JEE (Advanced) - 2014

Held on 25-05-2014

CODE 8 **PAPER-1**

P1-14-8
1603908

Time : 3 Hours **Maximum Marks : 180**

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

INSTRUCTIONS

A. General

1. This booklet is your Question Paper. Do not break the seal of this booklet before being instructed to do so by the invigilators.
2. The question paper CODE is printed on the left hand top corner of this sheet and on the back cover page of this booklet.
3. Blank spaces and blank pages are provided in the question paper for your rough work. No additional sheets will be provided for rough work.
4. Blank papers, clipboards, log tables, slide rules, calculators, cameras, cellular phones, pagers and electronic gadget of any kind are NOT allowed inside the examination hall.
5. Write your Name and Roll number in the space provided on the back cover of this booklet.
6. Answers to the questions and personal details are to be filled on an Optical Response Sheet, which is provided separately. The ORS is a doublet of two sheets - upper and lower, having identical layout. The upper sheet is a machine-gradable Objective Response Sheet (ORS) which will be collected by the invigilator at the end of the examination. The upper sheet is designed in such a way that darkening the bubble with a ball point pen will leave an identical impression at the corresponding place on the lower sheet. You will be allowed to take away the lower sheet at the end of the examination. (**see Figure-1** on the back cover page for the correct way of darkening the bubbles for valid answers).
7. **Use a black ball point pen only to darken the bubbles on the upper original sheet.** Apply sufficient pressure so that the impression is created on the lower sheet. **See Figure-1** on the back cover page for appropriate way of darkening the bubbles for valid answers.
8. DO NOT TAMPER WITH / MUTILATE THE ORS OR THIS BOOKLET.
9. On breaking the seal of the booklet check that it contains **28** pages and all the **60** questions and corresponding answer choices are legible. Read carefully the instruction printed at the beginning of each section.

B. Filling the right part of the ORS

10. The ORS also has a **CODE** printed on its left and right parts.
11. Verify that the CODE printed on the ORS (on both the left and right parts) is the same as that on this booklet and put your signature in the Box designated as R4.
12. **IF THE CODES DO NOT MATCH, ASK FOR A CHANGE OF THE BOOKLET / ORS AS APPLICABLE.**
13. Write your Name, Roll No. and the name of centre and sign with pen in the boxes provided on the upper sheet of ORS. **Do not write any of this anywhere else.** Darken the appropriate bubble **UNDER** each digit of your Roll No. in such way that the impression is created on the bottom sheet. (**see example in Figure 2** on the back cover)

C. Question Paper Format

The question paper consists of **three parts** (Physics, Chemistry and Mathematics). Each part consists of two sections.

14. **Section 1** contains **10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE THAN ONE are correct**.
15. **Section 2** contains **10 questions**. The answer to each of the questions is a single-digit integer, ranging from 0 to 9 (both inclusive).

DO NOT BREAK THE SEAL WITHOUT BEING INSTRUCTED TO DO SO BY THE INVIGILATOR

Please read the last page of this booklet for rest of the instructions.

PART I : PHYSICS

SECTION – 1 : (One or More Than One Options Correct Type)

This section contains 10 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONE or MORE THAN ONE are correct.

1. Heater of an electric kettle is made of a wire of length L and diameter d. It takes 4 minutes to raise the temperature of 0.5 kg water by 40 K. This heater is replaced by a new heater having two wires of the same material, each of length L and diameter $2d$. The way these wires are connected is given in the options. How much time in minutes will it take to raise the temperature of the same amount of water by 40 K?

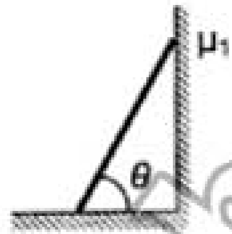
   (A) 4 if wires are in parallel
   (B) 2 if wires are in series
   (C) 1 if wires are in series
   (D) 0.5 if wires are in parallel

2. One end of a taut string of length $3m$ along the x axis is fixed at $x = 0$. The speed of the waves in the string is $100\ ms^{-1}$. The other end of the string is vibrating in the y direction so that stationary waves are set up in the string. The possible waveform(s) of these stationary waves is(are)

   (A) $y(t) = A \sin\frac{\pi x}{6} \cos\frac{50\pi t}{3}$
   (B) $y(t) = A \sin\frac{\pi x}{3} \cos\frac{100\pi t}{3}$
   (C) $y(t) = A \sin\frac{5\pi x}{6} \cos\frac{250\pi t}{3}$
   (D) $y(t) = A \sin\frac{5\pi x}{2} \cos 250\pi t$

Space for Rough Work

3. In the figure, a ladder of mass m is shown leaning against a wall. It is in static equilibrium making an angle θ with the horizontal floor. The coefficient of friction between the wall and the ladder is μ_1 and that between the floor and the ladder is μ_2. The normal reaction of the wall on the ladder is N_1 and that of the floor is N_2. If the ladder is about to slip, then



(A) $\mu_1 = 0 \quad \mu_2 \neq 0$ and $N_2 \tan\theta = \frac{mg}{2}$

(B) $\mu_1 \neq 0 \quad \mu_2 = 0$ and $N_1 \tan\theta = \frac{mg}{2}$

(C) $\mu_1 \neq 0 \quad \mu_2 \neq 0$ and $N_2 = \frac{mg}{1+\mu_1\mu_2}$

(D) $\mu_1 = 0 \quad \mu_2 \neq 0$ and $N_1 \tan\theta = \frac{mg}{2}$

4. A light source, which emits two wavelengths $\lambda_1 = 400\ nm$ and $\lambda_2 = 600\ nm$, is used in a Young's double slit experiment. If recorded fringe widths for λ_1 and λ_2 are β_1 and β_2 and the number of fringes for them within a distance y on one side of the central maximum are m_1 and m_2, respectively, then

(A) $\beta_2 > \beta_1$

(B) $m_1 > m_2$

(C) From the central maximum, 3rd maximum of λ_2 overlaps with 5th minimum of λ_1

(D) The angular separation of fringes for λ_1 is greater than λ_2

Space for Rough Work

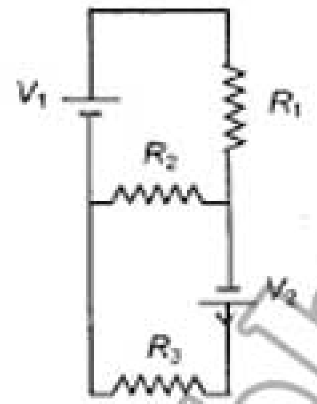
5. Two ideal batteries of *emf* V_1 and V_2 and three resistances R_1, R_2 and R_3 are connected as shown in the figure. The current in resistance R_2 would be zero if

(A) $V_1 = V_2$ and $R_1 = R_2 = R_3$

(B) $V_1 = V_2$ and $R_1 = 2R_2 = R_3$

(C) $V_1 = 2V_2$ and $2R_1 = 2R_2 = R_3$

(D) $2V_1 = V_2$ and $2R_1 = R_2 = R_3$



6. Let $E_1(r)$, $E_2(r)$ and $E_3(r)$ be the respective electric fields at a distance r from a point charge Q, an infinitely long wire with constant linear charge density λ, and an infinite plane with uniform surface charge density σ. If $E_1(r_0) = E_2(r_0) = E_3(r_0)$ at a given distance r_0, then

(A) $Q = 4\sigma\pi r_0^2$

(B) $r_0 = \frac{\lambda}{2\pi\sigma}$

(C) $E_1(r_0/2) = 2E_2(r_0/2)$

(D) $E_2(r_0/2) = 4E_3(r_0/2)$

Space for Rough Work

7. A student is performing an experiment using a resonance column and a tuning fork of frequency $244\ s^{-1}$. He is told that the air in the tube has been replaced by another gas (assume that the column remains filled with the gas). If the minimum height at which resonance occurs is $(0.350 \pm 0.005)\ m$, the gas in the tube is

(**Useful information** : $\sqrt{167RT} = 640\ J^{1/2}mole^{-1/2}$; $\sqrt{140RT} = 590\ J^{1/2}mole^{-1/2}$. The molar masses M in grams are given in the options. Take the values of $\sqrt{\frac{10}{M}}$ for each gas as given there.)

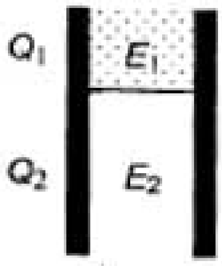
(A) Neon$\left(M = 20, \sqrt{\frac{10}{20}} = \frac{7}{10}\right)$

(B) Nitrogen$\left(M = 28, \sqrt{\frac{10}{28}} = \frac{3}{5}\right)$

(C) Oxygen$\left(M = 32, \sqrt{\frac{10}{32}} = \frac{9}{16}\right)$

(D) Argon$\left(M = 36, \sqrt{\frac{10}{36}} = \frac{17}{32}\right)$

A parallel plate capacitor has a dielectric slab of dielectric constant K between its plates that covers 1/3 of the area of its plates, as shown in the figure. The total capacitance of the capacitor is C while that of the portion with dielectric in between is C_1. When the capacitor is charged, the plate area covered by the dielectric gets charge Q_1 and the rest of the area gets charge Q_2. The electric field in the dielectric is E_1 and that in the other portion is E_2. Choose the correct option/options, ignoring edge effects.



(A) $\frac{E_1}{E_2} = 1$

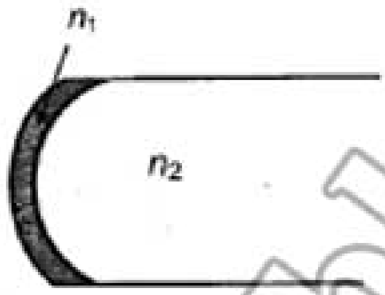
(B) $\frac{E_1}{E_2} = \frac{1}{K}$

(C) $\frac{Q_1}{Q_2} = \frac{3}{K}$

(D) $\frac{C}{C_1} = \frac{2+K}{K}$

Space for Rough Work

9. A transparent thin film of uniform thickness and refractive index $n_1 = 1.4$ is coated on the convex spherical surface of radius R at one end of a long solid glass cylinder of refractive index $n_2 = 1.5$, as shown in the figure. Rays of light parallel to the axis of the cylinder traversing through the film from air to glass get focused at distance f_1 from the film, while rays of light traversing from glass to air get focused at distance f_2 from the film. Then



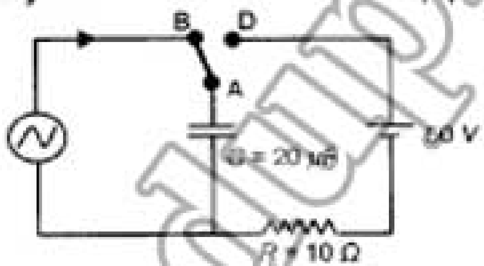
(A) $|f_1| = 3R$

(B) $|f_1| = 2.8R$

(C) $|f_2| = 2R$

(D) $|f_2| = 1.4R$

10. At time $t = 0$, terminal A in the circuit shown in the figure is connected to B by a key and an alternating current $I(t) = I_0 \cos(\omega t)$, with $I_0 = 1A$ and $\omega = 500\ rad\ s^{-1}$ starts flowing in it with the initial direction shown in the figure. At $t = \frac{7\pi}{6\omega}$, the key is switched from B to D. Now onwards only A and D are connected. A total charge Q flows from the battery to charge the capacitor fully. If $C = 20\mu F$, $R = 10\ \Omega$ and the battery is ideal with *emf* of $50V$, identify the correct statement (s).



(A) Magnitude of the maximum charge on the capacitor before $t = \frac{7\pi}{6\omega}$ is $1 \times 10^{-3} C$.

(B) The current in the left part of the circuit just before $t = \frac{7\pi}{6\omega}$ is clockwise.

(C) Immediately after A is connected to D, the current in R is $10A$.

(D) $Q = 2 \times 10^{-3} C$.

Space for Rough Work

SECTION – 2 : (One Integer Value Correct Type)

This section contains 10 questions. Each question, when worked out will result in one integer from 0 to 9 (both inclusive).

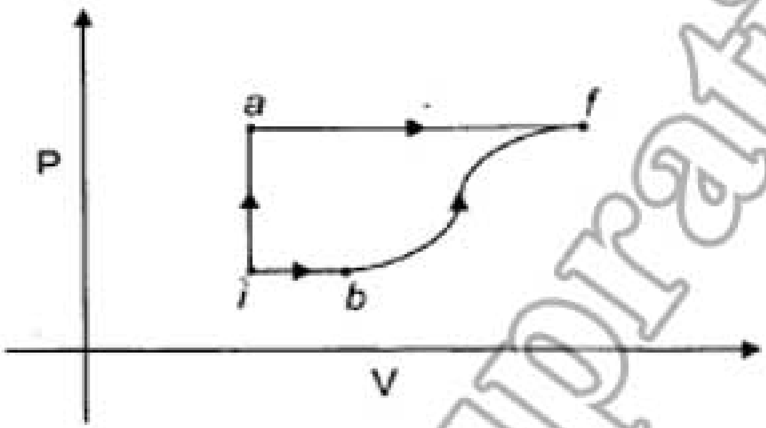
11. Two parallel wires in the plane of the paper are distance X_0 apart. A point charge is moving with speed u between the wires in the same plane at a distance X_1 from one of the wires. When the wires carry current of magnitude I in the same direction, the radius of curvature of the path of the point charge is R_1. In contrast, if the currents I in the two wires have directions opposite to each other, the radius of curvature of the path is R_2. If $\frac{X_0}{X_1} = 3$, the value of $\frac{R_1}{R_2}$ is 3

12. During Searle's experiment, zero of the Vernier scale lies between $3.20 \times 10^{-2}\ m$ and $3.25 \times 10^{-2}\ m$ of the main scale. The 20^{th} division of the Vernier scale exactly coincides with one of the main scale divisions. When an additional load of 2 kg is applied to the wire, the zero of the Vernier scale still lies between $3.20 \times 10^{-2}\ m$ and $3.25 \times 10^{-2}\ m$ of the main scale but now the 45^{th} division of Vernier scale coincides with one of the main scale divisions. The length of the thin metallic wire is 2 m and its cross-sectional area is $8 \times 10^{-7} m^2$. The least count of the Vernier scale is $1.0 \times 10^{-5}\ m$. The maximum percentage error in the Young's modulus of the wire is (

Space for Rough Work

13. To find the distance d over which a signal can be seen clearly in foggy conditions, a railways engineer uses dimensional analysis and assumes that the distance depends on the mass density ρ of the fog, intensity (power/area) S of the light from the signal and its frequency f. The engineer finds that d is proportional to $S^{1/n}$. The value of n is 2

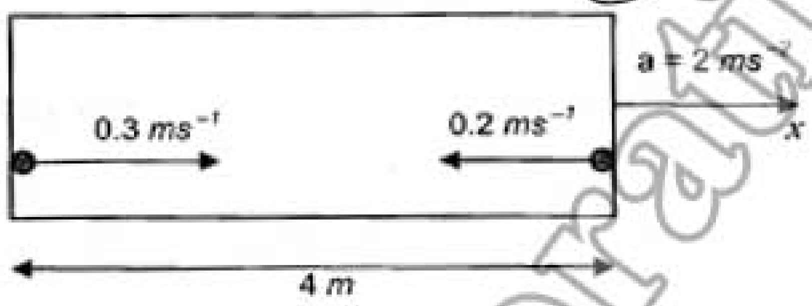
14. A thermodynamic system is taken from an initial state i with internal energy $U_i = 100\,J$ to the final state f along two different paths iaf and ibf, as schematically shown in the figure. The work done by the system along the paths af, ib and bf are $W_{af} = 200\,J, W_{ib} = 50\,J$ and $W_{bf} = 100\,J$ respectively. The heat supplied to the system along the path iaf, ib and bf are Q_{iaf}, Q_{ib} and Q_{bf} respectively. If the internal energy of the system in the state b is $U_b = 200\,J$ and $Q_{iaf} = 500\,J$, the ratio Q_{bf}/Q_{ib} is 3



Space for Rough Work

15. A galvanometer gives full scale deflection with 0.006 A current. By connecting it to a 4990 Ω resistance, it can be converted into a voltmeter of range 0 - 30 V. If connected to a $\frac{2n}{249}\Omega$ resistance, it becomes an ammeter of range 0 - 1.5 A. The value of n is 3

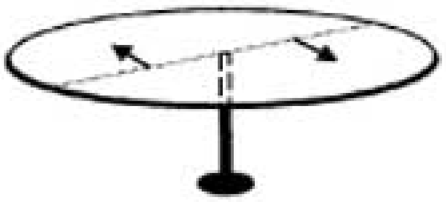
16. A rocket is moving in a gravity free space with a constant acceleration of 2 ms^{-2} along + x direction (see figure). The length of a chamber inside the rocket is 4 m. A ball is thrown from the left end of the chamber in + x direction with a speed of 0.3 ms^{-1} relative to the rocket. At the same time, another ball is thrown in $-x$ direction with a speed of 0.2 ms^{-1} from its right end relative to the rocket. The time in seconds when the two balls hit each other is 1



Space for Rough Work

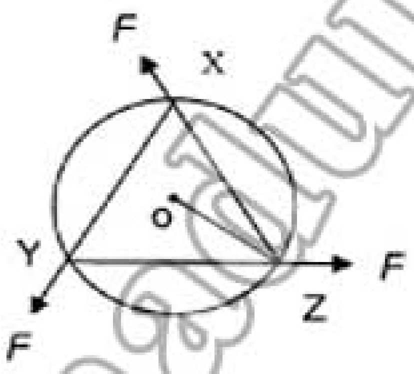
17. A horizontal circular platform of radius 0.5 m and mass 0.45 kg is free to rotate about its axis. Two massless spring toy-guns, each carrying a steel ball of mass 0.05 kg are attached to the platform at a distance 0.25 m from the centre on its either sides along its diameter (see figure). Each gun simultaneously fires the balls horizontally and perpendicular to the diameter in opposite directions. After leaving the platform, the balls have horizontal speed of 9 ms^{-1} with respect to the ground. The rotational speed of the platform in $rad\ s^{-1}$ after the balls leave the platform is

(2)



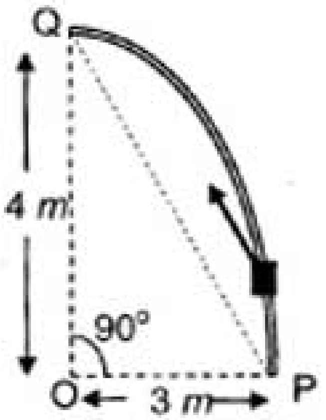
18. A uniform circular disc of mass 1.5 kg and radius 0.5 m is initially at rest on a horizontal frictionless surface. Three forces of equal magnitude F = 0.5 N are applied simultaneously along the three sides of an equilateral triangle XYZ with its vertices on the perimeter of the disc (see figure). One second after applying the forces, the angular speed of the disc in $rad\ s^{-1}$ is

(4)



Space for Rough Work

19. Consider an elliptically shaped rail PQ in the vertical plane with OP = 3 *m* and OQ = 4 *m*. A block of mass 1 *kg* is pulled along the rail from P to Q with a force of 18 *N*, which is always parallel to line PQ (see the figure given). Assuming no frictional losses, the kinetic energy of the block when it reaches Q is $(n \times 10)$Joules. The value of *n* is (take acceleration due to gravity = $10\ ms^{-2}$)



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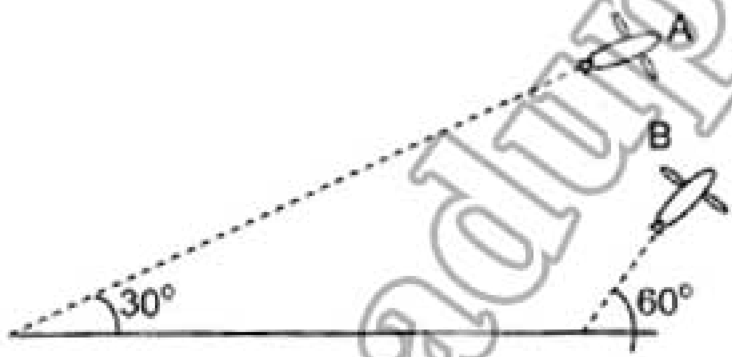
20. Airplanes A and B are flying with constant velocity in the same vertical plane at angles 30° and 60° with respect to the horizontal respectively as shown in figure. The speed of A is $100\sqrt{3}\ ms^{-1}$. At time $t = 0\ s$, an observer in A finds B at a distance of 500 *m*. This observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at $t = t_0$, A just escapes being hit by B, t_0 in seconds is

3



Space for Rough Work

PART II : CHEMISTRY

SECTION – 1 : (One or More Than One Options Correct Type)

This section contains 10 multiple choice type questions. Each question has four choices (A), (B), (C) and (D) out of which ONE or MORE THAN ONE are correct.

21. For the reaction :

$$I^- + ClO_3^- + H_2SO_4 \rightarrow Cl^- + HSO_4^- + I_2$$

The correct statement(s) in the balanced equation is/are :

(A) Stoichiometric coefficient of HSO_4^- is 6.

(B) Iodide is oxidized.

(C) Sulphur is reduced.

(D) H_2O is one of the products.

22. The pair(s) of reagents that yield paramagnetic species is/are

(A) Na and excess of NH_3 $Na + NH_3 \rightarrow NaNH_2$

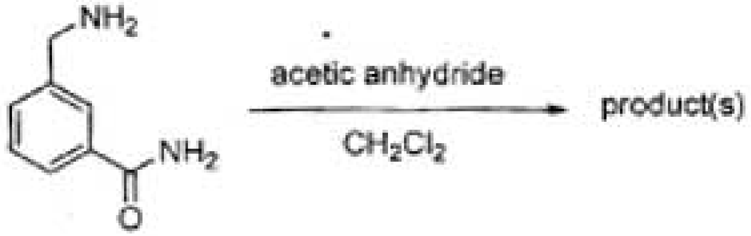
(B) K and excess of O_2

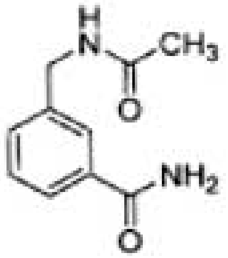
(C) Cu and dilute HNO_3

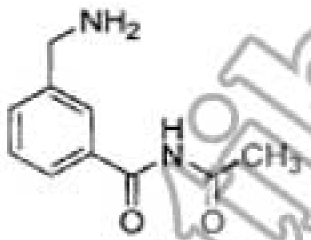
(D) O_2 and 2-ethylanthraquinol

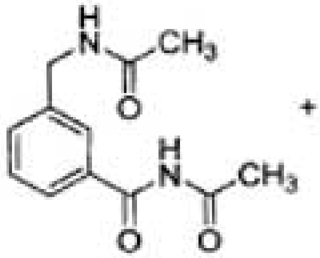
Space for Rough Work

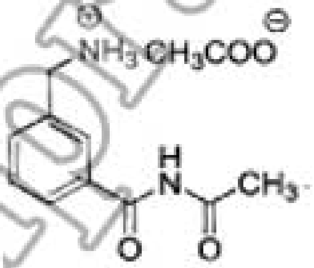
23. In the reaction shown below, the major product(s) formed is/are



(A)  + CH_3COOH

(B)  + CH_3COOH

(C)  + H_2O

(D) 

Space for Rough Work

24. In a galvanic cell, the salt bridge

(A) does not participate chemically in the cell reaction.

(B) stops the diffusion of ions from one electrode to another.

(C) is necessary for the occurrence of the cell reaction.

(D) ensures mixing of the two electrolytic solutions.

25. Upon heating with Cu_2S, the reagent(s) that give copper metal is/are

(A) $CuFeS_2$

(B) CuO

(C) Cu_2O

(D) $CuSO_4$

Space for Rough Work

26. Hydrogen bonding plays a central role in the following phenomena:

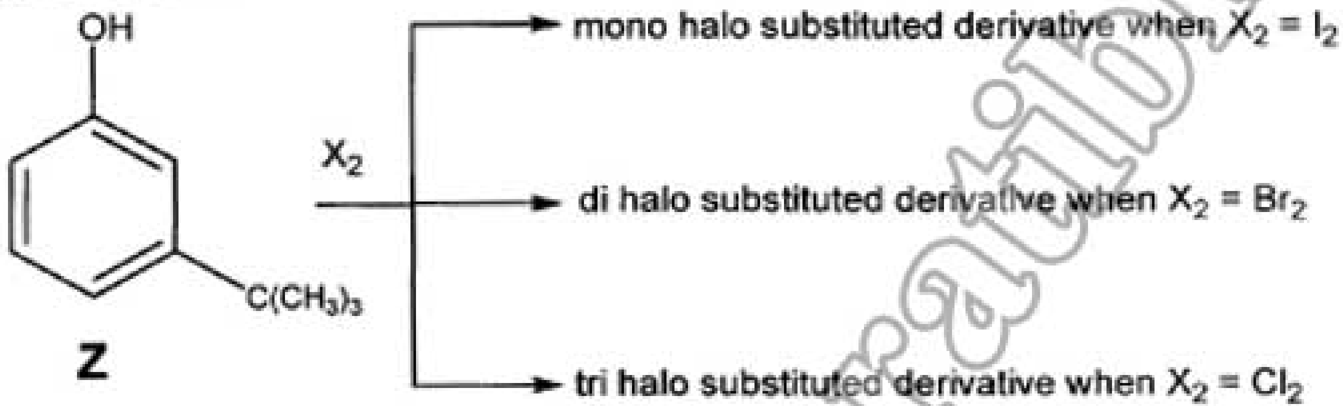
(A) Ice floats in water.

(B) Higher Lewis basicity of primary amines than tertiary amines in aqueous solutions.

(C) Formic acid is more acidic than acetic acid.

(D) Dimerisation of acetic acid in benzene.

27. The reactivity of compound **Z** with different halogens under appropriate conditions is given below :



Z

The observed pattern of electrophilic substitution can be explained by

(A) the steric effect of the halogen

(B) the steric effect of the *tert*-butyl group

(C) the electronic effect of the phenolic group

(D) the electronic effect of the *tert*-butyl group

Space for Rough Work

28. The correct combination of names for isomeric alcohols with molecular formula $C_4H_{10}O$ is/are

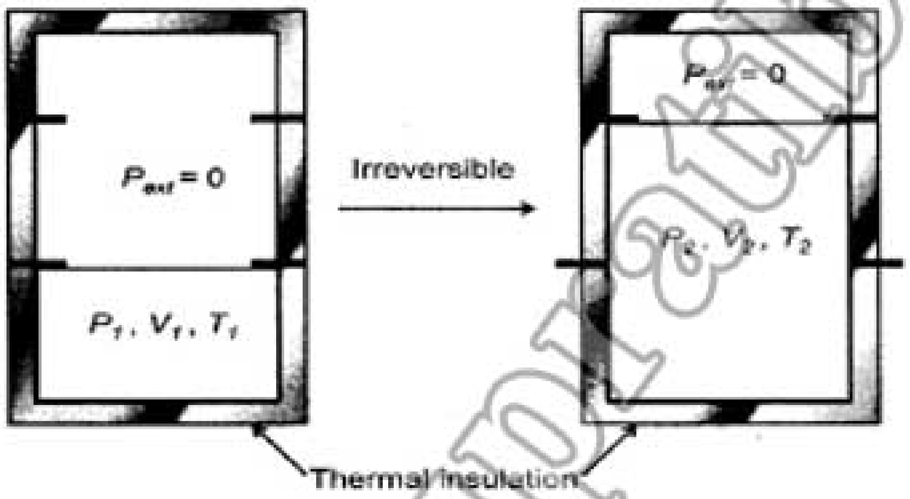
(A) *tert*-butanol and 2-methylpropan-2-ol

(B) *tert*-butanol and 1, 1-dimethylethan-1-ol

(C) *n*-butanol and butan-1-ol

(D) isobutyl alcohol and 2-methylpropan-1-ol

29. An ideal gas in a thermally insulated vessel at internal pressure = P_1, volume = V_1 and absolute temperature = T_1 expands irreversibly against zero external pressure, as shown in the diagram. The final internal pressure, volume and absolute temperature of the gas are P_2, V_2 and T_2, respectively. For this expansion,



(A) $q = 0$

(B) $T_2 = T_1$

(C) $P_2V_2 = P_1V_1$

(D) $P_2V_2^{\gamma} = P_1V_1^{\gamma}$

30. The correct statement(s) for orthoboric acid is/are

(A) It behaves as a weak acid in water due to self ionization.

(B) Acidity of its aqueous solution increases upon addition of ethylene glycol.

(C) It has a three dimensional structure due to hydrogen bonding.

(D) It is a weak electrolyte in water.

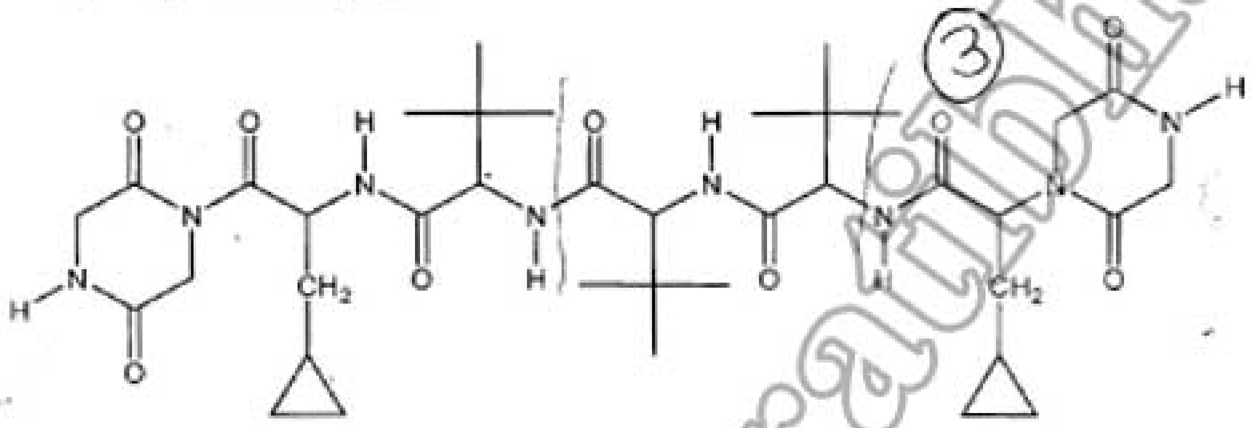
Space for Rough Work

SECTION – 2 : (One Integer Value Correct Type)

This section contains 10 questions. Each question, when worked out will result in one integer from 0 to 9 (both inclusive).

31. In an atom, the total number of electrons having quantum numbers $n = 4$, $|m_\ell| = 1$ and $m_s = -1/2$ is ~~4~~ 2

32. The total number of ***distinct naturally occurring amino acids*** obtained by complete acidic hydrolysis of the peptide shown below is 3



33. If the value of Avogadro number is $6.023 \times 10^{23}\ mol^{-1}$ and the value of Boltzmann constant is $1.380 \times 10^{-23} J\ K^{-1}$, then the number of significant digits in the calculated value of the universal gas constant is 4

$K = \frac{R}{N_A}$

34. A compound $\mathbf{H_2X}$ with molar weight of 80 g is dissolved in a solvent having density of 0.4 $g\ ml^{-1}$. Assuming no change in volume upon dissolution, the **molality** of a 3.2 molar solution is 8

$N_A \times K$

35. $\mathbf{MX_2}$ dissociates into $\mathbf{M^{2+}}$ and $\mathbf{X^-}$ ions in an aqueous solution, with a degree of dissociation (α) of 0.5. The ratio of the observed depression of freezing point of the aqueous solution to the value of the depression of freezing point in the absence of ionic dissociation is 2

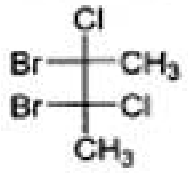
Space for Rough Work

36. Consider the following list of reagents :

Acidified $K_2Cr_2O_7$, alkaline $KMnO_4$, $CuSO_4$, H_2O_2, Cl_2, O_3, $FeCl_3$, HNO_3 and $Na_2S_2O_3$.

The total number of reagents that can oxidise aqueous iodide to iodine is 7

37. The total number(s) of **stable** conformers with **non-zero** dipole moment for the following compound is (are)



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38. Among PbS, CuS, HgS, MnS, Ag_2S, NiS, CoS, Bi_2S_3 and SnS_2, the total number of **BLACK** coloured sulphides is 4

39. Consider all possible isomeric ketones, including stereoisomers of MW = 100. All these isomers are independently reacted with $NaBH_4$ (**NOTE**: stereoisomers are also reacted separately). The total number of ketones that give a racemic product(s) is/are 0

40. A list of species having the formula XZ_4 is given below.

XeF_4, SF_4, SiF_4, BF_4^-, BrF_4^-, $[Cu(NH_3)_4]^{2+}$, $[FeCl_4]^{2-}$, $[CoCl_4]^{2-}$ and $[PtCl_4]^{2-}$.

Defining shape on the basis of the location of X and Z atoms, the total number of species having a square planar shape is 3

Space for Rough Work

PART III : MATHEMATICS

SECTION – 1 : (One or More Than One Options Correct Type)

This section contains 10 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONE or MORE THAN ONE are correct.

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41. Let $f:[a,b] \to [1, \infty)$ be a continuous function and let $g: \mathbb{R} \to \mathbb{R}$ be defined as

$$g(x) = \begin{cases} 0 & \text{if } x < a, \\ \int_a^x f(t)dt & \text{if } a \le x \le b, \\ \int_a^b f(t)dt & \text{if } x > b. \end{cases}$$

Then

(A) $g(x)$ is continuous but not differentiable at a

(B) $g(x)$ is differentiable on $\mathbb{R}$

(C) $g(x)$ is continuous but not differentiable at b

(D) $g(x)$ is continuous and differentiable at either a or b but not both

42. For every pair of continuous functions $f, g: [0,1] \to \mathbb{R}$ such that

$\max\{f(x): x \in [0,1]\} = \max\{g(x): x \in [0,1]\}$,

the correct statement(s) is(are) :

(A) $(f(c))^2 + 3f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0,1]$

(B) $(f(c))^2 + f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0,1]$

(C) $(f(c))^2 + 3f(c) = (g(c))^2 + g(c)$ for some $c \in [0,1]$

(D) $(f(c))^2 = (g(c))^2$ for some $c \in [0,1]$

Space for Rough Work

43. Let M be a 2×2 symmetric matrix with integer entries. Then M is invertible if

(A) the first column of M is the transpose of the second row of M

(B) the second row of M is the transpose of the first column of M

(C) M is a diagonal matrix with nonzero entries in the main diagonal

(D) the product of entries in the main diagonal of M is not the square of an integer

44. Let $\vec{x}$, $\vec{y}$ and $\vec{z}$ be three vectors each of magnitude $\sqrt{2}$ and the angle between each pair of them is $\frac{\pi}{3}$. If $\vec{a}$ is a nonzero vector perpendicular to $\vec{x}$ and $\vec{y} \times \vec{z}$ and $\vec{b}$ is a nonzero vector perpendicular to $\vec{y}$ and $\vec{z} \times \vec{x}$, then

(A) $\vec{b} = (\vec{b} \cdot \vec{z})(\vec{z} - \vec{x})$

(B) $\vec{a} = (\vec{a} \cdot \vec{y})(\vec{y} - \vec{z})$

(C) $\vec{a} \cdot \vec{b} = -(\vec{a} \cdot \vec{y})(\vec{b} \cdot \vec{z})$

(D) $\vec{a} = (\vec{a} \cdot \vec{y})(\vec{z} - \vec{y})$

45. From a point $P(\lambda, \lambda, \lambda)$, perpendiculars PQ and PR are drawn respectively on the lines $y = x$, $z = 1$ and $y = -x$, $z = -1$. If P is such that $\angle QPR$ is a right angle, then the possible value(s) of λ is(are)

(A) $\sqrt{2}$ (B) 1 (C) -1 (D) $-\sqrt{2}$

Space for Rough Work

46. Let M and N be two 3×3 matrices such that $MN = NM$. Further, if $M \neq N^2$ and $M^2 = N^4$, then

(A) determinant of $(M^2 + MN^2)$ is 0

(B) there is a 3×3 non-zero matrix U such that $(M^2 + MN^2)U$ is the zero matrix

(C) determinant of $(M^2 + MN^2) \geq 1$

(D) for a 3×3 matrix U, if $(M^2 + MN^2)U$ equals the zero matrix then U is the zero matrix

47. Let $f: (0, \infty) \to \mathbb{R}$ be given by

$$f(x) = \int_{\frac{1}{x}}^{x} e^{-\left(t+\frac{1}{t}\right)} \frac{dt}{t}.$$

Then

(A) $f(x)$ is monotonically increasing on $[1, \infty)$

(B) $f(x)$ is monotonically decreasing on $(0, 1)$

(C) $f(x) + f\left(\frac{1}{x}\right) = 0$, for all $x \in (0, \infty)$

(D) $f(2^x)$ is an odd function of x on $\mathbb{R}$

Space for Rough Work

48. Let $f: (-\frac{\pi}{2}, \frac{\pi}{2}) \to \mathbb{R}$ be given by

$$f(x) = (\log(\sec x + \tan x))^3.$$

Then

(A) $f(x)$ is an odd function

(B) $f(x)$ is a one-one function

(C) $f(x)$ is an onto function

(D) $f(x)$ is an even function

49. A circle S passes through the point $(0,1)$ and is orthogonal to the circles $(x-1)^2 + y^2 = 16$ and $x^2 + y^2 = 1$. Then

(A) radius of S is 8

(B) radius of S is 7

(C) centre of S is $(-7,1)$

(D) centre of S is $(-8,1)$

50. Let $a \in \mathbb{R}$ and let $f: \mathbb{R} \to \mathbb{R}$ be given by

$$f(x) = x^5 - 5x + a.$$

Then

(A) $f(x)$ has three real roots if $a > 4$

(B) $f(x)$ has only one real root if $a > 4$

(C) $f(x)$ has three real roots if $a < -4$

(D) $f(x)$ has three real roots if $-4 < a < 4$

Space for Rough Work

SECTION – 2 : (One Integer Value Correct Type)

This section contains 10 questions. Each question, when worked out will result in one integer from 0 to 9 (both inclusive).

51. The slope of the tangent to the curve $(y - x^5)^2 = x(1 + x^2)^2$ at the point $(1, 3)$ is

9

52. Let $f : [0, 4\pi] \to [0, \pi]$ be defined by $f(x) = \cos^{-1}(\cos x)$. The number of points $x \in [0, 4\pi]$ satisfying the equation

$$f(x) = \frac{10-x}{10}$$

is

53. The largest value of the non-negative integer a for which

$$\lim_{x \to 1} \left\{\frac{-ax + \sin(x-1) + a}{x + \sin(x-1) - 1}\right\}^{\frac{1-x}{1-\sqrt{x}}} = \frac{1}{4}$$

is

54. Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be respectively given by $f(x) = |x| + 1$ and $g(x) = x^2 + 1$. Define $h : \mathbb{R} \to \mathbb{R}$ by

$$h(x) = \begin{cases} \max \ \{f(x), g(x)\} & \text{if } x \le 0, \\ \min \ \{f(x), g(x)\} & \text{if } x > 0. \end{cases}$$

The number of points at which $h(x)$ is not differentiable is

Space for Rough Work

55. For a point P in the plane, let $d_1(P)$ and $d_2(P)$ be the distances of the point P from the lines $x - y = 0$ and $x + y = 0$ respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane and satisfying $2 \leq d_1(P) + d_2(P) \leq 4$, is

56. Let $n_1 < n_2 < n_3 < n_4 < n_5$ be positive integers such that $n_1 + n_2 + n_3 + n_4 + n_5 = 20$. Then the number of such distinct arrangements $(n_1, n_2, n_3, n_4, n_5)$ is 4

57. The value of

$$\int_0^1 4x^3 \left\{\frac{d^2}{dx^2}(1 - x^2)^5\right\} dx \_$$

is

Space for Rough Work

58. Let $\vec{a}, \vec{b}$, and $\vec{c}$ be three non-coplanar unit vectors such that the angle between every pair of them is $\frac{\pi}{3}$. If $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} = p\vec{a} + q\vec{b} + r\vec{c}$, where p, q and r are scalars, then the value of $\frac{p^2 + 2q^2 + r^2}{q^2}$ is 1

59. Let a, b, c be positive integers such that $\frac{b}{a}$ is an integer. If a, b, c are in geometric progression and the arithmetic mean of a, b, c is $b + 2$, then the value of

$$\frac{a^2 + a - 14}{a + 1}$$

is 3

60. Let $n \geq 2$ be an integer. Take n distinct points on a circle and join each pair of points by a line segment. Colour the line segment joining every pair of adjacent points by blue and the rest by red. If the number of red and blue line segments are equal, then the value of n is 2

Space for Rough Work

JEE (ADVANCED)-2014 'PAPER-1' KEY

Q.No.	Code-0	Code-1	Code-2	Code-3	Code-4	Code-5	Code-6	Code-7	Code-8	Code-9
1	C	CD	ABD	AC	AD	D	CD	ABC	BD	AC
2	BD	ABC	AC	C	CD	CD	ABD	AD	AC	D
3	AC	AC	CD	ABC	ABD	AD	AC	CD	CD	AD
4	D	AD	BD	CD	AC	AC	BD	C	ABC	AC
5	CD	C	ABC	CD	C	AC	AD	D	ABD	CD
6	ABD	D	CD	AC	AC	BD	CD	AC	C	ABC
7	ABC	BD	AD	ABD	BD	ABD	ABC	AC	D	CD
8	CD	CD	AC	D	ABC	C	D	BD	AD	ABD
9	AC	AC	D	AD	D	ABC	C	ABD	AC	C
10	AD	ABD	C	BD	CD	CD	AC	CD	CD	BD
11	5	5	3	2	5	4	3	8	3	4
12	3	4	2	5	3	5	4	5	4	2
13	4	2	8	2	4	2	5	4	3	5
14	4	3	5	5	8	3	4	5	2	5
15	2	3	4	8	5	3	2	2	5	2
16	5	5	5	4	2	8	5	3	8	3
17	2	5	4	3	2	5	3	3	4	5
18	3	8	2	5	4	2	5	5	2	8
19	5	4	5	4	3	4	8	2	5	3
20	8	2	3	3	5	5	2	4	5	4
21	BCD	ACD	AC	A	ABC	ACD	A	ABC	ABD	ACD
22	AC	ABC	ABC	ABD	ABD	ABC	BD	ABD	ABC	ABC
23	ABD	A	ABD	AC	ABC	ABD	ABD	BCD	A	ABC
24	ACD	ABC	BCD	ABC	A	AC	ABC	ACD	AC	BD
25	ABC	ABC	BD	ABC	BCD	ABD	ABC	ABC	BCD	ABC
26	ABC	AC	ACD	ABC	AC	ABC	AC	ABC	ABD	A
27	ABD	BCD	ABC	BD	ACD	BD	ABD	BD	ABC	AC
28	A	BD	ABC	ACD	ABC	BCD	ABC	A	ACD	BCD
29	ABC	ABD	ABD	BCD	ABD	ABC	ACD	AC	ABC	ABD
30	BD	ABC	A	BCD	BD	A	BCD	ABD	BD	ABD
31	3	5	2	7	8	6	4	3	6	4
32	2	4	6	3	4	3	8	6	3	3
33	4	6	3	6	2	7	2	4	4	7
34	8	3	4	4	6	4	6	5	8	6
35	6	7	8	5	3	5	3	7	2	5
36	7	3	3	2	4	6	6	8	7	2
37	4	6	4	8	7	4	7	6	3	6
38	5	4	7	4	3	2	5	2	6	8
39	3	8	5	6	6	3	4	3	5	4
40	6	2	6	3	5	8	3	4	4	3
41	AD	AB	CD	BD	ABC	ACD	BC	ACD	AC	ABC
42	BC	AD	BC	AC	C	BD	ABC	AB	AD	BD
43	C	ACD	ABC	ACD	AD	AD	ABC	BD	CD	ACD
44	ABC	BD	C	CD	AB	BC	AD	C	ABC	AD
45	ABC	AC	AD	BC	BD	ABC	BD	CD	C	AB
46	AC	ABC	AB	ABC	ACD	C	AC	ABC	AB	BC
47	ACD	C	BD	C	AC	CD	ACD	AD	ACD	CD
48	CD	ABC	ACD	ABC	ABC	AB	AB	AC	ABC	ABC
49	BD	BC	ABC	AD	CD	AC	C	ABC	BC	AC
50	AB	CD	AC	AB	BC	ABC	CD	BC	BD	C
51	6	4	4	3	2	7	3	5	8	2
52	4	5	3	8	6	5	2	2	3	3
53	2	7	6	5	3	3	8	7	2	5
54	3	3	2	4	5	4	6	4	3	8
55	3	2	5	6	3	4	3	8	6	4
56	7	8	4	2	4	8	5	3	7	4
57	5	2	7	4	7	2	4	4	2	3
58	8	3	3	7	2	2	2	2	4	2
59	4	6	8	2	4	3	7	3	4	6
60	2	4	2	3	8	6	4	6	5	7

జెఇఇ (అడ్వాన్స్‌డ్)
2014 'కీ'
అందించిన వారు

SRIGAYATRI
EDUCATIONAL INSTITUTIONS

S

JEE ADVANCED 2014

Paper 1
Question Paper with Solutions

CODE-6

INSTRUCTIONS

A. General

1. This booklet is your Question Paper. Do not break the seal of this booklet before being instructed to do so by the invigilators.
2. The question paper CODE is printed on the left hand top corner of this sheet and on the back cover page of this booklet
3. Blank spaces and blank pages are provided in the question paper for your rough work. No additional sheets will be provided for rough work.
4. Blank papers, clipboards, log tables, slide rules, calculators, cameras, cellular phones, pagers and electronic gadget of any kind are NOT allowed inside the examination hall.
5. Write your name and Roll number in the space provided on the back cover of this booklet.
6. Answers to the questions and personal details are to be filled on an Optical Response Sheet, which is provided separately. The ORS is a doublet of two sheets - upper and lower having identical layout. The upper sheet is a machine-gradable. Objective Response Sheet (ORS) which will be collected by the invigilator at the end of the examination. The upper sheet is designed in such a way that darkening the bubble with a ball point pen will leave an identical impresssion at the corresponding place on the lower sheet. You will be allowed to take away the lower sheet at the end of the examination (see figure-1 on the back cover page for the correct way of darkening the bubbles for valid answers)
7. Use a black ball point pen only to darken the bubbles on the upper original sheet. Apply sufficient pressure so that the impression is crated on the lower sheet. See Figure-1 on the back cover page for appropriate way of darkening the bubbles for valid answers.
8. DO NOT TAMPER WITH/MUTULATE THE ORS OR THIS BOOKLE
9. On breaking the seal of the booklet check that it contains 28 pages and all the 60 questions and corresponding answer choice are legible. Read carefully the instruction printed at the beinning of each section.

B. Filling the right part of the ORS

10. The ORS also has a CODE printed on its left and right parts.
11. Verify that the CODE printed on the ORS (on both the left and right parts) is the same as that on this booklet and put your signature in the Box designated as R4.
12. IF THE CODES DO NOT MATCH ASK FOR A CHANGE OF THE BOOKLET/ORS AS APPLICABLE.
13. Write your Name, Roll No. and the name of centre and sign with pen in the boxes provided on the upper sheet of ORS. Do not write any of thi anywhere else. Darken the appropriate bubble UNDER each digi of your Roll No. in such way that the impression is created on the bottom sheet. (see example in figure 2 on the back cover)

C. Question Paper Format

The question paper consists of three parts (Physics, Chemistry and Mathematics). Each part consists of two sections.

14. Section 1 contains 10 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONE OR MORE THAN ONE are correct
15. Section 2 contains 10 questions. The answer to each of the questions is a single-digit integer, ranging from 0 to 9 (both inclusive)

	Subject	Section	
Part 1	Physics	1	One or More Than One Option Correct Type
		2	One integer Value correct Type
Part II	Chemistry	1	One or More Than One Option Correct Type
		2	One integer Value correct Type
Part III	Mathematics	1	One or More Than One Option Correct Type
		2	One integer Value correct Type

PHYSICS

SECTION - 1 : (One or More Than One Options Correct Type)

This section contains 10 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONE or MORE THAN ONE are correct.

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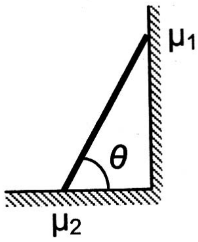
1. In the figure, a ladder of mass m is shown leaning against a wall. It is in static equilibrium making an angle θ with the horizontal floor. The coefficient of friction between the wall and the ladder is μ_1 and that between the floor and the ladder is μ_2. The normal reaction of the wall on the ladder is N_1 and that of the floor is N_2. If the ladder is about to slip, then



(A) $\mu_1 = 0 \quad \mu_2 \neq 0$ and $N_2 \tan\theta = \frac{mg}{2}$

(B) $\mu_1 \neq 0 \quad \mu_2 = 0$ and $N_1 \tan\theta = \frac{mg}{2}$

(C) $\mu_1 \neq 0 \quad \mu_2 \neq 0$ and $N_2 = \frac{mg}{1+\mu_1\mu_2}$

(D) $\mu_1 = 0 \quad \mu_2 \neq 0$ and $N_1 \tan\theta = \frac{mg}{2}$

Key : C, D

Sol : $f_1 = \mu_1 N_1; \quad f_2 = \mu_2 N_2$

$N_1 = f_2 \ ; \quad mg = N_2 + f_1$

$mg = N_2 + \mu_1 N_1$

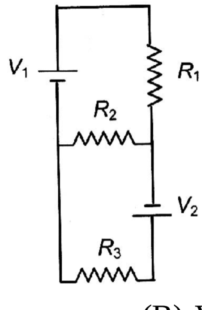
$$mg\cos\theta\frac{l}{2} = N_1 \sin\theta l + f_1 \cos\theta l$$

$$\Rightarrow \frac{mg}{2} = N_1\tan\theta + \mu_1 N_1 \qquad \Rightarrow N_1 = \frac{mg}{2(\tan\theta + \mu_1)}$$

$$N_2 = \frac{N_1}{\mu_2} = \frac{mg}{2\mu_2(\tan\theta + \mu_1)}$$

$$\therefore \quad \tan\theta = \frac{1-\mu_1\mu_2}{2\mu_2}$$

2. Two ideal batteries of emf V_1 and V_2 and three resistances R_1, R_2 and R_3 are connected as shown in the figure. The current in resistance R_2 would be zero if



(A) $V_1 = V_2$ and $R_1 = R_2 = R_3$

(B) $V_1 = V_2$ and $R_1 = 2R_2 = R_3$

(C) $V_1 = 2V_2$ and $2R_1 = 2R_2 = R_3$

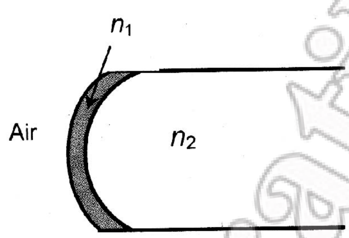
(D) $2V_1 = V_2$ and $2R_1 = R_2 = R_3$

Key : A, B, D

Sol : $i_2 = \dfrac{\dfrac{V_1}{R_1} - \dfrac{V_2}{R_3}}{R_2 + \dfrac{R_1 R_3}{R_1 + R_3}} = 0$

$\Rightarrow \dfrac{V_1}{R_1} = \dfrac{V_2}{R_3}$

3. **A transparent thin film of uniform thickness and refractive index $n_1 = 1.4$ is coated on the convex spherical surface of radius R at one end of a long solid glass cylinder of refractive index $n_2 = 1.5$, as shown in the figure. Rays of light parallel to the axis of the cylinder traversing through the film from air to glass get focused at distance f_1 from the film, while rays of light traversing from glass to air get focused at distance f_2 from the film. Then**



(A) $|f_1| = 3R$ (B) $|f_1| = 2.8R$ (C) $|f_2| = 2R$ (D) $|f_2| = 1.4R$

Key : A, C

Sol : $P = \dfrac{1.4-1}{R} + \dfrac{1.5-1.4}{R} = \dfrac{0.4+0.1}{R} = \dfrac{1}{2R}$

$f_{air} = \dfrac{1}{\dfrac{1}{2R}} = 2R = |f_2|$

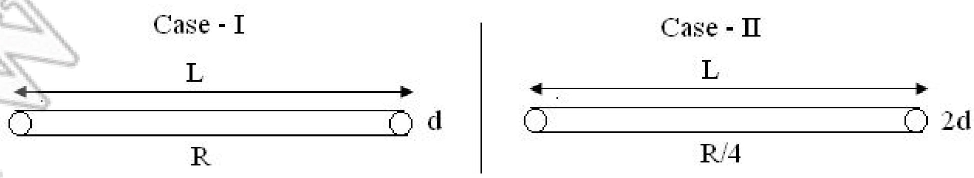
$|f_1| = 1.5 \times 2R = 3R$

4. **Heater of an electric kettle is made of a wire of length L and diameter d. It takes 4 minutes to raise the temperature of 0.5 kg water by 40K. This heater is replaced by a new heater having two wires of the same material, each of length L and diameter 2d. The way these wires are connected is given in the options. How much time in minutes will it take to raise the temperature of the same amount of water by 40K?**

(A) 4 if wires are in parallel
(B) 2 if wires are in series
(C) 1 if wires are in series
(D) 0.5 if wires are in parallel

Key : B, D

Sol :



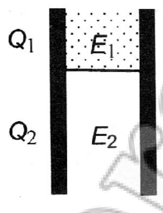
$$mc(40) = \frac{E^2}{R} \times 4$$

Series $mc(40) = \frac{E^2}{(R/2)} \times x$

$\Rightarrow 4 = 2x \Rightarrow x = 2 \text{ min}$

Parallel $mc(40) = \frac{E^2}{R/8} \times x \Rightarrow 4 = 8x \Rightarrow x = 0.5 \text{ min}$

5. A parallel plate capacitor has a dielectric slab of dielectric constant K between its plates that covers 1/3 of the area of its plates, as shown in the figure. The total capacitance of the capacitor is C while that of the portion with dielectric in between is C_1. When the capacitor is charged, the plate area covered by the dielectric gets charge Q_1 and the rest of the area gets charge Q_2. The electric field in the dielectric is E_1 and that in the other portion is E_2. Choose the correct option/options, ignoring edge effects.



(A) $\frac{E_1}{E_2} = 1$ (B) $\frac{E_1}{E_2} = \frac{1}{K}$ (C) $\frac{Q_1}{Q_2} = \frac{3}{K}$ (D) $\frac{C}{C_1} = \frac{2+K}{K}$

Key: A, D

Sol: $C = \frac{\epsilon_0}{d}\left(\frac{A}{3}K + \frac{2A}{3}\right)$

$C_1 = \frac{k \epsilon_0 A}{3d}$ $C_2 = \frac{2 \epsilon_0 A}{3d}$

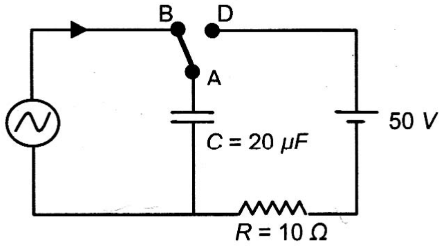
$$\frac{Q_1}{Q_2} = \frac{C_1}{C_2} = \frac{k}{3} \times \frac{3}{2} = \frac{k}{2}$$

$$\frac{C}{C_1} = \frac{\frac{\epsilon_0}{d}\frac{A}{3}(k+2)}{\frac{k \epsilon_0 A}{3d}} = \frac{k+2}{k}$$

$$\frac{E_1}{E_2} = \frac{\sigma_1}{\sigma_2} = \frac{Q_1 \, 2A/3}{A/3 Q_2 k} = \frac{2Q_1}{Q_2 k} = \frac{2}{k}\left[\frac{k}{2}\right] = 1$$

6. At time t = 0, terminal A in the circuit shown in the figure is conncted to B by a key and an alternating current $I(t) = I_0 \cos(\omega t)$, with $I_0 = 1A$ and $\omega = 500 \text{ rad s}^{-1}$ starts flowing in it with the initial direction shown in the figure. At $t = \frac{7\pi}{6\omega}$, the key is switched from B to D. Now onwards only A and D are connected . A total charge Q flows fom the battery to charge the capacitor fully.

If $C = 20\mu F$**,** $R = 10\Omega$ **and the battery is ideal with emf of 50V, identify the correct statement(s).**



(A) Magnitude of the maximum charge on the capacitor before $t = \frac{7\pi}{6\omega}$ is $1\times10^{-3}C$.

(B) The current in the left part of the circuit just before $t = \frac{7\pi}{6\omega}$ is clockwise.

(C) immediately after A is connected to D, the current in R is 10A

(D) $Q = 2\times10^{-3}C$

Key: C, D

Sol: $\frac{dq}{dt} = I_0 \cos\omega t$

$$\int dq = \int I_0 \cos\omega t \; dt$$

$$q = \frac{I_0}{w}\left[\sin\omega t\right]_0^{\frac{7\pi}{6\omega}}$$

$$q = \frac{I_0}{w}\sin\left[\cancel{w}\times\frac{7\pi}{6\cancel{w}}\right]$$

$$= -\frac{I_0}{w}\sin\left(\frac{\pi}{6}\right)$$

$$q = -\frac{1}{500}\times\frac{1}{2} = -\frac{1}{1000}\ C$$

$$\therefore\quad V = \frac{q}{c} = \frac{10^{-3}}{20\times10^{-6}} = \frac{100\cancel{0}}{2\cancel{0}} = 50V$$

max charge on capacitor $= \frac{I_0}{w} = \frac{1}{500}C = \frac{1000}{500}mc = 2\,mc$

(B) Wrong

(C) $I = \frac{100}{10} = 10A$

7. **A light source, which emits two wavelengths** $\lambda_1 = 400$ nm and $\lambda_2 = 600$ nm**, is used in a Young's double slit experiment. If recorded fringe widths for** λ_1 **and** λ_2 **are** β_1 **and** β_1 **and the number of fringes for them within a distance y on one side of the central maximum are** m_1 **and** m_2**, respectively, then**

(A) $\beta_2 > \beta_1$ (B) $m_1 > m_2$

(C) From the central maximum, 3rd maximum of λ_2 overlaps with 5th minimum of λ_1

(D) The angular separation of fringes for λ_1 is greater than λ_2

Key : A, B, C

Sol: $\beta = \frac{\lambda D}{d}$ $\qquad$ $\beta_2 > \beta_1$

$m_2 < m_1$

$$\left(\frac{2n_1 - 1}{2}\right)\lambda_1 = n_2\lambda_2$$

$$\frac{2n_1 - 1}{2n_2} = \frac{\lambda_2}{\lambda_1} = \frac{6}{4} = \frac{3}{2}$$

$$2n_1 - 1 = 3n_2$$

$$2\times5 - 1 = 3\times3$$

8. A student is perfoeming an experiment using a resonance column and a tuning fork of frequency $244\,s^{-1}$. He is told that the air in the tube has been replaced by another gas (assume that the column remains filled with the gas). If the minimum height at which resonance occurs is (0.350 ± 0.005) m, the gas in the tube is

(Useful information: $\sqrt{167RT} = 640J^{1/2}\ \text{mole}^{-1/2}$; $\sqrt{140RT} = 590J^{1/2}\ \text{mole}^{-1/2}$. The molar masses M in grams are given in the options. Take the values of $\sqrt{\frac{10}{M}}$ for each gas as given there.)

(A) Neon$\left(M = 20, \sqrt{\frac{10}{20}} = \frac{7}{10}\right)$

(B) Nitrogen$\left(M = 28, \sqrt{\frac{10}{28}} = \frac{3}{5}\right)$

(C) Oxygen$\left(M = 32, \sqrt{\frac{10}{32}} = \frac{9}{16}\right)$

(D) Argon$\left(M = 36, \sqrt{\frac{10}{36}} = \frac{17}{32}\right)$

Key: D

Sol: $n = 244\,s^{-1}$

$v = n\lambda;$

$\lambda = 4l$

$v = 244 \times 4 \times (0.35 \pm 0.005) = 336\ m/s\ \ to\ \ 346\ m/s$

by calculation only Option D is Correct.

9. Let $E_1(r), E_2(r)$ and $E_3(r)$ be the respective electric fields at a distance r from a point charge Q, an infifnitely long wire with constant linear charge density λ, and an infinite plane with uniform surface charge density σ. If $E_1(r_0) = E_2(r_0) = E_3(r_0)$ at a given distance r_0, then

(A) $Q = 4\sigma\pi r_0^2$

(B) $r_0 = \frac{\lambda}{2\pi\sigma}$

(C) $E_1(r_0/2) = 2E_2(r_0/2)$

(D) $E_2(r_0/2) = 4E_3(r_0/2)$

Key : C

Sol : $E_1(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

$$E_2(r) = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$E_3(r)=\frac{\sigma}{2\varepsilon_0}$$

$$E_1(r_0)=E_2(r_0)=E_3(r_0)$$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_o^2}=\frac{\lambda}{2\pi\varepsilon_0 r_0}=\frac{\sigma}{2\varepsilon_0}$$

$$\Rightarrow Q=2\pi\sigma r_0^2;$$

$$r_0=\frac{\lambda}{\pi\sigma}$$

By verification, key: (C)

10. One end of a taut string of length 3m along the x - axis is fixed at x=0. The speed of the waves in the string is $100\ \text{ms}^{-1}$. The other end of the string is vibrating in the y direction so that stationary waves are set up in the string. The possible waveform(s) of these stationary waves is (are)

(A) $y(t)=A\sin\frac{\pi x}{6}\cos\frac{50\pi t}{3}$

(B) $y(t)=A\sin\frac{\pi x}{3}\cos\frac{100\pi t}{3}$

(C) $y(t)=A\sin\frac{5\pi x}{6}\cos\frac{250\pi t}{3}$

(D) $y(t)=A\sin\frac{5\pi x}{2}\cos 250\pi t$

Key : A, C

Sol : x =0 is Node & x = 3 meter is antinode

By verification

SECTION - 2 : (One Integer Value Correct Type)

This section contains 10 questions. Each question, when worked out will result in one integer from 0 to 9 (both inclusive).

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11. To find the distance d over which a signal can be seen clearly in foggy conditions, a railways engineer uses dimensional analysis and asumes that the distance depends on the mass density ρ of the fog, intensity (power/area) S of the light from the signal and its frequency f. The engineer finds that d is proportinal to $S^{1/n}$. The value of n is

Key : 3

Sol : $d=K\rho^a S^{1/b} f^c$

using dimensional analysing

$$[L]=K\left[ML^{-3}\right]^a\left[\frac{ML^2T^{-3}}{L^2}\right]^{1/b}\left[T^{-1}\right]^c$$

$$\Rightarrow a+\frac{1}{b}=0 \quad \rightarrow (1)$$

-3a = 1

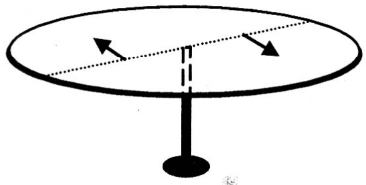
a = -1/3

$$-\frac{3}{b}-c=0$$

$$\frac{1}{b}=-a=\frac{1}{3}\Rightarrow b=3$$

$\therefore$ The value of $n = 3$

12. A horizontal circular platform of radius 0.5 m and mass 0.45 kg is free to rotate about its axis. Two massless spring toy-guns, each carrying a steel ball of mass 0.05 kg are attached to the platform at a distance 0.25 m from the centre on its either sides along its diameter (see figure). Each gun simulataneously fires the balls horizontally and perpendicular to the diameter in opposite directions. After leaving the platform, the balls have horizontal speed of 9 ms^{-1} with respect to the ground. The rotational speed of the platform in rad s^{-1} after the balls leave the platform is



Key : 4

Sol : change in momentum of each steel ball = mv = 0.05×9

$$= \frac{5}{100} \times 9 = 0.45\,\text{kg m/s}$$

Therefore the platform receives two impules in opposite direction

Each of value J = 0.45 kg m/s.

Agular impulse = 2 × J × L

$$= 2 \times 0.45 \times 0.25$$

$$= 2 \times \frac{45}{100} \times \frac{25}{100} = \frac{90}{100} \times \frac{25}{100} = \frac{9}{40}\,\text{kg m}^2/\text{s}$$

Angular impulse = change in angular momentum of platform

$$2J = \left(\frac{MR^2}{2}\right)\omega$$

$$\frac{9}{40} = 0.45 \times \left(\frac{0.5 \times 0.5}{2}\right) \times \omega$$

$$2 \times 0.45 \times 0.25 = 0.45 \times \frac{(0.5)^2}{2} \times \omega$$

$$\omega = 4\ \text{rad/s}$$

$\therefore$ angular speed of flatform $\omega = 4\,\text{rad/s}$.

13. A galvanometer gives full scale deflection with 0.006 A current. By connecting it to a 4990 Ω resistance, it can be converted into a voltmeter of range 0 - 30 V. If connected to a $\frac{2n}{249}\Omega$ resistance, it becomes an ammter of range 0 - 1.5 A. The value of n is

Key: 5

Sol: $i_g = 0.006\,\text{A} = 6 \times 10^{-3}\,\text{A}$

$R_s = 4990\Omega$

As volt meter:

V = 30 V

$V = i_g(G + R_s)$

$$30 = 6\times10^{-3}(G+4990)$$

$$G+4990 = \frac{30}{6}\times10^{3}$$

$$G + 4990 = 5000$$

$$G = 10\Omega$$

As ammeter:

$$S = \frac{G}{\left(\frac{i}{ig}-1\right)}$$

i = 1.5 A

$$S = \frac{2n}{249}$$

$$\frac{2n}{249} = \frac{10}{\left(\frac{1.5}{6\times10^{-3}}-1\right)}$$

$$\frac{2n}{249} = \frac{10}{\left(\frac{1}{4}\times10^{3}-1\right)}$$

$$\frac{2n}{249} = \frac{10}{250-1}$$

2n = 10

n = 5

14. During Searle's experiment, zero of the Vernier scale lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale. The 20th division of the vernier scale exactly coincides with one of the main scale divisions. When an additional load of 2 kg is applied to the wire, the zero of the Vernier scale still lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale but now the 45th division of Vernier scale coincides with one of the main scale divisions. The length of the thin metallic wire is 2 m and its cross-sectional area is 8×10^{-7} m^2. The least count of the Vernier scale is 1.0×10^{-5} m. The maximum percentage error in the Youngs modulus of the wire is

Key: 4

Sol: $\Delta l = -\left(3.25\times10^{-2}+20\times1.0\times10^{-5}\right)+\left(3.25\times10^{-2}+45\times1.0\times10^{-5}\right)$

$$= 25\times10^{-5}m$$

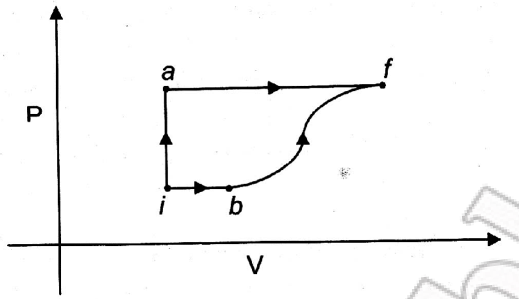
The error in Y is only due to error in Δl.

$$Y = \frac{Fl}{A\Delta l}$$

$$\frac{\Delta Y}{Y} = \frac{\Delta(\Delta l)}{\Delta l} = \frac{1.0\times10^{-5}}{25\times10^{-5}}$$

$$= 4\%$$

15. **A thermodynamic system is taken from an state i with internal energy** $U_i = 100J$ **to the final state f along two different paths iaf and ibf, as schematically shown in the figure. The work done by the system along the paths af, ib and bf are** $W_{af} = 200\,J$, $W_{ib} = 50\,J$, and $W_{bf} = 100\,J$ **respectively. The heat supplied to the system along the path iaf, ib and** Q_{iaf}, Q_{ib} **and** Q_{bf} **respectively. If the internal energy of the system in the state b is** U_b **= 200 J and** Q_{iaf} **= 500 J, the ratio** Q_{bf} / Q_{ib} **is**



Key: 2

Sol: $U_i = 100J$

$W_{af} = 200J$

$W_{ib} = 50\,J$

$W_{bf} = 100J$

$U_b = 200J$

$Q_{iaf} = 500J$

$Q_{bf} = (U_f - U_b) + W_{bf}$

$Q_{iaf} = (U_f - 100) + \text{zero} + 200$

$Q_{iaf} = U_f + 100 \quad \rightarrow (1)$

$500 = U_f + 100$

$\Rightarrow U_f = 400$

$Q_{bf} = W_{bf} + (U_f - U_b)$

$Q_{bf} = 100 + (400 - 200)$
$= 300\text{ J}$

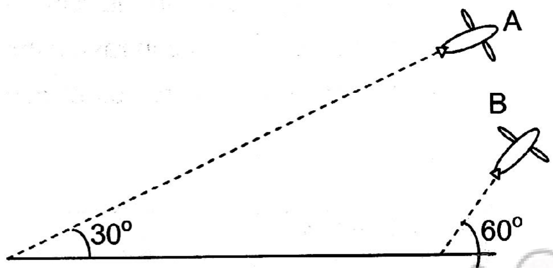
$Q_{ib} = W_{ib} + (U_b - U_i)$

$Q_{ib} = 50 + (200 - 100)$

$Q_{ib} = 150\,J$

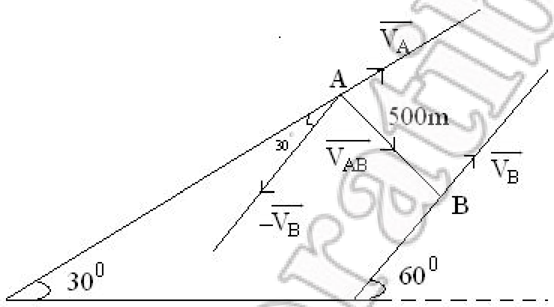
$\therefore \frac{Q_{bf}}{Q_{ib}} = \frac{300\,J}{150\,J} = 2$

16. Airplanes A and B are flying with constant velocity in the same vertical plane at angles 30^0 and 60^0 with respect to the horizontal respectively as shown in figure. The speed of A is $100\sqrt{3}ms^{-1}$. At time t = 0s, an observer in A finds B at a distance of 500 m. This observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at $t = t_0$, A just escapes being hit by B, t_0 in seconds is



Key : 5

Sol:



Relative velocity of A w.r.to B

= - (Relative velocity of B w.r.to A)

$= \overrightarrow{V_A} + \left(-\overrightarrow{V_B}\right)$

From figure:

$V_B \cos 30^0 = V_A$

$\Rightarrow V_B \frac{\sqrt{3}}{2} = 100\sqrt{3}$

$\Rightarrow V_B = 200\,m/s$

$V_{AB} = V_B \sin 30^0$

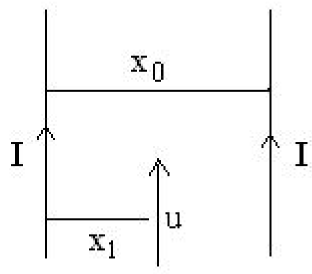
$= 200 \times \frac{1}{2} = 100\ m/s$

$\therefore t_0 = \frac{500}{V_{AB}} = \frac{500}{100} = 5\ \text{seconds}$

17. Two parallel wires in the plane of the paper are distance X_0 apart. A point charge is moving with speed u between the wires in the same plane at a distance X_1 from one of the wires. When the wires carry current of magnitude *l* in the same direction, the radius of curvature of the path of the point charge is R_1. In contrast, if the currents *l* in the two wires have directions opposite to each other, the radius of curvature of the path R_2. If $\frac{X_0}{X_1} = 3$, the value of $\frac{R_1}{R_2}$ is

Key: 3

Sol:



$$\vec{B}_1 = \frac{\mu_0 I}{2\pi x_1}\left(-\hat{k}\right);\ \vec{B}_2 = \frac{\mu_0 I}{2\pi(x_0 - x_1)}\left(\hat{k}\right)$$

$$\vec{B}_{net(1)} = \frac{\mu_0 I}{2\pi}\left[\frac{1}{x_0 - x_1} - \frac{1}{x_1}\right]\ \hat{k}$$

$$BVq = \frac{mv^2}{r}$$

$$r = \frac{mv}{Bq}$$

$$r \propto \frac{1}{B_{net}}$$

$$= \vec{B}_{net(2)} = \frac{\mu_0 I}{2\pi}\left[\frac{1}{x_0 - x_1} + \frac{1}{x_1}\right]\ \hat{k}$$

$$\frac{R_1}{R_2} = \frac{B_{net(2)}}{B_{net(1)}}$$

$$\frac{R_1}{R_2} = \frac{\frac{\mu_0 I}{2\pi}\left[\frac{1}{x_0 - x_1} + \frac{1}{x_1}\right]}{\frac{\mu_0 I}{2\pi}\left[\frac{1}{x_0 - x_1} - \frac{1}{x_1}\right]}$$

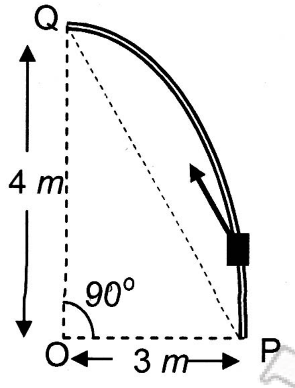
$$= \frac{\left[\frac{x_1 + x_0 - x_1}{(x_0 - x_1).x_1}\right]}{\left[\frac{x_1 - x_0 + x_1}{(x_0 - x_1).x_1}\right]} = \frac{x_0}{2x_1 - x_0}$$

$$\frac{x_0}{x_1} = 3, \qquad x_0 = 3x_1$$

$$= \frac{3x_1}{2x_1 - 3x_1} = \frac{3}{-1} = 3$$

$$\frac{R_1}{R_2} = 3.$$

18. Consider an elliptically shaped rail PQ in the vertical plane with OP = 3 m and OQ = 4 m. A block of mass 1 kg is pulled along the rail from P to Q with a force of 18 N, which is always parallel to line PQ (see the figure given). Assuming no frictional losses, the kinetic energy of the block when it reaches Q is (n×10) Joules. The value of n is (take acceleration due to gravity = 10 ms^{-2}).



Key: 5

Sol: Work done = Change in P.E + Change in K.E

$$F\times r=\left(P.E_{final}-P.E_{initial}\right)+\left(K.E_{final}-K.E_{initial}\right)$$

$$18\times 5=\left(mgh-0\right)+\left(K.E_{final}-0\right)$$

$$18\times 5=1\times 10\times 4+K.E_{f}$$

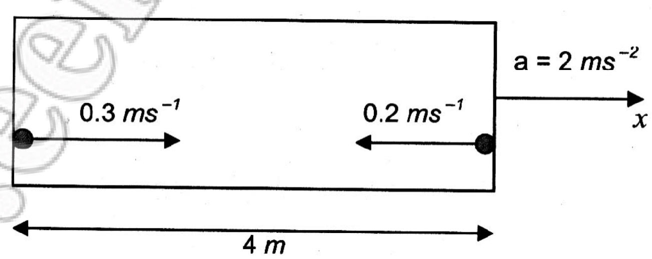
$$K.E_{f}=90-40=50J$$

$$K.E_{f}=5\times 10J$$

$$K.E_{f}=n\times 10J$$

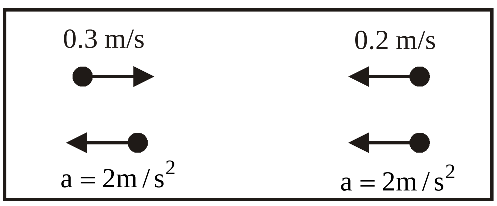
n = 5

19. A rocket is moving in a gravity free space with a constant acceleration of 2 ms^{-2} along + x derection (see figure). The length of a chamber inside the rocket is 4 m. A ball is thrown from the left end of the chamber in + x direction wiht a speed of 0.3 ms^{-1} relative to the rocket. At the same time, another ball is thrown in -x direction with a speed of 0.2 ms^{-1} from its right end relative to the rocket. The time in seconds when the two balls hit each other is



key: 8

Sol:



The accelaration of balls w.r.to rcket = $2m/s^2$ (towards left)

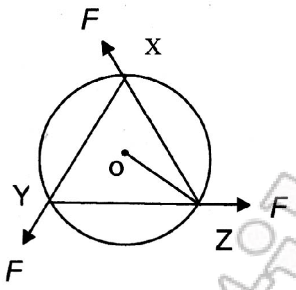
If t is the time in seconds then the balls hit each other.

$$0.3t - \frac{1}{2}\times 2\times t^2 + 0.2t + \frac{1}{2}\times 2\times t^2 = 4$$

$\therefore 0.5t = 4$

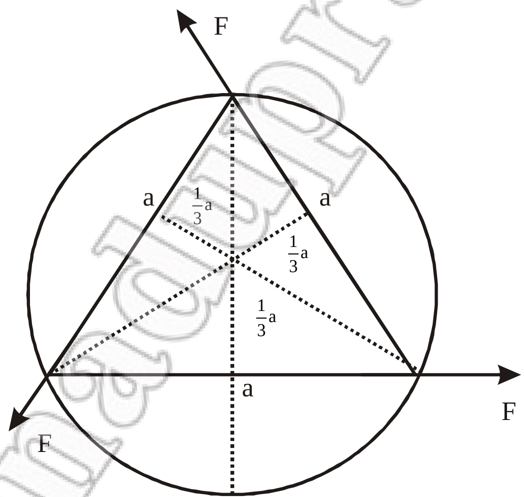
or t = 8 seconds

20. A uniform circular disc of mass 1.5 kg and radius 0.5 m is initially at rest on a horizontal frictionless surface. Three forces of equal magnitude F = 0.5 N are applied simultaneously along the three sides of an equilateral triagle XYZ with its vertices on the perimeter of the disc (see figure). One second after applying the forces, the angular speed of the disc in rad s^{-1} is



Key: 2

Sol:



Net Torque = $F\times r_1 + F\times r_2 + F\times r_3$

$$\tau_{net} = F\times\frac{1}{3}a + F\times\frac{1}{3}a + F\times\frac{1}{3}a$$

$$\tau_{net} = 3\times\frac{F}{3}a$$

$$\tau_{net} = F\times a$$

but $\frac{2}{3}a = r \Rightarrow a = \frac{3r}{2}$

$$\tau_{net} = \frac{2}{3}a = r \Rightarrow a = \frac{3r}{2}$$

$$\tau_{net} = F\times\frac{3}{2}r$$

$$\tau_{net} = \frac{1}{2}\times\frac{3}{2}\times\frac{1}{2} = \frac{3}{8}\,Nm$$

Torque = $I\alpha$

$$\frac{3}{8} = \frac{MR^2}{2}\times\alpha$$

$$\frac{3}{8} = \frac{1.5\times0.5\times0.5}{2}\times\alpha$$

$$\alpha = 2\ \text{rad s}^{-2}$$

CHEMISTRY

SECTION - 1

(One or More Tahn One Options Correct Type)

This section contains 10 multiple choice type questions. Each question has four choices (A), (B), (C) and (D) out of which ONE or MORE THAN ONE are correct :

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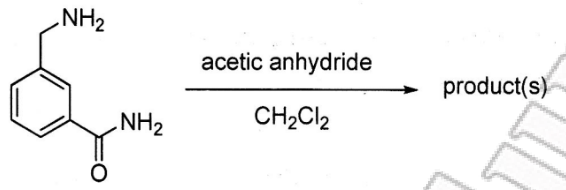
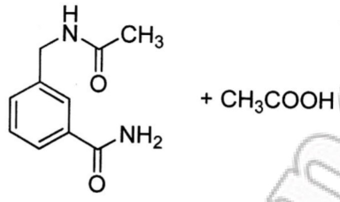
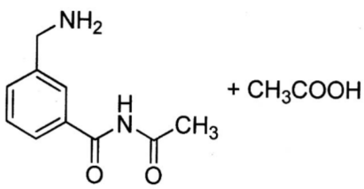
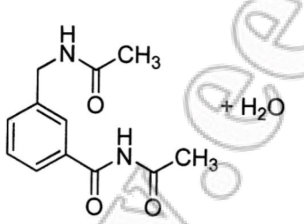
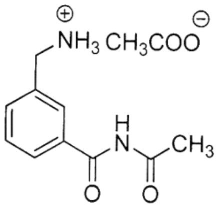
21. In the reacation shown below, the major product(s) formed is/are



(A) 

(B) 

(C) 

(D) 

Key : A

Sol : Conceptual

22. The correct statement(s) for orthoboric acid is/are

(A) It behaves as a weak acid in water due to self ionization

(B) Acidity of its aqueous solution increases upon addition of ethylene glycol

(C) It has a three dimensional structure due to hydrogen bonding

(D) It is a weak electrolyte in water

Key B, C, D

Sol: Conceptual

23. For the reaction :

$I^- + ClO_3^- + H_2SO_4 \rightarrow Cl^- + HSO_4^- + I_2$

The correct statement(s) in the balanced equation is/are

(A) Stoichiometric coefficient of HSO_4^- is 6

(B) Iodide is oxidized

(C) Sulphur is reduced

(D) H_2O is one of the product

Key : A, B, D

Sol : $2I^- + ClO_3^- + H_2SO_4 \rightarrow Cl^- + HSO_4^- + I_2$

$6I^- + ClO_3^- + H_2SO_4 + 5H^+ \rightarrow Cl^- + HSO_4^- + 3I_2 + 3H_2O$

24. The pair(s) of reagents that yield paramagnetic species is/are

(A) Na and excess of NH_3

(B) K and excess of O_2

(C) Cu and dilute HNO_3

(D) O_2 and 2-ethylanthraquinol

Key:A, B, C

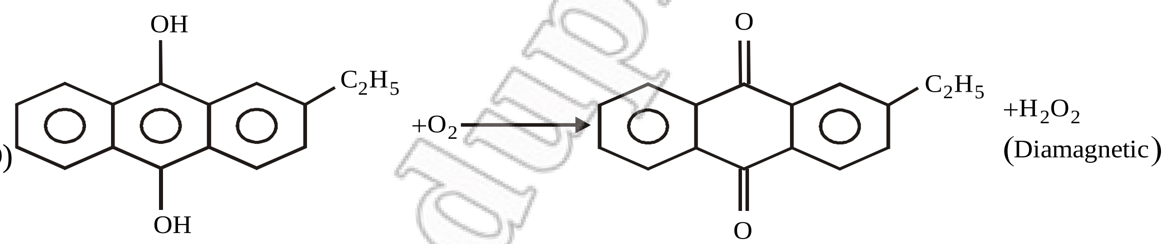
Sol: A) $Na + excess(X+Y)NH_3 \rightarrow [Na(NH_3)_x] + [e^-(NH_3)_y]$

Paramagnetic.

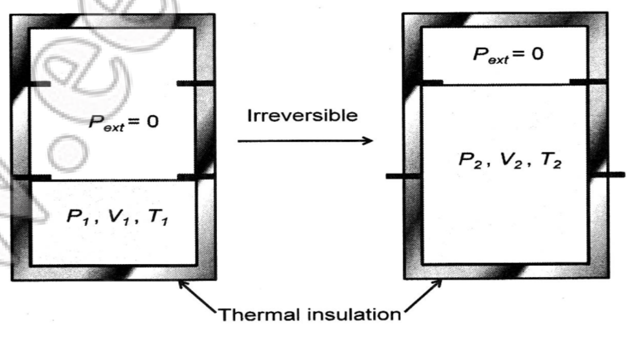
B) For $K + O_2(excess) \rightarrow KO_2$

$O_2^- \rightarrow$ paramagnetic

C) $Cu + di/HNO_3 \rightarrow Cu(NO_3)_2 + NO(paramagnetic) + H_2O$

D) 

25. An ideal gas in a termally insulated vessel at internal pressure = P_1, volume = V_1 and absolute temperature = T_1 expands irreversibly against zero external presue, as shown in the diagram. The final internak pressure, volume and absolute temperature of the gas are P_2, V_2 and T_2 respectively. For this expansion,



(A) q = 0

(B) $T_2 = T_1$

(C) $P_2V_2 = P_1V_1$

(D) $P_2V_2^\gamma = P_1V_1^\gamma$

Key : A, B, C

Sol : Since adiabatic process

Q=0

For the first law of Thermodynamics $\Delta U = Q + W$

$\Delta U = 0 + W$

$\Delta U = W$

$\Delta U = -P_{ext}(V_2 - V_1)$

But $P_{ext} = 0$

$\therefore \Delta U = 0$

$\therefore$ Temperature constnat

Hence A,B, D correct

26. In a galvanic cell, the salt bridge

(A) does not participate chemically in the cell reaction

(B) stops the diffusion of ions from one electrode to another

(C) is necesary for the occurrence of the cell reaction

(D) ensures mixing of the two electrolytic solutions

Key :A, C, B

Sol : Conceptual

27. Hydrogen bonding plays a central role in the following phenomena

(A) Ice floats in water

(B) Higher Lewis basicity of primary amines than tertiary amines in aqueous solutions

(C) Formic acid is more acidic than acetic acid

(D) Dimerisation of acetic acid in benzene

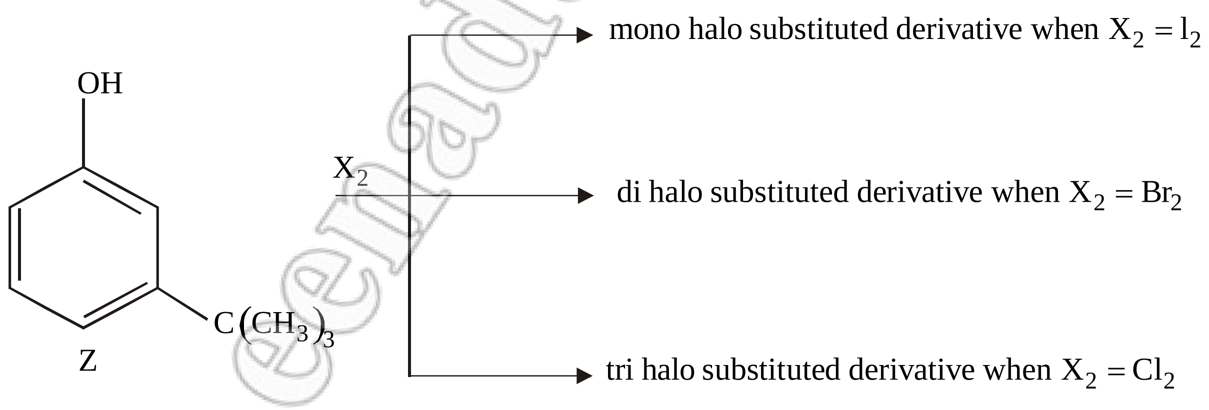
Key : A, B, D

Sol : A) Anomolous expansion of water

B) Lewis basicity order for different amines due to hydrogen bonding.

D) $2CH_3COOH = (CH_3COOH)_2$ due to hydrogen bonding.

28. The reactivity of compound Z with different halogens under appropriate condition is given below :



The observed pattern of electrophilic substitution can be explained by

(A) the steric effect of the halogen

(B) the steric effect of the tert-butyl group

(C) the electronic effect of the phenoic group

(D) the electronic effect of the tert-butyl group

Key:A, B, C

Sol: Conceptual

29. The correct combination of names for isomeric alcohols with molecular formula $C_4H_{10}O$ is/are

A) tert- butanol and 2- methylpropan -2- ol

B) tert- butanol and 1, 1-dimethylethan -1-ol

C) n- butanol and butan -1-ol

D) isobutyl alcohol and 2- methylpropan-1-ol

Key :A, C, D

Sol : A) $\begin{array}{c} CH_3 \\ | \\ H_3C-C-OH \\ | \\ CH_3 \end{array}$

C) $CH_3-CH_2-CH_2-CH_2-OH$

D) $\begin{array}{l} CH_3-CH-CH_2-OH \\ \quad\quad\;\; | \\ \quad\quad CH_3 \end{array}$

30. Upon heating with Cu_2S, the reagent(s) that give copper metal is/are

A) $CuFeS_2$ B) CuO C) Cu_2O D) $CuSO_4$

Key: B, C, D

Sol: B) $\Rightarrow CuFeS_2 + Cu_2S \xrightarrow{\Delta} \text{No reaction}$

C) $\Rightarrow 2CuO \xrightarrow{1100^0C} Cu_2O + \frac{1}{2}O_2$

$2Cu_2S + Cu_2O \rightarrow 6Cu + SO_2$

D) $CuSO_4 \xrightarrow{\Delta} CuO + SO_2 + \frac{1}{2}O_2$

$2CuO \xrightarrow{1100^0C} Cu_2O + \frac{1}{2}O_2$.

SECTION-2 (One Integer Value Correct Type)

This section contains 10 questions. Each question, when worked out will result in one integer fgrom 0 to 9 (both inclusive)

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31. If the value of Avogadro number is $6.023\times10^{23}mol^{-1}$ and the value of Boltzmann constant is $1.380\times10^{-23}JK^{-1}$, then the number of significant digits in the calculated value of the universal gas constant is.

Key:4

Sol: Boltzman constant $K = \frac{R}{N}$

$R = KN$

$=1.380\times10^{-23}\times6.023\times10^{23}$

$=1.380\times6.023$

$\Rightarrow 8.31174$

$\therefore$ No. of significant figures =4

32. A Compound H_2X with molar weight of 80 g is dissolved in a solvent having density of $0.4\ g\ ml^{-1}$. Assuming no change in volume upon dissolution, the molality of a 3.2 molar solution is.

Key:8

Sol: $\because$ density of solvent = 0.4 gm / ml

that is 1 ml solvent =0.4 gm

1000 ml solvent = 400 gm

$$\text{molality} = \frac{\text{given weight}}{\text{gmw}} \times \frac{1000}{\text{wt.of solvent in gms}}$$

$$= \frac{80 \times 3.2}{80} \times \frac{1000}{400}$$

$$= \frac{32}{4} = 8$$

33. MX_2 **dissociates into** M^{2+} **and** X^- **ions in an aqueous solution, with a degree of dissociation** (α) **of 0.5. The ratio of the observed depression of freezing point of the aqueous solution to the value of the depression fo freezing point in the absence of ionic dissociation is .**

Key: 2

Sol: $MX_2 \to M^{2+} + 2X^-$

$\therefore \alpha = 0.5$

$\therefore i = 1 + (n-1)\alpha$

$= 1 + (3-1) \times 0.5$

$= 2$

$$\because i = \frac{-\Delta Tf_{(obs)}}{\Delta Tf_{(theoritical)}}.$$

34. In an atom, the total number of electrons having quantum numbers $n = 4, |m_l| = 1$ **and** $m_s = -\frac{1}{2}$ **is**

Key: 6

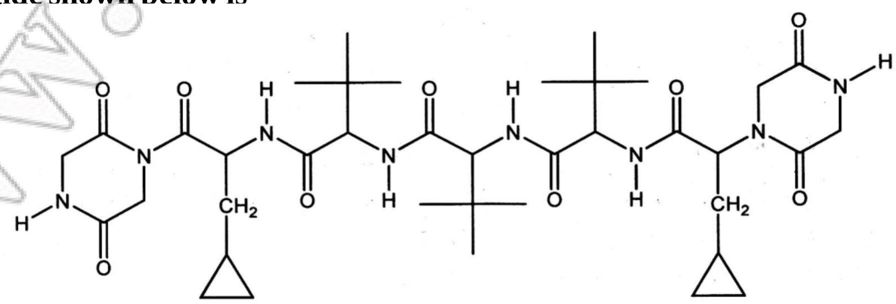
Sol: n=4 it has s, p, d, f

m, l=1 $\downarrow$ 0 $\downarrow$ 1 $\downarrow$ 1 $\downarrow$ 1

$s = -\frac{1}{2}$

$\Rightarrow 3e^-$

35. The total number of distinct naturally occurring amino acids obtained by complete acidic hydrolysis of the peptide shown below is



Key: 3

Sol: Conceptual

36. **Among** $PbS, CuS, HgS, MnS, Ag_2S, NiS, CoS, Bi_2S_3$ and SnS_2**, the total number of BLACK coloured sulphides is.**

Key:6

Sol: The black ppt's are PbS, CuS, HgS, NiS, CoS, Bi_2S_3

37. **Consider the following list of reagents:**

Acidified $K_2Cr_2O_7$**, alkaline** $KMnO_4, CuSO_4, H_2O_2, Cl_2, O_3$ **,** $FeCl_3, HNO_3$ **and** $Na_2S_2O_3$**. The total number of reagents that can oxidise aqueous iodide to iodine is**

Key: 7

Sol: $K_2Cr_2O_7 + KI \longrightarrow Cr_2\left(SO_4\right)_3 + K_2SO_4 + I_2$

$KMnO_4 + KI \xrightarrow{\text{Alkalin}} 2MnO_2 + I_2 + 4KCl$

$CuSO_4 + KI \longrightarrow CuI_2 + K_2SO_4$

$H_2O_2 + KI \longrightarrow 2KOH + I_2$

$Cl_2 + 2KI \longrightarrow KCl + I_2$

$H_2O + O_3 + KI \longrightarrow 2KOH + O_2 + I_2$

$FeCl_3 + KI \longrightarrow FeI_3 + 3KCl$

$HNO_3 + KI \longrightarrow FeI_3 + 3KCl$

$HNO_3 + KI \longrightarrow I_3 + 2NO_2 + 2H\ O$

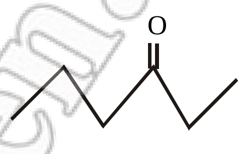
$Na_2S_2O_3 + KI$.

38. **Consider all possible isomeric ketones, including stereoisomere of MW= 100. All these isomers are independently reacted with** $NaBH_4$ **(NOTE: stereoisomers are also reacted separately). The total number of ketones that give a racemic product(s) is/are.**

Key:5

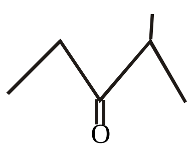
Sol:

(A)

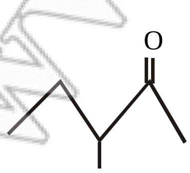


(B)

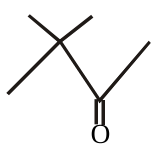
(C)



(D)



(E)



(F)

39. A list of species having the formula XZ_4 is given below.

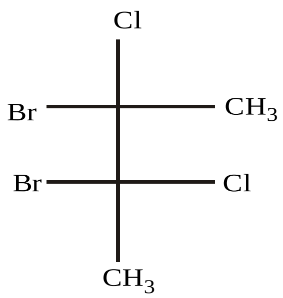
$XeF_4, SF_4, SiF_4, BF_4^-, BrF_4^-, \left[Cu\left(NH_3\right)_4\right]^{2+}, \left[FeCl_4\right]^{2-}, \left[CoCl_4\right]^{2-}$ **and** $\left[PtCl_4\right]^{2-}$

Defining shape on the basis of the location of X and Z atoms, the total number of species having a square planar shape is

Key: 4

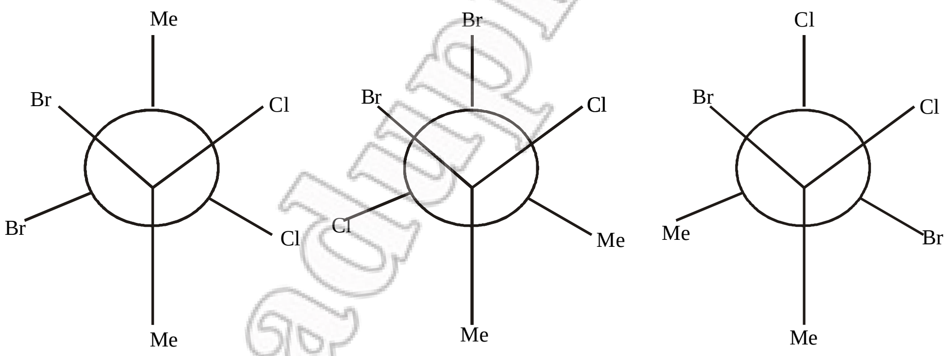
Sol: $XeF_4, BrF_4^- \left[Cu\left(NH_3\right)_4\right]^{2+}, \left[PtCl_4\right]^{2-} \Rightarrow$ Z effect more for platinum

40. The total number(s) of <u>stable</u> conformers with non- zero dipole moment for the following compound is (are).



Key:3

Sol:



MATHEMATICS

SECTION - 1 : (One or More Than One Options Correct Type)

This section contains 10 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONE or MORE THAN ONE are correct.

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41. A circle S passes through the point (0, 1) and is orthogonal to the circles $\left(x-1\right)^2+y^2=16$ and $x^2+y^2=1$. Then

1) radius of S is 8 2) radius of S is 7 3) centre of S is (-7, 1) 4) centre of S is (-8, 1)

Key : BC

Sol : $x^2+y^2+2gx+2fy+c=0$ -----(1)

$\left(0,1\right) \in (1) \Rightarrow 1+2f+c=0$

$x^2+y^2-1=0$ ----(2)

(1), (2) orthogonal $\Rightarrow c = 1$

$x^2 + y^2 - 2x - 15 = 0$ ----(3)

(1), (3) orthogonal $\Rightarrow g = 7$

Centre = (-7, 1)

$r = 7$

42. Let $\vec{x}, \vec{y}$ and $\vec{z}$ be three vectors each of magnitude $\sqrt{2}$ and the angle between each pair of them is $\frac{\pi}{3}$. If $\vec{a}$ is a nonzero vector perpendicular to $\vec{x}$ and $\vec{y} \times \vec{z}$ and $\vec{b}$ is a nonzero vector perpendicular to $\vec{y}$ and $\vec{z} \times \vec{x}$, then

1) $\vec{b} = (\vec{b}.\vec{z})(\vec{z} - \vec{x})$ 2) $\vec{a} = (\vec{a}.\vec{y})(\vec{y} - \vec{z})$ 3) $\vec{a}.\vec{b} = -(\vec{a}.\vec{y})(\vec{b}.\vec{z})$ 4) $\vec{a} = (\vec{a}.\vec{y})(\vec{z} - \vec{y})$

Key : ABC

Sol : $\vec{a} = \vec{x} \times (\vec{y} \times \vec{z})$

$\vec{a} = \vec{y} - \vec{z}$ ----(1)

$\vec{b} = \vec{y} \times (\vec{z} \times \vec{x})$

$\vec{b} = \vec{z} - \vec{x}$ ----(2)

A) $\vec{b}.\vec{z} = (\vec{z} - \vec{x}) = 1$

B) $\bar{a}.\bar{y} = 1$

C) $\vec{a}.\vec{b} = (\vec{y} - \vec{z}).(\vec{z} - \vec{x}) = 1$

43. Let $f : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to R$ be given by $f(x) = (\log(\sec x + \tan x))^3$. Then

(A) f(x) is an odd function **(B) f(x) is a one-one function**

(C) f(x) is an onto function **(D) f(x) is an even function**

Key : **ABC**

Sol : $f(-x) = -f(x)$

$f'(x) = 3\sec x (\log(\sec x + \tan x))^2 > 0$

$\Rightarrow f'(x)$ is monotonic

$\Rightarrow$ Every monotonic functioin is one to one

$\Rightarrow f(x)$ is on to function

44. For every pair of continuous functions f , g : [0, 1] $\to$ R such that max {f(x) : x $\in$ [0, 1]} = max {g(x) : x $\in$ [0, 1]} the correct statement(s) is(are)

(A) $(f(c))^2 + 3f(c) = (g(c))^2 + 3g(c)$ for some c $\in$ [0, 1]

(B) $(f(c))^2 + f(c) = (g(c))^2 + 3g(c)$ for some c $\in$ [0, 1]

(C) $(f(c))^2 + 3f(c) = (g(c))^2 + g(c)$ for some c $\in$ [0, 1]

(D) $(f(c))^2 = (g(c))^2$ for some c $\in$ [0, 1]

Key : BCD

Sol : $f(x) = g(x)$ has atleast one solution in $[0,1]$

They are continuous and having maximum value $C_1 \in [0,1]$ and $C_2 \in [0,1]$

$f(x) - g(x) > 0$

$f(y) - g(y) < 0 \Rightarrow f(x) = g(x)$

45. Let $a \in R$ and let $f : R \to R$ be given by f(x) = $x^5 - 5x + a$. Then

(A) f(x) has three real roots if a > 4
(B) f(x) has only one real root if a > 4
(C) f(x) has three real roots if a < –4
(D) f(x) has three only real roots if –4 < a < 4

Key : BD

Sol : $f(x) = x^5 - 5x + a$

$f'(x) = 5x^4 - 5$

$f(x)$ has three real roots of $f(-1)f(1) < 0$

$(a+4)(a-4) < 0$

$-4 < a < 4$

$f(x)$ has only one real root if $f(-1)f(1) > 0$

$(a+4)(a-4) > 0$

$a = -4$ or $a > 4$

46. Let f : [a, b] → $[1, \infty)$ be a continuous function and let g : R → R be defined as

$$\mathbf{g(x)} = \begin{cases} 0 & if\ x < a \\ \int_a^x f(t)\,dt & if\ a \le x \le b \\ \int_a^b f(t)\,dt & if\ x > b \end{cases}$$

Then

(A) g(x) is continuous but not differentiable at a
(B) g(x) is differentiable on R
(C) g(x) is continuous but not differentiable at b
(D) g(x) is continuous and differentiable at either a or b but not both

Key : AC

Sol : $g^1(x) = \begin{cases} 0, & x < a \\ f(x), & a \le x \le b \\ 0 & x > b \end{cases}$

$g^1(a-) \ne g^1(a+)$

$g(x)$ is not differentiable at $x = a$

$\underset{x \to a-}{\text{Lt}}\ g(x) = \underset{x \to a+}{\text{Lt}}\ g(x)$

$g(x)$ is Continuous at $x = a$

$g^1(b-) \neq g^1(b+)$

not differentiable at $x = b$

$\therefore \underset{x\to b-}{\text{Lt}} g(x) = \underset{x\to b+}{\text{Lt}} g(x)$

$g(x)$ is continuous at $x = b$.

47. Let f : $f:(0,\infty)\to\mathbb{R}$ be given by

$$f(x)=\int_{\frac{1}{x}}^{x} e^{-\left(t+\frac{1}{t}\right)}\frac{dt}{t}$$ **. Then**

(A) f(x) is monotonically increasing on $[1,\infty)$

(B) f(x) is monotonically decreasing on (0, 1)

(C) f(x) = $f\left(\frac{1}{x}\right)$ = 0, for all $x\in(0,\infty)$

(D) $f(2^x)$ is an odd function of x on $\mathbb{R}$

Key : ACD

Sol : $f^1(x) = 2.\dfrac{e^{-\left(x+\frac{1}{x}\right)}}{x} > 0, \forall x \in (1,\infty)$

$f(x)$ is monotonically increasing

$f\left(\frac{1}{x}\right) = -f(x)$

$f(x)+f\left(\frac{1}{x}\right) = 0, \forall x \in (0,\infty)$

$f\left(2^{-x}\right) = -f\left(2^{-x}\right)$

$\therefore$ Odd function

48 Let M and N be two 3 $\times$ 3 matrices such that MN = NM. Furher, if M $\neq$ N^2 and $M^2 = N^4$, then

(A) determinant of $(M^2 + MN^2)$ is 0

(B) there is a 3 $\times$ 3 non-zero matrix u such that $(M^2 + MN^2)U$ is the zero matrix

(C) determinant of $(M^2 + MN^2) \geq 1$

(D) for a 3 $\times$ 3 matrix u, if $(M^2 + MN^2)U$ equals the zero matrix then u is the zero

Key : AB

Sol : $M^2 = N^4$

$M^2 - N^4 = 0$

$\left(M-N^2\right)\left(M+N^2\right) = 0$ $\qquad (\because MN = NM)$

If $M+N^2$ is non singular

then $M-N^2 = 0$

$\Rightarrow M = N^2$

which is a contriduction

$M+N^2$ is singular

$\Rightarrow$ determinent $\left(M+N^2\right)=0$

$\det\left(M^2+MN^2\right)=\det(M)\det\left(M+N^2\right)$

$=\det M(0)$

$=0$

$M\left(M+N^2\right)U=0$

$\Rightarrow MU=0$

$\left(M+N^2\right)U=0$

$U=0$

49. From a point $P(\lambda,\lambda,\lambda)$, perpendiculars PQ and PR are drawn respectively on the lines $y=x, z=1$ and $y=-x, z=-1$. If P is such that $\angle QPR$ is a right angle then the possible value(s) of λ is (are)

(A) $\sqrt{2}$ (B) 1 (C) -1 (D) $-\sqrt{2}$

Key : C

Sol : Equations of lines are

$$\frac{x-1}{1}=\frac{y-1}{1}=\frac{z-1}{0}=t$$

$$\frac{x-1}{1}=\frac{y-1}{-1}=\frac{z+1}{0}=s$$

Q & R are $(\lambda,\lambda,1)$ & $(0,0,-1)$

D.R's of $\overleftrightarrow{PQ}$ & $\overleftrightarrow{PR}$ are $(0,0,\lambda-1)$ & $(\lambda,\lambda,\lambda+1)$

$$\angle QPR=\frac{\pi}{2}$$

When $\lambda=1$ point P lies on the line

$\therefore \lambda=-1$

50. Let M be a 2×2 symmetrix matrix with integer entries. Then M is invertible if

(A) the first column of M is the transpose of the second row of M

(B) the second row of M is the transpose of the first column of M

(C) M is a diagonal matrix with nonzero entries in the main diagonal

(D) the product of entries in the main diagonal of M is not the square of an integer

Key : CD

Sol : $m=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

Since m is symmetric $\Rightarrow m^J=m$

$$\Rightarrow \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix}=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$a_{21}=a_{12}$ ----(1)

Verify options

SECTION - 2 : (One Integer Value Correct Type)

This section contains 10 questions. Each question, when worked out will result in one integer from 0 to 9 (both inclusive).

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51. **Let** $f:[0,4\pi]\to[0,\pi]$ **be defined by** $f(x)=\cos^{-1}(\cos x)$**. The number of points** $x\in[0,4\pi]$ **satisfying the equation** $f(x)=\frac{10-x}{10}$ **is**

Key : 3

Sol : $f:[0,4\pi]\to[0,\pi]$

$$f(x)=1-\frac{x}{10}$$

$$\cos^{-1}(\cos x)=1-\frac{x}{10}$$

$$x\in[0,\pi],\cos^{-1}(\cos x)=x$$

$$1-\frac{x}{10}=x\Rightarrow 1=\frac{11x}{10}$$

$$1-\frac{x}{10}=x\Rightarrow 1=\frac{11x}{10}$$

$x\in[\pi,2\pi]$ $\qquad$ $\cos^{-1}(\cos(2\pi-x))=2\pi-x$

$$2\pi-x=1-\frac{x}{10}$$

$$2\pi-1=\frac{9x}{10}$$

$$x=\frac{10}{9}(2\pi-1)\in[\pi,2\pi]$$

$x\in[2\pi,3\pi]$ $\qquad$ $\cos^{-1}(\cos x)=1-\frac{x}{10}$

$$2\pi\le x\le 3\pi$$
$$-2\pi\ge -x\ge -3\pi$$
$$0\ge 2\pi-x\ge -\pi$$
$$0\le x-2\pi\le \pi$$

$$x-2\pi=1-\frac{x}{10}$$

$$\frac{11x}{10}=2\pi+1$$

$$x=\frac{10}{11}(2\pi+1)\in[2\pi,3\pi]$$

$x\in[3\pi,4\pi]$ $\qquad$ $\cos^{-1}(\cos x)=1-\frac{x}{10}$

$3x \le x \le 4\pi$

$-3\pi \ge -4 \ge -4\pi$

$\pi \ge 4\pi - x \ge 0$

$4\pi - x = 1 - \frac{x}{10}$

$\frac{9x}{10} = 4\pi - 1$

$x = (4\pi - 1)\frac{10}{9} \notin [3\pi, 4\pi]$

52. The largest value of the non-negative integer a for which $\lim_{x\to 1}\left\{\frac{-ax+\sin(x-1)+a}{x+\sin(x-1)-1}\right\}^{\frac{1-x}{1-\sqrt{x}}} = \frac{1}{4}$ **is**

Key : 2

Sol. $\lim_{x\to 1}\left(\frac{\frac{\sin(x-1)+a(1+x)}{x-1}}{\frac{(x-1)+\sin(x-1)}{x-1}}\right)^{\frac{1-x}{1+\sqrt{x}}}$

$\lim_{x\to 1}\left(\frac{1-a}{2}\right)^{1+\sqrt{x}} = \frac{1}{4}$

Ans : 2

53. The slope of the tangent to the curve $\left(y - x^5\right)^2 = x\left(1 + x^2\right)^2$ **at the point (1, 3) is**

Key : 8

Sol : $\left(y - x^5\right)^2 = x\left(1 + x^2\right)^2$

$2\left(y - x^5\right)\left(y^1 - 5x^4\right) = \left(1 + x^2\right)^2 + 4x^2\left(1 + x^2\right)$

at (1,3)

Slope : $\frac{dy}{dx} = 8$

54. For a point P in the plane, let d_1(P) and d_2(P) be the distances of the point P from the lines $x - y = 0$ **and x + y = 0 respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane and satisfying** $2 \le d_1(P) + d_2(P) \le 4$**, is**

Key : 6

Sol : $d_1(p) = \frac{|x - y|}{\sqrt{2}}$

$d_2(p) = \frac{|x + y|}{\sqrt{2}}$

$$2 \le \frac{|x-y|}{\sqrt{2}} + \frac{|x+y|}{\sqrt{2}} \le 4$$

$$2\sqrt{2} \le |x-y| + |x+y| \le 4\sqrt{2}$$

$x > 0, y > 0 \,\&\, x \ge y \qquad \sqrt{2} \le x \le 2\sqrt{2}$

$x > 0, y > 0 \,\&\, x < y \qquad \sqrt{2} \le y \le 2\sqrt{2}$

Area $= A_1 + A_2$

$= \left(2\sqrt{2}\right)^2 - \left(\sqrt{2}\right)^2$

$= 6$ Sq. Units

55. Let $f: R \to R$ and $g: R \to R$ be respectively given by $f(x) = |x| + 1$ and g(x) = x^2 + 1. Define h : R
$\to$ R by

h(x) = $\begin{cases} \max\{f(x), g(x)\} & \text{if } x \le 0, \\ \min\{f(x), g(x)\} & \text{if } x > 0, \end{cases}$

The number of points at which h(x) is not differentiable is

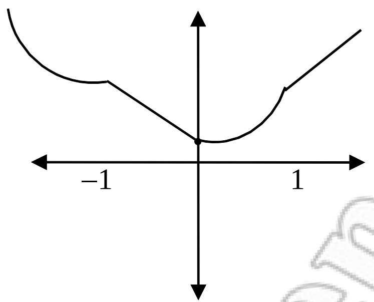
Key : 3

Sol : $h(x) = x^2 + 1 \qquad x < -1$

$= 1 - x \qquad -1 \le x < 0$

$= x^2 + 1 \qquad 0 \le x < 1$

$= 1 + x \qquad x \ge 1$



Not differentiable $x = \pm 1, 0$

56. Let $n \ge 2$ be an integer. Take n distinct points on a circle and join each pair of points by a line segment. Colour the line segment joining every pair of adjacent points by blue and the rest by red. If the number of red and blue line segments are equal, then the value of n is

Key : 5

Sol : No. of sides $=$ no. of diagrams

$n = {}^nC_2 - n$

$2n = \frac{n(n-1)}{2}$

$n - 1 = 4$

$n = 5$

57. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-coplanar unit vectors such that the angle between every pair of them is $\frac{\pi}{3}$. If $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}=p\vec{a}+q\vec{b}+r\vec{c}$, where p, q and r are scalars, then the value of $\frac{p^2+2q^2+r^2}{q^2}$ is

Key : 4

Sol : $\left[\bar{a}\,\bar{b}\,\bar{c}\right]^2=\begin{vmatrix}\bar{a}.\bar{a} & \bar{a}.\bar{b} & \bar{a}.\bar{c}\\ \bar{b}.\bar{a} & \bar{b}.\bar{b} & \bar{b}.\bar{c}\\ \bar{c}.\bar{a} & \bar{c}.\bar{b} & \bar{c}.\bar{c}\end{vmatrix}=\frac{1}{2}$

$2p+q+r=\sqrt{2}$

$p+2q+r=0$

$p+q+2r=\sqrt{2}$

$p=\frac{1}{\sqrt{2}}, q=-\frac{1}{\sqrt{2}}, r=\frac{1}{\sqrt{2}}$

$\frac{p^2+2q^2+r^2}{q^2}=4$

58. The value of $\int_0^1 4x^3\left\{\frac{d^2}{dx^2}\left(1-x^2\right)^5\right\}dx$ is

Key : 2

Sol : $\int_0^1 4x^3\frac{d^2}{dx^2}\left(1-x^2\right)^5 dx$

$=\left[4x^3\left[5\left(1-x^2\right)^4(-2x)\right]\right]_0^1+120\int_0^1 x^3\left(1-x^2\right)^4 dx$

$=0+120\int_0^1 x^3\left(1-x^2\right)^4 dx$

Put $x=\sin\theta$

$=120\int_0^{\pi/2}\sin^3\theta\cos^9\theta d\theta$

$=120\times\frac{2}{120}=2$

59. **Let** $n_1 < n_2 < n_3 < n_4 < n_5$ **be positive integers such that** $n_1 + n_2 + n_3 + n_4 + n_5 = 20$ **. Then the number of such distinct arragements** $(n_1, n_2, n_3, n_4, n_5)$ **is**

Key : 7

Sol : $(1,2,3,4,10)$

$(1,2,3,5,9)$

$(1,2,3,6,8)$

$(1,2,4,5,8)$

$(1,2,4,6,7)$

$(1,3,4,5,7)$

$(2,3,4,5,6)$

Ans : 7

60. **Let a, b, c be positive integers such that** $\frac{b}{a}$ **is an integer. If a, b, c are in geometric progression and the arithmetic mean of a, b, c is b + 2, then the value of** $\frac{a^2 + a - 14}{a+1}$ **is**

Key : 4

Sol : $\frac{a + ar + ar^2}{3} = (ar + 2)$

$a(r-1)^2 = 6$

r is an integer

where $r = 2, a = 6$

$\frac{a^2 + a - 14}{a+1} = 4$

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