JEE Advanced - 2016

Held on 22-5-2016

P1-16-3-0

PAPER-1

CODE **0**

Time : 3 Hours

Maximum Marks : 186

READ THE INSTRUCTIONS CAREFULLY

GENERAL

1. This sealed booklet is your Question Paper. Do not break the seal till you are told to do so.
2. The paper CODE is printed on the right hand top corner of this sheet and the right hand top corner of the back cover of this booklet.
3. Use the Optical Response Sheet (ORS) provided separately for answering the questions.
4. The paper CODE is printed on the left part as well as the right part of the ORS. Ensure that both these codes are identical and same as that on the question paper booklet. If not, contact the invigilator for change of ORS.
5. Blank spaces are provided within this booklet for rough work.
6. Write your name, roll number and sign in the space provided on the back cover of this booklet.
7. After breaking the seal of the booklet at **9:00 am**, verify that the booklet contains **36** pages and that all the **54** questions along with the options are legible. If not, contact the invigilator for replacement of the booklet.
8. You are allowed to take away the Question Paper at the end of the examination.

OPTICAL RESPONSE SHEET

9. The ORS (top sheet) will be provided with an attached Candidate's Sheet (bottom sheet). The Candidate's Sheet is a carbon-less copy of the ORS.
10. Darken the appropriate bubbles on the ORS by applying sufficient pressure. This will leave an impression at the corresponding place on the Candidate's Sheet.
11. The ORS will be collected by the invigilator at the end of the examination.
12. You will be allowed to take away the Candidate's Sheet at the end of the examination.
13. Do not tamper with or mutilate the ORS. **Do not use the ORS for rough work.**
14. Write your name, roll number and code of the examination center, and sign with pen in the space provided for this purpose on the ORS. **Do not write any of these details anywhere else** on the ORS. Darken the appropriate bubble under each digit of your roll number.

DARKENING THE BUBBLES ON THE ORS

15. Use a **BLACK BALL POINT PEN** to darken the bubbles on the ORS.
16. Darken the bubble ◯ **COMPLETELY**.
17. The correct way of darkening a bubble is as : ●
18. The ORS is machine-gradable. Ensure that the bubbles are darkened in the correct way.
19. Darken the bubbles **ONLY IF** you are sure of the answer. There is **NO WAY** to erase or "un-darken" a darkened bubble.

Please see the last page of this booklet for rest of the instructions.

317170

DO NOT BREAK THE **SEALS** WITHOUT BEING INSTRUCTED TO DO SO BY THE INVIGILATOR

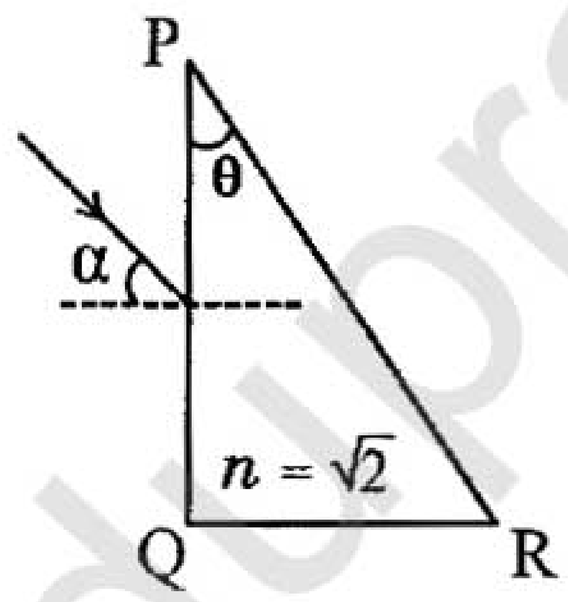
PART I : PHYSICS

SECTION 1 (Maximum Marks: 15)

- This section contains **FIVE** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks : +3 If only the bubble corresponding to the correct option is darkened.
Zero Marks : 0 If none of the bubbles is darkened.
Negative Marks : −1 In all other cases.

Q.1 A parallel beam of light is incident from air at an angle α on the side PQ of a right angled triangular prism of refractive index $n = \sqrt{2}$. Light undergoes total internal reflection in the prism at the face PR when α has a minimum value of 45°. The angle θ of the prism is



(A) 15° (B) 22.5° (C) 30° (D) 45°

Space for rough work

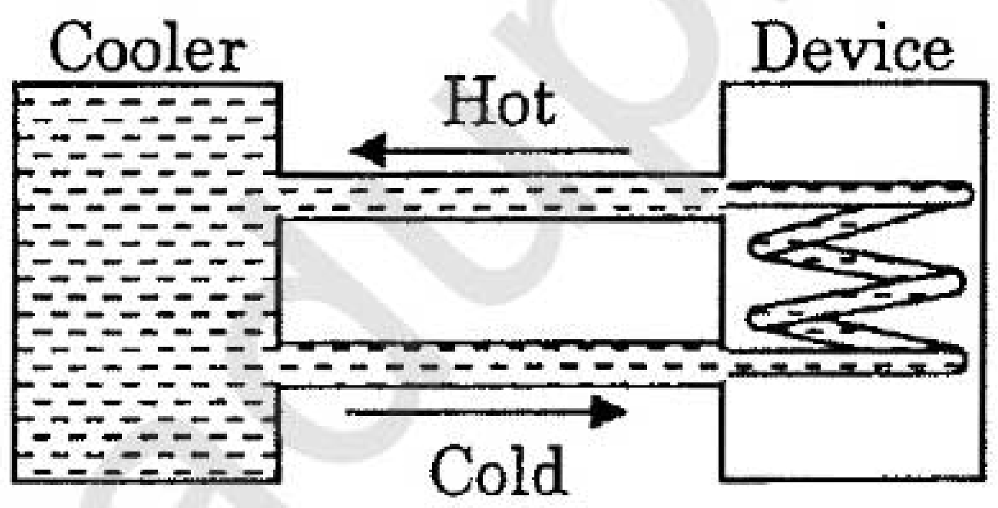
Q.2 In a historical experiment to determine Planck's constant, a metal surface was irradiated with light of different wavelengths. The emitted photoelectron energies were measured by applying a stopping potential. The relevant data for the wavelength (λ) of incident light and the corresponding stopping potential (V_0) are given below :

λ (μm)	V_0 (Volt)
0.3	2.0
0.4	1.0
0.5	0.4

Given that $c = 3 \times 10^8$ m s^{-1} and $e = 1.6 \times 10^{-19}$ C, Planck's constant (in units of J s) found from such an experiment is

(A) 6.0×10^{-34} (B) 6.4×10^{-34} (C) 6.6×10^{-34} (D) 6.8×10^{-34}

Q.3 A water cooler of storage capacity 120 litres can cool water at a constant rate of P watts. In a closed circulation system (as shown schematically in the figure), the water from the cooler is used to cool an external device that generates constantly 3 kW of heat (thermal load). The temperature of water fed into the device cannot exceed 30 °C and the entire stored 120 litres of water is initially cooled to 10 °C. The entire system is thermally insulated. The minimum value of P (in watts) for which the device can be operated for 3 hours is



(Specific heat of water is 4.2 kJ kg^{-1} K^{-1} and the density of water is 1000 kg m^{-3})

(A) 1600 (B) 2067 (C) 2533 (D) 3933

Space for rough work

Q.4 A uniform wooden stick of mass 1.6 kg and length l rests in an inclined manner on a smooth, vertical wall of height $h(<l)$ such that a small portion of the stick extends beyond the wall. The reaction force of the wall on the stick is perpendicular to the stick. The stick makes an angle of 30° with the wall and the bottom of the stick is on a rough floor. The reaction of the wall on the stick is equal in magnitude to the reaction of the floor on the stick. The ratio h/l and the frictional force f at the bottom of the stick are

$(g = 10 \text{ m s}^{-2})$

(A) $\frac{h}{l}=\frac{\sqrt{3}}{16}, f=\frac{16\sqrt{3}}{3}\text{N}$

(B) $\frac{h}{l}=\frac{3}{16}, f=\frac{16\sqrt{3}}{3}\text{N}$

(C) $\frac{h}{l}=\frac{3\sqrt{3}}{16}, f=\frac{8\sqrt{3}}{3}\text{N}$

(D) $\frac{h}{l}=\frac{3\sqrt{3}}{16}, f=\frac{16\sqrt{3}}{3}\text{N}$

Space for rough work

Q.5 An infinite line charge of uniform electric charge density λ lies along the axis of an electrically conducting infinite cylindrical shell of radius R. At time $t=0$, the space inside the cylinder is filled with a material of permittivity ε and electrical conductivity σ. The electrical conduction in the material follows Ohm's law. Which one of the following graphs best describes the subsequent variation of the magnitude of current density $j(t)$ at any point in the material?

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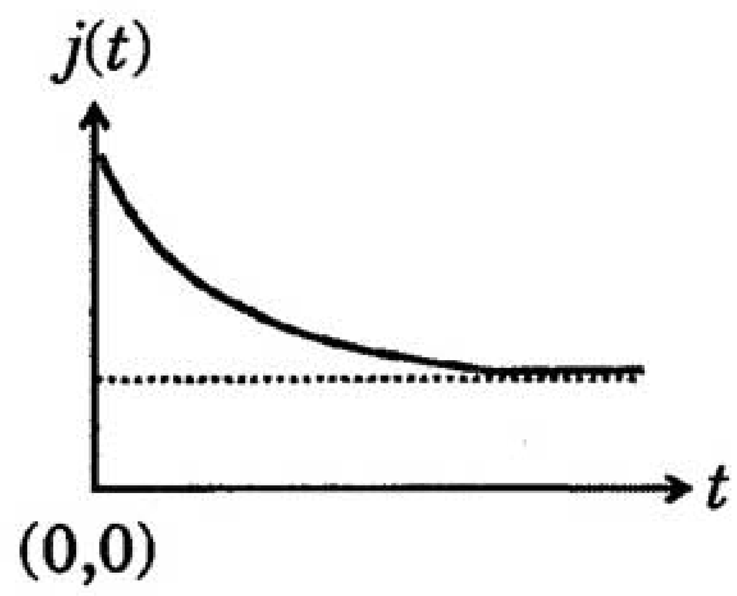

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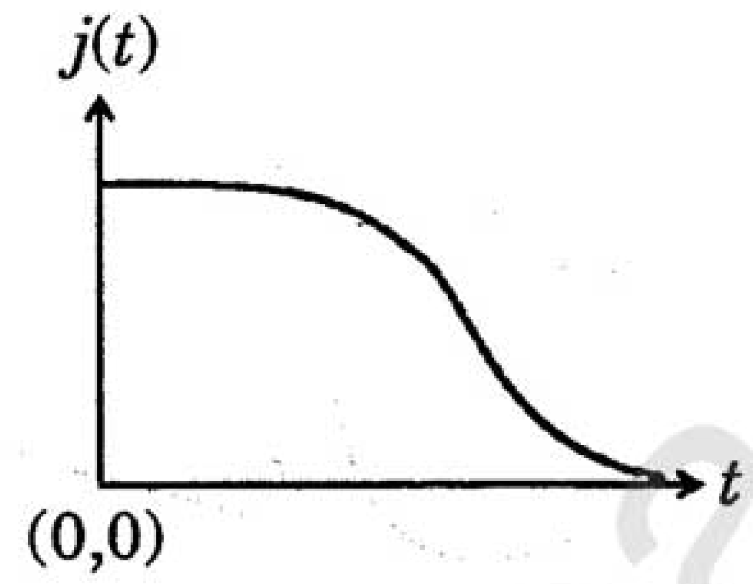

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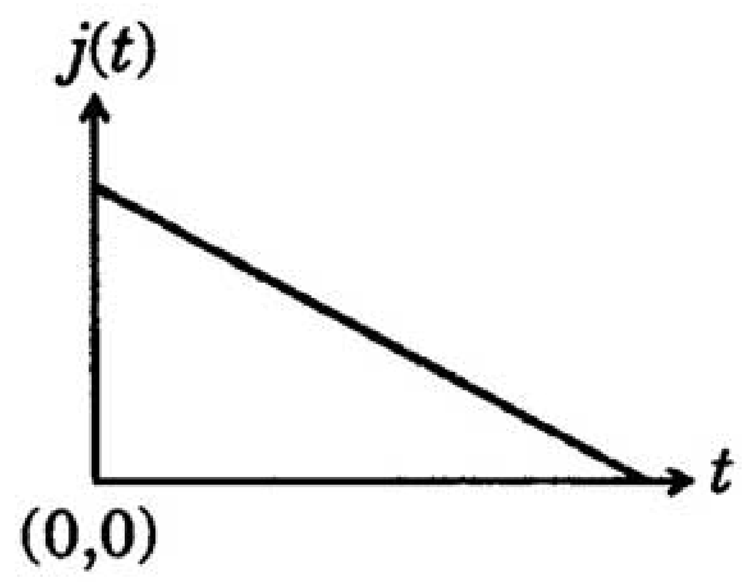

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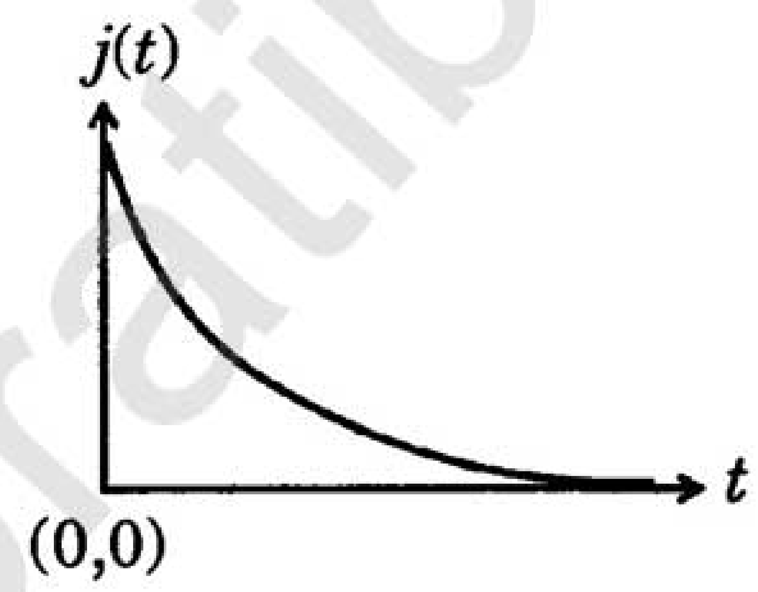


Space for rough work

SECTION 2 (Maximum Marks: 32)

- This section contains **EIGHT** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks	:	+4	If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.
Partial Marks	:	+1	For darkening a bubble corresponding **to each correct option**, provided NO incorrect option is darkened.
Zero Marks	:	0	If none of the bubbles is darkened.
Negative Marks	:	−2	In all other cases.

- For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (A) and (D) will result in +2 marks; and darkening (A) and (B) will result in −2 marks, as a wrong option is also darkened.

Q.6 A plano-convex lens is made of a material of refractive index n. When a small object is placed 30 cm away in front of the curved surface of the lens, an image of double the size of the object is produced. Due to reflection from the convex surface of the lens, another faint image is observed at a distance of 10 cm away from the lens. Which of the following statement(s) is(are) true?

(A) The refractive index of the lens is 2.5

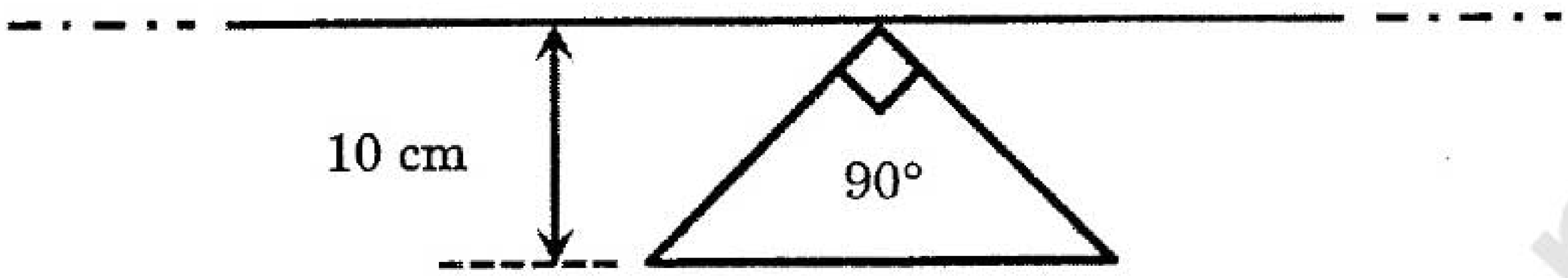
(B) The radius of curvature of the convex surface is 45 cm

(C) The faint image is erect and real

(D) The focal length of the lens is 20 cm

Space for rough work

Q.7 A conducting loop in the shape of a right angled isosceles triangle of height 10 cm is kept such that the 90° vertex is very close to an infinitely long conducting wire (see the figure). The wire is electrically insulated from the loop. The hypotenuse of the triangle is parallel to the wire. The current in the triangular loop is in counterclockwise direction and increased at a constant rate of 10 $\mathrm{A\,s^{-1}}$. Which of the following statement(s) is(are) true?



(A) There is a repulsive force between the wire and the loop

(B) If the loop is rotated at a constant angular speed about the wire, an additional *emf* of $\left(\frac{\mu_0}{\pi}\right)$ volt is induced in the wire

(C) The magnitude of induced *emf* in the wire is $\left(\frac{\mu_0}{\pi}\right)$ volt

(D) The induced current in the wire is in opposite direction to the current along the hypotenuse

Space for rough work

Q.8 The position vector $\vec{r}$ of a particle of mass m is given by the following equation

$$\vec{r}(t)=\alpha t^3\,\hat{i}+\beta t^2\,\hat{j},$$

where $\alpha=10/3\ \mathrm{m\,s^{-3}}$, $\beta=5\ \mathrm{m\,s^{-2}}$ and $m=0.1$ kg. At $t=1$ s, which of the following statement(s) is(are) true about the particle?

(A) The velocity $\vec{v}$ is given by $\vec{v}=(10\hat{i}+10\hat{j})\ \mathrm{m\,s^{-1}}$

(B) The angular momentum $\vec{L}$ with respect to the origin is given by $\vec{L}=-(5/3)\hat{k}$ N m s

(C) The force $\vec{F}$ is given by $\vec{F}=(\hat{i}+2\hat{j})$N

(D) The torque $\vec{\tau}$ with respect to the origin is given by $\vec{\tau}=-(20/3)\hat{k}$ N m

Q.9 A length-scale (l) depends on the permittivity (ε) of a dielectric material, Boltzmann constant (k_B), the absolute temperature (T), the number per unit volume (n) of certain charged particles, and the charge (q) carried by each of the particles. Which of the following expression(s) for l is(are) dimensionally correct?

(A) $l=\sqrt{\left(\dfrac{nq^2}{\varepsilon k_B T}\right)}$

(B) $l=\sqrt{\left(\dfrac{\varepsilon k_B T}{nq^2}\right)}$

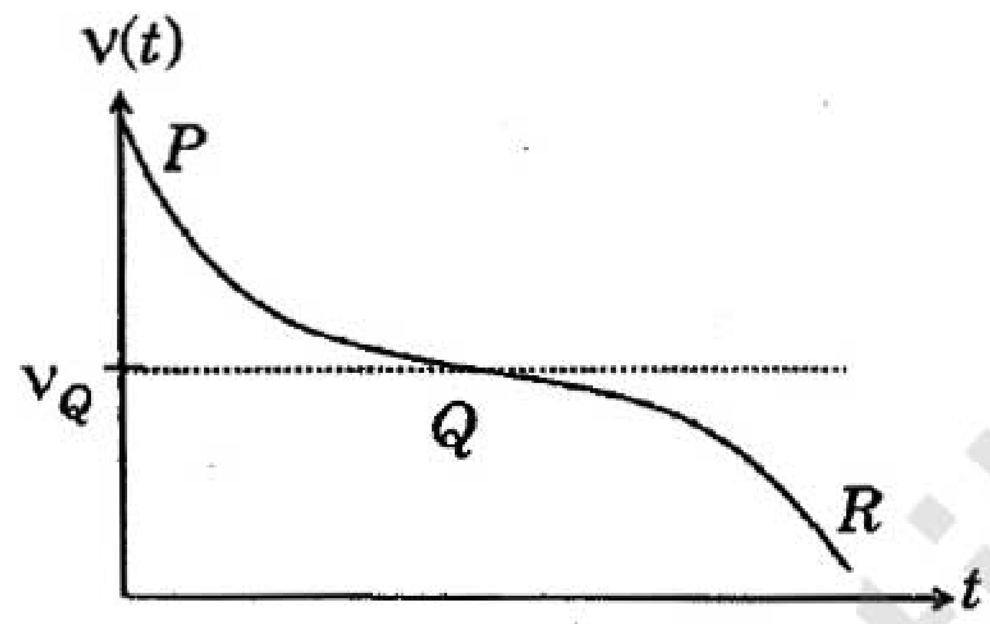
(C) $l=\sqrt{\left(\dfrac{q^2}{\varepsilon n^{2/3} k_B T}\right)}$

(D) $l=\sqrt{\left(\dfrac{q^2}{\varepsilon n^{1/3} k_B T}\right)}$

Space for rough work

Q.10 Two loudspeakers M and N are located 20 m apart and emit sound at frequencies 118 Hz and 121 Hz, respectively. A car is initially at a point P, 1800 m away from the midpoint Q of the line MN and moves towards Q constantly at 60 km/hr along the perpendicular bisector of MN. It crosses Q and eventually reaches a point R, 1800 m away from Q. Let $\nu(t)$ represent the beat frequency measured by a person sitting in the car at time t. Let ν_P, ν_Q and ν_R be the beat frequencies measured at locations P, Q and R, respectively. The speed of sound in air is 330 m s^{-1}. Which of the following statement(s) is(are) true regarding the sound heard by the person?

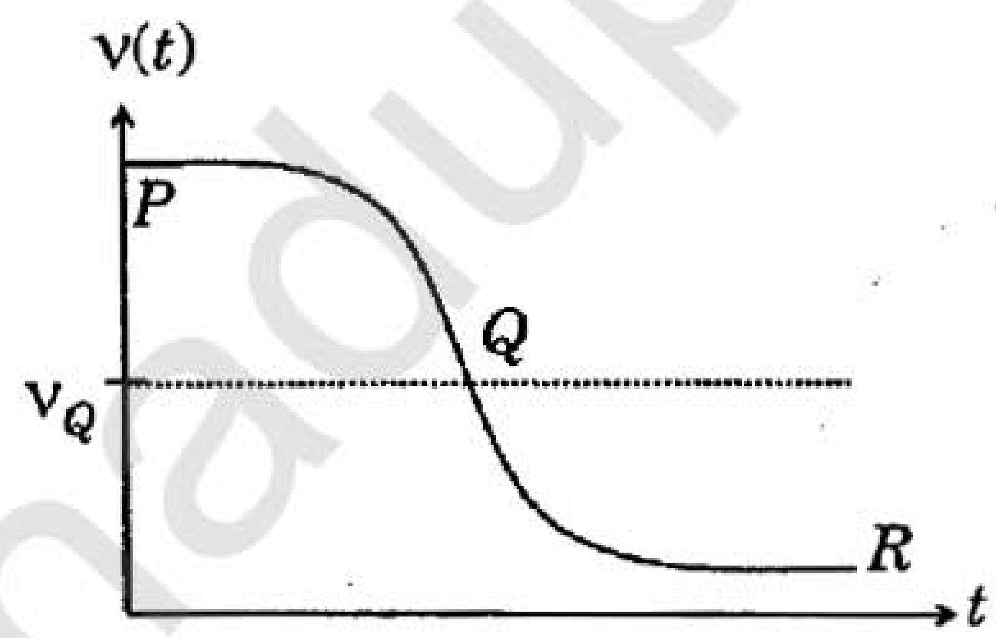
(A) The plot below represents schematically the variation of beat frequency with time



(B) The rate of change in beat frequency is maximum when the car passes through Q

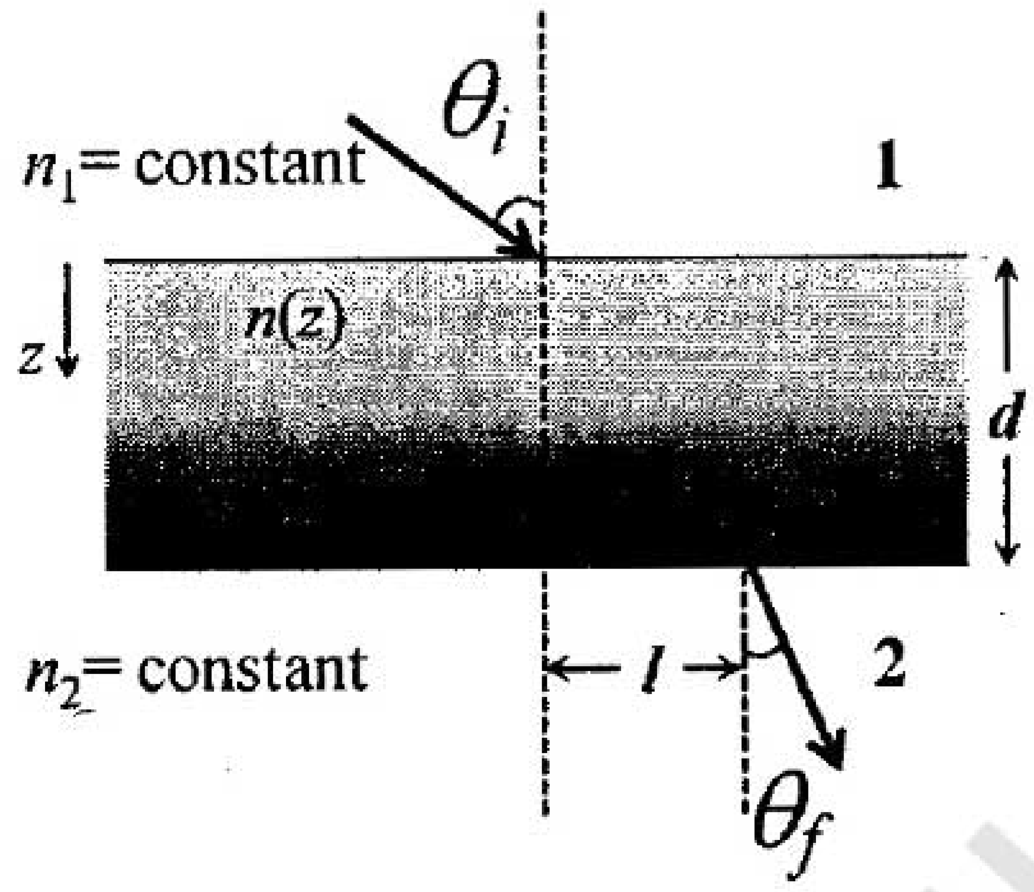
(C) $\nu_P + \nu_R = 2\,\nu_Q$

(D) The plot below represents schematically the variation of beat frequency with time



Space for rough work

Q.11 A transparent slab of thickness d has a refractive index $n(z)$ that increases with z. Here z is the vertical distance inside the slab, measured from the top. The slab is placed between two media with uniform refractive indices n_1 and $n_2 (> n_1)$, as shown in the figure. A ray of light is incident with angle θ_i from medium 1 and emerges in medium 2 with refraction angle θ_f with a lateral displacement l.



Which of the following statement(s) is(are) true?

(A) l is dependent on $n(z)$

(B) $n_1 \sin\theta_i = (n_2 - n_1)\sin\theta_f$

(C) $n_1 \sin\theta_i = n_2 \sin\theta_f$

(D) l is independent of n_2

Space for rough work

Q.12 Highly excited states for hydrogen-like atoms (also called Rydberg states) with nuclear charge Ze are defined by their principal quantum number n, where $n >> 1$. Which of the following statement(s) is(are) true?

(A) Relative change in the radii of two consecutive orbitals does not depend on Z

(B) Relative change in the radii of two consecutive orbitals varies as $1/n$

(C) Relative change in the energy of two consecutive orbitals varies as $1/n^3$

(D) Relative change in the angular momenta of two consecutive orbitals varies as $1/n$

Space for rough work

Q.13 An incandescent bulb has a thin filament of tungsten that is heated to high temperature by passing an electric current. The hot filament emits black-body radiation. The filament is observed to break up at random locations after a sufficiently long time of operation due to non-uniform evaporation of tungsten from the filament. If the bulb is powered at constant voltage, which of the following statement(s) is(are) true?

(A) The temperature distribution over the filament is uniform

(B) The resistance over small sections of the filament decreases with time

(C) The filament emits more light at higher band of frequencies before it breaks up

(D) The filament consumes less electrical power towards the end of the life of the bulb

Space for rough work

SECTION 3 (Maximum Marks: 15)

- This section contains **FIVE** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of the following categories:

  Full Marks : +3 If only the bubble corresponding to the correct answer is darkened.

  Zero Marks : 0 In all other cases.

Q.14 A hydrogen atom in its ground state is irradiated by light of wavelength 970 Å. Taking $hc/e = 1.237 \times 10^{-6}$ eV m and the ground state energy of hydrogen atom as −13.6 eV, the number of lines present in the emission spectrum is

Q.15 The isotope ${}^{12}_{5}\mathrm{B}$ having a mass 12.014 u undergoes β-decay to ${}^{12}_{6}\mathrm{C}$. ${}^{12}_{6}\mathrm{C}$ has an excited state of the nucleus (${}^{12}_{6}\mathrm{C}^*$) at 4.041 MeV above its ground state. If ${}^{12}_{5}\mathrm{B}$ decays to ${}^{12}_{6}\mathrm{C}^*$, the maximum kinetic energy of the β-particle in units of MeV is
(1 u = 931.5 MeV/c^2, where c is the speed of light in vacuum).

Space for rough work

Q.16 Consider two solid spheres P and Q each of density 8 gm cm^{-3} and diameters 1 cm and 0.5 cm, respectively. Sphere P is dropped into a liquid of density 0.8 gm cm^{-3} and viscosity $\eta = 3$ poiseulles. Sphere Q is dropped into a liquid of density 1.6 gm cm^{-3} and viscosity $\eta = 2$ poiseulles. The ratio of the terminal velocities of P and Q is

Q.17 Two inductors L_1 (inductance 1 mH, internal resistance 3 Ω) and L_2 (inductance 2 mH, internal resistance 4 Ω), and a resistor R (resistance 12 Ω) are all connected in parallel across a 5 V battery. The circuit is switched on at time $t = 0$. The ratio of the maximum to the minimum current (I_{max} / I_{min}) drawn from the battery is

Q.18 A metal is heated in a furnace where a sensor is kept above the metal surface to read the power radiated (P) by the metal. The sensor has a scale that displays $\log_2 (P/P_0)$, where P_0 is a constant. When the metal surface is at a temperature of 487 °C, the sensor shows a value 1. Assume that the emissivity of the metallic surface remains constant. What is the value displayed by the sensor when the temperature of the metal surface is raised to 2767 °C?

END OF PART I : PHYSICS

Space for rough work

PART II : CHEMISTRY

SECTION 1 (Maximum Marks: 15)

- This section contains **FIVE** questions.
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  Zero Marks : 0 If none of the bubbles is darkened.
  Negative Marks : –1 In all other cases.

Q.19 The increasing order of atomic radii of the following Group 13 elements is

(A) Al < Ga < In < Tl
(B) Ga < Al < In < Tl
(C) Al < In < Ga < Tl
(D) Al < Ga < Tl < In

Q.20 Among $[Ni(CO)_4]$, $[NiCl_4]^{2-}$, $[Co(NH_3)_4Cl_2]Cl$, $Na_3[CoF_6]$, Na_2O_2 and CsO_2, the total number of paramagnetic compounds is

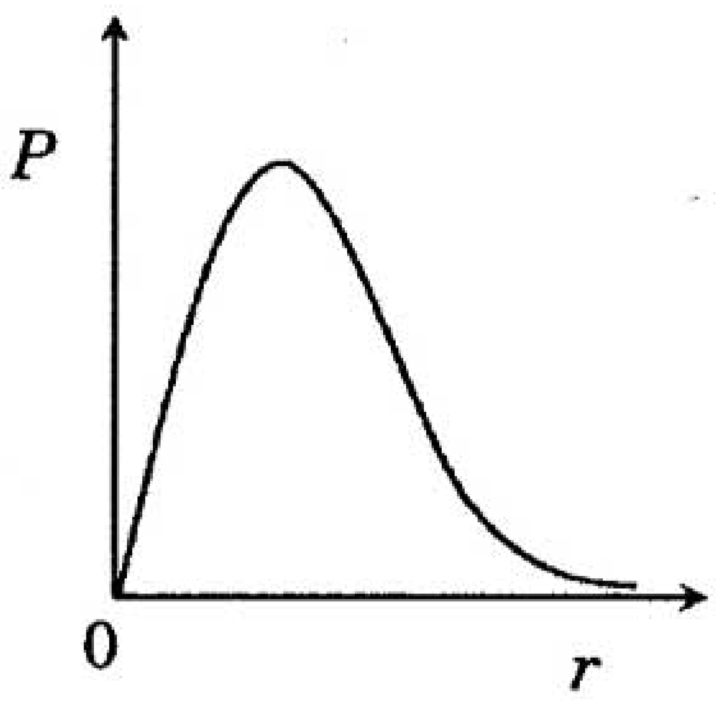
(A) 2
(B) 3
(C) 4
(D) 5

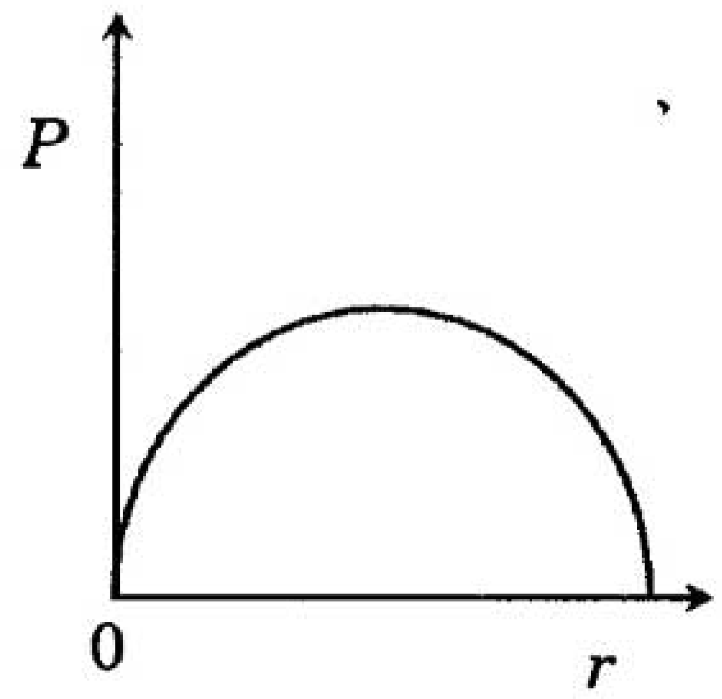
Q.21 On complete hydrogenation, natural rubber produces

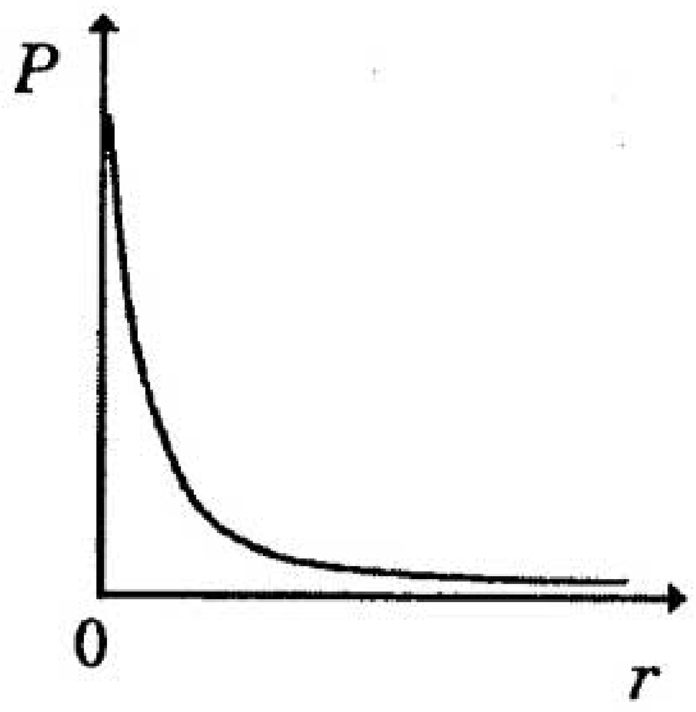
(A) ethylene-propylene copolymer
(B) vulcanised rubber
(C) polypropylene
(D) polybutylene

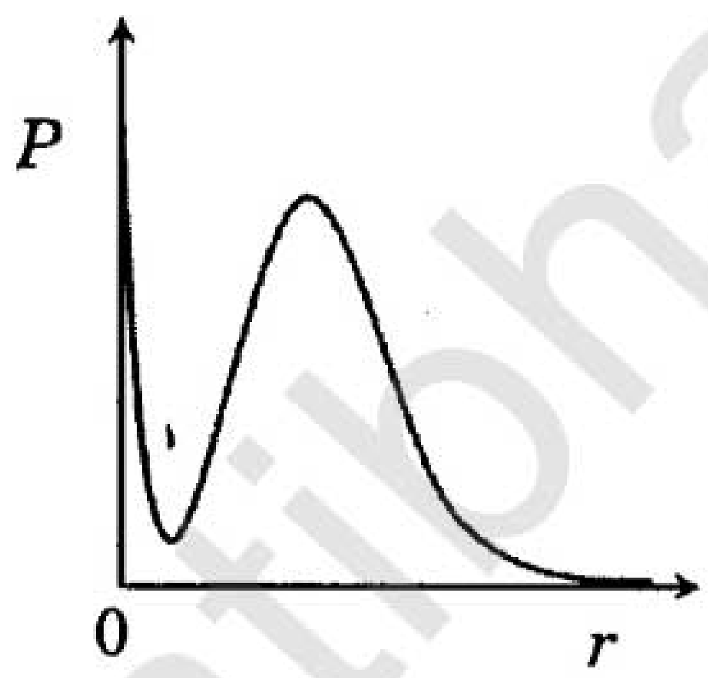
Space for rough work

Q.22 P is the probability of finding the 1s electron of hydrogen atom in a spherical shell of infinitesimal thickness, dr, at a distance r from the nucleus. The volume of this shell is $4\pi r^2 dr$. The qualitative sketch of the dependence of P on r is

(A) 

(B) 

(C) 

(D) 

Q.23 One mole of an ideal gas at 300 K in thermal contact with surroundings expands isothermally from 1.0 L to 2.0 L against a constant pressure of 3.0 atm. In this process, the change in entropy of surroundings (ΔS_{surr}) in J K^{-1} is

(1 L atm = 101.3 J)

(A) 5.763 (B) 1.013 (C) −1.013 (D) −5.763

Space for rough work

SECTION 2 (Maximum Marks: 32)

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  Partial Marks : +1 For darkening a bubble corresponding **to each correct option**, provided NO incorrect option is darkened.

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Q.24 The correct statement(s) about the following reaction sequence is(are)

$$\text{Cumene } (C_9H_{12}) \xrightarrow[\text{ii) } H_3O^+]{\text{i) } O_2} \mathbf{P} \xrightarrow{CHCl_3/NaOH} \mathbf{Q}\text{ (major)} + \mathbf{R}\text{ (minor)}$$

$$\mathbf{Q} \xrightarrow[PhCH_2Br]{NaOH} \mathbf{S}$$

(A) **R** is steam volatile

(B) **Q** gives dark violet coloration with 1% aqueous $FeCl_3$ solution

(C) **S** gives yellow precipitate with 2, 4-dinitrophenylhydrazine

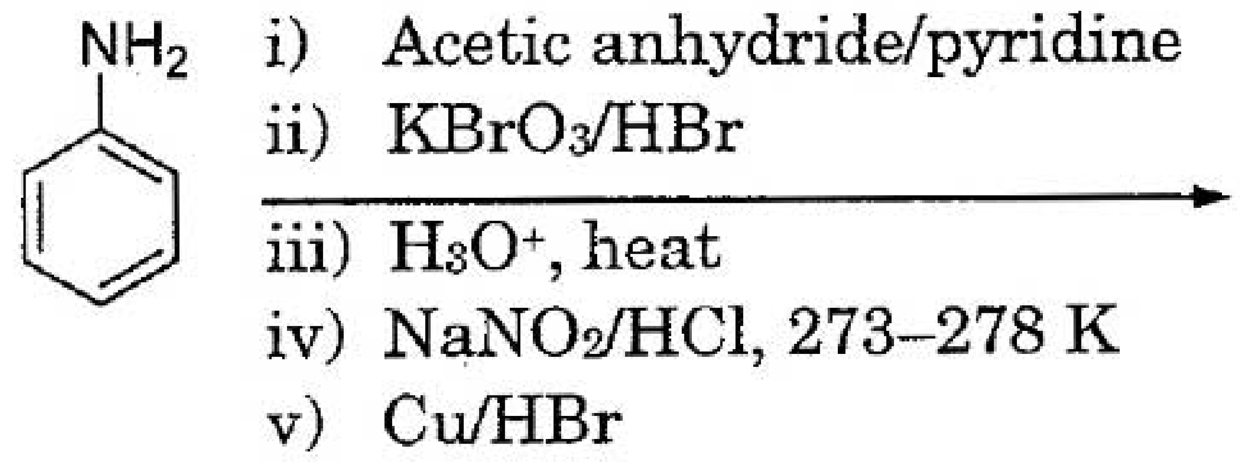
(D) **S** gives dark violet coloration with 1% aqueous $FeCl_3$ solution

Space for rough work

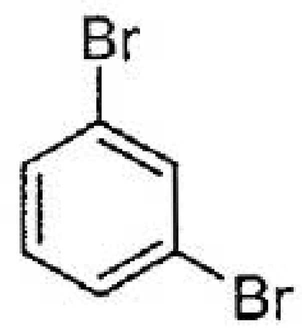
Q.25 The compound(s) with TWO lone pairs of electrons on the central atom is(are)

(A) BrF_5 (B) ClF_3 (C) XeF_4 (D) SF_4

Q.26 The product(s) of the following reaction sequence is(are)



(A)



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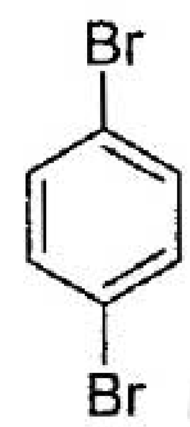

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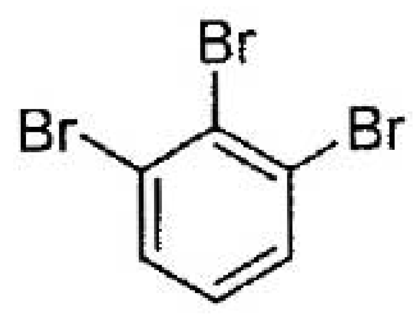


(D)

Space for rough work

Q.27 According to the Arrhenius equation,

(A) a high activation energy usually implies a fast reaction.

(B) rate constant increases with increase in temperature. This is due to a greater number of collisions whose energy exceeds the activation energy.

(C) higher the magnitude of activation energy, stronger is the temperature dependence of the rate constant.

(D) the pre-exponential factor is a measure of the rate at which collisions occur, irrespective of their energy.

Q.28 The crystalline form of borax has

(A) tetranuclear $[B_4O_5(OH)_4]^{2-}$ unit

(B) all boron atoms in the same plane

(C) equal number of sp^2 and sp^3 hybridized boron atoms

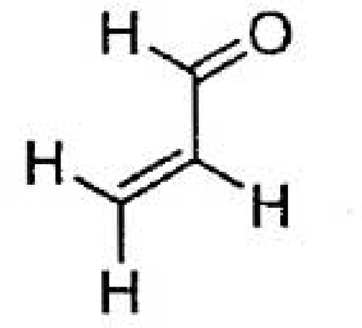
(D) one terminal hydroxide per boron atom

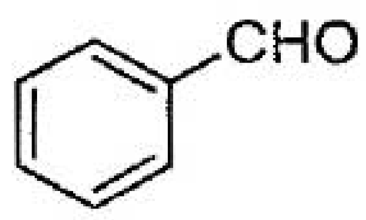
Q.29 The reagent(s) that can selectively precipitate S^{2-} from a mixture of S^{2-} and SO_4^{2-} in aqueous solution is(are)

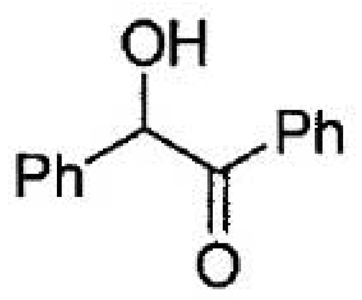
(A) $CuCl_2$ (B) $BaCl_2$ (C) $Pb(OOCCH_3)_2$ (D) $Na_2[Fe(CN)_5NO]$

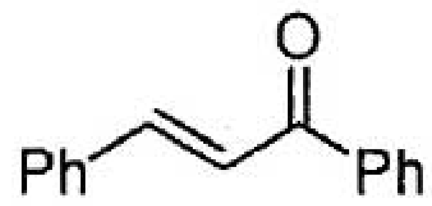
Space for rough work

Q.30 Positive Tollen's test is observed for

(A) 

(B) 

(C) 

(D) 

Q.31 A plot of the number of neutrons (N) against the number of protons (P) of stable nuclei exhibits upward deviation from linearity for atomic number, $Z > 20$. For an unstable nucleus having N/P ratio less than 1, the possible mode(s) of decay is(are)

(A) β^--decay (β emission)

(B) orbital or K-electron capture

(C) neutron emission

(D) β^+-decay (positron emission)

Space for rough work

SECTION 3 (Maximum Marks: 15)

- This section contains **FIVE** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in <u>one of the following categories</u>:
  Full Marks : +3 If only the bubble corresponding to the correct answer is darkened.
  Zero Marks : 0 In all other cases.

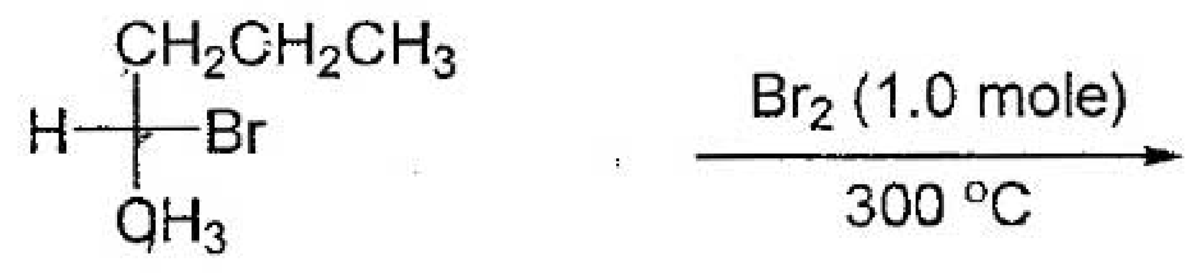
Q.32 The diffusion coefficient of an ideal gas is proportional to its mean free path and mean speed. The absolute temperature of an ideal gas is increased 4 times and its pressure is increased 2 times. As a result, the diffusion coefficient of this gas increases x times. The value of x is

Q.33 The number of geometric isomers possible for the complex $[CoL_2Cl_2]^-$ $(L = H_2NCH_2CH_2O^-)$ is

Q.34 The mole fraction of a solute in a solution is 0.1. At 298 K, molarity of this solution is the same as its molality. Density of this solution at 298 K is 2.0 g cm^{-3}. The ratio of the molecular weights of the solute and solvent, $\left(\frac{MW_{solute}}{MW_{solvent}}\right)$, is

Space for rough work

Q.35 In the following monobromination reaction, the number of possible chiral products is



(1.0 mole)

(enantiomerically pure)

Q.36 In neutral or faintly alkaline solution, 8 moles of permanganate anion quantitatively oxidize thiosulphate anions to produce **X** moles of a sulphur containing product. The magnitude of **X** is

END OF PART II : CHEMISTRY

Space for rough work

PART III : MATHEMATICS

SECTION 1 (Maximum Marks: 15)

- This section contains **FIVE** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks : +3 If only the bubble corresponding to the correct option is darkened.
Zero Marks : 0 If none of the bubbles is darkened.
Negative Marks : –1 In all other cases.

Q.37 A computer producing factory has only two plants T_1 and T_2. Plant T_1 produces 20% and plant T_2 produces 80% of the total computers produced. 7% of computers produced in the factory turn out to be defective. It is known that

P (computer turns out to be defective given that it is produced in plant T_1)

$=10P$ (computer turns out to be defective given that it is produced in plant T_2),

where $P(E)$ denotes the probability of an event E. A computer produced in the factory is randomly selected and it does not turn out to be defective. Then the probability that it is produced in plant T_2 is

(A) $\frac{36}{73}$ (B) $\frac{47}{79}$ (C) $\frac{78}{93}$ (D) $\frac{75}{83}$

Q.38 A debate club consists of 6 girls and 4 boys. A team of 4 members is to be selected from this club including the selection of a captain (from among these 4 members) for the team. If the team has to include at most one boy, then the number of ways of selecting the team is

(A) 380 (B) 320 (C) 260 (D) 95

Space for rough work

Q.39 Let $-\frac{\pi}{6} < \theta < -\frac{\pi}{12}$. Suppose α_1 and β_1 are the roots of the equation $x^2 - 2x\sec\theta + 1 = 0$ and α_2 and β_2 are the roots of the equation $x^2 + 2x\tan\theta - 1 = 0$. If $\alpha_1 > \beta_1$ and $\alpha_2 > \beta_2$, then $\alpha_1 + \beta_2$ equals

(A) $2(\sec\theta - \tan\theta)$ (B) $2\sec\theta$

(C) $-2\tan\theta$ (D) 0

Q.40 Let $S = \left\{x \in (-\pi, \pi) : x \neq 0, \pm\frac{\pi}{2}\right\}$. The sum of all distinct solutions of the equation $\sqrt{3}\sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0$ in the set S is equal to

(A) $-\frac{7\pi}{9}$ (B) $-\frac{2\pi}{9}$ (C) 0 (D) $\frac{5\pi}{9}$

Q.41 The least value of $\alpha \in \mathbb{R}$ for which $4\alpha x^2 + \frac{1}{x} \geq 1$, for all $x > 0$, is

(A) $\frac{1}{64}$ (B) $\frac{1}{32}$ (C) $\frac{1}{27}$ (D) $\frac{1}{25}$

Space for rough work

SECTION 2 (Maximum Marks: 32)

- This section contains **EIGHT** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks	: +4	If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.
Partial Marks	: +1	For darkening a bubble corresponding **to each correct option**, provided NO incorrect option is darkened.
Zero Marks	: 0	If none of the bubbles is darkened.
Negative Marks	: –2	In all other cases.

- For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (A) and (D) will result in +2 marks; and darkening (A) and (B) will result in –2 marks, as a wrong option is also darkened.

Q.42 A solution curve of the differential equation $(x^2+xy+4x+2y+4)\frac{dy}{dx}-y^2=0$, $x>0$, passes through the point $(1,3)$. Then the solution curve

(A) intersects $y=x+2$ exactly at one point

(B) intersects $y=x+2$ exactly at two points

(C) intersects $y=(x+2)^2$

(D) does **NOT** intersect $y=(x+3)^2$

Space for rough work

Q.43 Consider a pyramid $OPQRS$ located in the first octant $(x \geq 0, y \geq 0, z \geq 0)$ with O as origin, and OP and OR along the x-axis and the y-axis, respectively. The base $OPQR$ of the pyramid is a square with $OP = 3$. The point S is directly above the mid-point T of diagonal OQ such that $TS = 3$. Then

(A) the acute angle between OQ and OS is $\frac{\pi}{3}$

(B) the equation of the plane containing the triangle OQS is $x - y = 0$

(C) the length of the perpendicular from P to the plane containing the triangle OQS is $\frac{3}{\sqrt{2}}$

(D) the perpendicular distance from O to the straight line containing RS is $\sqrt{\frac{15}{2}}$

Q.44 The circle $C_1 : x^2 + y^2 = 3$, with centre at O, intersects the parabola $x^2 = 2y$ at the point P in the first quadrant. Let the tangent to the circle C_1 at P touches other two circles C_2 and C_3 at R_2 and R_3, respectively. Suppose C_2 and C_3 have equal radii $2\sqrt{3}$ and centres Q_2 and Q_3, respectively. If Q_2 and Q_3 lie on the y-axis, then

(A) $Q_2Q_3 = 12$

(B) $R_2R_3 = 4\sqrt{6}$

(C) area of the triangle OR_2R_3 is $6\sqrt{2}$

(D) area of the triangle PQ_2Q_3 is $4\sqrt{2}$

Space for rough work

Q.45 Let $f:\mathbb{R}\to\mathbb{R}$, $g:\mathbb{R}\to\mathbb{R}$ and $h:\mathbb{R}\to\mathbb{R}$ be differentiable functions such that $f(x)=x^3+3x+2$, $g(f(x))=x$ and $h(g(g(x)))=x$ for all $x\in\mathbb{R}$. Then

(A) $g'(2)=\frac{1}{15}$

(B) $h'(1)=666$

(C) $h(0)=16$

(D) $h(g(3))=36$

Q.46 Let $P=\begin{bmatrix}3 & -1 & -2\\ 2 & 0 & \alpha\\ 3 & -5 & 0\end{bmatrix}$, where $\alpha\in\mathbb{R}$. Suppose $Q=[q_{ij}]$ is a matrix such that $PQ=kI$, where $k\in\mathbb{R}$, $k\neq 0$ and I is the identity matrix of order 3. If $q_{23}=-\frac{k}{8}$ and $\det(Q)=\frac{k^2}{2}$, then

(A) $\alpha=0, k=8$

(B) $4\alpha-k+8=0$

(C) $\det(P\,\mathrm{adj}(Q))=2^9$

(D) $\det(Q\,\mathrm{adj}(P))=2^{13}$

Q.47 Let RS be the diameter of the circle $x^2+y^2=1$, where S is the point $(1,0)$. Let P be a variable point (other than R and S) on the circle and tangents to the circle at S and P meet at the point Q. The normal to the circle at P intersects a line drawn through Q parallel to RS at point E. Then the locus of E passes through the point(s)

(A) $\left(\frac{1}{3},\frac{1}{\sqrt{3}}\right)$

(B) $\left(\frac{1}{4},\frac{1}{2}\right)$

(C) $\left(\frac{1}{3},-\frac{1}{\sqrt{3}}\right)$

(D) $\left(\frac{1}{4},-\frac{1}{2}\right)$

Space for rough work

Q.48 Let $f:(0,\infty)\rightarrow \mathbb{R}$ be a differentiable function such that $f'(x)=2-\dfrac{f(x)}{x}$ for all $x\in(0,\infty)$ and $f(1)\neq 1$. Then

(A) $\lim\limits_{x\to 0+} f'\left(\dfrac{1}{x}\right)=1$

(B) $\lim\limits_{x\to 0+} x f\left(\dfrac{1}{x}\right)=2$

(C) $\lim\limits_{x\to 0+} x^2 f'(x)=0$

(D) $|f(x)|\leq 2$ for all $x\in(0,2)$

Q.49 In a triangle XYZ, let x, y, z be the lengths of sides opposite to the angles X, Y, Z, respectively, and $2s=x+y+z$. If $\dfrac{s-x}{4}=\dfrac{s-y}{3}=\dfrac{s-z}{2}$ and area of incircle of the triangle XYZ is $\dfrac{8\pi}{3}$, then

(A) area of the triangle XYZ is $6\sqrt{6}$

(B) the radius of circumcircle of the triangle XYZ is $\dfrac{35}{6}\sqrt{6}$

(C) $\sin\dfrac{X}{2}\sin\dfrac{Y}{2}\sin\dfrac{Z}{2}=\dfrac{4}{35}$

(D) $\sin^2\left(\dfrac{X+Y}{2}\right)=\dfrac{3}{5}$

Space for rough work

SECTION 3 (Maximum Marks: 15)

- This section contains **FIVE** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of the following categories:
  Full Marks : +3 If only the bubble corresponding to the correct answer is darkened.
  Zero Marks : 0 In all other cases.

Q.50 Let m be the smallest positive integer such that the coefficient of x^2 in the expansion of $(1+x)^2+(1+x)^3+\cdots+(1+x)^{49}+(1+mx)^{50}$ is $(3n+1)\ {}^{51}C_3$ for some positive integer n. Then the value of n is

Q.51 Let $\alpha, \beta \in \mathbb{R}$ be such that $\lim\limits_{x\to 0}\dfrac{x^2\sin(\beta x)}{\alpha x-\sin x}=1$. Then $6(\alpha+\beta)$ equals

Space for rough work

Q.52 Let $z = \frac{-1+\sqrt{3}i}{2}$, where $i = \sqrt{-1}$, and $r, s \in \{1, 2, 3\}$. Let $P = \begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix}$ and I be the identity matrix of order 2. Then the total number of ordered pairs (r, s) for which $P^2 = -I$ is

Q.53 The total number of distinct $x \in \mathbb{R}$ for which $\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 10$ is

Q.54 The total number of distinct $x \in [0, 1]$ for which $\int_0^x \frac{t^2}{1+t^4} dt = 2x - 1$ is

END OF THE QUESTION PAPER

Space for rough work

CODE **0**

QUESTION PAPER FORMAT AND MARKING SCHEME

20. The question paper has three parts: Physics, Chemistry and Mathematics.

21. Each part has three sections as detailed in the following table:

Section	Question Type	Number of Questions	Category-wise Marks for Each Question				Maximum Marks of the Section
			Full Marks	Partial Marks	Zero Marks	Negative Marks	
1	Single Correct Option	5	+3 If only the bubble corresponding to the correct option is darkened	—	0 If none of the bubbles is darkened	−1 In all other cases	15
2	One or more correct option(s)	8	+4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened	+1 For darkening a bubble corresponding to each correct option, provided NO incorrect option is darkened	0 If none of the bubbles is darkened	−2 In all other cases	32
3	Single digit Integer (0-9)	5	+3 If only the bubble corresponding to the correct answer is darkened	—	0 In all other cases	—	15

NAME OF THE CANDIDATE ...

ROLL NO ......6052260.....................

I have read all the instructions and shall abide by them. --------------------------- Signature of the Candidate	I have verified the identity, name and roll number of the candidate, and that question paper and ORS codes are the same. --------------------------- Signature of the Invigilator

JEE (ADVANCED)-2016 'PAPER-1' KEY

Q.No.	Code-0	Code-1	Code-2	Code-3	Code-4	Code-5	Code-6	Code-7	Code-8	Code-9
1	A	B	B	A	D	D	A	B	A	A
2	B	D	D	B	A	B	B	B	B	B
3	B	B	B	B	D	B	B	A	B	D
4	D	A	D	D	B	A	A	D	D	B
5	D	C	A	A	B	C	B	B	B	A
6	A, D	A, B, D	B, C, D	B, C	A, B, D	C, D	A, B, D	A, B, C	B, C, D	A, B, D
7	A, C	A, B, C	B, D	B, D	B, D	A, B, D	A, B, C	C, D	A, D	B, D
8	A, B, D	C, D	C, D	A, C, D	A, B	B, D	A, D	C, D	A, B, D	A, D
9	B, D	A, D	A, B, D	A, D	B, C, D	B, C, D	A, B, D	A, B, D	C, D	C, D
10	B, C, D	B, D	A, D	A, B, D	A, B, D	B, D	B, D	B, D	B, D	B, D
11	A, C, D	A, D	B, C	A, B, C	A, B, D	A, B, D	A, B, D	B, C, D	A, B, D	A, B, D
12	A, B, D	A, B, D	A, B, D	C, D	C, D	A, D	B, C	A, D	A, B, D	A, B, D
13	C, D	A, C, D	A, B, D	A, B, D	A, D	A, B, D	C, D	A, B, D	A, D	A, C, D
14	6	9	9	9	8	9	8	6	9	3
15	9	9	9	8	6	3	9	3	3	8
16	3	6	3	3	9	9	6	9	9	9
17	8	3	6	9	9	8	3	9	8	6
18	9	8	8	6	3	6	9	8	6	9
19	B	D	A	B	B	C	C	A	A	B
20	B	C	B	B	A	B	C	C	B	C
21	A	B	C	A	C	B	B	B	C	B
22	A	B	C	C	B	A	B	A	D	A
23	C	A	B	B	A	B	A	B	B	B
24	B, C	B, C, D	A, B	A, C, D	B, C	B, D	B	B, D	A, C, D	B, D
25	B, C	B, D	B, C	B, D	B, C	A, C, D	B, C	B, C	A, B	B, C
26	B	A, C, D	B, D	A, B	A, C, D	A, C, D	A, C, D	A, C, D	B, C	A, B
27	B, C, D	B, C	A, C, D	B, C	A, C, D	A, B	A, C, D	B, C, D	B, D	B
28	A, C, D	A, C, D	A, C, D	A, C, D	A, B	B, C	B, D	B, C	B	B, C
29	A, C, D	A, B	B, C	B, C	B	B, C, D	A, B	B	B, C, D	A, C, D
30	A, B	B	B	B, C, D	B, C, D	B	B, C	A, B	A, C, D	A, C, D
31	B, D	B, C	B, C, D	B	B, D	B, C	B, C, D	A, C, D	B, C	B, C, D
32	4	9	5	4	4	5	5	5	5	9
33	5	4	6	5	5	4	9	5	9	5
34	9	6	9	5	5	6	5	6	5	4
35	5	5	4	9	6	5	6	4	4	5
36	6	5	5	6	9	9	4	9	6	6
37	C	C	C	C	A	C	C	C	C	A
38	A	A	C	C	C	C	A	C	C	C
39	C	C	A	C	C	C	C	A	A	C
40	C	C	C	C	C	C	C	C	C	C
41	C	C	C	A	C	A	C	C	C	C
42	D	B, C, D	A, C	A, C, D	C	A, C, D	A	A, C, D	B, C, D	D
43	B, C, D	A	A, B, C	D	B, C, D	B, C, D	A, B, C	B, C	A, B, C	B, C, D
44	A, B, C	B, C	B, C	B, C, D	A, C, D	D	D	D	A	A, C, D
45	B, C	A, C, D	D	A, C	D	A	A, C, D	A, C	B, C	A, C
46	B, C	D	B, C	B, C	B, C	B, C	A, C	A, B, C	A, C	A, B, C
47	A, C	B, C	A, C, D	A	B, C	A, B, C	B, C	B, C	D	B, C
48	A	A, B, C	B, C, D	B, C	A	A, C	B, C	B, C, D	A, C, D	A
49	A, C, D	A, C	A	A, B, C	A, C	B, C	B, C, D	A	B, C	B, C
50	5	2	1	1	1	1	1	1	1	5
51	7	5	2	5	7	5	5	7	7	2
52	1	1	7	1	1	7	1	1	5	1
53	2	7	5	7	5	2	2	2	2	1
54	1	1	1	2	2	1	7	5	1	7

JEE ADVANCED 2016

Paper 1 Question Paper with Solutions

CODE-0

READ THE INSTRUCTIONS CAREFULLY

GENERAL :

1. This sealed booklet is your Question Paper. Do not break the seal till you are told to do so.
2. The paper CODE is printed on the right hand top corner of this sheet and the right hand top corner of the back cover of this booklet.
3. Use the Optical Response Sheet (ORS) provided separately for answering the questions.
4. The paper CODE is printed on its left part as well as the right part of the ORS. Ensure that both these codes are identical and same as that on the question paper booklet. If not, contact the invigilator for change of ORS.
5. Blank spaces are provided within this booklet for rough work.
6. Write your name and roll number and sign in the space provided on the back cover of this booklet.
7. After breaking the seal of the booklet at 9:00 am, verify that the booklet contains **36** pages and that all the **54** questions along with the options are legible. If not, contact the invigilator for replacement of the booklet.
8. You are allowed to take away the Question Paper at the end of the examination.

OPTICAL RESPONSE SHEET :

9. The ORS (top sheet) will be provided with an attahced Candidate's Sheet (bottom sheet). The Candidate's Sheet is a carbon-less copy of the ORS.
10. Darken the appropriate bubbles on the ORS by applying sufficient pressure. This will leave an impression at the corresponding place on the Candidate's Sheet.
11. The ORS will be collected by the invigilator at the end of the examination.
12. You will be allowed to take away the Candidate's Sheet at the end of the examination.
13. Do not tamper with or mutilate the ORS. **Do not use the ORS for rough work**.
14. Write your name, roll number and code of the examination center and sign with pen in the space provided for this purpose on the ORS. **Do not write any of these details anywhere else** on the ORS. Darken the appropriate bubble under each digit of your roll number.

DARKENING THE BUBBLES ON THE ORS :

15. Use a **BLACK BALL POINT PEN** to darken the bubbles on the ORS.
16. Darken the bubble ○ **COMPLETELY.**
17. The correct way of darkening a bubble is as : ●
18. The ORS is machine-gradable. Ensure that the bubbles are darkened in the correct way.
19. Darken the bubbles **ONLY IF** you are sure of the answer. There is **NO WAY** to erase or "un-darken" a darkened bubble.

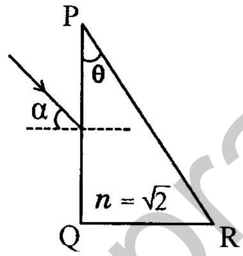
PART I : PHYSICS

SECTION 1 (Maximum Marks:15)

* This section contains FIVE questions.
* Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is correct.
* For each question, darken the bubble corresponding to the ccorrect option in the ORS.
* For each question, makrs will be awarded in one of the following categories:

Full marks : +3 If only the bubble corresponding to hte correct option is darkened.
Zero Marks : 0 If none of the bubbles is darkened.
Negative Marks : -1 In all other cases.

1. A parallel beam of light is incident from air at an angle α on the side PQ of a right angled triangular prism of refractive index $n = \sqrt{2}.$ Light undergoes total internal reflection in the prism at the face PR when α has a minimum value of 45^0. The angle θ of the prism is



A) 15^0 B) 22.5^0 C) 30^0 D) 45^0

Key : A

Sol : In $\triangle^{le}$ PAB

$\theta + 90 + r + 45 = 180$

$\Rightarrow r = 45 - \theta$

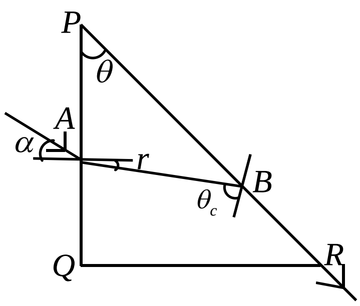
At A, Snell's law gives

$\sin\alpha = \sqrt{2}\sin(45-\theta)$

as $\alpha = 45$

$\Rightarrow \frac{1}{\sqrt{2}} = \sqrt{2}\sin(45-\theta)$

$\Rightarrow \theta = 15^0$



2. In a historical experiment to determine Planck's constant, a metal surface was irradiated with light of different wavelengths. The emitted photoelectron energies were measured by applying a stopping potential. The relevant data for the wavelength (λ) of incident light and the corresponding stopping potential (V_0) are given below:

$\lambda(\mu m)$	$V_0(Volt)$
0.3	2.0
0.4	1.0
0.5	0.4

Given that $c = 3\times10^{8} ms^{-1}$ and $e = 1.6\times10^{-19} C,$ Planck's constant (in units of J s) found from such an experiment is

A) 6.0×10^{-34} B) 6.4×10^{-34} C) 6.6×10^{-34} D) 6.8×10^{-34}

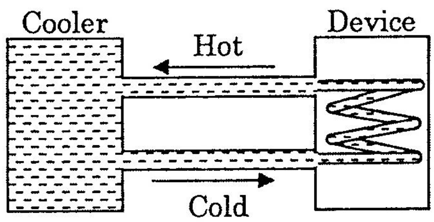
Key : B

Sol : Slope of V_0 $vs\frac{1}{\lambda}$ graph

is $\frac{hc}{e}$

$$\frac{2-1}{\left(\frac{1}{0.3}-\frac{1}{0.4}\right)\times10^{6}} = 12\times10^{-7} = \frac{hc}{e} \Rightarrow h = 6.4\times10^{-34}$$

3. A water cooler of storage capacity 120 litres can cool water at a constant rate of P watts. In a closed circulation system (as shown schematically in the figure), the water from the cooler is used to cool an external device that generates constantly 3kW of heat (thermal load). The temperature of water fed into the device cannot exceed $30^{0}C$ and the entire stored 120 litres of water is initially cooled to $10^{0}C.$ The entire system is thermally insulated. The minimum value of P (in watts) for which the deevice can be operated for 3 hours is



(Specific heat of water is $4.2kJ\,kg^{-1}K^{-1}$ and the density of water is $1000\,km\,m^{-3}$)

A) 1600 B) 2067 C) 2533 D) 3933

Key : B

Sol : $mc\frac{d\theta}{dt} = 3000 - P$

$$\frac{120\times4.2\times10^{3}\times20}{3\times60\times60} = 3000 - P$$

$\Rightarrow P = 2067\ W$

4. **A uniform wooden stick of mass 1.6 kg and length l rests in an inclined manner on a smooth, vertical wall of height $h(<l)$ such that a small portion of the stick extends beyond the wall. The reaction force of the wall on the stick is perpendicular to the stick. The stick makes an angle of 30^0 with the wall and the bottom of the stick is on a rough floor. The reaction of the wall on the stick is equal in magnitude to the reaction of the floor on the stick. The ratio h/l and the frictional force f at the bottom of th stick are $\left(g = 10ms^{-2}\right)$**

A) $\frac{h}{l} = \frac{\sqrt{3}}{16}, f = \frac{16\sqrt{3}}{3}N$

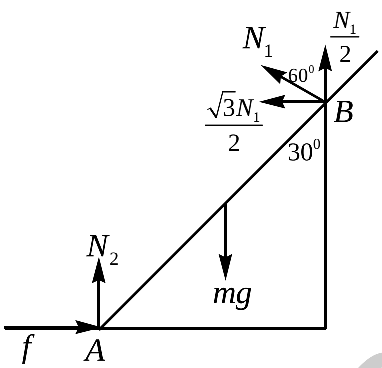
B) $\frac{h}{l} = \frac{3}{16}, f = \frac{16\sqrt{3}}{3}N$

C) $\frac{h}{l} = \frac{3\sqrt{3}}{16}, f = \frac{8\sqrt{3}}{3}N$

D) $\frac{h}{l} = \frac{3\sqrt{3}}{16}, f = \frac{16\sqrt{3}}{3}N$

Key : D

Sol :



Given $N_1 = N_2$ (1)

Translational equilibrium

$$mg = N_2 + \frac{N_1}{2} \quad (2)$$

$$f = \frac{N_1\sqrt{3}}{2} \quad (3)$$

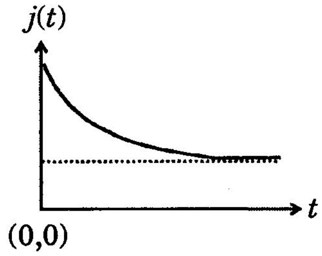
From (1) and (2) $N_1 = \frac{32}{3} \Rightarrow f = \frac{N_1\sqrt{3}}{2} = \frac{16\sqrt{3}}{3}N$

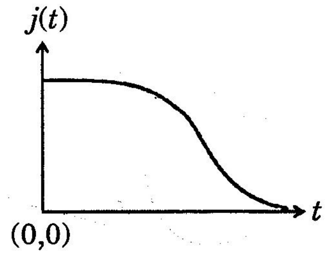
and Torque about A = 0

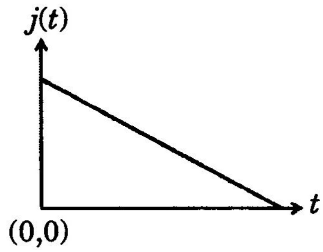
$$\Rightarrow N_1 \frac{h}{\cos 30^0} = mg \sin 30^0 \times \frac{l}{2}$$

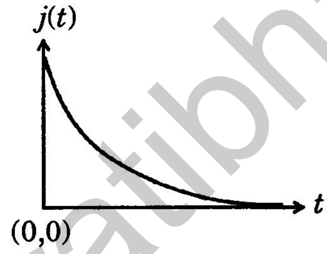
$$\Rightarrow \frac{h}{l} = \frac{2\sqrt{3}}{N_1} = \frac{3\sqrt{3}}{16}$$

5. **An infinte line charge of uniform electric charge density λ lies along the axis of an electrically conducting infinite cylindrical shell of radius R. At time t = 0, the space inside the cylinder is filled with a material of permittivity ε and electrical conductivity σ. The electrical conduction in the material follows Ohm's law. Which one of the following graphs best describes the subsequent variation of the magnitude of current density $j(t)$ at any point in the material?**

A)


B)


C)


D)


Key : D

Sol : It is like an RC circuit with $\tau = RC = \sigma\varepsilon$

$q = q_0 e^{-t/\tau}$

$\therefore i = \frac{-q_0}{\tau} e^{-t/\tau}$ & $j = \frac{i}{A} = \frac{-q_0}{\tau A} e^{-t/\tau}$

$\therefore$ with time its an exponential decay.

SECTION 2 (Maximum Marks:32)

- **This section contains EIGHT questions.**
- **Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is (ARE) correct.**
- **For each question, darken the bubble(S) corresponding to all the ccorrect option(s) in the ORS.**
- **For each question, makrs will be awarded in one of the following categories:**

Full marks : +4 If only the bubble(s) corresponding to the correct option(s) is (are)darkened.

Partical Marks : +1 For darkening a bubble corresponding to each correct option, provided NO incorret option is darkened.

Zero Marks : 0 If none of the bubbles is darkened.

Negative Marks : -2 In all other cases.

6. **A plano-convex lens is made of a material of refractive index n. When a small object is placed 30cm away in front of the curved surface of the lens, an image of double the size of the object is produced. Due to reflection from the convex surface of the lens, another faint image is observed at a distance of 10cm away from the lens. Which of the following statement(s) is (are) true?**

A) The refractive index of the lens is 2.5

B) The radius of curvature of the convex surface is 45cm

C) The faint image is erect and real

D) The focal length of the lens is 20 cm

Key : A,D

Sol : Convex surface of lens acts like a convex mirror

For this surface

$u=-30cm \quad v=+10cm$

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}=\frac{2}{R}$$

$$\frac{1}{10}-\frac{1}{30}=\frac{2}{R}$$

$$\frac{20}{10\times 30}=\frac{2}{R}$$

R = 30 cm

For plano convex lens:-

$$\frac{1}{f}=(n-1)\left(\frac{1}{R}\right)\rightarrow(1) \quad \rightarrow 1$$

$$m=-2=\frac{v}{u} \quad \Rightarrow v=-2u$$

v = 60 cm

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \quad \Rightarrow \frac{1}{60}+\frac{1}{30}=\frac{1}{f}$$

f = 20 cm

$$\frac{1}{20}=(n-1)\left(\frac{1}{30}\right)$$

$$n-1=\frac{3}{2}$$

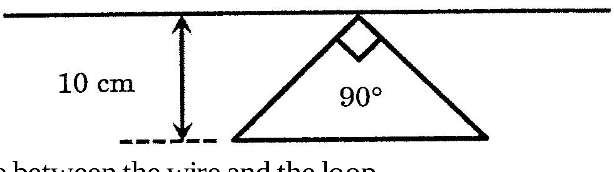
n = 2.5

Case 2:- Virtual erect image

$$m=+2=\frac{v}{u} \quad = -\ 60\text{ cm}$$

f turns out to be negative
which in not possible

7. A conducting loop in the shape of a right angled isosceles triangle of height 10cm is kept such that the 90^0 vertex is very close to an infinitely long conducting wire (see the figure). The wire is electrically insulated from the loop. The hypotenuse of the triangle is parallel to the wire. The current in the triangular loop is in counterclockwise direction and increased at a constant rate of $10\,As^{-1}$. Which of the following statement(s) is (are) true?



A) There is repulsive force between the wire and the loop

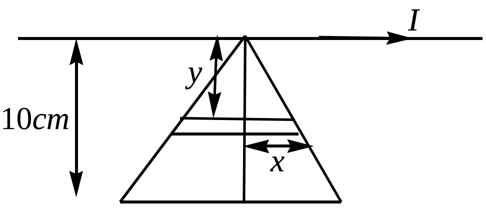
B) If the loop is rotated at a constant angular speed about the wire, an additional emf of $\left(\frac{\mu_0}{\pi}\right)$ volt is induced in the wire

C) The magnitude of induced emf in th wire is $\left(\frac{\mu_0}{\pi}\right)$ volt

D) The induced current in the wire is in opposite direction to the current along the hypotenuse

Key : A,C

Sol :



Assume current in straight wire & find flux through triangular loop:

Flux through shaded part

$$d\phi = \frac{\mu_0 I}{2\pi y} \times 2x dy \;\; \& \; x = y$$

$$\Rightarrow d\phi = \frac{\mu_0 I}{\pi} dy$$

$$\Rightarrow \phi = \frac{\mu_0 I}{\pi} \times 0.1 \Rightarrow M = \frac{\phi}{I} = \frac{\mu_0}{10\pi}$$

$$\therefore \varepsilon = -M\frac{di}{dt} = \frac{-\mu_0}{10\pi} \times 10 = \frac{\mu_0}{\pi} \; V$$

As the triangle would try to move such that flux through it decreases, it moves away from wire

8. The position vector $\vec{r}$ of a particle of mass m is given by the following equation $\vec{r}(t) = \alpha t^3 \hat{i} + \beta t^2 \hat{j}$, where $\alpha = 10/3 ms^3, \beta = 5 ms^{-2}$ and m=0.1 kg. At t=1 s, which of the following statement(s) is (are) true about the particle?

A) The velocity $\vec{v}$ is given by $\vec{u} = \left(10\hat{i} + 10\hat{j}\right) ms^{-1}$

B) The angular momentum $\vec{L}$ with respect to the origin is given by $\vec{L} = -(5/3)\hat{k}$ N m s

C) The force $\vec{F}$ is given by $\vec{F} = \left(\hat{i} + 2\hat{j}\right) N$

D) The torque $\vec{\tau}$ with respect to the origin is given by $\vec{\tau} = -(20/3)\hat{k} \; N\,m$

Key : A,B,D

Sol : $\vec{r}(t) = \alpha t^3 \hat{i} + \beta t^2 \hat{j}$

$\vec{v}(t) = 3\alpha t^2 \hat{i} + 2\beta t\hat{j}$

At t = 1,

$\vec{v} = (10\hat{i} + 10\hat{j}) ms^{-1}$

$\vec{L} = \vec{r} \times m\vec{v} \qquad \vec{r} = \frac{10}{3}\hat{i} + 5\hat{j}$ at t = 1

$\vec{L} = m(\vec{r} \times \vec{v})$

$= 0.1\left(\frac{10}{3}\hat{i} + 5\hat{j}\right) \times \left(10\hat{i} + 10\hat{j}\right)$

$\vec{L} = \left(\frac{1}{3}\hat{i} + \frac{1}{2}\hat{j}\right) \times \left(10\hat{i} + 10\hat{j}\right)$

$\vec{L} = \frac{10}{3}\hat{k} - 5\hat{k} = \left(\frac{-5}{3}\right)\hat{k}$

acceleration $\vec{a} = 6\alpha t\hat{i} + 2\beta\hat{j}$

at t = 1,

$\vec{a} = 20\hat{i} + 10\hat{j}$

$F = m\vec{a} = \left(2\vec{i} + \hat{j}\right)N$

$\vec{\tau} = \vec{r} \times \vec{F} = \left(\frac{10}{3}\hat{i} + 5\hat{j}\right) \times \left(2\hat{i} + \hat{j}\right)$

or $\vec{\tau} = \frac{10}{3}\hat{k} - 10\hat{k} = \frac{-20}{3}\hat{k}$ N-m

9. A length-scale (l) depends on the permittivity (ε) of a dielectric material, Boltzmann constant (k_B), the absolute temperature (T), the number per unit volume (n) of certain charged particles, and the charge (q) carried by each of the particles. Which of the following expression(s) for l is (are) dimensionally correct?

A) $l = \sqrt{\left(\frac{nq^2}{\varepsilon k_B T}\right)}$ B) $l = \sqrt{\left(\frac{\varepsilon k_B T}{nq^2}\right)}$ C) $l = \sqrt{\left(\frac{q^2}{\varepsilon n^{2/3} k_B T}\right)}$ D) $l = \sqrt{\left(\frac{q^2}{\varepsilon n^{1/3} k_B T}\right)}$

Key : B,D

Sol : $\ell = [L^1]$; $k_B = \left[M^1 L^2 T^{-2} K^{-1}\right]$

$T = [K^1]$; $n = \left[L^{-3}\right]$

$q = \left[I^1 T^1\right]$

$\varepsilon = farad / meter$

$= \left[M^{-1} L^{-1} T^4 I^2\right]$

A) $\sqrt{\frac{nq^2}{\varepsilon k_B T}} = \sqrt{\frac{\left[L^{-3}\right] \times \left[I^1 T^1\right]^2}{\left[M^{-1} L^{-3} T^4 I^2\right]\left[M^1 L^2 T^{-2} K^{-1}\right]\left[K^1\right]}}$

$= \sqrt{\frac{\left[T^2 I^2\right]}{M^0 L^1 T^2 I^2 K^0}} = \sqrt{\frac{I^2}{L}}$

$\therefore (A)$ is not correct

B) $\sqrt{\frac{\varepsilon k_B T}{nq^2}} = \sqrt{\frac{\left[M^{-1} L^{-3} T^4 I^2\right] \times \left[M^1 L^2 T^{-2} K^{-1}\right]\left[K^1\right]}{\left[L^{-3}\right]\left[I^1 T^1\right]^2}}$

$= \sqrt{\frac{M^0 L^{-1} T^2 I^2 K^0}{L^{-3} I^1 T^2}} = \sqrt{L^2} = L$

$\therefore (B)$ is correct

C) $\sqrt{\dfrac{q^2}{\varepsilon n^{2/3} k_B T}} = \sqrt{\dfrac{(IT)^2}{(M^1L^{-3}T^4I^2)(L^{-3})^{2/3}(M^1L^2T^{-2}K^{-1})(K^1)}}$

$= \sqrt{\dfrac{I^2T^2}{M^2L^{-3}T^2I^2}} = \sqrt{\dfrac{L^3}{M^2}} = \left[M^{-1}L^{3/2}\right]$

$\therefore (C)$ is not correct

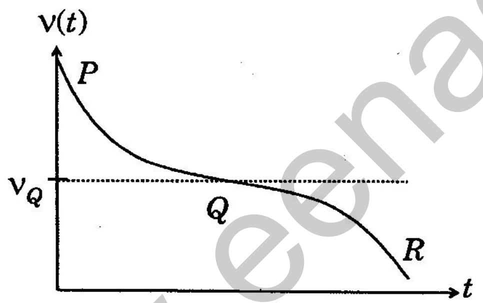
D) $\sqrt{\dfrac{q^2}{\varepsilon n^{1/3} k_B T}} = \sqrt{\dfrac{[IT]^2}{[M^{-1}L^{-3}T^4I^2][L^{-3}]^{1/3}[M^1L^2T^{-2}K^{-1}][K^{+1}]}}$

$= \sqrt{\dfrac{I^2T^2L^2}{T^2I^2K^0}} = [L]$

$\therefore (D)$ is correct

10. Two loudspeakers M and N are located 20 m apart and emit sound at frequencies 118 Hz and 121 Hz, respectively. A car is initially at a point P, 1800m away from the midpoint Q of the line MN and moves towards Q constantly at 60 km/hr along the perpendicular bisector of MN. It crosses Q and eventually reaches a point R, 1800 m away from . Let $v(t)$ represent the beat frequency measured by a person sitting in the car at time t. Let v_p, v_Q and v_R be the beat frequencies measured at locations P, Q and R, respectively. The speed of sound in air is $330 ms^{-1}$. Which of the following statement(s) is (are) true regarding the sound heard by the person?

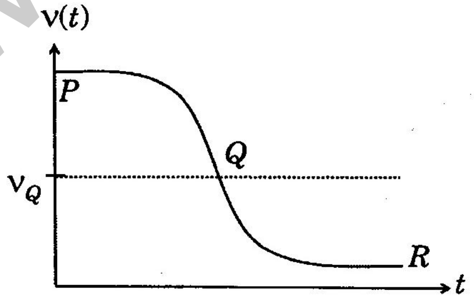
A) The plot below represents schematically the variation of beat frequency with time



B) The rate of change in beat frequency is maximum when the car passes through Q

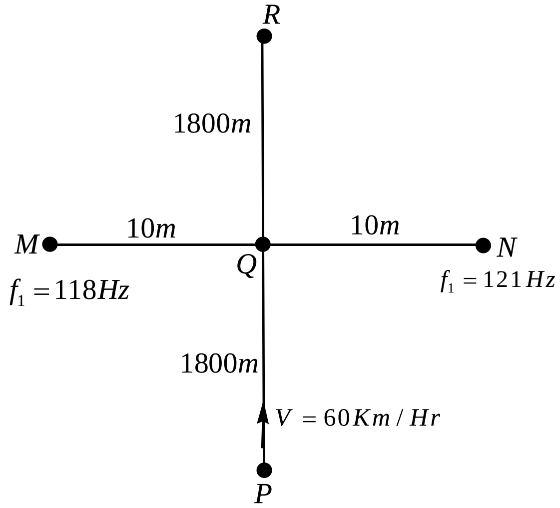
C) $v_P + v_R = 2v_Q$

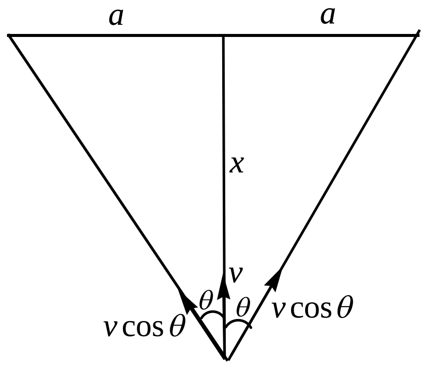
D) The plot below represents schematically the variation of beat frequency with time



Key : B,C,D

Sol:





$$f_1' = f_1\left(\frac{V_s + v\cos\theta}{V_s}\right) \qquad f_2' = f_2\left(\frac{V_s + v\cos\theta}{V_s}\right)$$

Δf = Beat frequency $= f_2' - f_1' = 4\left(1 + \frac{v}{Vs}\cos\theta\right)$

$$\cos\theta = \frac{x}{\sqrt{a^2 + x^2}}$$

$$\Delta f = 4\left(1 + \frac{v}{Vs}\frac{x}{\sqrt{a^2 + x^2}}\right) where \ \ x = 1800 - vt$$

As $x \to \infty, v_Q \to constant$

Hence correct plot is (D)

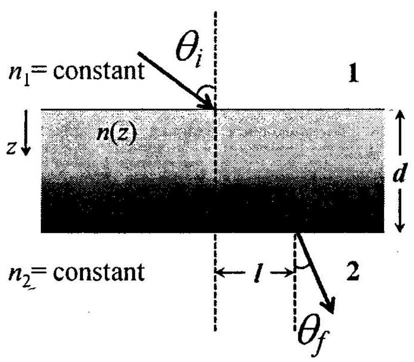
As the car moves away, $\cos\theta$ is negative

Therfore $v_P + v_R = 2v_Q$

(C) is correct.

From plot (D), rate of change of beat frequency is maximum at Q.

11. A transparent slab of thickeness d has a refractive index n(z) that increases with z. Here z is the vertical distance inside the slab, measured from the top. The slab is placed between two media with uniform refractive indices n_1 and $n_2 (> n_1)$, as shown in the figure.A ray of light is incident with angle θ_i from medium 1 and emerges in medium 2 with refraction angle θ_f with a lateral displacement l.



Which of the following statement (s) is (are) true?

A) l is dependent on $n(z)$

B) $n_1 \sin\theta_i = (n_2 - n_1)\sin\theta_f$

C) $n_1 \sin\theta_i = n_2 \sin\theta_f$

D) l is independent of n_2

Key: A,C,D

Sol:

With in slab l depends on refractive index of the slab

$\therefore$ statement 'A' is correct

According to snell's law when mediam are parallel $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\therefore$ statement 'C' is correct

is l only depends on n(z)

$\therefore$ statement 'D' is correct

12. Highly excited states for hydrogen-like atoms (also called Rydberg states) with nuclear charge Ze are defined by their principal quantum number n, where n>>1. Which of the following statement(s) is (are) true ?

A) Relative change in the radii of two consecutive orbitals does not depend on Z

B) Relative change in the radii of two consecutive orbitals varies as $1/n$

C) Relative change in the energy of two consecutive orbitals varies as $1/n^3$

D) Relative change in the angular momenta of two consecutive orbitals varies as $1/n$

Key : A,B,D

Sol :

$$\frac{r_{n+1}-r_n}{r_n} = \frac{\frac{0.51(n+1)^2}{Z} - \frac{(0.51)n^2}{Z}}{\frac{(0.51)n^2}{Z}}$$

$$= \frac{(n+1)^2 - n^2}{n^2} = \frac{1}{n^2} - \frac{2}{n}$$

as n>>1

$$\frac{r_{n+1}-r_n}{r_n} \approx \frac{2}{n}$$

and $$\frac{L_{n+1}-L_n}{L_n} = \frac{(n+1)\frac{h}{2\pi} - \frac{nh}{2\pi}}{\frac{nh}{2\pi}} = \frac{1}{n}$$

$$\frac{E_{n+1}-E_n}{E_n} = \frac{-\frac{13.6.Z^2}{(n+1)^2} + \frac{13.6Z^2}{n^2}}{-\frac{13.6Z^2}{n^2}}$$

$$\frac{\frac{1}{(n+1)^2} - \frac{1}{n^2}}{\frac{1}{n^2}}$$

$$\frac{n^2}{(n+1)^2} - 1 \simeq \frac{1}{n}(\text{as } n >> 1)$$

13. **An incandescent bulb has a thin filament of tungsten that is heated to high temperature by passing an electtric current.The hot filament emits black-body radiation.The filament is observed to break up at random locations after a suffciently long time of operation due to non-uniform evaporation of tungsten from the filament. IF the bulb is powered at constant voltage, which of the followign statement(s) is (are) true?**

A) The temperature distribution over the filament is uniform
B) The resistance over small sections of the filament dectreases with time
C) The filament emits more light at higher band of frequencies before it breaks up
D) The filament consumes less electrical power towards the end of the life of the bulb
Key: C,D
Sol: Conceptual

SECTION 3 (Maximum Marks: 15)

* **This section contains FIVE question.**
* **The answer to each question is a SINGLE DIGIT INTEGERranging from 0 to 9, both inclusive.**
* **For each question, darken the bubble corresponding to the correct integer in the ORS.**
* **For each questionm, marks will be awarded in one of the following categories :**
Full Marks : +3 If only the bubble corresponding to the correct answer is darkened.
Zero Marks : 0 In all other cases.

14. **A hydrogen atom in its ground state is irradiated by light of wavelength $970\,\overset{o}{A}$ .Taking $hc/e = 1.237\times10^{-6}\,eV$ m and the ground state energy of hydrogen atom as-13.6 eV,the number of lines present in the emission spectrum is**

Key: 6

Sol : $\lambda = 970\,\overset{o}{A}$

$E_2 - E_1 = \left(\frac{hc}{\lambda e}\right)$ electron - volt

$E_2 - E_1 = \frac{1.237\times10^{-6}}{970\times10^{-10}} = \frac{1237}{97}$

$= 12.75eV$

$\therefore n = 4$

$\therefore$ no. of emission lines $= {}^{n}C_{2} = {}^{4}C_{2}$

$= 6$

15. **The isotope ${}^{12}_{5}B$ having a mass 12.014 u undergoes $\beta - \text{decay}$ to ${}^{12}_{6}C$ has an excited state of the nucleus $\left({}^{12}_{6}C^*\right)$ at 4.041 MeV above its ground state.If ${}^{12}_{5}B$ decays to ${}^{12}_{6}C^*$,the maximum kinetic energy of the β-particle in units of MeV is ($1u = 931.5MeV/c^2$, where c is the speed of light in vacuum.)**

Key: 9

Sol. : Mass of ${}^{12}_{6}C$ nucleus $= 12$ amu

There fore mass defect $= [12.014 - 12]$ amu.

Energy released $= [0.014\times931.5]$ MeV

There fore, maximum KE of β - particle

$= [0.014\times931.5] - 4.041$

$= 9MeV$

16. Consider two solid spheres P and Q each of density 8 gm cm^{-3} and diameters 1 cm 8 and 0.5 cm, respectively. Sphere P is dropped into a liquid of density 0.8gm cm^{-3} and viscosity $\eta = 3$ poiseulles.Sphere Q is dropped into a liquid of density 1.6 gm cm^{-3} and viscosity $\eta = 2$ poiseulles.The ratio of the terminal velocities of P and Q is

Key: 3

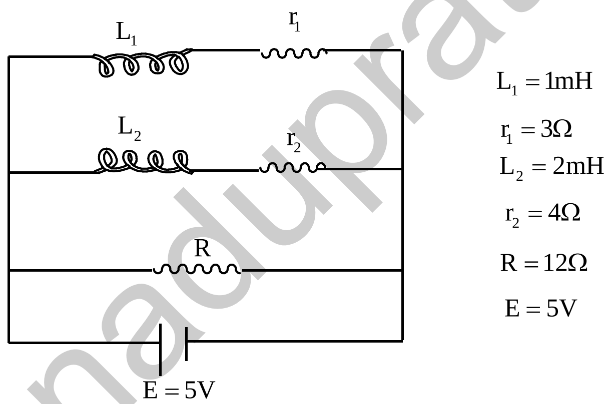
$$\text{Sol}: V_T = \frac{2}{9}\frac{r^2 g}{\eta}\left(d_s - d_L\right)$$

$$\frac{V_{TP}}{V_{TQ}} = \left(\frac{1}{0.5}\right)^2 \times \frac{2}{3} \times \frac{(8-0.8)}{(8-1.6)} = \frac{4\times 2}{3} \times \frac{7.2}{6.4} = 3$$

17. Two inductors L_1 (inductance 1mH, internal resistance 3 Ω) and L_2 (inductance 2 mH, internal resistance 4 Ω), and a resistor R (resistance 12 Ω) are all connected in parallel across a 5 V battery. The circuit is switched on at time $t = 0$.The ratio of the maximum to the minimum current $\left(I_{max} / I_{min}\right)$ drawn from the battery is

Key: 8

Sol:



At $t = 0$, There is minimum current

At $t = \infty$, There is maximum current

At $t = 0$ inductor acts like open circuit

At $t = \infty$ inductor acats like short circuit

$$I_{min} = \frac{5}{12}A$$

$$I_{max} = \frac{\frac{5}{1}}{\frac{1}{3}+\frac{1}{4}+\frac{1}{12}} = \frac{5\times 2}{3} = \frac{10}{3}A$$

$$\frac{I_{max}}{I_{min}} = \frac{10}{3}\times\frac{12}{5} = 8$$

18. A metal is heated in a furnace where a sensor is kept above the metal surface to read the power radiated (P) by the metal.The sensor has a scale that displays $\log_2(P/P_0)$, where P_0 is a constant. When the metal surface is at a temperature of 487 $^\circ C$, the sensor shows a value 1.Assume that the emissivity of the metallic surface remains constant. What is the value displayed by the sensor when the temperature of the metal surface is raised to $2767^\circ C$?

Key: 9

Sol:

$P \alpha T^4$ (T = alosolute temperature)

$$1 = \log_2\left(\frac{KT_1^4}{P_0}\right) \quad K = \cos\tan t$$

$$x = \log_2\left(\frac{kT_2^4}{P_0}\right)$$

$$\frac{x-1}{1} = \frac{\log\left(\frac{\left(kT_2^4\right)}{P_0}\right) - \log\left(\frac{KT_1^4}{P_0}\right)}{\log\left(\frac{KT_1^4}{P_0}\right)}$$

$$\frac{x-1}{1} = \frac{\log\frac{T_2^4}{T_1^4}}{\log\left(\frac{kT_1^4}{P_0}\right)} = \log_2\left(\frac{T_2^4}{T_1^4}\right)$$

$$x-1 = \log_2\left(\frac{3040}{760}\right)^4 = 4\log_2\left(\frac{304}{760}\right)$$

$$x = 1 + 4\log_2^4 = 1 + 8 = 9$$

PART - II CHEMISTRY

SECTION 1 (Maximum Marks:15)

* This section contains **FIVE** questions.
* Each questions has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
* For each question, darken the bubble corresponding to the correct option is the **ORS**.
* For each question, marks will be awarded in one of the following categories:

Full marks : +3 If only the bubble corresponding to the correct option is darkened.
Zero marks : 0 if none of the bubbles is darkened.
Negative Marks : - 1 In all other cases.

19. The incresing order of atomic radii of the following Group 13 elements is

A) Al < Ga < In < Tl B) Ga < Al < In < Tl C) Al < In < Ga < Tl D) Al < Ga < Tl < In

Key : B

Sol : Ga < Al < In < Tl

20. Among $\left[Ni(CO)_4\right]$, $\left[NiCl_4\right]^{2-}$, $\left[Co(NH)_3Cl_2\right]Cl$, $Na_3\left[CoF_6\right]$, Na_2O_2 and CsO_2, the total number of paramagnetic comopound is

A) 2 B) 3 C) 4 D) 5

Key : B

Sol: $Ni(CO)_4 - SP^3$ – diamagnetic

$[NiCl_4]^{-2} - SP^3 -$ paramagnetic (2 u.p)

$[CO(NH_3)_4 Cl_2]Cl - d^2sp^3$ – diamagnetic

$Na_3[COF_6] - SP^3d^2 -$ paramagnetic (4 u.p)

$Na_2O_2 - [O_2^{-2}] -$ diamagnetic

$CSO_2 - (O_2^{-1}) -$ paramagnetic

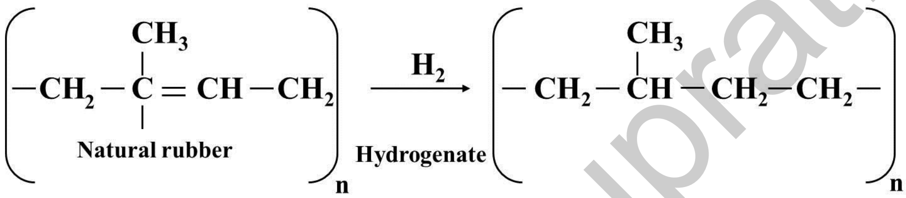
21. On complete hydrogenation, natural rubber produces

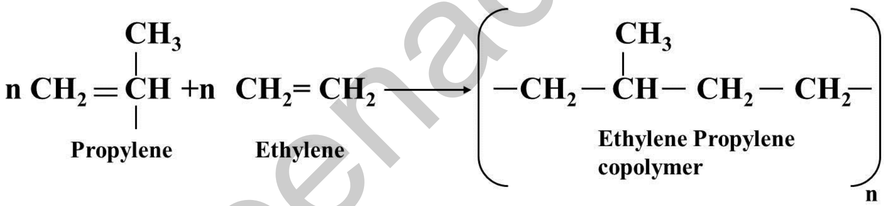
A) ethylene - propylene copolymer B) vulcanised rubber

C) polypropylene D) polybutylene

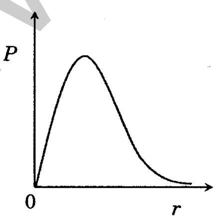
Key : A

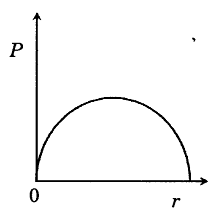
Sol :

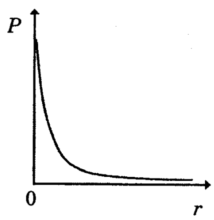


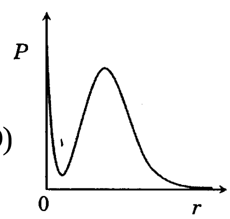


22. P is the probability of finding the 1s electrons of hydrogen atom in a spherical shell of infinitesimal thickness, dr, at a distance r from the nucleus. The vol ume of this shell is $4\pi r^2 dr.$ The qualitative sketch of the dependence of P on r is

A) 

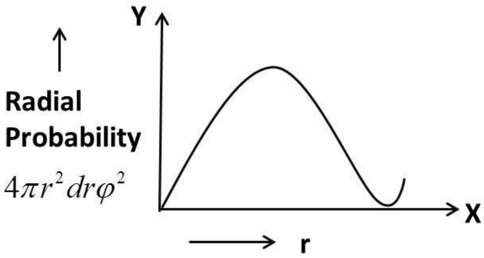
B) 

C) 

D) 

Key : A

Sol : Radial probability (VS) Distance (r) Curve for '1s' orbital



23. One mole of an ideal gas at 300K in thermal contact with surroundings expands isothermally from 1.0 L to 2.0 L against a constant pressure of 3.0 atm. It this process, the change in entropy of surroundings $\left(\Delta S_{surr}\right)$ in JK^{-1} is

(1 L atm = 101.3 J)

A) 5.763 B) 1.013 C) -1.013 D) -5.763

Key : C

Sol : $W = -P_{ext}.\left(V_2 - V_1\right)$

$= -3\left(2-1\right)$

$= -3\left(101.3\right)$

$\therefore\ \Delta S_{(surroundings)} = \dfrac{-3\left(101.3\right)}{300} = -1.013$

<u>SECTION 2 (Maximum Marks: 32)</u>

* This section contains EIGHT questions.
* Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is (are) correct.
* For each questions, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
* For each questions, marks will be awarded in one of the following categories:

Full Marks : +4 If only darkening a bubble corresponding to each correct option, provided NO incorrect option is darkened.

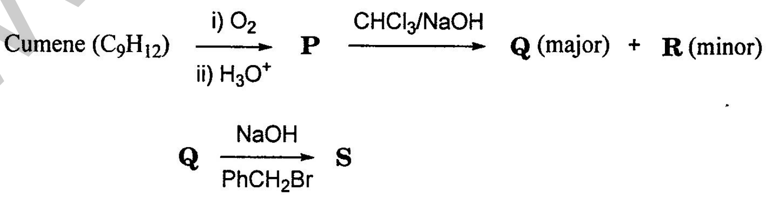
Partial Marks : +1 For darkening a bubble corresponding to each correct option, provided NO incorrect option is darkened.

Zero Marks : 0 If none of the bubbles is darkened.

Negative Marks : -2 In all other cases.

* For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks: darkening only (A) and (D) will result in +2 marks; and darkening (A) and (B) will result in - 2 marks, as a wrong option is also darkened.

24. The correct statement(s) about the following reaction sequence is (are)



A) R is steam volatile

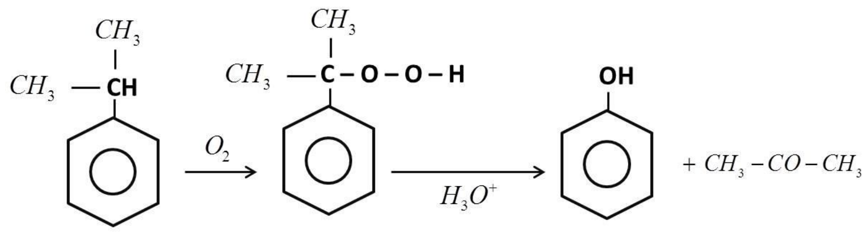
B) Q gives dark violet coloration with 1% aqueous $FeCl_3$ solution

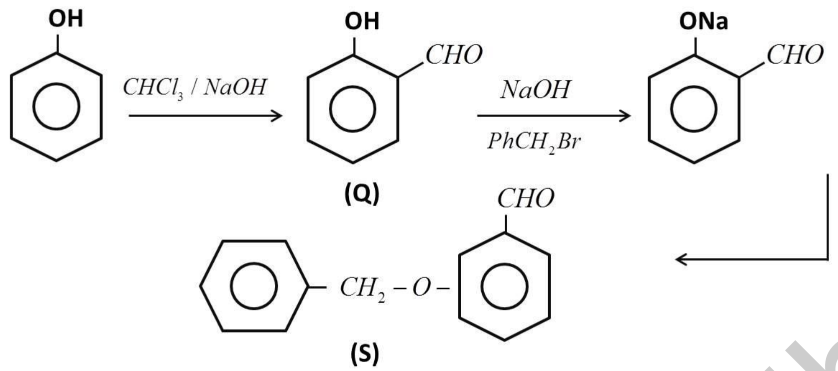
C) S gives yellow precipitate with 2, 4 - dinitrophenylhydrazine

D) S gives dark violet coloration with 1% aqueous $FeCl_3$ solution

Key : B, C

Sol :





25. The compoud(s) with TWO lone pairs of electrons on the central atom is (are)

A) BrF_5 B) ClF_3 C) XeF_4 D) SF_4

Key : B, C

Sol : BrF_5

No. of lone pairs $= \frac{7-5}{2} = 1$

ClF_3

No. of lone pairs $= \frac{7-3}{2} = 2$

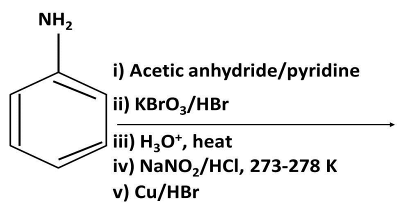
XeF_4

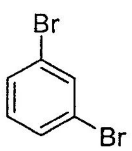
No. of lone pairs $= \frac{8-4}{2} = 2$

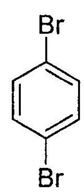
SF_4

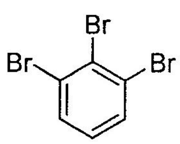
No. of lone pairs $= \frac{6-4}{2} = 1$

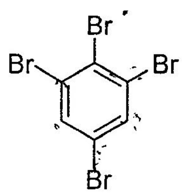
26. The product(s) of the following reaction sequence is (are)



A) 

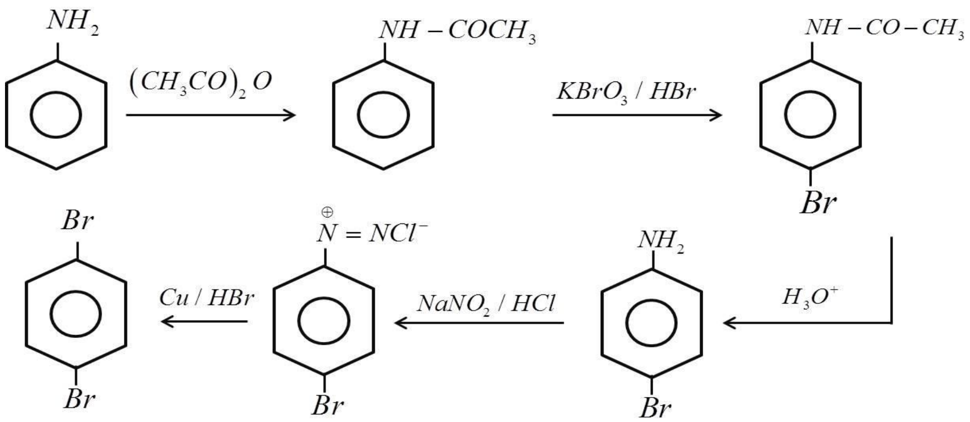
B) 

C) 

D) 

Key : B

Sol :



27. According to the Arrhenius equation,

A) a high activation energy usually implies a fast reaction.

B) rate constant increases with increase in temperature. This is due to a greater number of collisions whose energy exceeds the activation energy.

C) higher the magnitude of actiavtion energy, stronger is the temperature dependence of the rate constant.

D) the pre-exponential factor is measure of the rate at which collisions occur, irrespective of their energy.

Key : B, C, D

Sol : A) $K = Ae^{-Ea/RT}$

E_a increases $\rightarrow$ 'K' decreases

Hence, Rate of reaction decreases

B) As Temparature increases

Activation energy decreases

Rate constant increases

C) If $E_a = 0$ then K = A

i.e., 'K' is independent of temparature

At higher activation energy value effect of temparature more on rate constant

D) Since $\frac{K}{A}$ indiactes fraction of the molecules which lead a chemical reaction

28. The crystalline form of borax has

A) tetranuclear $\left[B_4O_5(OH)_4\right]^{2-}$ unit

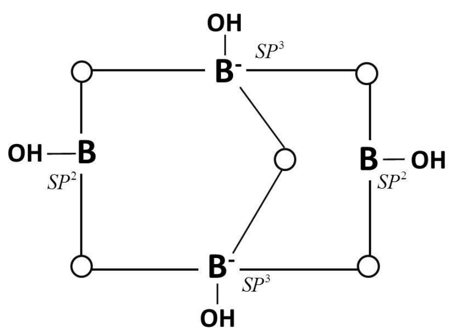
B) all boron atoms in the same plane

C) euqal number of sp^2 and sp^3 hybridized boron atoms

D) one terminal hydroxide per boron atom

Key : A, C, D

Sol :



29. The reagent(s) that can selectively precipitate S^{2-} from a mixture of S^{2-} and SO_4^{2-} in aqueous solution is (are)

A) $CuCl_2$ **B)** $BaCl_2$

C) $Pb\left(OOCCH_3\right)_2$ **D)** $Na_2\left[Fe\left(CN\right)_5 NO\right]$

Key : A, C, D

Sol :

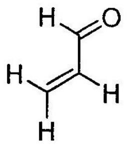
$$CuCl_2 + S^{-2} \longrightarrow \underset{(Black\ ppt)}{Cus} + 2Cl^-$$

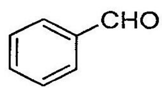
$$BaCl_2 + S^{-2} \longrightarrow \underset{(not\ ppt)}{Bas} + 2Cl^-$$

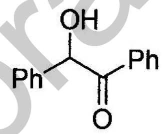
$$Pb(CH_3COO)_2 + S^{-2} \longrightarrow \underset{(Black\ ppt)}{Pbs} + 2CH_3COO^{\odot}$$

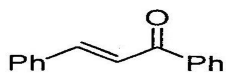
$$S^{-2} + \left[Fe\left(CN\right)_5 NO\right]^{-2} \longrightarrow \left[Fe\left(CN\right)_5 NOS\right]\left(\text{violet ppt}\right)$$

30. Positive Tollen's test is observed for

A) 

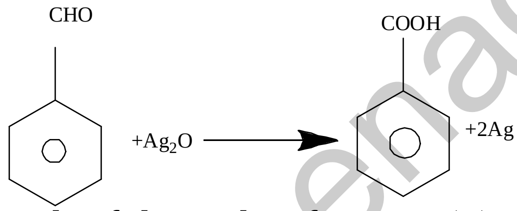
B) 

C) 

D) 

Key : A,B

Sol : $A) CH_2 = CH - CHO + Ag_2O \rightarrow CH_2 = CH - COOH + 2Ag$



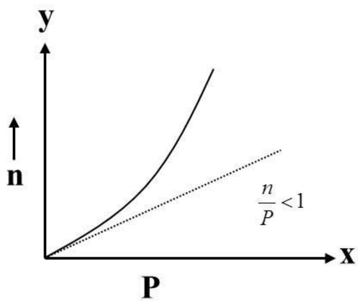
31. A plot of the number of neutrons (N) against the number of protons (P) of stable nuclei exhibits upwrd deviation from linearity for atomic number, Z>20. For an unstable nucleus having N/P ratio less than 1, the possible mode (s) of decay is (are)

A) β^- -decay ($(\beta\, emission)$ B) orbital or K-electron capture

C) neutron emission D) β^+ -de cay $\left(positron\, emission\right)$

Key : B,D

Sol : ${}_1P^1 \rightarrow {}_0n^1 + {}_{+1}e^0$



SECTION 3 (Maximum Marks: 15)

- This section contains **FIVE** questions
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.
- This section contains **FIVE** questions
- For each questions, darken the bubble corresponding to the correct integer in the ORS>
- For each question, marks will be awarded in one of the following categories

Full marks : +3 If only the bubble corresponding to the correct answer is darkened.
Zero marks : 0 In all other cases.

32. The diffusion coefficient of an ideal gas is proportional to its mean free path and mean speed. The absolute temperature of an ideal gas is increased 4 times and its pressure is increased 2 times. As a result, the diffusion coefficient of this gas increases x times. The value of x is

Key : 4

Sol:

$$Diffusion\,Coefficient(\eta) \propto \frac{T}{P}$$

$$\propto \sqrt{T}$$

$$\therefore \eta \propto \frac{T^{3/2}}{P}$$

$$\frac{\eta_1}{\eta_2} = \left(\frac{T_1}{T_2}\right)^{3/2} \cdot \left(\frac{P_2}{P_1}\right)$$

$$\frac{\eta_1}{\eta_2} = \left(\frac{T}{4T}\right)^{3/2} \cdot \left(\frac{2P}{P_1}\right)$$

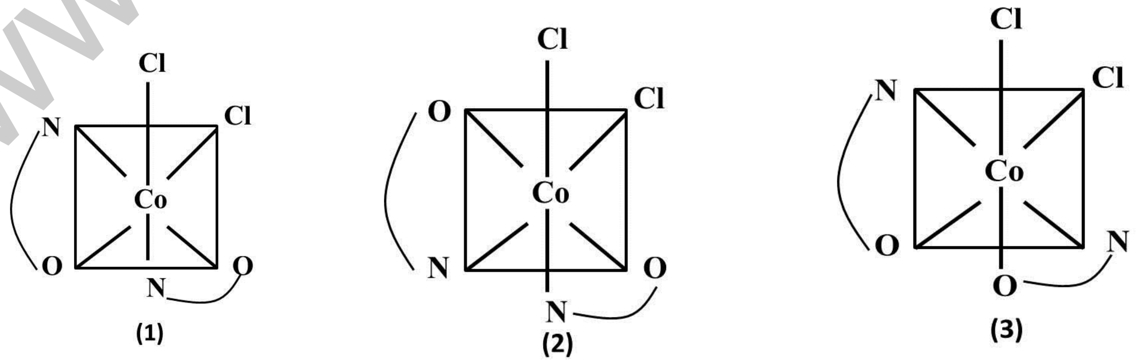
$$= \frac{1}{2^3} . 2$$

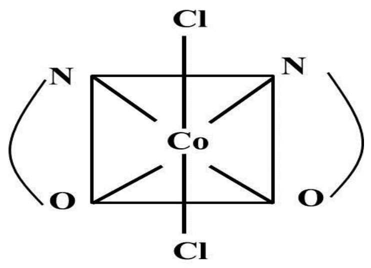
$$\frac{\eta_1}{\eta_2} = \frac{1}{4}$$

$$\therefore \eta_1 = 4.\eta_2$$

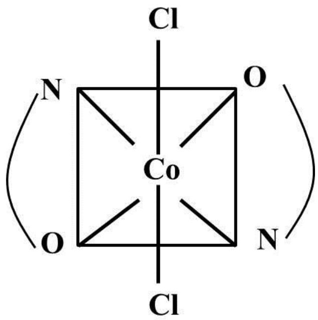
33. The number of geometric isomers possible for the example $\left[CoL_2Cl_2\right]^-\left(L = H_2NCH_2CH_2O^-\right)$ **is**

Key : 5
Sol:





(4)



(5)

34. The mole fraction of a solute in a solution is 0.1. At 298K, molarity of this solution is the same as its molarity. Density of this solution at 298K is $2.0\, g\, cm^{-3}$. The ratio of the molecular weights of the solute and solvent, $\left(\frac{MW_{solute}}{MW_{solvent}}\right)$, is

Key : 9

$$\text{Sol}: \frac{d}{M} = \frac{1}{m} + \frac{GMW\ of\ solute}{1000}$$

$$But\, M \simeq m$$

$$\frac{d}{m} = \frac{1}{m} + \frac{GMW\ of\ solute}{1000}$$

$$\frac{2}{m} - \frac{1}{m} = \frac{GMW\ of\ solute}{1000} \rightarrow (1)$$

$$0.1 = \frac{m}{m + \frac{1000}{MW\ of\ solvent}}$$

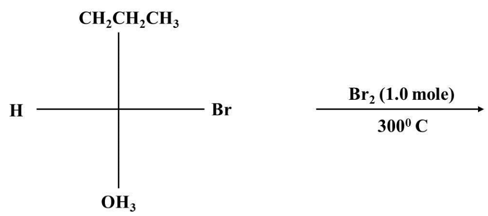
$$10 = 1 + \frac{1000}{MW\ of\ solvent \times m}$$

$$9 = \frac{1000}{MW\ of\ solvent} \times \frac{1}{m} \rightarrow (2)$$

$$9 = \frac{1000}{MW\ of\ solvent} \times \frac{MW\ of\ solute}{1000}$$

$$\therefore \frac{MW\ of\ solute}{MW\ of\ solvent} = 9$$

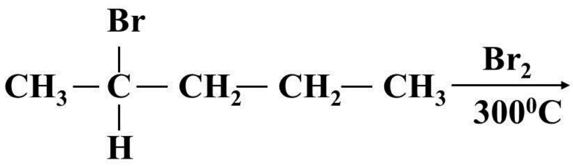
35. In the followig monobromination reaction, the number of possible chiral product is

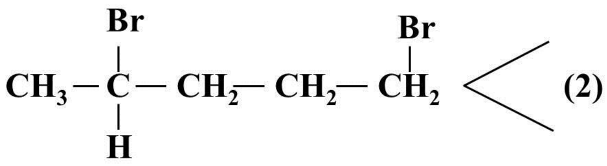


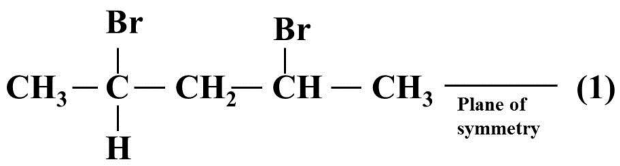
(1.0 mole)

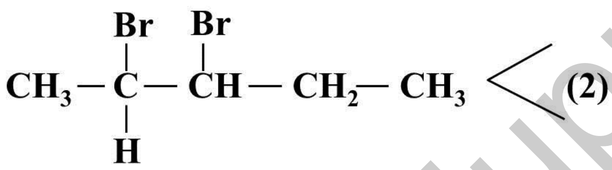
(enatiomerically pure)

Key : 5
Sol :



a) 

b) 

c) 

36. In neutral or faintly alkaline solution, 8 moles of permanganate anion quantitavely oxidze thiosulphate anions to produce X moles of a sulphur containing product. The magniture of X is

Key :
Sol : 6

$$n-factor = 4$$

$$MnO_4^- + S_2O_3^{-2} \longrightarrow MnO_2 + 2S$$

$$n-factor = 3$$

$$4MnO_4^- + 3S_2O_3^{-2} \longrightarrow 4MnO_2 + 3S_4O_6^{-2}$$

$$4\,moles \rightarrow 3\,moles$$

$$8\,moles \rightarrow 6\,moles$$

PART-3 : MATHEMATICS

SECTION 1 (Maximum Marks : 15)

- **This section contains FIVE questions.**
- **Each question has FOUR options (A), (B), (C) and (D), ONLY ONE of these four options is correct.**
- **For each question, darken the bubble corresponding to the correct option in the ORS.**
- **For each question, marks will be awarded in one of the following categories :**

Full Marks : +3 If only the bubble corresponding to the correct option is darkened
Zero Marks : 0 If none of the bubbles is darkened.
Negative Marks : -1 In all other cases.

37. A computer producing factor has only two plants T_1 and T_2. Plant T_1 produces 20% and plant T_2 produces 80% of the total computers produced. 7% of computers produced in the factory turn out to be defective. It is known that

P (computer turns out to be defective given that it is produced in plant T_1)

$=10P$ (computer turns out to be defective given that it is produced in plant T_2),

where $P(E)$ denotes the probability of an event E. A computer produced in the factory is randomly selected and it does ot turn out to be defective. Then the probability that it is produced in plant T_2 is

A) $\frac{36}{73}$ B) $\frac{47}{79}$ C) $\frac{78}{93}$ D) $\frac{75}{83}$

Key : C

Sol: $T_1(20\%), T_2(80\%)$

7% Defective

$P(D/T_1) = 10 \times P(D/T_2)$

$$\frac{\frac{x}{100}}{\frac{20}{100}} = 10 \times \frac{\frac{(7-x)}{100}}{\frac{80}{100}}$$

$$\Rightarrow \frac{x}{20} = \frac{7-x}{8}$$

$\Rightarrow x = 5$

T_1	T_2
(15%)	(78%)

$$P(E) = \frac{\frac{78}{100}}{\frac{15}{100} + \frac{78}{100}} = \frac{78}{93}$$

38. A debate club consists of 6 girls and 4 boys. A team of 4 members is to be selected from this club including the selection of a captain (from among these 4 member) for the team. If the team has to include at most one boy, then the number of ways of selecting the team is

A) 380 B) 320 C) 260 D) 95

Key : A

Sol.

6(G)	4(B)	No. of Selections
3	1	${}^6C_3 \times {}^4C_1 \longrightarrow 80$
4	0	${}^6C_4 \times {}^4C_0 \longrightarrow 15$
	Total	95

Total no. of selections $95 \times 44 = 380$.

39. Let $-\frac{\pi}{6}<\theta<-\frac{\pi}{12}$. Suppose α_1 and β_1 are the roots of the equation $x^2-2x\sec\theta+1=0$ and α_2 and β_2 are the roots of the equation $x^2+2x\tan\theta-1=0$. If $\alpha_1>\beta_1$ amd $\alpha_2>\beta_2$, then $\alpha_1+\beta_2$ equals

A) $2(\sec\theta-\tan\theta)$ B) $2\sec\theta$ C) $-2\tan\theta$ D) 0

Key : C

Sol: $x^2-2x\sec\theta+1=0$

$\alpha_1+\beta_1=2\sec\theta \quad \Rightarrow \quad |\alpha_1-\beta_1|=\frac{\sqrt{4\sec\theta-4}}{1}$

$\alpha_1\beta_1=1 \qquad \alpha_1-\beta_1=-2\tan\theta \rightarrow (1)$

$x^2+2x\tan\theta-1=0$

$\alpha_2+\beta_2=-2\tan\theta \qquad |\alpha_2-\beta_2|=\frac{\sqrt{4\tan^2\theta+4}}{1}$

$\alpha_2\beta_2=-1 \qquad \alpha_2-\beta_2=2\sec\theta\rightarrow(2)$

$\alpha_1-\beta_1=-2\tan\theta \qquad \alpha_2+\beta_2=-2\tan\theta$

$\alpha_1+\beta_1=2\sec\theta \qquad \alpha_2+\beta_2=2\sec\theta$

$2\alpha_1=2\sec\theta-2\tan\theta \qquad 2\beta_2=-2\tan\theta-2\sec\theta$

$\alpha_1=\sec\theta-\tan\theta \qquad \beta_2=-\tan\theta-\sec\theta$

$\alpha_1+\beta_2=\sec\theta-\tan\theta-\tan\theta-\sec\theta$

$=-2\tan\theta$

40. Let $S=\left\{x\in(-\pi,\pi):x\neq 0,\pm\frac{\pi}{2}\right\}$. The sum of all distinct solutions of the equation $\sqrt{3}\sec x+\operatorname{cos}ecx+2(\tan x-\cot x)=0$ in the set S is equal to

A) $-\frac{7\pi}{9}$ B) $-\frac{2\pi}{9}$ C) 0 D) $\frac{5\pi}{9}$

Key : C

Sol: $\cos\left(x-\frac{\pi}{6}\right)=\cos 2x$

$x=2n\pi-\frac{\pi}{6} \quad and \quad 3x=2n\pi+\frac{\pi}{6}$

$\therefore x=\frac{-\pi}{6},\frac{\pi}{18},\frac{13\pi}{18},\frac{-11\pi}{18}$

$Sum=\frac{-\pi}{6}+\frac{\pi}{18}+\frac{13\pi}{18}-\frac{11\pi}{18}$

$=0$

41. **The least value of** $\alpha \in R$ **for which** $4\alpha x^2 + \frac{1}{x} \geq 1$, **for all x>0,is**

A) $\frac{1}{64}$ B) $\frac{1}{32}$ C) $\frac{1}{27}$ D) $\frac{1}{25}$

Key : C

Sol: $\frac{4\alpha x^2 + \frac{2}{\alpha} + \frac{2}{x}}{3} \geq \sqrt[3]{\alpha}$

$\Rightarrow \sqrt[3]{\alpha} \geq \frac{1}{3}$

$\Rightarrow \alpha \geq \frac{1}{27}$

SECTION 2 (Maximum Marks : 32)

- **This section contains EIGHT questions.**
- **Each question has FOUR options (A), (B), (C) and (D), ONE OR MORE THAN ONE of these four option(s) is(are) correct.**
- **For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.**
- **For each question, marks will be awarded in one of the following categories :**

Full Marks : +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened

Partial Marks : +1 For darkening a bubble corresponding to each correct option, provided NO incorrect option is darkened.

Zero Marks : 0 If none of the bubbles is darkened.

Negative Marks : -2 In all other cases.

For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (A) and (D) will result in +2 marks; and darkening (A) and (B) will result in -2 marks, as a wrong option is also darkened.

42. **A solution curve of the differential equation** $\left(x^2 + xy + 4x + 2y + 4\right)\frac{dy}{dx} - y^2 = 0, x > 0$, **pases through the point** $(1, 3)$**. Then the solution curve**

A) intersects $y = x + 2$ exactly at one point

B) intersects $y = x + 2$ exactly at two points

C) intersects $y = (x+2)^2$

D) does NOT intersect $y = (x+3)^2$

Key : D

Sol: $(x^2 + xy + 4x + 2y + 4)\frac{dy}{dx} - y^2 = 0$

$y^2 \frac{dx}{dy} = x^2 + xy + 4x + 2y + 4$

$y^2 \frac{dx}{dy} = (x+2)^2 + y(x+2)$

$$\frac{1}{(x+2)^2}\frac{dx}{dy}=\frac{1}{y^2}+\frac{1}{x+2}.\frac{1}{y}$$

$$\frac{1}{(x+2)^2}\frac{dx}{dy}-\frac{1}{y(x+2)}=\frac{1}{y^2}$$

Put $\frac{-1}{x+2}=z$

$$\frac{1}{(x+2)^2}\frac{dx}{dy}=\frac{dz}{dy}$$

$$\frac{dz}{dy}+\frac{z}{y}=\frac{1}{y^2}$$

I.E = $e^{\int\frac{1}{y}dy}=e^{\log e^{y}}=y$

$$zy=\int\frac{1}{y}dy$$

=log(y)+log c

$$cy=e^{zy}$$

$$cy=e^{\frac{-y}{x+2}}$$

$$3c=e^{-1}$$

$$c=\frac{1}{3e}$$

$$y=3e\left(e^{\frac{-y}{x+2}}\right)$$

when y=x+2

$$x+2=3e.e^{-1}$$

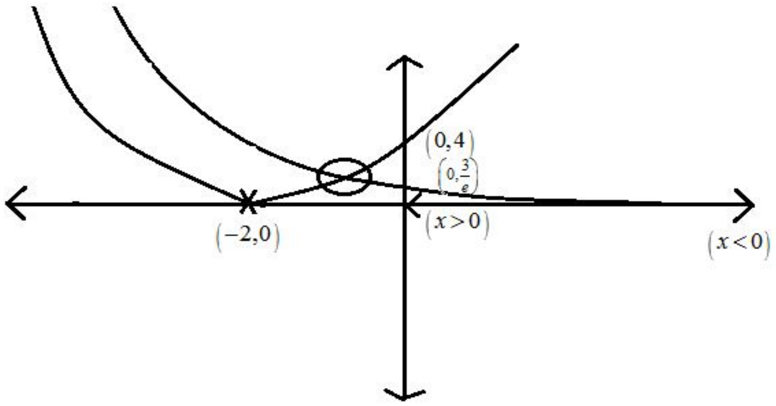
$$x=1,y=3$$

$$y=(x+2)^2$$

$$(x+2)^2=3e^1.e^{-(x+2)}$$

$$(x+2)^2=3e^{-1-x}$$

$$(x+2)^2=\frac{3}{e^{x+1}}$$

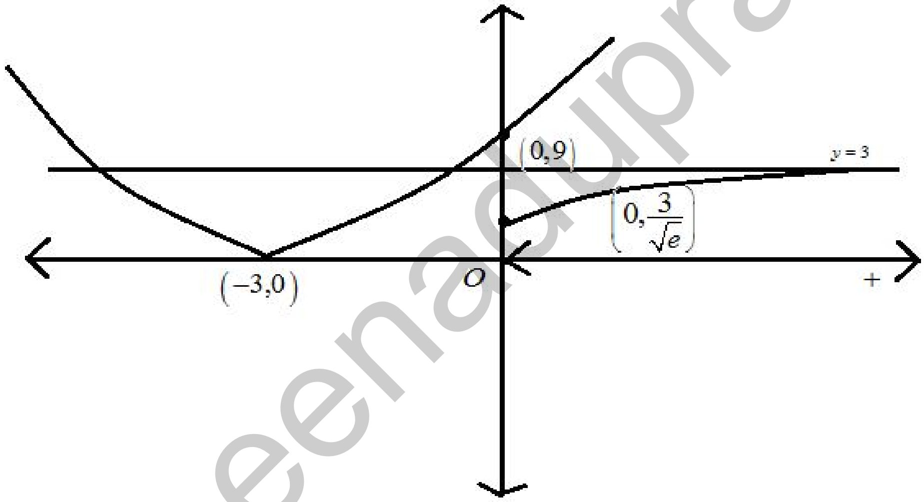


$y=\left(x+3\right)^{2} \qquad y=3e \quad e^{-\frac{y}{x+2}}$

$\left(x+3\right)^{2}=3e\ e^{-\frac{(y+3)}{x+2}}$

$=3e.e^{-1}.e.e^{-\frac{1}{4+2}}$

$\left(x+3\right)^{2}=\frac{3}{e^{\frac{1}{x+2}}}=3e^{-\frac{1}{x+2}}$



Ans : No solution

43. Consider a pyramid OPQRS located in the first octant $(x\geq 0, y\geq 0, z\geq 0)$ **with O as origin, and OP and OR along the x-axis and the y-axis, respectively. The base OPQR of the pyramid is a square of OP = 3. The point S is directly above the mid-point T of diagonal OQ such that TS = 3. Then**

A) the acute angle between OQ and OS is $\frac{\pi}{3}$

B) the equation of the plane containing the triangle OQS is $x-y=0$

C) the length of the perpendicular from P to the plane containing the triangle OQS is $\frac{3}{\sqrt{2}}$

D) the perpendicullar distance from O to the straight line containing RS is $\sqrt{\frac{15}{2}}$

Key : B, C, D

Sol. : $\overline{OS}=\left(\frac{3}{2},\frac{3}{2},3\right)$

$\overline{OQ}=(3,3,0)$

$\overline{OR}=(0,3,0)$

Eqn. to $OQS=\begin{vmatrix} x & y & z \\ 3 & 3 & 0 \\ \frac{3}{2} & \frac{3}{2} & 3 \end{vmatrix}=0$

$x-y=0$ .... (B)

Length of perpendicular from P to the plane containing the triangle OQS = $\frac{|3-0|}{\sqrt{2}}=\frac{3}{\sqrt{2}}$ .... (C)

D) $R=(0,3,0)$

$S=\left(\frac{3}{2},\frac{3}{2},3\right)$

$\frac{3}{2}:\frac{-3}{2}:3=1:-1:2$

$\frac{x-0}{1}=\frac{y-3}{-1}=\frac{z-0}{2}$

$(\lambda,3-\lambda,2\lambda)$

$\Rightarrow \lambda-3+\lambda+2\lambda=0$

$\lambda=\frac{1}{2}$

Perpendicular distance from O to straight line containing RS = $\sqrt{\frac{1}{4}+\frac{15}{4}+1}=\sqrt{\frac{15}{2}}$..........$(D)$

44. The circle $C_1: x^2+y^2=3$, with centre at O, intersects the parabola $x^2=2y$ at the point P in the first quadrant. Let the tangent to the circle C_1 at P touches other two circles C_2 and C_3 at R_2 and R_3, respectively. Suppose C_2 and C_3 have equal radii $2\sqrt{3}$ and centres Q_2 and Q_3, respectively. If Q_2 and Q_3 lie on the -axis, then

A) $Q_2Q_3=12$

B) $R_2R_3=4\sqrt{6}$

C) area of the triangle OR_2R_3 is $6\sqrt{2}$

D) area of the triangle PQ_2Q_3 is $4\sqrt{2}$

Key : A, B, C

Sol : $P(\sqrt{2},1)$

$Q_2(0,9)$

$Q_3(0,-3)$

$Q_2Q_3 = 12........(A)$

$R_2(-2\sqrt{2}, 7)$

$R_3 = (-2\sqrt{2}, -1)$

$R_2R_3 = 4\sqrt{6}............(B)$

Area of $\Delta OR_2R_3 = 6\sqrt{2}$

Area of $PQ_2Q_3 = \frac{1}{2}\begin{vmatrix} \sqrt{2} & 0 \\ 4 & 12 \end{vmatrix}$

$$= \frac{1}{2}(12\sqrt{2}) = 6\sqrt{2}........(C)$$

45. Let $f:\mathbb{R}\to\mathbb{R}$, $g:\mathbb{R}\to\mathbb{R}$ and $h:\mathbb{R}\to\mathbb{R}$ be differentiable functions such that $f(x) = x^2 + 3x + 2$, $g(f(x)) = x$ and $h(g(g(x))) = x$ for all $x \in \mathbb{R}$. Then

A) $g'(2) = \frac{1}{15}$ B) $h'(1) = 666$ C) $h(0) = 16$ D) $h(g(3)) = 36$

Key : B, C

Sol: $f(c) = x^3 + 3x + 2$ $f^1(x) = 3x^2 + 3 > 0$

$g(f(x) = x$ $f : R \to R$ is bijection

$f^1 = g$

A) $g^1(f(x))f^1(x) = 1$

$g^1(f(x)) = \frac{1}{f^1(x)}$

$g^1(f(0)) = \frac{1}{f^1(0)}$

$g^1(2) = \frac{1}{3}$

B) $h^1(g(g(x)))g^1(g(x))g^1(x) = 1$

$h^1(1)g^1(g(236))g^1(236) = 1$

$h^1(1) \times \frac{1}{6} \times \frac{1}{111} = 1$

$h^1(1) = 666$

C) $h(g(g(x))) = h(0)\measuredangle$

$g(g(x)) = 0$

$g(x) = g^{-1}(0) = f(0) = 2$

$x = g^{-1}(2)$

$= f(2)$

$= 16$

$h(g(g(x))) = x$

$h(g(g(16)) = 16$

$h(0) = 16$

D) $h(g(g(x)) = x$

$g(x) = 3$

$f^{-1}(x) = 3$

$x = f(3)$

=38

h(g(3))=38

46. Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$**, where** $\alpha \in \mathbb{R}$ **. Suppose** $Q = [q_{ij}]$ **is a matrix such that PQ = kI, where** $k \in \mathbb{R}$**,** $k \neq 0$ **and I is the identity matrix of order 3. If** $q_{23} = -\frac{k}{8}$ **and** $\det(Q) = \frac{k^2}{2}$**, then**

A) $\alpha = 0, k = 8$ B) $4\alpha - k + 8 = 0$ C) $\det(P\,\mathrm{adj}(Q)) = 2^9$ D) $\det(Q\,\mathrm{adj}(P)) = 2^{13}$

Key : B,C

Sol : $PQ = KI$

$\Rightarrow |p||Q| = K^3$

$\Rightarrow |p| \times \frac{k^2}{2} = k^3$

$\Rightarrow |p| = 2k$

$\because p(2Q) = 2KI$

$\Rightarrow 2Q = adj(p)$

cofactor of -s is $\frac{-k}{4}$

$\Rightarrow -3\alpha - 4 = -\frac{k}{4}$

$\Rightarrow 3\alpha + 4 = \frac{k}{4}$

$\Rightarrow 12\alpha + 16 = k \Rightarrow 12\alpha + 16 = 6\alpha + 10$

$\alpha = -1$

$\alpha = -1, k = 4$ it satisficed option----------(2)

det (p adj Q) = $|p||Adj\,Q|$

$= |p||Q|^2$

$= 2k.\frac{k^4}{4}$

$= \frac{k^5}{2} = \frac{2^{10}}{2} = 2^9$

47. Let RS be the diameter of the circle $x^2+y^2=1$, where S is the point (1, 0). Let P be a variable point (other than R and S) on the circle and tangents to the circle at S and P meet at the point Q. The normal to the circle at P intersects a line drawn through Q parallel to RS at point E. Then the loocus of E passes through the point(s)

A) $\left(\frac{1}{3},\frac{1}{\sqrt{3}}\right)$ B) $\left(\frac{1}{4},\frac{1}{2}\right)$ C) $\left(\frac{1}{3},-\frac{1}{\sqrt{3}}\right)$ D) $\left(\frac{1}{4},-\frac{1}{2}\right)$

Key : A, C

Sol : Equation of the tangent at P is

$x\cos\theta+y\sin\theta=1$

when $x=1$

$y=\frac{1-\cos\theta}{\sin\theta}$

$y=\operatorname{cosec}\theta-\cot\theta$ & $x\sin\theta-y\cos\theta=0$

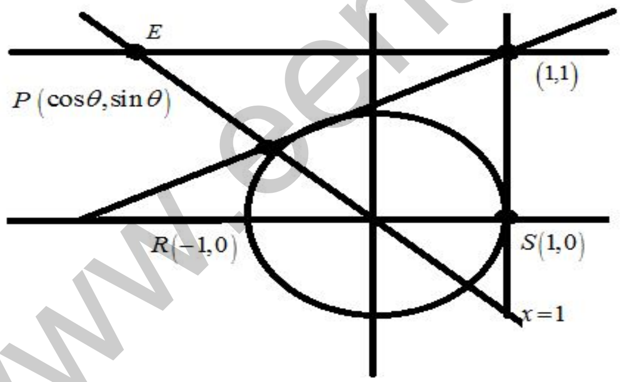
$\tan\theta=\frac{y}{x}$

Locus point of intersection of $\frac{1}{y}=\operatorname{cosec}\theta+\cot\theta$

$\frac{1}{y}-y=2\cot\theta$

$\frac{1-y^2}{y}=2\frac{x}{y}$

$y^2=1-2x$



48. Let $f:(0,\infty)\to R$ be a differentiable function such that. $f'(x)=2-\frac{f(x)}{x}$ for all $x\in(0,\infty)$ and $f(1)\neq 1$. Then

A) $\lim\limits_{x\to 0+} f'\left(\frac{1}{x}\right)=1$ B) $\lim\limits_{x\to 0+} x\,f\left(\frac{1}{x}\right)=2$

C) $\lim\limits_{x\to 0+} x^2 f'(x)=0$ D) $|f(x)|\le 2$ for all $x\in(0,2)$

Key : A

Sol: $f'(x) = z - \frac{f(x)}{x} \qquad x \in (0,a)$

$$\frac{dy}{dx} = z - \frac{y}{x}$$

Put $y = vx$

$$v + x\frac{dv}{dx} = z - v$$

$$x\frac{dv}{dx} = 2 - 2v$$

$$\frac{dv}{v-1} = -\frac{2dv}{x}$$

$$\log|v-1| + \log x^2 = \log C$$

$$x^2\left(\frac{y}{x} - 1\right) = C$$

$$x(y-x) = C \qquad (C \neq 0)$$

$$y = x + \frac{C}{x}$$

$$f(x) = x + \frac{C}{x}$$

$$f'(x) = 1 - \frac{C}{x^2}$$

$$\lim_{x\to 0+} x^2 f'(x) = x^2\left(1 - \frac{C}{x^2}\right) = x^2 - C = -C$$

$$\lim_{x\to 0+} f'\left(\frac{1}{x}\right) = \lim_{x\to 0+}\left(1 - Cx^2\right) = 1$$

$$\lim_{x\to 0+} n\, f\left(\frac{1}{x}\right) = \lim_{x\to 0+} n.\left(\frac{1}{x} + Cx\right)$$

$$\lim_{x\to 0+} 1 + Cx^2$$

$$= 1$$

$$x + \frac{C}{x} \geq 2\sqrt{C}$$

$$|f(x)| \geq 2\sqrt{C}$$

49. In a triangle XYZ, let x, y, z be the lengths of sides opposite to the angles X, Y, Z, respectively, and $2s = x + y + z$. If $\frac{s-x}{4} = \frac{s-y}{3} = \frac{s-z}{2}$ and area of incircle of the triangle XYZ is $\frac{8\pi}{3}$, then

A) area of the triangle XYZ is $6\sqrt{6}$

B) the radius of circumcircle of the triangle XYZ is $\frac{35}{6}\sqrt{6}$

C) $\sin\frac{X}{2}\sin\frac{Y}{2}\sin\frac{Z}{2} = \frac{4}{35}$

D) $\sin^2\left(\frac{X+Y}{2}\right) = \frac{3}{5}$

Key : A, C, D

Sol. : $\frac{s-x}{y} = \frac{s-y}{z} = \frac{s-z}{x} = \lambda$

$s-x = 4\lambda$, $s-y = 3\lambda$, $s-z = 2\lambda$

$s = 9\lambda$

$x = 5\lambda$, $y = 6\lambda$, $z = 7\lambda$

$\Rightarrow$ x : y : z = 5 : 6 : 7

$\pi r^2 = \frac{8\pi}{3}$

$r^2 = \frac{8}{3}$ $r^2 = \frac{8}{3}$ $r = \frac{2\sqrt{2}}{\sqrt{3}}$

Δxyz $\Delta = rs$

$= \frac{2\sqrt{2}}{\sqrt{3}} > 9\lambda$

$= 6\sqrt{6}\,\lambda$

$\Delta = \sqrt{9\lambda(4\lambda)(3\lambda)(2\lambda)}$

$= \sqrt{8\times 27\lambda^4}$

$= 2\sqrt{2}.3\sqrt{3}\lambda^2$

$= 6\sqrt{6}\,\lambda^2$

$s = 9\lambda$

$r = \frac{\Delta}{s} = \frac{6\sqrt{6}\lambda}{9} = \frac{2\sqrt{2}}{\sqrt{3}}$

$\lambda = \frac{2\sqrt{2}}{\sqrt{3}} \times \frac{9}{6\sqrt{6}}$

$\lambda = 1$

x = 5, y = 6, z = 7

(A) $\Delta xyz = 6\sqrt{6}$

(B) $R=\frac{xyz}{4\Delta}=\frac{35}{4\sqrt{6}}$

(C) $r=4R\sin\frac{x}{2}\sin\frac{y}{2}\sin\frac{z}{2}$

$\sin\frac{x}{2}\sin\frac{y}{2}\sin\frac{z}{2}=\frac{4}{4R}\quad=\frac{\left(\frac{2\sqrt{2}}{\sqrt{3}}\right)}{4\times\frac{35}{4\sqrt{6}}}$

$=\frac{2\sqrt{2}}{\sqrt{3}}\times\frac{\sqrt{6}}{35}=\frac{4}{35}$

(D) $\sin^2\left(\frac{x+y}{2}\right)=\cos^2\frac{z}{2}$

$=\frac{1+\cos z}{2}$

$\cos z=\frac{x^2+y^2-z^2}{2xy}$

$=\frac{25+36-49}{2(5)(6)}\quad=\frac{1}{5}$

$\sin^2\left(\frac{x\times y}{z}\right)=\frac{1+\frac{1}{5}}{2}\quad=\frac{3}{5}$

SECTION 3 (Maximum Marks : 15)

* **This section contains FIVE questions.**
* **The answer to each question is a SINGLE DIGIT INTEGER ranging from 0 to 9, both iclusive**
* **For each question, darken the bubble corresponding to the correct integer in the ORS.**
* **For each question, marks will be awarded in one of the following categories:**
  Full Marks : +3 If only the bubble corresponding to the correct answer is darkened.
  Zero Marks : 0 In all other cases.

50. Let m be the smallest positive integer such that the coefficient of x^2 in the expansion of $(1+x)^2+(1+x)^3+.....+(1+x)^{49}+(1+mx)^{50}$ is $(3n+1)\,{}^{51}C_3$ for some positive integer n. Then the value of n is

Key : 5

Sol: $2c_2+3c_2+..+49c_2+50c_2m^2=(3n+1)51c_3$

$50c_3+50c_2m^2=(3n+1)\frac{51}{3}50c_2$

$\frac{48}{3}+m^2=17(3n+1)$

$16+m^2=51n+17$

$m^2=51n+1$

n=5

51. Let $\alpha, \beta \in \mathbb{R}$ be such that $\lim_{x\to 0}\frac{x^2 \sin(\beta x)}{\alpha x - \sin x} = 1$. Then $6(\alpha+\beta)$ equals

Key : 7

Sol: $x \xrightarrow{\lim} 0 \frac{x^2 \sin \beta x}{\alpha x - \sin x} = 1$

$$x \xrightarrow{lit} 0 \frac{\beta x^3}{\alpha x - \left(\frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} + ...\right)} = 1$$

$\alpha - 1 = 0 \quad \Rightarrow \alpha = 1$

$$\frac{\beta}{\left(\frac{1}{6}\right)} = 1$$

$6\beta = 1$

$6(\alpha+\beta) = 6\alpha + 6\beta$

$=6+1$

$6(\alpha+\beta) = 7$

52. Let $z = \frac{-1+\sqrt{3}i}{2}$, where $i = \sqrt{-1}$, and $r, s \in \{1,2,3\}$. Let $P = \begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix}$ and I be the identity matrix of order 2. Then the total number of ordered pairs (r,s) for which $P^2 = -I$ is

Key : 1

Sol: $Z = \frac{-1+\sqrt{3}i}{2} = \omega$

$$p^2 = \begin{bmatrix} (-1)^r z^r & z^{2r} \\ z^{2r} & z^r \end{bmatrix}\begin{bmatrix} (-1)^r z^r & z^{2r} \\ z^{2r} & z^r \end{bmatrix}$$

$z^{2r} + z^{4r} = -1$

r=1,3

$z^{r+2r}((-1)^r + 1) = 0$

r is odd

when r=1,

$z^2 + z^{4r} = -1$

$\omega^2 + \omega^r = -1$

r=1

(1,1)

No of ordered pairs =1

53. The total number of distinct $x \in \mathbb{R}$ for which $\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 10$ is.

Key : 2

Sol. : $\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 10$

$x^3 \begin{vmatrix} 1 & 1 & 1+x^3 \\ 2 & 4 & 1+8x^3 \\ 3 & 9 & 1+27x^3 \end{vmatrix} = 10$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - R_2$

$x^3 \begin{vmatrix} 1 & 1 & 1+x^3 \\ 1 & 3 & 7x^3 \\ 1 & 5 & 19x^3 \end{vmatrix} = 10$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - R_2$

$x^3 \begin{vmatrix} 1 & 1 & 1+x^3 \\ 0 & 2 & 6x^3-1 \\ 0 & 4 & 18x^3-1 \end{vmatrix} = 10$

$x^3\left[\left(36x^3-2\right)-\left(24x^3-4\right)\right]=10$

$x^3\left(6x^3+1\right)=5$

$6x^6+x^3-5=0$

Put $x^3=t$

$6t^2+t-5=0$

$6t^2+6t-5t-5=0$

$6t(t+1)-5(t+1)=0$

$t=\frac{5}{6};\ t=-1$

$x^3=\frac{5}{6};\ x^3=-1$

$x\in R \Rightarrow x=\sqrt[3]{\frac{5}{6}},\ -1$

No. of real values of x is 2.

54. The total number of distinct $x\in[0,1]$ for which $\int_0^x \frac{t^2}{1+t^4}dt = 2x-1$ is.

Key : 1

Sol : $g(x)=\int_0^x \frac{t^2}{1+t^4}\,dt$

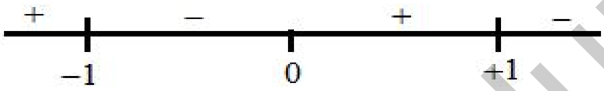
$g^1.(x)=\frac{x^2}{1+x^4}>0$

$g(x)\uparrow$

$$g^{11}(x)=\frac{(1+x^4)2x-x^2(4x^3)}{(1+x^4)^2}$$

$$=\frac{2x+2x^5-4x^5}{(1+x^4)^2}$$

$$=\frac{2x+2x^5}{(1+x^4)^2}$$



$$=\frac{2x(1+x)(1-x)(1+x^2)}{(1+x^4)2}$$

y(x)$\uparrow$ & concave up

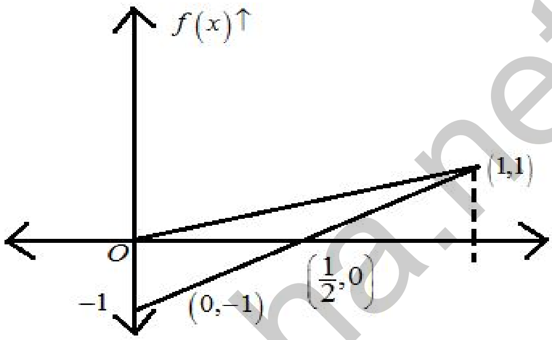
In the interval

$x\in(0,1)$ it has one solutions

$f(x)=2x-1$

$f^1(x)=2>0$

$f(x)\uparrow$



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