\% efficiency}} \underset{\text{(Acetanilide)}}{\overset{\boxed{M=135}}{C_6H_5-NH-\overset{\overset{O}{||}}{C}-CH_3}}$$

Given 1.86 g

⇒ 1 mol $C_6H_5NH_2$ give 1 mol $C_6H_5\,NH\overset{\overset{O}{||}}{C}CH_3$

∴ moles of $C_6H_5\ NH_2$ = moles of $C_6H_5\,NH\overset{\overset{O}{||}}{C}CH_3$

$$\Rightarrow \frac{1.86}{93} = \frac{W_{\text{ace tan ilide}}}{135}$$

$$\Rightarrow W_{\text{acelanilide}} = \frac{1.86\times 135}{93}g = 2.70g$$

But efficiency of reaction is 90% only

$$\therefore \text{Mass of acetanilide produced} = 2.70\times\frac{90}{100}g$$

= 2.43 g

= 243 × 10^{-2}g

⇒ x = 243

10. Assuming ideal behaviour, the magnitude of log K for the following reaction at 25°C is x × 10^{-1}. The value of x is _______. (Integer answer)

$$3HC\equiv CH_{(g)} \rightleftharpoons C_6H_{6(\ell)}$$

[Given: $\Delta_f G^o(HC\equiv CH) = -2.04\times 10^5\ J\ mol^{-1}$; $\Delta_f G^o(C_6H_6) = -1.24\times 10^5\ J\ mol^{-1}$; R = 8.314 J K^{-1} mol^{-1}]

Official Ans. by NTA (855)

Sol. $3HC\equiv CH_{(g)} \rightarrow C_6H_6(\ell) : \Delta G^0 = -RT\ln k$

$$\Delta G_f^0\ -2.04\times 10^5\frac{J}{mol} - 1.24\times 10^5 J/mol$$

$$\Rightarrow \Delta G^0 = \sum\left(\Delta G_f^0\right)_P - \sum\left(\Delta G_f^0\right)_R$$

$$\Rightarrow -RT\ \ell nk = 1\times\left(-124\times 10^5\right) - \left(-3\times 2.04\times 10^5\right)$$

$$\Rightarrow -2.303\times R\times T\log k = 4.88\times 10^5$$

$$\Rightarrow \log k = -\frac{4.88\times 10^5}{2.303\times R\times T} = -\frac{488000}{5705.848} = -85.52$$

= 855 × 10^{-1}

⇒ x = 855

MATHEMATICS

SECTION-A

1. For the statements p and q, consider the following compound statements :

(a) $(\sim q \wedge (p \to q)) \to \sim p$

(b) $((p \vee q) \wedge \sim p) \to q$

Then which of the following statements is correct?

(1) (a) and (b) both are not tautologies.

(2) (a) and (b) both are tautologies.

(3) (a) is a tautology but not (b).

(4) (b) is a tautology but not (a).

Official Ans. by NTA (2)

Sol. (A)

p	q	$\sim q$	$p \to q$	$\sim p$	$(\sim q \wedge (p \to q))$	
T	T	F	T	F	F	T
T	F	T	F	F	F	T
F	T	F	T	T	F	T
F	F	T	T	T	T	T

(B)

p	q	$p \vee q$	$\sim p$	$(p \vee q) \wedge \sim p$	
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

Both are tautologies

2. Let a, b $\in$ R. If the mirror image of the point P(a, 6, 9) with respect to the line $\frac{x-3}{7}=\frac{y-2}{5}=\frac{z-1}{-9}$ is (20, b, –a –9), then |a + b| is equal to :

(1) 88 (2) 86

(3) 84 (4) 90

Official Ans. by NTA (1)

Sol. P(9, 6, 9)

$$\frac{x-3}{7}=\frac{y-2}{5}=\frac{z-1}{-9}$$

Q = (20, b, –a – 9)

$$\frac{\frac{20+a}{2}-3}{7}=\frac{\frac{b+6}{2}-2}{5}=\frac{-\frac{9}{2}-1}{-9}$$

$$\frac{14+9}{14}=\frac{b+2}{10}=\frac{a+2}{18}$$

$\Rightarrow$ a = –56 and b = –32

$\Rightarrow$ |a + b| = 88

3. The vector equation of the plane passing through the intersection of the planes $\vec{r}.(\hat{i}+\hat{j}+\hat{k})=1$ and $\vec{r}.(\hat{i}-2\hat{j})=-2$, and the point (1, 0, 2) is :

(1) $\vec{r}.(\hat{i}+7\hat{j}+3\hat{k})=\frac{7}{3}$

(2) $\vec{r}.(3\hat{i}+7\hat{j}+3\hat{k})=7$

(3) $\vec{r}.(\hat{i}+7\hat{j}+3\hat{k})=7$

(4) $\vec{r}.(\hat{i}-7\hat{j}+3\hat{k})=\frac{7}{3}$

Official Ans. by NTA (3)

Sol. $\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)=1$

$\vec{r}.(\hat{i}-2\hat{j})=-2$

point (1, 0, 2)

Eq^n of plane

$$\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)-1+\lambda\left\{r.(\hat{i}-2\hat{j})+2\right\}=0$$

$$\vec{r}.\left\{\hat{i}\left(1+\lambda\right)+\hat{j}\left(1-2\lambda\right)+\hat{k}(1)\right\}-1+2\lambda=0$$

Point $\hat{i}+0\hat{j}+2\hat{k}=\vec{r}$

$$\therefore\ (\hat{i}+2\hat{k}).\left\{\hat{i}(1+\lambda)+\hat{j}(1-2\lambda)+\hat{k}(1)\right\}-1+2\lambda=0$$

$1+\lambda+2-1+2\lambda=0$

$\lambda=-\frac{2}{3}$

$$\therefore\quad \vec{r}.\left[\hat{i}\left(\frac{1}{3}\right)+\hat{j}\left(\frac{7}{3}\right)+\hat{k}\right]=\frac{7}{3}$$

$$r.\left[\hat{i}+7\hat{j}+3\hat{k}\right]=7$$

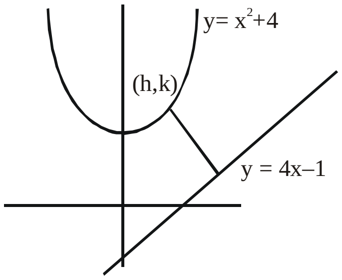
Ans. 3

4. If P is a point on the parabola $y = x^2 + 4$ which is closest to the straight line $y = 4x - 1$, then the co-ordinates of P are :

(1) (3, 13)	(2) (1, 5)
(3) (–2, 8)	(4) (2, 8)

Official Ans. by NTA (4)

Sol. **Ans. (4)**



P :$y = x^2 + 4$

$k = h^2 + 4$

L : $y = 4x - 1$

$y - 4x + 1 = 0$

$$d = AB = \left|\frac{k-4h+1}{\sqrt{5}}\right| = \left|\frac{h^2-4-4h+1}{\sqrt{5}}\right|$$

$$\frac{d(d)}{dh} = \frac{2h-4}{\sqrt{5}} = 0$$

$h = 2$

$$\frac{d^2(d)}{dh^2} = \frac{2}{\sqrt{5}} > 0$$

$\therefore$ $k = 4 + 4 = 8$

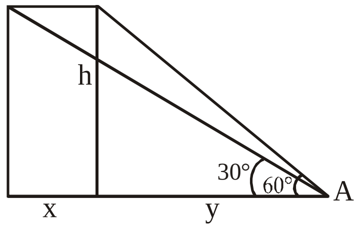
$\therefore$ Point (2, 8)

5. The angle of elevation of a jet plane from a point A on the ground is 60°. After a flight of 20 seconds at the speed of 432 km/hour, the angle of elevation changes to 30°. If the jet plane is flying at a constant height, then its height is :

(1) $1800\sqrt{3}$ m	(2) $3600\sqrt{3}$ m
(3) $2400\sqrt{3}$ m	(4) $1200\sqrt{3}$ m

Official Ans. by NTA (4)

Sol.



$$\tan 60° = \frac{h}{y}$$

$$\sqrt{3} = \frac{h}{y} \Rightarrow h = \sqrt{3}y \qquad .....(1)$$

$$\tan 30° = \frac{h}{x+y}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x+y} \Rightarrow \sqrt{3}h = x + y \quad ......(2)$$

Speed 432 km/h $\Rightarrow \dfrac{432 \times 20}{60 \times 60} \Rightarrow \dfrac{12}{5}$ km

$$\sqrt{3}h = \frac{12}{5} + y$$

$$\sqrt{3}h - \frac{12}{5} = y$$

from (1)

$$h = \sqrt{3}\left[\sqrt{3}h - \frac{12}{5}\right]$$

$$h = 3h - \frac{12\sqrt{3}}{5}$$

$$h = \frac{6\sqrt{3}}{5}\text{ km}$$

$$h = 1200\sqrt{3}\text{ m}$$

6. If $n \geq 2$ is a positive integer, then the sum of the series ${}^{n+1}C_2 + 2({}^2C_2 + {}^3C_2 + {}^4C_2 + ... + {}^nC_2)$ is:

(1) $\dfrac{n(n-1)(2n+1)}{6}$	(2) $\dfrac{n(n+1)(2n+1)}{6}$
(3) $\dfrac{n(2n+1)(3n+1)}{6}$	(4) $\dfrac{n(n+1)^2(n+2)}{12}$

Official Ans. by NTA (2)

Sol. $${}^{n+1}C_2 + 2\left({}^2C_2 + {}^3C_2 + {}^4C_2 + ... + {}^nC_2\right)$$

$${}^{n+1}C_2 + 2\left({}^3C_3 + {}^3C_2 + {}^4C_2 + ... + {}^nC_2\right)$$

$$\left\{\text{use } {}^nC_{r+1} + {}^nC_r = {}^{n+1}C_r\right\}$$

$$= {}^{n+1}C_2 + 2\left({}^4C_3 + {}^4C_2 + {}^5C_3 + ... + {}^nC_2\right)$$

$$= {}^{n+1}C_2 + 2\left({}^5C_3 + {}^5C_2 + ... + {}^nC_2\right)$$

$$\vdots \quad \vdots \quad \vdots \quad \vdots$$

$$= {}^{n+1}C_2 + 2\left({}^nC_3 + {}^nC_2\right)$$

$$= {}^{n+1}C_2 + 2.{}^{n+1}C_3$$

$$= \frac{(n+1)n}{2} + 2.\frac{(n+1)(n)(n-1)}{2.3}$$

$$= \frac{n(n+1)(2n+1)}{6}$$

7. Let f **:** **R** → **R** be defined as,

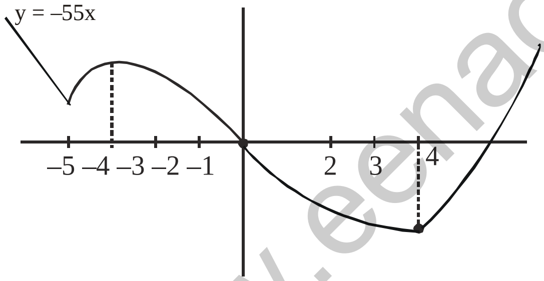
$$f(x) = \begin{cases} -55\,x, & \text{if } x < -5 \\ 2x^3 - 3x^2 - 120x, & \text{if } -5 \le x \le 4 \\ 2x^3 - 3x^2 - 36x - 336, & \text{if } x > 4, \end{cases}$$

Let A = {**x** ∈ **R :** f is increasing}. Then A is equal to :

(1) $(-\infty, -5) \cup (4, \infty)$
(2) $(-5, \infty)$
(3) $(-\infty, -5) \cup (-4, \infty)$
(4) $(-5, -4) \cup (4, \infty)$

Official Ans. by NTA (4)

Sol.



$$f'(x) = \begin{cases} -55\,; & x < -5 \\ 6(x-5)(x+4)\,; & -5 < x < 4 \\ 6(x-3)(x+2)\,; & x > 4 \end{cases}$$

f(x) is increasing in
$x \in (-5, -4) \cup (4, \infty)$

8. Let f be a twice differentiable function defined on R such that f(0) = 1, f '(0) = 2 and f '(x) ≠ 0 for all x ∈ R. If $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix} = 0$, for all x ∈ R, then the value of f(1) lies in the interval:

(1) (9, 12)
(2) (6, 9)
(3) (0, 3)
(4) (3,6)

Official Ans. by NTA (2)

Sol. $f(x)f''(x) - (f'(x))^2 = 0$

$$\frac{f''(x)}{f'(x)} = \frac{f'(x)}{f(x)}$$

$$\ln(f'(x)) = \ln f(x) + \ln c$$

$$f'(x) = cf(x)$$

$$\frac{f'(x)}{f(x)} = c$$

$\ln f(x) = cx + k_1$
$f(x) = ke^{cx}$
$f(0) = 1 = k$
$f'(0) = c = 2$
$f(x) = e^{2x}$
$f(1) = e^2 \in (6, 9)$

9. For which of the following curves, the line $x + \sqrt{3}y = 2\sqrt{3}$ is the tangent at the point $\left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)$?

(1) $x^2 + y^2 = 7$
(2) $y^2 = \frac{1}{6\sqrt{3}}x$
(3) $2x^2 - 18y^2 = 9$
(4) $x^2 + 9y^2 = 9$

Official Ans. by NTA (4)

Sol. $m = -\frac{1}{\sqrt{3}}, c = 2$

(1) $c = a\sqrt{1+m^2}$

$c = \sqrt{7}\,\frac{2}{\sqrt{3}}$ (incorrect)

(2) $c = \frac{a}{m} = \frac{\frac{1}{24\sqrt{3}}}{\frac{-1}{\sqrt{3}}} = -\frac{1}{24}$ (incorrect)

(3) $c = \sqrt{a^2m^2 - b^2}$

$c = \sqrt{\frac{9}{2}\cdot\frac{1}{3} - \frac{1}{2}} = 1$ (incorrect)

(4) $c = \sqrt{a^2m^2 + b^2}$

$c = \sqrt{9\cdot\frac{1}{3} + 1} = 2$ (correct)

10. The value of the integral, $\int_{1}^{3}[x^2-2x-2]\,dx$, where [x] denotes the greatest integer less than or equal to x, is :

(1) $-\sqrt{2}-\sqrt{3}+1$ (2) $-\sqrt{2}-\sqrt{3}-1$

(3) –5 (4) –4

Official Ans. by NTA (2)

Sol. $\int_{1}^{3}\left(\left[\left(x-1\right)^{2}\right]-3\right)dx$

$$=\int_{1}^{2}[x^2]-3\int_{1}^{3}dx$$

$$=\int_{1}^{3}0\,dx+\int_{1}^{\sqrt{2}}1.dx+\int_{\sqrt{2}}^{\sqrt{3}}2.dx+\int_{\sqrt{3}}^{2}3.dx-6$$

$$=\sqrt{2}-1+2(\sqrt{3}-\sqrt{2})+3(2-\sqrt{3})-6$$

$$=-\sqrt{2}-\sqrt{3}-1$$

11. A possible value of $\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$ is :

(1) $\frac{1}{\sqrt{7}}$ (2) $2\sqrt{2}-1$

(3) $\sqrt{7}-1$ (4) $\frac{1}{2\sqrt{2}}$

Official Ans. by NTA (1)

Sol. Let $\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}=\theta$

$$\sin 4\theta=\frac{\sqrt{63}}{8}$$

$$\cos 4\theta=\frac{1}{8}$$

$$2\cos^2 2\theta-1=\frac{1}{8}$$

$$\cos^2 2\theta=\frac{9}{16}$$

$$\cos 2\theta=\frac{3}{4}$$

$$2\cos^2\theta-1=\frac{3}{4}$$

$$\cos^2\theta=\frac{7}{8}$$

$$\cos\theta=\frac{\sqrt{7}}{2\sqrt{2}}$$

$$\tan\theta=\frac{1}{\sqrt{7}}$$

12. The negative of the statement ~p ∧ (p ∨ q) is

(1) ~p ∨ q (2) p ∨ ~ q

(3) ~p ∧ q (4) p ∧ ~ q

Official Ans. by NTA (2)

Sol. ~(~p ∧ (p ∨ q))

p ∨ (~p ∧ ~q)

$$\underbrace{(pv\sim p)}_{t}\wedge\left(pv\sim q\right)$$

p ∨ ~q

13. If the curve $y = ax^2 + bx + c$, $x \in R$, passes through the point (1,2) and the tangent line to this curve at origin is y = x, then the possible values of a, b, c are :

(1) a = $\frac{1}{2}$, b = $\frac{1}{2}$, c = 1

(2) a = l, b = 0, c= 1

(3) a = l, b = l, c = 0

(4) a = –1, b = l, c = l

Official Ans. by NTA (3)

Sol. a + b + c = 2 ...(1)

and $\left.\frac{dy}{dx}\right|_{(0,\,0)}=1$

$$\left.2ax+b\right|_{(0,\,0)}=1$$

b = 1

Curve passes through origin

So, c = 0

and a = 1

14. The area of the region :

R = {(x, y) : $5x^2 \le y \le 2x^2 + 9$} is :

(1) $11\sqrt{3}$ square units

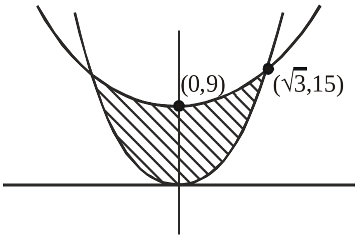
(2) $12\sqrt{3}$ square units

(3) $9\sqrt{3}$ square units

(4) $6\sqrt{3}$ square units

Official Ans. by NTA (2)

Sol.



Required area = $2\int_{0}^{\sqrt{3}}(2x^2+9-5x^2)\,dx$

$= 2\left[9x-x^3\right]_0^{\sqrt{3}}$

$= 2\left[9\sqrt{3}-3\sqrt{3}\right] = 12\sqrt{3}$

15. If a curve y = f(x) passes through the point (1, 2) and satisfies $x\frac{dy}{dx}+y=bx^4$, then for what value of b, $\int_1^2 f(x)dx=\frac{62}{5}$?

(1) 5
(2) 10
(3) $\frac{62}{5}$
(4) $\frac{31}{5}$

Official Ans. by NTA (2)

Sol. $\frac{dy}{dx}+\frac{y}{x}=bx^3$

I.F. = $e^{\int\frac{1}{x}dx}=x$

So, solution of D.E. is given by

y.x = $\int b.x^3.x\,dx+c$

y = $\frac{c}{x}+\frac{bx^4}{5}$

Passes through (1, 2)

2 = c + $\frac{b}{5}$ ...(1)

$\int_1^2 f(x)dx=\frac{62}{5}$

$\left[c\ln x+\frac{bx^5}{25}\right]_1^2=\frac{62}{5}$

$c\ln 2+\frac{31\,b}{25}=\frac{62}{5}$ ...(2)

By equation (1) & (2)

c = 0 and b = 10

16. Let f(x) be a differentiable function defined on [0, 2] such that f '(x) = f '(2 – x) for all x ∈ (0, 2), f(0) = 1 and f(2) = e^2. Then the value of $\int_0^2 f(x)\,dx$ is :

(1) 1 – e^2
(2) 1 + e^2
(3) 2(1 – e^2)
(4) 2(1 + e^2)

Official Ans. by NTA (2)

Sol. $f'(x)=f'(2-x)$

f(x) = – f(2 – x) + c

put x = 0

$f'(0)=-f'(2)+c$

c = f(0) + f(2) = 1 + e^2

so, f(x) + f(2 – x) = 1 + e^2

$I=\int_0^2 f(x)\,dx$

$I=\int_0^2 f(2-x)\,dx$

$2I=\int_0^2\left(f(x)+f(2-x)\right)dx$

$2I=(1+e^2)\int_0^2 dx$

I = 1 + e^2

17. Let A and B be 3 × 3 real matrices such that A is symmetric matrix and B is skew-symmetric matrix. Then the system of linear equations $(A^2B^2 - B^2A^2)X = O$, where X is a 3 × 1 column matrix of unknown variables and O is a 3 × l null matrix, has :

(1) no solution
(2) exactly two solutions
(3) infinitely many solutions
(4) a unique solution

Official Ans. by NTA (3)

Sol. Let $A^T = A$ and $B^T = -B$

$C = A^2B^2 - B^2A^2$

$C^T = (A^2B^2)^T - (B^2A^2)^T$

$= (B^2)^T(A^2)^T - (A^2)^T(B^2)^T$

$= B^2A^2 - A^2B^2$

$C^T = -C$

C is skew symmetric.

So det(C) = 0

so system have infinite solutions.

18. Let a, b, c be in arithmetic progression. Let the centroid of the triangle with vertices (a, c), (2, b) and (a, b) be $\left(\frac{10}{3}, \frac{7}{3}\right)$. If α, β are the roots of the equation $ax^2 + bx + 1 = 0$, then the value of $\alpha^2 + \beta^2 - \alpha\beta$ is :

(1) $\frac{71}{256}$ (2) $\frac{69}{256}$

(3) $-\frac{69}{256}$ (4) $-\frac{71}{256}$

Official Ans. by NTA (4)

Sol. $\frac{a+2+a}{3} = \frac{10}{3}$

a = 4

and $\frac{c+b+b}{3} = \frac{7}{3}$

c + 2b = 7

also 2b = a + c

2b – a + 2b = 7

b = $\frac{11}{4}$

now $4x^2 + \frac{11}{4}x + 1 = 0 \begin{smallmatrix}\alpha \\ \beta\end{smallmatrix}$

$\alpha^2 + \beta^2 - \alpha\beta = (\alpha + \beta)^2 - 3\alpha\beta$

$= \left(\frac{-11}{16}\right)^2 - 3\left(\frac{1}{4}\right)$

$= \frac{121}{256} - \frac{3}{4} = \frac{-71}{256}$

19. For the system of linear equations :

x – 2y = l, x – y + kz = –2, ky + 4z = 6, k ∈ R,

consider the following statements :

(A) The system has unique solution if k ≠ 2, k ≠ – 2.

(B) The system has unique solution if k = – 2.

(C) The system has unique solution if k = 2.

(D) The system has no-solution if k = 2.

(E) The system has infinite number of solutions if k ≠ –2.

Which of the following statements are correct ?

(1) (C) and (D) only

(2) (B) and (E) only

(3) (A) and (E) only

(4) (A) and (D) only

Official Ans. by NTA (4)

Sol. $D = \begin{vmatrix} 1 & -2 & 0 \\ 1 & -1 & k \\ 0 & k & 4 \end{vmatrix} = 4 - k^2$

so, A is correct and B, C, E are incorrect.

If k = 2

$D_1 = \begin{vmatrix} 1 & -2 & 0 \\ -2 & -1 & 2 \\ 6 & 2 & 4 \end{vmatrix} = -48 \neq 0$

So no solution

D is correct.

20. The probability that two randomly selected subsets of the set {1, 2, 3, 4, 5} have exactly two elements in their intersection, is :

(1) $\frac{65}{2^7}$ (2) $\frac{65}{2^8}$

(3) $\frac{135}{2^9}$ (4) $\frac{35}{2^7}$

Official Ans. by NTA (3)

Sol. Total subsets = 2^5 = 32

Probability = $\frac{{}^5C_2 \times 3^3}{32 \times 32} = \frac{10 \times 27}{12^{10}} = \frac{135}{2^9}$

SECTION-B

1. For integers n and r, let $\binom{n}{r} = \begin{cases} {}^nC_r, & \text{if } n \geq r \geq 0 \\ 0, & \text{otherwise} \end{cases}$

The maximum value of k for which the sum

$\sum_{i=0}^{k} \binom{10}{i}\binom{15}{k-i} + \sum_{i=0}^{k+1} \binom{12}{i}\binom{13}{k+1-i}$ exists, is equal to ________.

Official Ans. by NTA (12)

Ans. by ALLEN (BONUS)

Sol. **Bonus**

$\sum_{i=0}^{k} \binom{10}{i}\binom{15}{k-i} + \sum_{i=0}^{k+1} \binom{12}{i}\binom{13}{k+1-i}$

${}^{25}C_k + {}^{25}C_{k+1}$

${}^{26}C_{k+1}$

as ${}^nC_r$ is defined for all values of n as will as r so ${}^{26}C_{k+1}$ always exists

Now k is unbounded so maximum value is not defined.

2. Let λ be an interger. If the shortest distance between the lines $x - \lambda = 2y - 1 = -2z$ and $x = y + 2\lambda = z - \lambda$ is $\frac{\sqrt{7}}{2\sqrt{2}}$, then the value of $|\lambda|$ is _______.

Official Ans. by NTA (1)

Sol. $\frac{x-\lambda}{1} = \frac{y-\frac{1}{2}}{\frac{1}{2}} = \frac{z-0}{-\frac{1}{2}}$

$\frac{x-0}{1} = \frac{y+2\lambda}{1} = \frac{z-\lambda}{1}$

Shortest distance = $\frac{(a_2-a_1).(b_1\times b_2)}{|b_1\times b_2|}$

$$b_1\times b_2 = \begin{vmatrix} i & j & k \\ 1 & \frac{1}{2} & -\frac{1}{2} \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \hat{i}\left(\frac{1}{2}+\frac{1}{2}\right) - \hat{j}\left(1+\frac{1}{2}\right) + \hat{k}\left(1-\frac{1}{2}\right)$$

$$= \hat{i} - \frac{3}{2}\hat{j} + \frac{\hat{k}}{2} = \frac{2\hat{i}-3\hat{j}+\hat{k}}{2}$$

$$\frac{b_1\times b_2}{|b_1\times b_2|} = \frac{2\hat{i}-3\hat{j}+\hat{k}}{\sqrt{14}}$$

$$\frac{(a_2-a_1).(b_1\times b_2)}{|b_1\times b_2|} = \left(-\lambda\hat{i}+\left(-2\lambda+\frac{1}{2}\right)+\lambda\hat{k}\right)$$

$$\left(\frac{2\hat{i}-3\hat{j}+\hat{k}}{\sqrt{14}}\right)$$

$$= \left|\frac{-2\lambda+6\lambda-\frac{3}{2}+\lambda}{\sqrt{14}}\right| = \frac{\sqrt{7}}{2\sqrt{2}}$$

$$\left|5\lambda-\frac{3}{2}\right| = \frac{7}{2}$$

$$5\lambda = \frac{3}{2}\pm\frac{7}{2}$$

$5\lambda = 5, -2$

$\lambda = 1, -\frac{2}{5}$

3. If $a + \alpha = 1$, $b + \beta = 2$ and $af(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}$, $x \neq 0$, then the value of expression $\frac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}}$ is _______.

Official Ans. by NTA (2)

Sol. $af(x)+\alpha f\left(\frac{1}{x}\right) = bx+\frac{\beta}{x}$ ....(1)

replace x by $\frac{1}{x}$

$af\left(\frac{1}{x}\right)+\alpha f(x) = \frac{b}{x}+\beta x$ ....(2)

(1) + (2)

$$(a+\alpha)f(x)+(a+\alpha)f\left(\frac{1}{x}\right) = x(b+\beta)+(b+\beta)\frac{1}{x}$$

$$\frac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}} = \frac{b+\beta}{a+\alpha} = \frac{2}{1} = 2$$

4. Let a point P be such that its distance from the point (5, 0) is thrice the distance of P from the point (– 5, 0). If the locus of the point P is a circle of radius r, then $4r^2$ is equal to ________.

Official Ans. by NTA (56)

Sol. Let point is (h, k)

So, $\sqrt{(h-5)^2+k^2} = 3\sqrt{(h+5)^2+k^2}$

$8x^2 + 8y^2 + 100x + 200 = 0$

$x^2 + y^2 + \frac{25}{2}x + 25 = 0$

$$r^2 = \frac{(25)^2}{4^2} - 25$$

$$4r^2 = \frac{25^2}{4} - 100$$

$4r^2 = 156.25 - 100$

$4r^2 = 56.25$

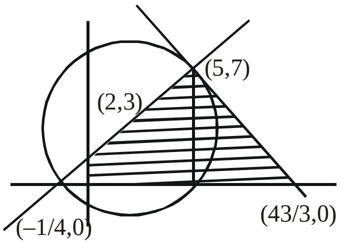
After round of $4r^2 = 56$

5. If the area of the triangle formed by the positive x-axis, the normal and the tangent to the circle $(x - 2)^2 + (y - 3)^2 = 25$ at the point (5, 7) is A, then 24A is equal to_____.

Official Ans. by NTA (1225)

Ans. by ALLEN (1225 / BONUS)

Sol.



Equation of normal

$4x - 3y + 1 = 0$

and equation of tangents

$3x + 4y - 43 = 0$

$$\text{Area of triangle} = \frac{1}{2}\left(\frac{43}{3}+\frac{1}{4}\right)\times(7)$$

$$= \frac{1}{2}\left(\frac{172+3}{12}\right)\times 7$$

$$A = \frac{1225}{24}$$

$24A = 1225$

* as positive x-axis is given in the question so question should be bonus.

6. If the variance of 10 natural numbers 1, 1, 1,...., 1, k is less than 10, then the maximum possible value of k is ______.

Official Ans. by NTA (11)

Sol. $$\sigma^2 = \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2$$

$$= \frac{9+k^2}{10} - \left(\frac{9+k}{10}\right)^2 < 10$$

$90 + 10k^2 - 81 - k^2 - 18\,k < 1000$

$9k^2 - 18k - 991 < 0$

$$k^2 - 2k < \frac{991}{9}$$

$$(k - 1)^2 < \frac{1000}{9}$$

$$\frac{-10\sqrt{10}}{3} < k - 1 < \frac{10\sqrt{10}}{3}$$

$$k < \frac{10\sqrt{10}}{3} + 1$$

$k \leq 11$

Maximum value of k is 11.

7. The sum of first four terms of a geometric progression (G.P.) is $\frac{65}{12}$ and the sum of their respective reciprocals is $\frac{65}{18}$. If the product of first three terms of the G.P. is 1, and the third term is α, then 2α is _______.

Official Ans. by NTA (3)

Sol. Let number are a, ar, ar^2, ar^3

$$a\frac{(r^4-1)}{r-1} = \frac{65}{12} \qquad ...(1)$$

$$\frac{1}{a}\frac{\left(\frac{1}{r^4}-1\right)}{\frac{1}{r}-1} = \frac{65}{18}$$

$$\frac{1}{ar^3}\left(\frac{1-r^3}{1-r}\right) = \frac{65}{18} \qquad ....(2)$$

$$\frac{(1)}{(2)} \Rightarrow a^2r^3 = \frac{3}{2}$$

and $\quad a^3.r^3 = 1$

$\qquad ar = 1$

$$(ar)^2.r = \frac{3}{2}$$

$$r = \frac{3}{2},\ a = \frac{2}{3}$$

$$\text{So, third term} = ar^2 = \frac{2}{3}\times\frac{9}{4}$$

$$\alpha = \frac{3}{2}$$

$2\alpha = 3$

8. The students S_1, S_2,....., S_{10} are to be divided into 3 groups A, B and C such that each group has at least one student and the group C has at most 3 students. Then the total number of possibilities of forming such groups is _____.

Official Ans. by NTA (31650)

Sol. If group C has one student then number of groups

$^{10}C_1[2^9 - 2] = 5100$

If group C has two students then number of groups

$^{10}C_2[2^8 - 2] = 11430$

If group C has three students then number of groups

$= {}^{10}C_3 \times [2^7 - 2] = 15120$

So total groups = 31650

9. Let $i=\sqrt{-1}$. If $\frac{(-1+i\sqrt{3})^{21}}{(1-i)^{24}}+\frac{(1+i\sqrt{3})^{21}}{(1+i)^{24}}$ = k, and n = [| k |] be the greatest integral part of | k |. Then $\sum_{j=0}^{n+5}(j+5)^2-\sum_{j=0}^{n+5}(j+5)$ is equal to __________.

Official Ans. by NTA (310)

Sol. $$K = \frac{1}{2^9}\left[\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{21}}{\left(\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i\right)^{24}}+\frac{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{21}}{\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right)^{24}}\right]$$

$$K = \frac{1}{512}\left[\frac{\left(e^{i\frac{2\pi}{3}}\right)^{21}}{\left(e^{-\frac{i\pi}{4}}\right)^{24}}+\frac{\left(e^{\frac{i\pi}{3}}\right)^{21}}{\left(e^{\frac{i\pi}{4}}\right)^{24}}\right]$$

$$K = \frac{1}{512}\left[e^{i(14\pi+6\pi)}+e^{i(7\pi-6\pi)}\right]$$

$$K = \frac{1}{512}\left[e^{20\pi i}+e^{\pi i}\right]$$

$$K = \frac{1}{512}[1+(-1)]=0$$

n = [|k|] = 0

$$\sum_{j=0}^{5}(j+5)^2-\sum_{j=0}^{5}(j+5)$$

$$\sum_{j=0}^{5}(j^2+25+10j-j-5)$$

$$\sum_{j=0}^{5}(j^2+9j+20)$$

$$\sum_{j=0}^{5}j^2+9\sum_{j=0}^{5}j+20\sum_{j=0}^{5}1$$

$$\frac{5\times6\times11}{6}+9\left(\frac{5\times6}{2}\right)+20\times6$$

= 55 + 135 + 120

= 310

10. The number of the real roots of the equation $(x+1)^2+|x-5|=\frac{27}{4}$ is ________.

Official Ans. by NTA (2)

Sol. **Case-I**

$x \le 5$

$$(x+1)^2-(x-5)=\frac{27}{4}$$

$$(x+1)^2-(x+1)-\frac{3}{4}=0$$

$$x+1=\frac{3}{2},-\frac{1}{2}$$

$$x=\frac{1}{2},-\frac{3}{2}$$

Case-II

$x > 5$

$$(x+1)+(x-5)=\frac{27}{4}$$

$$(x+1)^2+(x+1)-\frac{51}{4}=0$$

$$x=\frac{-1\pm\sqrt{52}}{2}$$ (rejected as x > 5)

So, the equation have two real root.