EAMCET-TS 2016

ENGINEERING

Question Paper with Solutions

CODE-A

MATHS

1. If $f(x)=x^2-2x+4$ then the set of values of x satisfying $f(x-1)=f(x+1)$ is

1) $\{-1\}$ 2) $\{-1,1\}$ 3) $\{1\}$ 4) $\{1,2\}$

Key: 3

Sol: . $f(x-1)=f(x+1)$

$(x-1)^2-2(x-1)+4=(x+1)^2-2(x+1)+4$

$-4x+7=3$

$-4x=-4$

$\boxed{x=1}$

2. The number of real linear functions $f(x)$ satisfying $f(f(x))=x+f(x)$ is

1) 0 2) 4 3) 5 4) 2

Key: 4

Sol: . $f(x)=ax+b$

$f(f(x))=x+f(x)$

$a(ax+b)+b=x+ax+b$

$a^2x+ab+b=x(a+1)+b$

$a+1=a^2$ | $ab+b=b$

$a^2-a-1=0$ | $ab=0$

$\Delta>0$ | $a=0\ (or)\ b=0$

$a=\frac{1\pm\sqrt{5}}{2}$ | $a\neq 0$, $b=0$

$f(x)=\left(\frac{1+\sqrt{5}}{2}\right)^n,\left(\frac{1-\sqrt{5}}{2}\right)^n$

3. The remainder when $7^n-6n-50\,(n\in N)$ is divided by 36, is

1) 22 2) 23 3) 1 4) 21

Key: 2

Sol: . $7^n-6n-50=(1+6)^n-6n-50$

$=1+6n+36(\text{integer})-6n-50$

$=36(\text{integer})-49-23+23$

$=36(\text{integer})+23$

4. **Consider the system of equations**

$ax+by+cz=2$

$bx+cy+az=2$

$cx+ay+bz=2$

where a,b,c **are real numbers such that** $a+b+c=0$

Then the system

1) has two solutions
2) is inconsistent
3) has unique solution
4) has infinitely many solutions

Key: 2

Sol: . $a+b+c=0$

$$\begin{bmatrix} 1 & 1 & -2 & 2 \\ 1 & -2 & 1 & 2 \\ -2 & 1 & 1 & 2 \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 + 2R_1$

$$\sim \begin{bmatrix} 1 & 1 & -2 & 2 \\ 0 & -3 & 3 & 0 \\ 0 & 3 & -3 & 6 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\sim \begin{bmatrix} 1 & 1 & -2 & 2 \\ 0 & -3 & 3 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$

$r\left(Al \neq r\left(\left[AB\right]\right)\right)$

5. **Suppose A and B are two square matrices of same order. If A, B are symmetric matrices then AB - BA is**

1) a symmetric matrix
2) a skew symmetric
3) a scalar matrix
4) a triangular matrix

Key: 2

Sol: . $A^T = A, B^T = B$

$\left(AB-BA\right)^T = \left(AB\right)^T - \left(BA\right)^T$

$= B^T A^T - A^T B^T$

$= BA - AB$

$= -\left(AB - BA\right)$

6. **If** $A(x)=\begin{vmatrix} x+1 & 2x+1 & 3x+1 \\ 2x+1 & 3x+1 & x+1 \\ 3x+1 & x+1 & 2x+1 \end{vmatrix}$ **then** $\int_0^1 A(x)\,dx =$

1) -15 2) $\frac{-15}{2}$ 3) -30 4) -5

Key: 2

Sol: . $A(x)=\begin{vmatrix} x+1 & 2x+1 & 3x+1 \\ 2x+1 & 3x+1 & x+1 \\ 3x+1 & x+1 & 2x+1 \end{vmatrix}$

$C_1 \to C_1 + C_2 + C_3$

$$=\begin{vmatrix} 6x+3 & 2x+1 & 3x+1 \\ 6x+3 & 3x+1 & x+1 \\ 6x+3 & x+1 & 2x+1 \end{vmatrix}$$

$$=(6x+3)\begin{vmatrix} 1 & 2x+1 & 3x+1 \\ 1 & 3x+1 & x+1 \\ 1 & x+1 & 2x+1 \end{vmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$=(6x+3)\begin{vmatrix} 1 & 2x+1 & 3x+1 \\ 0 & x & -2x \\ 0 & -x & -x \end{vmatrix}$$

$$=3(2x+1)\left(-x^2-2x^2\right)$$

$$=-9(2x+1)x^2$$

$$=-9\left(2x^3+x^2\right)$$

$$\int_0^1 A(x)\,dx = -9\int_0^1\left(2x^3+x^2\right)dx$$

$$=-9\left[\frac{2x^4}{4}+\frac{x^3}{3}\right]_0^1$$

$$=-9\left[\frac{1}{2}+\frac{1}{3}\right]$$

$$=-9\frac{5}{6}$$

$$= -\frac{15}{2}$$

7. If $z = x + iy$ is a complex number such that $\bar{z}^{\frac{1}{3}} = a + ib$, then the value of $\frac{1}{a^2+b^2}\left(\frac{x}{a}+\frac{y}{b}\right) =$

1) -1 2) -2 3) 0 4) 2

Key: 2

Sol: . $z^{\frac{1}{3}} = a + ib$

$\bar{z} = (a+ib)^3$

$= a^3 - 3ab^2 + i(3a^2b - b^3)$

$x = a^3 - 3ab^2$ | $y = b^3 - 3a^2b$

$\frac{x}{a} = a^2 - 3b^2$ | $\frac{y}{b} = b^2 - 3a^2$

$\frac{x}{a} + \frac{y}{b} = -2(a^2+b^2)$

$\frac{1}{(a^2+b^2)}\left(\frac{x}{a}+\frac{y}{b}\right) = -2$

8. The locus of z satisfying $|z| + |z-1| = 3$ is

1) a circle 2) a pair of straight lines 3) an ellipse 4) a parabola

Key: 3

Sol: . $|z| + |z-1| = 3$

$PA + PB = K$

$A(0,0)$ $B(1,0)$

$AB = 1$ $k = 3$

$e = \frac{AB}{k} = \frac{1}{3} < 1$

ellipse

9. If the point $z = (1+i)(1+2i)(1+3i).......(1+10i)$ lies on a circle with centre at origin and radius r, then $r^2 =$

1) $10!$ 2) $2\times3\times4\times......\times10$ 3) $2\times5\times10\times......\times101$ 4) $11!$

Key: 3

Sol: . $z = (1+i)(1+2i)(1+3i).............(1+10i)$

$|z|^2 = 2\times5\times10\times.............\times101$

10. The minimum value of $|z-1| + |z-5|$ is

1) 5 2) 4 3) 3 4) 2

Key: 2

Sol: . $|z-1|+|z-5|$

Minima of $PA+PB=AB$

$A(1,0)\quad B(5,0)$

$AB=4$

11. The number of real roots $|x|^2-5|x|+6=0$ is

1) 2 2) 3 3) 4 4) 1

Key: 3

Sol: . $|x|^2-5|x|+6=0$

$(|x|-2)(|x|-3)=0$

$|x|=2,\ |x|=3$

$x=\pm2\quad x=\pm3$

12. If α,β are the roots of $x^2-x+1=0$ then the quadratic equation whose roots are $\alpha^{2015},\beta^{2015}$ is

1) $x^2-x+1=0$ 2) $x^2+x+1=0$ 3) $x^2+x-1=0$ 4) $x^2-x-1=0$

Key: 1

Sol: . $x^2-x+1=0$

$x=\dfrac{1\pm i\sqrt{3}}{2}$

$x=-\omega,-\omega^2$

$\alpha^{2015}=(-\omega)^{2015}=-(\omega^{2015})=-\omega^2$

$\beta^{2015}=(-\omega^2)^{2015}=-(\omega^{4030})=-\omega^{4029}.\omega$

$=-\omega$

$sum=-\omega-\omega^2=1$

Product = 1

$x^2-x+1=0$

13. If α,β,γ are roots of $x^3-5x+4=0$ then $\left(\alpha^3+\beta^3+\gamma^3\right)^2=$

1) 12 2) 13 3) 169 4) 144

Key: 4

Sol: . $x^3-5x+4=0$

$\alpha+\beta+\gamma=0\qquad s_1=0,\ s_2=-5,\ s_3=-4$

$\alpha^3+\beta^3+\gamma^3=3\alpha\beta\gamma$

$\left(\alpha^3+\beta^3+\gamma^3\right)^2=9(\alpha\beta\gamma)^2$

$=9(-4)^2$

$=16(9)=144$

14. Suppose α,β,γ are roots of $x^3+x^2+2x+3=0$. If $f(x)=0$ is a cubic polynomial equation whose roots are $\alpha+\beta,\beta+\gamma,\gamma+\alpha$ then $f(x)=$

1) x^3+2x^2-3x-1 2) x^3+2x^2-3x+1 3) x^3+2x^2+3x-1 4) x^3+2x^2+3x+1

Key: 3

Sol: . $y=\alpha+\beta+\gamma-x$

$=-1-x$

$y=-1-x$

$x=-(1+y)$

$x^3+x^2+2x+3=0$

$-(1+y)^3+(1+y)^2-2(1+y)+3=0$

$-y^3-3y^2-3y-1+1+y^2+2y-2-2y+3=0$

$-y^3-2y^2-3y+1=0$

$\boxed{y^3+2y^2+3y-1=0}$

$\boxed{x^3+2x^2+3x-1=0}$

15. The number of 4 letters words that can be formed with the letters in the word EQUATION with at least one letter repeated is

1) 2400 2) 2408 3) 2416 4) 2432

Key: 3

Sol: . $8^4-{}^8P_4$

$=2416$

16. The number of divisors of $7!$ is

1) 24 2) 72 3) 64 4) 60

Key: 4

Sol: . $7!=2\times3\times2^2\times5\times2\times3\times7$

$=2^4\times3^2\times5^1\times7^1$

Number of divisors $=(4+1)(2+1)(1+1)(1+1)$

= 60

17. The sum of the series

$$1+\frac{2}{3}\left(\frac{1}{8}\right)+\frac{2\times5}{3\times6}\left(\frac{1}{8}\right)^2+\frac{2\times5\times8}{3\times6\times9}\left(\frac{1}{8}\right)^3+..........$$

1) $\frac{4}{\sqrt[3]{49}}$ 2) $\frac{\sqrt[3]{49}}{4}$ 3) $\frac{4}{\sqrt[3]{81}}$ 4) $\frac{\sqrt[3]{81}}{4}$

Key: 1

Sol: . $S=1+2\left(\frac{1}{24}\right)+\frac{2\times5}{1.2}\left(\frac{1}{24}\right)^2+.......$

$p = 2, q = 3$

$\frac{x}{q} = \frac{1}{24}$

$x = \frac{1}{8}$

$= \left(1 - \frac{1}{8}\right)^{-\frac{2}{3}}$

$= \left(\frac{7}{8}\right)^{-\frac{2}{3}}$

$= \left(\frac{8}{7}\right)^{-\frac{2}{3}}$

$= \frac{4}{\sqrt[3]{49}}$

18. If C_r denotes the binomial coefficient ${}^nC_r$ then $(-1)C_0^2 + 2C_1^2 + 5C_2^2 + ... + (3n-1)C_n^2 =$

1) $(3n-2)\,{}^{2n}C_n$ 2) $\left(\frac{3n-2}{2}\right){}^{2n}C_n$ 3) $(5+3n)\,{}^{2n}C_n$ 4) $\left(\frac{3n-5}{2}\right){}^{2n}C_{n+1}$

Key: 2

Sol: . $2S = (3n-2)\left(C_0^2 + C_1^2 + ... + C_n^2\right)$

$S = \frac{(3n-2)}{2}\ {}^{2n}C_n$

19. $\frac{x+1}{x^4(x+2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x^4} + \frac{E}{x+2}$

$\Rightarrow B + D + E =$

1) $A + C$ 2) $A - C$ 3) $2A + C$ 4) $2A + 2C$

Key: 1

Sol: . $\frac{x+1}{x^4(x+2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x^4} + \frac{E}{x+2}$

$x + 1 = Ax^3(x+2) + B(x^2)(x+2) + Cx(x+2) + D(x+2) + Ex^4$

$A + E = 0$

$2D = 1$

$D = \frac{1}{2}$

$-1 = 16E \Rightarrow \boxed{E = -\frac{1}{16}}$

$2A+B=0 \qquad B=-2A=-\frac{1}{8}$

$B+D+E=-\frac{1}{8}+\frac{1}{2}-\frac{1}{16}=\frac{5}{16}$

coefficient x^2

$0=2B+C\Rightarrow C=-2B$

$=-2\left(-\frac{1}{8}\right)$

$=\frac{1}{4}$

$A=\frac{1}{16} \qquad C=\frac{1}{4}$

$A+C=B+D+E$

20. **If** $\cos^3\theta+\cos^3\left(\frac{2\pi}{3}+\theta\right)+\cos^3\left(\frac{4\pi}{3}+\theta\right)=a\cos 3\theta$ **, then** $a=$

1) $\frac{1}{4}$ 2) $\frac{3}{4}$ 3) $\frac{5}{4}$ 4) $\frac{7}{4}$

Key: 2

Sol: $\cos^3\theta+\cos^3\theta\left(\frac{2\pi}{3}+\theta\right)+\cos^3\left(\frac{4\pi}{3}+\theta\right)=a\cos^3\theta$

Put $\alpha=0^0$

$1-\frac{1}{8}-\frac{1}{8}=a$

$a=1-\frac{1}{4}$

$\boxed{a=\frac{3}{4}}$.

21. $\frac{\cos 13^0-\sin 13^0}{\cos 13^0+\sin 13^0}+\frac{1}{\cot 148^0}=$

1) 1 2) -1 3) 0 4) $\frac{1}{2}$

Key: 3

Sol: . $\frac{\cos 13^0-\sin 13^0}{\cos 13^0+\sin 13^0}+\frac{1}{\cot 148^0}$

$=\tan\left(45^0-13^0\right)+\frac{1}{-\cot 32^0}$

$=\tan 32^0-\tan 32^0 \qquad =0$

22. **If** $\cos x+\cos y+\cos\alpha=0$ **and** $\sin x+\sin y+\sin\alpha=0$**, then** $\cot\left(\frac{x+y}{2}\right)=$

1) $\sin\alpha$ 2) $\cos\alpha$ 3) $\tan\alpha$ 4) $\cot\alpha$

Key: 4

Sol: . $\cos x+\cos y=-\cos\alpha$

$\sin x+\sin y=-\sin\alpha$

$$2\cos\frac{x+y}{2}.\cos\frac{x-y}{2}=-\cos\alpha\ \rightarrow(1)$$

$$2\sin\frac{x+y}{2}.\cos\frac{x-y}{2}=-\sin\alpha\ \rightarrow(2)$$

by equations 1 and 2

$$\cot\left(\frac{x+y}{2}\right)=\cot\alpha$$

23. **If** $f(x)=\cos^2 x+\cos^2 2x+\cos^2 3x$**, then the number of values of** $x\in[0,2\pi]$ **for which** $f(x)=1$ **is**

1) 4 2) 6 3) 8 4) 10

Key: 4

Sol: . $\cos^2 x+\cos^2 2x+\cos^2 3x=1$

$\cos^2 x+\cos^2 2x-\sin^2 3x=0$

$\cos 4x\cos(2x)+\cos^2 2x=0$

$\cos 2x[\cos 4x+\cos 2x]=0$

$2\cos 2x\cos 3x\cos x=0$

$\cos x=0,\cos 2x=0,\cos 3x=0$

$$x=(2n+1)\frac{\pi}{2},(2n+1)\frac{\pi}{4},(2n+1)\frac{\pi}{6}$$

$$\left\{\frac{\pi}{6},\frac{\pi}{4},\frac{\pi}{2},\frac{3\pi}{4},\frac{3\pi}{2},\frac{5\pi}{4},\frac{7\pi}{4},\frac{5\pi}{6},\frac{7\pi}{6},\frac{11\pi}{6}\right\}$$

24. **The value of** x **which satisfies** $\sin\left(\cot^{-1}x\right)=\cos\left(\tan^{-1}(1+x)\right)$ **is**

1) $-\frac{1}{2}$ 2) $\frac{1}{2}$ 3) -1 4) 1

Key: 1

Sol: . $\sin\left(\cot^{-1}x\right)=\cos\left(\tan^{-1}(1+x)\right)$

$\cot^{-1}x=\alpha$, $\tan^{-1}(1+x)=\beta$

$x=\cot\alpha$ $\tan\beta=1+x$

$$\sin\alpha=\frac{1}{\sqrt{1+x^2}}\quad \cos\beta=\frac{1}{\sqrt{2+2x+x^2}}$$

$2+2x+x^2=1+x^2$

$2x=-1$

$\boxed{x=-\frac{1}{2}}$

25. **For** $\theta\in\left(0,\frac{\pi}{2}\right), \sec h^{-1}(\cos\theta)=$

1) $\log\left|\tan\left(\frac{\pi}{6}+\frac{\theta}{2}\right)\right|$ 2) $\log\left|\tan\left(\frac{\pi}{3}+\frac{\theta}{2}\right)\right|$ 3) $\log\left|\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right|$ 4) $\log\left|\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\right|$

Key: 3

Sol: . $\theta\in\left(0,\frac{\pi}{2}\right)$ $\sec h^{-1}(\cos\theta)=x$

$\sec h(x)=\cos\theta$

$\frac{2}{e^x+e^{-x}}=\cos\theta$

$\frac{2e^x}{(e^x)^2+1}=\cos\theta$

$(\cos\theta)(e^x)^2-2e^x+\cos\theta=0$

$e^x=\frac{2\pm\sqrt{4-4\cos^2\theta}}{2\cos\theta}$

$=\frac{2\pm 2\sin\theta}{2\cos\theta}$

$=\frac{1+\sin\theta}{\cos\theta}$

$=\frac{1+\cos\left(\frac{\pi}{2}-\theta\right)}{\sin\left(\frac{\pi}{2}-\theta\right)}=\frac{2\cos^2\left(\frac{\pi}{4}-\frac{\theta}{2}\right)}{2\sin\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\cos\left(\frac{\pi}{4}-\frac{\theta}{2}\right)}$

$x=\log_e\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$

26. **If** ΔABC **is such that** $\angle A=90^0, \angle B\neq\angle C$**, then** $\frac{b^2+c^2}{b^2-c^2}\sin(B-C)=$

1) $\frac{1}{3}$ 2) $\frac{1}{2}$ 3) 1 4) $\frac{3}{2}$

Key: 3

Sol: . $\angle A=90^0$ $\angle B\neq\angle C$ $b^2+c^2=a^2$

$$\frac{b^2+c^2}{b^2-c^2}\sin(B-C)=\frac{a^2}{b^2-c^2}\sin(B-C)$$

$$=\frac{\sin^2 A}{\sin(B+C)\sin(B-C)}\sin(B-C)$$

$$=\frac{\sin^2 A}{\sin A}$$

$$=\sin A$$

$$=1$$

27. In ΔABC, if $8R^2=a^2+b^2+c^2$, then the triangle is a

1) right angled triangle
2) equilateral triangle
3) scalene triangle
4) obtuse angled triangle

Key: 1

Sol: . $8R^2=a^2+b^2+c^2$

$\sin^2 A+\sin^2 B+\sin^2 C=2$

28. In ΔABC, if $2R+r=r_2$, then $\angle B=$

1) $\frac{\pi}{3}$
2) $\frac{\pi}{4}$
3) $\frac{\pi}{6}$
4) $\frac{\pi}{2}$

Key: 4

Sol: . $2R+r=r_2$

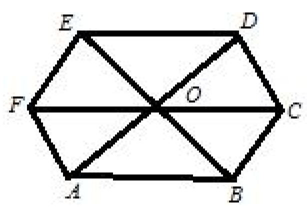
$2R=r_2-r$

$=\frac{\Delta}{s-b}-\frac{\Delta}{s}$

29. ABCDEF is a regular hexagon whose centre is O. Then $\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}$ is

1) $2\overrightarrow{AO}$
2) $3\overrightarrow{AO}$
3) $5\overrightarrow{AO}$
4) $6\overrightarrow{AO}$

Key: 4



Sol: . $\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}=6\overrightarrow{AO}$

$\overrightarrow{AD}=2\overrightarrow{AO}$

$\overrightarrow{AB}=\overrightarrow{ED}$

$\overrightarrow{AB}+\overrightarrow{AE}=\overrightarrow{AE}+\overrightarrow{ED}=\overrightarrow{AD}=2\overrightarrow{AO}$

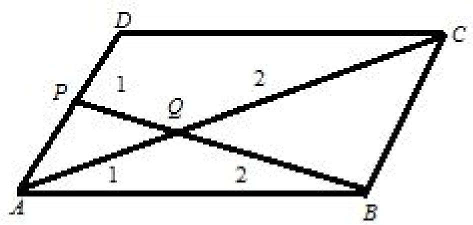
$\overrightarrow{AC}+\overrightarrow{AF}=\overrightarrow{AC}+\overrightarrow{CD}=\overrightarrow{AD}=2\overrightarrow{AO}$

30. ABCD is a parallelogram and P is the mid point of the side AD. The line BP meets the diagonal AC in Q. Then the ratio AQ : QC =

1) 1 : 2
2) 2 : 1
3) 1 : 3
4) 3 : 1

Key: 1

Sol: .



$\overrightarrow{AQ}:\overrightarrow{QC}=1:2$

$\overrightarrow{OP}=\frac{\overrightarrow{OA}+\overrightarrow{OD}}{2}$

$\overrightarrow{OQ}=\frac{\overrightarrow{OC}+2\overrightarrow{OA}}{3}$

$\overrightarrow{OQ}=\frac{2\overrightarrow{OP}+\overrightarrow{OB}}{3}$

$=\frac{\overrightarrow{OA}+\overrightarrow{OD}+\overrightarrow{OB}}{3}$

$=\frac{8\overrightarrow{OA}+2\overrightarrow{OD}}{3}$

31. **The vectors** $2\bar{i}-3\bar{j}+\bar{k},\bar{i}-2\bar{j}+3\bar{k},3\bar{i}+\bar{j}-2\bar{k}$

1) are linearly dependent 2) are linearly independent
3) form sides of a triangle 4) are coplanar

Key: 2

Sol: . $\left[\vec{a}\ \vec{b}\ \vec{c}\right]=\begin{vmatrix}2 & -3 & 1\\1 & -2 & 3\\3 & 1 & -2\end{vmatrix}$

$=2(4-3)+3(-2-9)+1(1+6)$

$=2-33+7$

$\neq 0$

32. $\bar{a},\bar{b},\bar{c}$ **are three vectors such that** $|\bar{a}|=1|\bar{b}|=2,|\bar{c}|=3$ **and** $\bar{b},\bar{c}$ **are perpendicular. If projection of** $\bar{b}$ **on** $\bar{a}$ **is the same as the projection of** $\bar{c}$ **on** $\bar{a}$ **, then** $|\bar{a}-\bar{b}+\bar{c}|=$

1) $\sqrt{2}$ 2) $\sqrt{7}$ 3) $\sqrt{14}$ 4)

Key: 3

Sol: . $|\vec{a}|=1,|\vec{b}|=2,|\vec{c}|=3$

$\vec{b}.\vec{c}=0$

$\frac{\vec{b}.\vec{a}}{|\vec{a}|}=\frac{\vec{c}.\vec{a}}{|\vec{a}|}\Rightarrow \vec{a}.\vec{b}=\vec{a}.\vec{c}$

$|\vec{a}-\vec{b}+\vec{c}|^2=1+4+9$

$=14$

$|\vec{a}-\vec{b}+\vec{c}|=\sqrt{14}$

33. If $\overline{a},\overline{b},\overline{c}$ are unit vectors satisfying the relation $\overline{a}+\overline{b}+\sqrt{3}\overline{c}=0$ then the angle between $\overline{a}$ and $\overline{b}$ is

1) $\frac{\pi}{6}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{2}$

Key: 3

Sol: . $\vec{a}+\vec{b}+\sqrt{3}\vec{c}=\vec{O}$

$\left|\vec{a}+\vec{b}\right|=\sqrt{3}\left|\vec{c}\right|$

$2+2\ \vec{a}.\vec{b}=3$

$\vec{a}.\vec{b}=\frac{1}{2}$

$\cos\left(\vec{a}.\vec{b}\right)=\frac{1}{2}$

$\left(\vec{a},\vec{c}\right)=\frac{\pi}{3}$

34. $\overline{a}$ is perpendicular to both $\overline{b}$ and $\overline{c}$. The angle between $\overline{b}$ and $\overline{c}$ is $\frac{2\pi}{3}$. If $\left|\overline{a}\right|=2,\left|\overline{b}\right|=3,\left|\overline{c}\right|=4$, then $\overline{c}.\left(\overline{a}\times\overline{b}\right)=$

1) $18\sqrt{3}$ 2) $12\sqrt{3}$ 3) $8\sqrt{3}$ 4) $6\sqrt{3}$

Key: 2

Sol: . $\begin{matrix}\vec{a}!\vec{b}=0\\ \vec{a}-\vec{c}=0\end{matrix}$ $\vec{a}\parallel\vec{b}\times\vec{c}$ $\left(\vec{b},\vec{c}\right)=\frac{2\pi}{3}$

$\left|\vec{a}\right|=2,\left|\vec{b}\right|=3,\left|\vec{c}\right|=4$

$\vec{c}.\left(\vec{a}\times\vec{b}\right)=\vec{a}.\left(\vec{b}\times\vec{c}\right)$

$=\left|\vec{a}\right|\left|\vec{b}\times\vec{c}\right|\cos\left(\vec{a},\vec{b}+\vec{c}\right)$

$=\pm\left|\vec{a}\right|\ \left|\vec{b}\right|\ \left|\vec{c}\right|\ \sin\left(\vec{b},\vec{c}\right)$

$=\pm 2\times 3\times 4\times\frac{\sqrt{3}}{2}$

$=\pm 12\sqrt{3}$

35. If the average of the first n numbers in the sequence 148, 146, 144,....... is 125, then $n=$

1) 18 2) 24 3) 30 4) 36

Key: 2

Sol: . Let $x=148$

The sequence is $x, x-2, ...upto\ \ 'n'\ terms$

$$\frac{x+(x-2)+(x-4)..........+[x-2(n-1)]}{n}=125$$

$$\frac{nx-2[(1+2............+(n-1))]}{n}=115$$

$$\frac{nx-\frac{2.(n-1)n}{2}}{n}=125$$

$x-(n-1)=125$

$n-1=148-125$

$=23$

$\boxed{n=24}$

36. The standard deviation of $a, a+d, a+2d, ..., a+2nd$ **is**

1) nd 2) n^2d 3) $\sqrt{\frac{n(n+1)}{3}}d$ 4) $\sqrt{\frac{n(n+3)}{3}}d$

Key: 3

Sol: . $n=2$

$a, a+d, a+2d, a+3d, a+4d$

$x=a+2d$

$\sigma^2=\frac{1}{5}\sum\left[4d^2+d^2+0+d^2+4d^2\right]$

$=2d^2$

$\sigma=\sqrt{2}d$

37. Two events A and B are such that

$P(A)=\frac{1}{4}, P(A|B)=\frac{1}{4}$ **and** $P(B|A)=\frac{1}{2}$

Consider the following statements :

I) $P(\bar{A}|\bar{B})=\frac{3}{4}$

II) A and B are mutually exclusive

III) $P(A|B)+P(A|\bar{B})=1$

Then

1) Only (I) is correct
2) Only (I) and (II) are correct
3) Only (I) and (III) are correct
4) Only (II) and (III) are correct

Key: 1

Sol: . $P(A)=\frac{1}{4}$ $P(A/B)=\frac{1}{4}P(B/A)=\frac{1}{2}$

$\frac{P(A\cap B)}{P(A)}=\frac{1}{2}$ $\frac{P(A\cap B)}{P(B)}=\frac{1}{4}$

$P(A\cap B)=\frac{1}{8}$ $P(B)=\frac{1}{2}$

I) $P(\bar{A}|\bar{B})=\frac{P(\bar{A}\cap\bar{B})}{P(\bar{B})}$

$=\frac{1-P(A\cup B)}{1-P(B)}$

$=\frac{1-\left[\frac{1}{4}+\frac{1}{2}-\frac{1}{8}\right]}{\frac{3}{2}}$

$=\frac{1-\frac{5}{8}}{\frac{1}{2}}$

$=\frac{3}{8}\times 2$

II) Wrong

III) $P(A|B)+P(A/B)$

$=\frac{1}{4}+\frac{P(A)-P(A\cap B)}{1-P(B)}$

$=\frac{1}{4}+\frac{\frac{1}{4}-\frac{1}{8}}{1-\frac{1}{2}}=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$

38. A five digit number is formed by the digits 1,2,3,4,5 with no digit being repeated. The probability that the number is divisible by 4, is

1) $\frac{1}{5}$ 2) $\frac{2}{5}$ 3) $\frac{3}{5}$ 4) $\frac{4}{5}$

Key: 1

Sol: . $n(S)={}^5P_5=120$

$n(A)=4\times{}^3P_3=2C_1$

$P(A)=\frac{14}{120}=\frac{3}{15}=\frac{1}{5}$

39. **When a pair of six faced fair dice are thrown, the probability that the sum of the numbers on the two dice is greater than 7, is**

1) $\frac{1}{3}$ 2) $\frac{5}{12}$ 3) $\frac{1}{2}$ 4) $\frac{1}{4}$

Key: 2

Sol: . $n(S)=36$

$n(A)=1+2+3+4+5$

$=15$

$P(A)=\frac{15}{36}=\frac{5}{12}$

40. **In a family with 4 children, the probability that there are at least two girls is**

1) $\frac{1}{2}$ 2) $\frac{9}{16}$ 3) $\frac{3}{4}$ 4) $\frac{11}{16}$

Key: 4

Sol: . $\frac{{}^4C_2+{}^4C_3+{}^4C_4}{2^4}$

$=\frac{11}{16}$

41. **On an average nine out of 10 ships that have departed at A reach B safely. The probability that out of five ships that have departed at A at least four will reach B safely is**

1) $14(0.9)^5$ 2) $1.4(0.9)^5$ 3) $0.14(0.9)^4$ 4) $1.4(0.9)^4$

Key: 4

Sol: $p=\frac{9}{10},\ q=\frac{1}{10}$

$p(x\geq 4)=p(x=4)+p(x=5)$

$={}^5C_4\left(\frac{1}{10}\right)\left(\frac{9}{10}\right)^4+{}^5c_5\left(\frac{9}{10}\right)^5$

$=\left(\frac{5}{10}\right)\left(\frac{9}{10}\right)^4+\left(\frac{9}{10}\right)^5$

$=\left[(0.5)+(0.9)\right]\left(\frac{9}{10}\right)^4$

$=(1.4)(0.9)^4$.

42. **If $A(5,-4)$ and $B(7,6)$ are points in a plane, then the set of all points $P(x,y)$ in the plane such that AP : PB = 2 : 3 is**

1) a circle 2) a hyperbola 3) an ellipse 4) a parabola

Key: 1

Sol: $3(AP)=2(PB)$

$\Rightarrow 9(AP^2)=4(PB)^2$

$\Rightarrow 9\left[(x-5^2)+(y+4)^2\right]=4\left[(x-7)^2+(y-6)^2\right]$

$\Rightarrow 9\left[x^2+y^2-10x+8y+4\right]=4\left[x^2-14x+49+y^2-12y+36\right]$

$\Rightarrow 9x^2+9y^2-90x+72y+369=4x^2-56x+196+4y^2-48y+144$

$\Rightarrow 5x^2+5y^2-36x+120y+29=0$

locus represents a circle..

43. If the axes are rotated anticlockwise through an angle 90^0 then the equation $x^2=4ay$ is changed to the equation.

1) $y^2=4ax$ 2) $x^2=-4ay$ 3) $y^2=-4ax$ 4) $x^2=4ay$

Key: 3

Sol:

	x	y
x	0	-1
y	1	0

$\theta=+90^0$

$x=-y$

$y=x$

$x^2=4ay$

$y^2=+4ax^2$

44. The combined equation of the straight lines of the form $y=kx+1$ (where k is an integer) such that the point of intersection of each with the line $3x+4y=9$ has an integer as its x-coordinate is

1) $(y+x+1)(y+2x-1)=0$ 2) $(y+x-1)(y+2x+1)=0$

3) $(y+x+1)(y+2x+1)=0$ 4) $(y+x-1)(y+2x-1)=0$

Key : 4

Sol : $3x+4(kx+1)=9$

$x(3+4k)=5$

$x=\dfrac{5}{3+4k}$

$3+4k=\pm 5$

$4k=8$

$k=-2$

$3+4k=\pm 1$

$4k=-4$

$k=-1$

$y = 2x + 1 \quad (x + y - 1)(2x - y - 2) = 0$

$y = -x + 1$

$(y + 2x - 1)(x + y - 1) = 0$

45. A value of k such that the straight lines $y - 3kx + 4 = 0$ **and** $(2k-1)x - (8k-1)y - 6 = 0$ **are perpendicular is**

1) $\frac{1}{6}$ 2) $-\frac{1}{6}$ 3) **1** 4) **0**

Key: **1**

Sol: $m_1 = 3k$

$m_2 = \frac{2k-1}{8k-1}$

$\frac{2k-1}{8k-1} \times \frac{-1}{3k}$

$6k^2 - 3k = -8k + 1$

$6k^2 + 5k - 1 = 0$

$6k^2 + 6k - k - 1 = 0$

$6l(k+1) - 1(k+1) = 0$

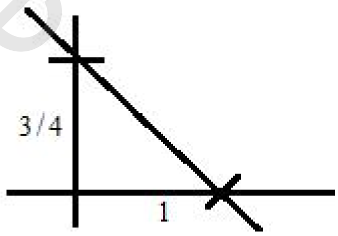
$k = \frac{1}{6}, -1$

46. The length of the segment of the straight line passing through (3, 3) and (7, 6) cut off by the coordinate axes is

1) $\frac{4}{5}$ 2) $\frac{5}{4}$ 3) $\frac{7}{4}$ 4) $\frac{4}{7}$

Key: 2

Sol: $m = \frac{3}{4}, (3, 3)$



$3x - 4y = 9 - 12$

$3x - 4y = -3$

$\frac{x}{1} + \frac{y}{3/4} = 1$

length $= \sqrt{1 + \frac{9}{16}} = 5/4$

47. The equation of the pair of straight lines through the point (1, 1) and perpendicular to the pair of straight lines $3x^2-8xy+5y^2=0$ is

1) $5x^2+8xy+3y^2-14x-18y+16=0$
2) $5x^2+8xy+3y^2-14x-18y+16=0$
3) $5x^2-8xy+3y^2-18x-14y+32=0$
4) $5x^2-8xy+3y^2-14x-18y+32=0$

Key : 2

Sol : $5(x-1)^2-8(x-1)(y-1)+3(y-1)^2=0$

$5(x^2-2x+1)+8(xy-x-y+1)+3(y^2-2y+1)=0$

$5x^2-10x+5+8xy-8x-8y+8+3y^2-6y+3=0$

$5x^2+8xy+3y^2-18x-14y+16=0$

48. The combined equation of the three sides of a triangle is $(x^2-y^2)(2x+3y-6)=0$. If the point $(0,\alpha)$ lies in the interior of this triangle then

1) $-2<\alpha<0$
2) $-2<\alpha<2$
3) $0<\alpha<2$
4) $\alpha\geq 2$

Key : 3

Sol : $(x^2-y^2)(2x+3y-6)=0$

$(x+y)(x-y)(2x+3y-6)=0$

$x+y=0$

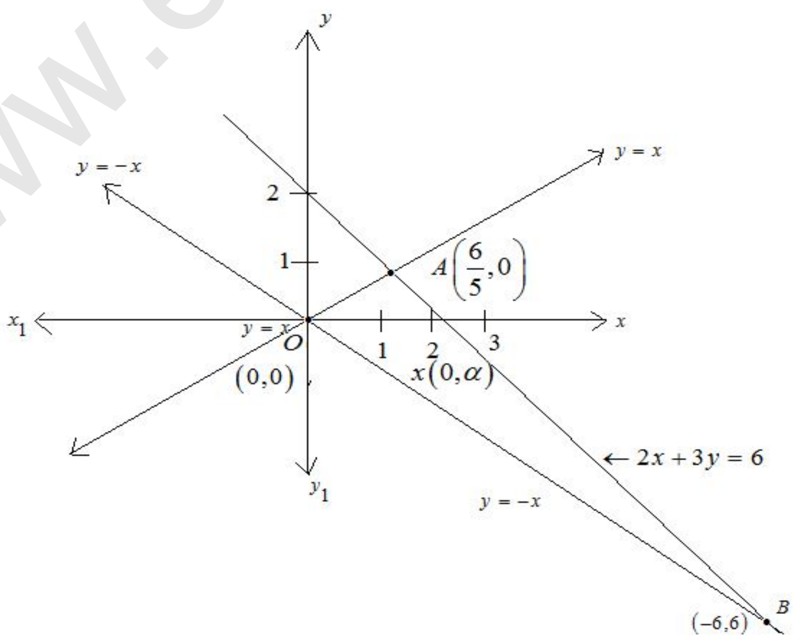
$x-y=0$

$2x+3y=6$

$\frac{x}{3}+\frac{y}{2}=1$

$(0+\alpha)\left(\frac{6}{5}+0\right)>0\Rightarrow\alpha>0$

$(-6-6)(0-\alpha)>0$

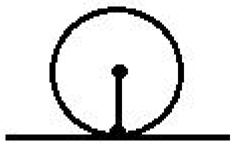


49. The point where the line $4x-3y+7=0$ touches the circle $x^2+y^2-6x+4y-12=0$ is

1) $(1,1)$ 2) $(1,-1)$ 3) $(-1,1)$ 4) $(-1,-1)$

Key: 3

Sol: Centre $(3,-2)$



$$\frac{h-3}{4}=\frac{k+2}{-3}=\frac{-[12+6+7]}{25}$$

$$\frac{h-3}{4}=\frac{k+2}{-3}=-1$$

$h-3=-4 \mid k+2=3$

$h=-1 \mid k=1$

$=(-1,1)$

50. The normal to the circle given by $x^2+y^2-6x+8y-144=0$ at (8, 8) meets thecircle again at the point

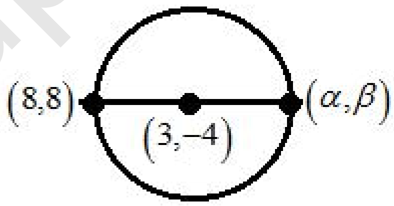
1) $(2,-16)$ 2) $(2,16)$ 3) $(-2,16)$ 4) $(-2,-16)$

Key: 4

Sol: $\frac{8+\alpha}{2}=3 \mid \frac{8+\beta}{2}=-4$

$8+\alpha=6 \mid 8+\beta=-8$

$\alpha=-2 \mid \beta=-16$



$(-2,-16)$

51. For all real values of k, the polar of the point $(2k, k-4)$ with respect to $x^2+y^2-4x-6y+1=0$ passes through the point

1) (1, 1) 2) $(1,-1)$ 3) $(-3,1)$ 4) $(3,1)$

Key: 4

Sol: $S_1=0$

$x(2k)+y(k-4)-2(x+2k)-3(y+k-4)+1=0$

$2kx+ky-4y-2x-4k-3y-3k+12+1=0$

$k(2x+y-4-3)+(-4y-2x-3y+13)=0$

$k(2x+y-7)+(-2x-7y+13)=0$

$8x+y-7=0$

$2x+7y+3=0$

$-6y+6=0 \Rightarrow y=1$

$\Rightarrow 2x-6=0$

$x=3$

(3,1)

52. If the circles $x^2+y^2-2\lambda x-2y-7=0$ and $3(x^2+y^2)-8x+29y=0$ are orthogonal then λ =

1) 4 2) 3 3) 2 4) 1

Key: 4

Sol: $x^2+y^2-2\lambda x-2y-7=0$ ....(1)

$x^2+y^2-\frac{8}{3}x+\frac{29}{3}y=0$ ....(2)

$-2\lambda\left(\frac{-4}{3}\right)+(-2)\left(\frac{29}{6}\right)=-7+0$

$\frac{8\lambda}{3}-\frac{29}{3}=-7$

$\frac{8\lambda}{3}=\frac{29}{3}-\frac{7}{1}=\frac{29-21}{3}$

$\frac{8\lambda}{3}=\frac{8}{3}$

$\Rightarrow \lambda=1$

53. The radical centre of the circles $x^2+y^2=1$, $x^2+y^2-2x-3=0$ and $x^2+y^2-2y-3=0$ is

1) (1, 1) 2) $(1,-1)$ 3) $(-1,1)$ 4) $(-1,-1)$

Key:4

Sol: $x^2+y^2-1=0$ .... (1)

$x^2+y^2-2x-3=0$ .... (2)

$x^2+y^2-2y-3=0$ .... (3)

(1) - (2) $\Rightarrow 2x+2=0 \Rightarrow =-1$

(1) - (3) $\Rightarrow 2y+2=0 \Rightarrow y=-1$

$(-1,-1)$

54. From a point (C, 0) three normals are drawn to the parabola $y^2=x$. Then

1) $C<\frac{1}{2}$ 2) $C=\frac{1}{2}$ 3) $C>\frac{1}{2}$ 4) $\frac{1}{2}>C>\frac{1}{4}$

Key : 3

Sol : $4a=1$

$a=\frac{1}{2}$ (c,0)

$2a=\frac{1}{2}$ (c>2a)

$\frac{2(16)}{5}=\frac{32}{5}$

55. The points of intersection of the parabolas $y^2 = 5x$ and $x^2 = 5y$ lie on the line

1) $x+y=10$ 2) $x-2y=0$ 3) $x-y=0$ 4) $2x-y=0$

Key: 3

Sol: $y^2 = 5x$

$x^2 = 5y$

$y = \frac{x^2}{5}$

$\frac{x^4}{25} = 5x$

$x^4 - 125x = 0$

$x\left(x^3 - 5^3\right) = 0$

$(0,0),(5,5)$

$x - y = 0$

56. For the ellipse given by $\frac{(x-3)^2}{25} + \frac{(y-2)^2}{16} = 1$, match the equation of the lines given in List i with those in the List II

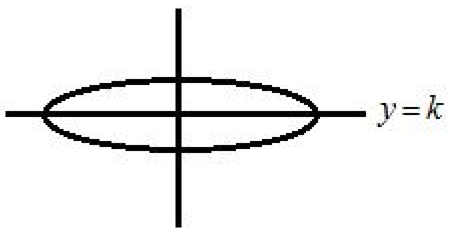
List I	**List II**
(i) The equation of the major axis	**(a) 3x = 34**
(ii) The equation of a directrix	**(b) y =2**
(ii) The equation of a latus rectum	**(c) x + y = 9**
	(d) x = 6
	(e) x = 3
	(f) 3y = 34

The correct matching is

1) i - e; ii - a; iii - d 2) i - b; ii - f; iii - e 3) i - b; ii - a; iii - e 4) i - b; ii - a; iii - d

Key 4

Sol: $(h,k) = (3,2)$



$a = 5$, $b = 4$

Equation of major axis $y = 2$

$e = \sqrt{1 - \frac{16}{25}} = 3/5$

$\frac{a}{e} = 5 \times \frac{5}{3} = \frac{25}{3}$

Equation of directrix $x = h \pm \frac{a}{e}$

$x = 3 + \frac{25}{3}$ | $x = 3 - \frac{25}{3}$

$3x = 9 + 25$

$3x - 34 = 0$

Equation of L.R. $x = h \pm ae$

$x = 3 \pm 5\left(\frac{3}{5}\right)$ $x = 6, 0$

$x = 6$

57. If S and S' are the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and if PSP' is a focal chord with SP = 8 then SS' =

1) $4 + S'P$ 2) $S'P - 1$ 3) $4 + SP$ 4) $SP - 1$

Key: 1

Sol:. $\frac{1}{sp} + \frac{1}{sp^1} = \frac{2}{l}$

$\frac{1}{SP} + \frac{1}{SP^1} = 2\left(\frac{5}{16}\right)$

$\frac{1}{sp} + \frac{1}{sp^1} = \frac{5}{8}$

$\frac{1}{8} + \frac{1}{sp^1} = \frac{5}{8}$

$\frac{1}{sp^1} = \frac{1}{2}$ $sp = 8$

$sp^1 = 2$

$SP + S'P = 2a$

$8 + S'P = 10$

$\boxed{S^1P = 2}$

58. Let $A(2\sec\theta, 3\tan\theta)$ and $B(2\sec\phi, 3\tan\phi)$, where $\theta + \phi = \frac{\pi}{2}$, be two poins on the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$. If (α, β) is the point of intersection of normals to the hyperbola at A and B, then β =

1) $\frac{-13}{3}$ 2) $\frac{13}{3}$ c) $\frac{3}{13}$ 4) $\frac{-3}{13}$

Key: 1

Sol: $\frac{2x}{\sec\theta}+\frac{3y}{\tan\theta}=13$

$\frac{2x}{\text{cosec}\theta}+\frac{3y}{\cot\theta}=13$

$2x\cos\theta+3y\frac{\cos\theta}{\sin\theta}=13$

$2x\sin\theta+3y\frac{\sin\theta}{\cos\theta}=13$

$2x\cos\theta\sin\theta+3y\cos\theta=13\sin\theta$

$2x\sin\theta\cos\theta+3y\sin\theta=13\cos\theta$

$3y(\cos\theta-\sin\theta)=13(\sin\theta-\cos\theta)$

3y=-13

$y=-\frac{13}{3}$

59. Points $A(3,2,4)$**,** $B\left(\frac{33}{5},\frac{28}{5},\frac{38}{5}\right)$**, and** $C(9,8,10)$ **are given. The ratio in which B divides** $\overline{\mathbf{AC}}$ **is**

1) 5 : 3 2) 2 : 1 3) 1 : 3 4) 3 : 2

Key: 4

Sol: $B=\left(\frac{33}{5},\frac{28}{5},\frac{38}{5}\right)=(\alpha,\beta,\gamma)$

$A=(3,2,4)=(x_1,y_1,z_1)$

$C=(9,8,10)=(x_2,y_2,z_2)$

$\text{Ratio}=(x_1-\alpha);(\alpha-x_2)$

$=\left(3-\frac{33}{5}\right);\left(\frac{33}{5}-9\right)$

$=\frac{15-33}{5};\frac{33-45}{5}$

$=\frac{-18}{5};\frac{12}{5}$

$=3:2$

60. If the angle between the lines whose direction cosines are $\left(-\frac{2}{\sqrt{21}},\frac{C}{\sqrt{21}},\frac{1}{\sqrt{21}}\right)$ **and** $\left(\frac{3}{\sqrt{54}},\frac{3}{\sqrt{54}},\frac{6}{\sqrt{54}}\right)$ **is** $\frac{\pi}{2}$**, then the value of C is**

1) 6 2) 4 3) -4 4) 2

Key: 2

Sol: $\left(\frac{-2}{\sqrt{21}}\right)\left(\frac{3}{\sqrt{54}}\right)+\left(\frac{1}{\sqrt{21}}\right)\left(\frac{3}{\sqrt{54}}\right)+\left(\frac{1}{\sqrt{21}}\right)\left(\frac{-6}{\sqrt{54}}\right)=0$

$$\frac{-6+3c-6}{\sqrt{1134}}=0$$

$$3c=12$$

$$c=2\cdot$$

61. The image of the points (5, 2, 6) with respect to the plane x + y + z = 9 is

1) $(3,-5,2)$ 2) $\left(\frac{7}{2},-1,5\right)$ 3) $\left(\frac{7}{3},-\frac{2}{3},\frac{10}{3}\right)$ 4) $\left(\frac{7}{3},-\frac{2}{3},-\frac{5}{3}\right)$

Key: 3

Sol: $\frac{h-5}{1}=\frac{k-2}{1}=\frac{t-6}{1}=\frac{-2(5+2+6-9)}{1+1+1}$

$$\frac{h-5}{1}=\frac{k-2}{1}=\frac{t-6}{1}=\frac{-8}{3}$$

$$h-5=-\frac{8}{3}$$

$$h=5-\frac{8}{3}$$

$$=\frac{7}{3}$$

$$\frac{k-2}{1}=\frac{-8}{3}$$

$$k=2-\frac{8}{3}$$

$$=\frac{-2}{3}$$

$$\frac{t-6}{1}=\frac{-8}{3}$$

$$t=6-\frac{8}{3}=\frac{10}{3}$$

$$\left(\frac{7}{3},\frac{-2}{3},\frac{10}{3}\right)$$

62. $\lim_{x\to\infty}\left[\frac{x^2+x+3}{x^2-x+2}\right]^x$ =

1) ∞ 2) e 3) e^4 4) e^2

Key: 4

Sol: $\underset{x\to\infty}{Lt}\left[\frac{x^2+x+3}{x^2-x+2}\right]^x=e^{1+1}=e^2$

63. The values of p and q so that the function

$$f(x)=\begin{cases}(1+|sixx|)^{\frac{p}{\sin x}}, & \frac{-\pi}{6}<x<0\\ q & , x=0\\ e^{\frac{\sin 2x}{\sin 3x}} & , 0<x<\frac{\pi}{6}\end{cases}$$

is continuous at x = 0 is

1) $p=\frac{1}{3},\ q=e^{2/3}$ 2) $p=0, q=e^{2/3}$ 3) $p=\frac{2}{3}, q=e^{-2/3}$ 4) $p=-\frac{2}{3}, q=e^{2/3}$

Key: 4

Sol: $e^{2/3}=q$ .... (1)

$e^{-p}=q=e^{2/3}$

$\Rightarrow p=-2/3$

$p=-2/3,\ q=e^{2/3}$.

64. **If** $y=\tan^{-1}\left[\frac{5\cos x-12\sin x}{12\cos x+5\sin x}\right]$**, then** $\frac{dy}{dx}=$

1) 1 2) -1 3) -2 4) $\frac{1}{2}$

Key: 2

Sol: $y=\tan^{-1}\left[\frac{5\cos x-12\sin x}{12\cos x+5\sin x}\right]$

$$=\tan^{-1}\left[\frac{\frac{5}{12}-\tan x}{1+\frac{5}{12}\tan x}\right]$$

$$=\tan^{-1}\left[\frac{\tan\alpha-\tan x}{1+\frac{5}{12}\tan x}\right]$$

= $=\tan^{-1}\left[\tan(\alpha-x)\right]$

$=\alpha-x$

$\frac{dy}{dx}=-1$

65. $\frac{d}{dx}\tan^{-1}\left[\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{\sqrt{1+\sin x}+\sqrt{1-\sin x}}\right]=$

B1) 1 2) $-\frac{1}{2}$ 3) $\frac{1}{2}$ 4) -1

Key: 3

Sol: $\frac{d}{dx}\tan^{-1}\left[\frac{\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)-\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)}{\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)+\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right]$

$$\frac{d}{dx}\tan^{-1}\left[\frac{1-\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)}{1+\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right]$$

$$=\frac{d}{dx}\left[\tan^{-1}\left(\tan\frac{x}{2}\right)\right]$$

$$=\frac{d}{dx}\left(\frac{x}{2}\right)=\frac{1}{2}$$

66. **If** $y = a\cos 9\sin 2x) + bs9n(sn2x)$ **, then** $y'' + (2\tan 2x)y' =$

1) 0 2) $4(\cos^2 2x)y$ 3) $-4(\cos^2 2x)y$ 4) $-(\cos^2 2x)y$

Key: 3

Sol: $y = a\cos(\sin 2x) + b\sin(\sin 2x)$

$$y^1 = -a\sin(\sin 2x).\cos 2x.2 + b\cos(\sin 2x).\cos 2x.2$$

$$y^{11} = -2a\left[\cos(\sin 2x).\cos^2 2x.2 + \sin(\sin 2x).x - \sin(2x)\ \ 2\right]$$

$$+2b\left[-\sin(\sin 2x).\cos^2 2x.2 + \cos(\sin 2x)\,x - \cos 2x \times 2\right]$$

$$=(-4a)\cos(\sin 2x).\cos^2(2x) + (-4a)\left[\sin(\sin 2x).\sin 2x\right]$$

$$-(4b)\sin(\sin 2x).\cos^2 2x - 4b\cos(\sin 2x)\cos(2x) \quad ..... (1)$$

$$y^1(2\tan 2x) = \frac{-4a\sin(\sin 2x).\sin 2x}{+4b\cos(\sin 2x).\cos 2x} \quad .... (2)$$

$$y^{11} + y^1(2\tan 2x) = -4\cos^2(2x)\left[a\cos(\sin 2x) + b\sin(\sin 2x)\right]$$

$$= -4y\left(\cos^2(2x)\right)$$

67. **The length of the segment of the tangent line to the curve** $x = a\cos^3 t$ **,** $y = a\sin^3 t$ **at any point on the curve cut off by the coordinate axes is**

1) $4a$ 2) a 3) a^2 4) $2a$

Key: 2

Sol: $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{a}\right)^{2/3} = 0$

$$\Rightarrow x^{2/3} + y^{2/3} = a^{2/3}$$

$$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}.\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-x^{-1/3}}{y^{-1/3}} = -\left(\frac{y}{x}\right)^{1/3}$$

$m = -\left(\frac{a\sin^3 t}{a\cos^3 t}\right)^{1/3}$

$m = \frac{-\sin t}{\cos t}$

Eqn. of tangent at p is

$y - a\sin^3 t = -\frac{\sin t}{\cos t}\left(x - a\cos^3 t\right)$

$\Rightarrow y\cos t - a\sin^3 t\cos t = -x\sin t + a\sin t\cos^3 t$

$\Rightarrow x\sin t + y\cos t = a\sin t\cos t\,(1)$

$\Rightarrow \frac{x}{a\cos t} + \frac{y}{a\sin t} = 1$

Length of inttercept

$= \sqrt{a^2\cos^2 t + a^2\sin^2 t}$

$= a$

68. The area of the triangle formed by the positive x-axis, the tangent and normal to the curve $x^2 + y^2 = 16a^2$ at the point $(2\sqrt{2}a, 2\sqrt{2}a)$ is

1) a^2 2) $16a^2$ 3) $4a^2$ 4) $8a^2$

Key: 4

Sol: $2x + 2y.\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{x}{y}$

$\Rightarrow m = \frac{-2\sqrt{2}a}{2\sqrt{2}a} = -1$

Area = $\frac{\frac{1}{2}\left|y_1^2\right|.\left(1+m^2\right)}{|m|}$

$= \frac{\frac{1}{2}\times 8a^2 \times 2}{1}$

$= 8a^2$

69. Define $f(x) = \frac{1}{2}[|\sin x| + \sin x],\ 0 < x \le 2\pi$. Then, f is

1) increasing in $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

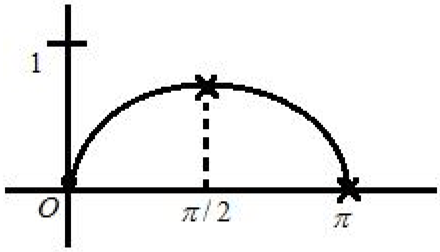
2) decreasing in $\left(0, \frac{\pi}{2}\right)$ and increasing in $\left(\frac{\pi}{2}, \pi\right)$

3) increasing in $\left(0, \frac{\pi}{2}\right)$ and decreasing in $\left(\frac{\pi}{2}, \pi\right)$

4) increasing in $\left(0, \frac{\pi}{4}\right)$ and decreasing in $\left(\frac{\pi}{4}, \pi\right)$

Key: 3

Sol: $f(x)=\frac{1}{2}\left[|\sin x|+\sin x\right],\ 0<x\le 2\pi$



$$f(x)=\begin{cases}\frac{1}{2}[\sin x+\sin x],0<x\le\pi\\ \frac{1}{2}[-\sin x+\sin x],\pi<x\le 2\pi\end{cases}$$

$$f(x)=\begin{cases}\sin x,0<x\le\pi\\ 0,\pi<x\le 2\pi\end{cases}$$

increasing in $\left(0,\frac{\pi}{2}\right)$

and decreasing in $(\pi/2,\pi)$

70. The smllest value of the constant $m>0$ for which $f(x)=9mx-1+\frac{1}{x}\ge 0$ for all x > 0 is

1) $\frac{1}{9}$ 2) $\frac{1}{16}$ 3) $\frac{1}{36}$ 4) $\frac{1}{81}$

Key: 3

Sol: $f(x)=9mx-1+\frac{1}{x}$

$$\frac{9mx+\frac{1}{x}}{2}=\sqrt{9m}$$

$$9mx+\frac{1}{x}\ge\sqrt[6]{m}$$

$$9mx+\frac{1}{x}-1\ge\sqrt[6]{m}-1\ge 0$$

$$m=\frac{1}{36}$$

71. $\int\frac{(x^2+1)}{x^4+7x^2+1}dx=$

1) $\frac{1}{3}\tan^{-1}\left(\frac{x^2-1}{3x}\right)+c$ 2) $\tan^{-1}\left(\frac{x^2-1}{x}\right)+c$ 3) $\frac{1}{3}\tan^{-1}\left(\frac{x^2-1}{x}\right)+c$ 4) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{3}x}\right)+c$

Key: 3

Sol: $\int \frac{2^2+1}{x^4+7x^2+1}$

$$\int \frac{x^2(1+\frac{1}{x^2})}{x^2\left[\left(x^2+\frac{1}{x^2}\right)+7\right]}dx$$

$$\int \frac{\left(1+\frac{1}{x^2}\right)}{\left(x-1/x\right)^2+9}dx$$

$$\left(\frac{1}{3}\right)Tan^{-1}\left(x-1/x\right)+c$$

$$\frac{1}{3}\tan^{-1}\left(\frac{x^2-1}{x}\right)+c$$

72. $\int \frac{x^3}{\sqrt{1+x^2}}dx =$

1) $\sqrt{1+x^2}-\frac{x}{3}(1+x^2)^{3/2}+c$

2) $x\sqrt{1+x^2}+\frac{2}{3}(1+x^2)^{3/2}+c$

3) $x^2\sqrt{1+x^2}-\frac{2}{3}(1+x^2)^{3/2}+c$

4) $x^2\sqrt{1+x^2}-\frac{1}{3}(1+x^2)^{1/2}+c$

Key: 3

Sol: $1+x^2=t$

$$\int \frac{x^2}{2}.\frac{2x}{\sqrt{1+x^2}}dx$$

$$= \frac{x^2}{2}.2\sqrt{1+x^2}-\int 2\sqrt{1+x^2}.x\,dx$$

$$= x^2\sqrt{1+x^2}-\int 2x\sqrt{1+x^2}dx$$

$$= x^2\sqrt{1+x^2}-\int t^{\frac{1}{2}}dt$$

$$= x^2\sqrt{1+x^2}-\frac{2}{3}t^{3/2}$$

$$= x^2\sqrt{1+x^2}-\frac{2}{3}\left(1+x^2\right)^{3/2}+c$$

73. $\int \frac{dx}{\cos(x+4)\cos(x+2)} =$

1) $\frac{1}{\sin 2}\log|\cos(x+4)^2|+c$

2) $\frac{1}{2}\log\left|\frac{\sec(x+2)}{\sec(x+4)}\right|+c$

3) $\frac{1}{\sin 2}\log\left|\frac{\sec(x+4)}{\sec(x+2)}\right|+c$

4) $\log\left|\frac{\sec(x+4)}{\sec(x+2)}\right|+c$

Key: 3

Sol: $\frac{1}{Sin2}\int \frac{Sin\left[(x+4)-(x+2)\right]}{Cos(x+4)Cos(x+2)}dx$

$\frac{1}{Sin2}\left[\int \tan(x+4)dx-\int \tan(x+2)dx\right]$

$\frac{1}{Sin2}\log\left|\frac{Sec(x+4)}{Sec(x+2)}\right|+c$

74. $\int \frac{2x+2}{\sqrt{x^2-4x-5}}dx =$

1) $\sqrt{x^2-4x-5}+\log\left|x+\sqrt{x^2-4x-5}\right|+c$

2) $\log\left|\sqrt{x^2-4x-5}\right|+\sqrt{x^2-4x-5}+c$

3) $\sqrt{x^2-4x-5}+6\log\left|(x-2)+\sqrt{x^2-4x-5}\right|+c$

4) $2\sqrt{x^2-4x-5}+6\log\left|(x+2)+\sqrt{x^2-4x-5}\right|+c$

Key: 4

Sol : $\int \frac{2x+2}{\sqrt{x^2-4x-5}}dx$

$\int \frac{2x-4+6}{\sqrt{x^2-4x-5}}dx$

$=2\sqrt{x^2-4x-5}+6\int \frac{1}{\sqrt{(x-2)^2-5-4}}dx$

$=2\sqrt{x^2-4x-5}+6\int \frac{1}{\sqrt{(x-2)^2-3^2}}dx$

$=2\sqrt{x^2-4x-5}+6\left|\log(x-2)+\sqrt{x^2-4x-5}\right|+c$

75. $\int_0^{\pi/4} \frac{\sin x+\cos x}{7+9\sin 2x}dx =$

1) $\frac{\log 3}{4}$ 2) $\frac{\log 3}{36}$ 3) $\frac{\log 7}{12}$ 4) $\frac{\log 7}{24}$

Key: 4

Sol: $\int_0^{\pi/4} \frac{\sin x + \cos x}{7 + 9\sin 2x} du$

$= \int_1^0 \frac{dt}{7 + 9\left(1 + t^2\right)}$

$= \int_1^0 \frac{1}{16 - 9t^2} - dt$

$= -\left(\frac{1}{9}\right) \int_1^0 \frac{1}{\left(\frac{16}{9}\right) - t^2} dt$

$= \left(\frac{1}{9}\right) = \int_1^0 \frac{1}{\left(\frac{4}{3}\right)^2 - t^2} dt$

$= -\frac{1}{9}\left[\frac{1}{2\times\left(\frac{4}{3}\right)} \log\left|\frac{\frac{4}{3}+t}{\frac{4}{3}-t}\right|\right]_1^0$

$= -\frac{1}{9\times\frac{8}{3}}\left[\log|1| - \log\left|\frac{\frac{7}{3}}{\frac{1}{3}}\right|\right]$

$= \left(\frac{1}{24}\right)[\log 7]$

76. $\int_0^{\pi/4} \left[\sqrt{\tan x} + \sqrt{\cot x}\right] dx =$

1) $\frac{\pi}{\sqrt{2}}$ 2) $\frac{\pi}{2}$ 3) $\frac{3\pi}{\sqrt{2}}$ 4) π

Key: 1

Sol: $\int_0^{\pi/4} \left[\sqrt{\tan x} + \sqrt{\cot x}\right] dx$

$= \int_0^{\pi/4} \left[\frac{\sin x + \cos x}{\sqrt{\sin x \cos x}}\right] dx$

$\sin x - \cos x = t$

$1 - \sin x \cos n = t^2$

$2\sin x \cos n = 1 - t^2$

$\sin x \cos x = \frac{1-t^2}{2}$

$= \sqrt{2}\int_{-1}^{0} \frac{dt}{\sqrt{1-t^2}}$

$= \sqrt{2}\left(\sin^1 t\right)_{-1}^{0}$

$= \sqrt{2}\left[0 + \frac{\pi}{2}\right]$

$= \frac{\pi}{\sqrt{2}}$

77. If the area bounded by the curves $y = ax^2$ and $x = ay^2$, $(a > 0)$ is 3 sq. units, then the value of a is

1) $\frac{2}{3}$ 2) $\frac{1}{3}$ 3) 1 4) 4

Key: 2

Sol: $x^2 = \frac{1}{a}y$ $y^2 = \frac{1}{a}x$

$x^2 = 4BX$ $y^2 = 4AX$

$4B = \frac{1}{a}$ $4A = \frac{1}{a}$

$B = \frac{1}{4a}$ $A = \frac{1}{4a}$

$Area = \frac{16AB}{3} = \frac{16}{3}\left(\frac{1}{4a}\right)\left(\frac{1}{4a}\right)$

$= \frac{1}{3a^2}$

$Area = 3$

$\frac{1}{3a^2} = 3$

$a^2 = \frac{1}{9} \Rightarrow a = \frac{1}{3}$

78. Let $p \in IR$, then the differential equation of the faily of curves $y = (\alpha + \beta x)e^{px}$, where α, β are arbitrary constants, is

1) $y'' + 4py' + p^2 y = 0$ 2) $y'' - 2py' + p^2 y = 0$ 3) $y'' + 2py' - p^2 y = 0$ 4) $y'' + 2py' + p^2 y = 0$

Key: 2

Sol: $y=(\alpha+\beta x)e^{px}$ ....(1)

$y^1=(\alpha+\beta x)e^{px}.p+e^{px}\beta$

$y^1=py+\beta.e^{px}$ ....(2)

$y^1-py=\beta e^{px}$ ....(3)

$y^{11}-py^1=\beta.pe^{px}$

$=p\left(y^1-py\right)$

$y^{11}-py^1=py^1-p^2y$

$y^{11}-2py^1+p^2y=0$

79. The solution of the differential equations $3xy'-3y+(x^2-y^2)^{1/2}=0,$ **satisfying the condition** $y(1)=1$ **is**

1) $3\cos^{-1}\left(\frac{y}{x}\right)=\ln|x|$ 2) $3\cos\left(\frac{y}{x}\right)=\ln|x|$ 3) $3\cos^{-1}\left(\frac{y}{x}\right)=2\ln|x|$ 4) $3\sin^{-1}\left(\frac{y}{x}\right)=\ln|x|$

Key: 2

Sol: $3x\frac{dy}{dx}-3y+\sqrt{x^2-y^2}=0$

$3x\frac{dy}{dx}-3y-\sqrt{x^2-y^2}$

$\frac{dy}{dx}=\frac{3y-\sqrt{x^2-y^2}}{3x}$

Let y=vx$\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$

$v+x.\frac{dv}{dx}=\frac{3vx-\sqrt{x^2-v^2x^2}}{3x}$

$=\frac{3v-\sqrt{1-v^2}}{3}$

$x\frac{dv}{dx}=\frac{3v-\sqrt{1-x^2}}{3}-v$

$=\frac{3v-\sqrt{1-x^2}-3v}{3}$

$=-\sqrt{\frac{1-v^2}{3}}$

$\Rightarrow\frac{-3dv}{\sqrt{1-v^2}}=\frac{dx}{x}$

$\Rightarrow 3\cos^{-1}(v)=\log x+k$

$$\Rightarrow 3\cos^{-1}\left(\frac{y}{x}\right) = \log x + k$$

$$y(1) = 1, 3\cos^{-1}(1) = 0 + k$$

$$3\cos^{-1}\left(\frac{y}{x}\right) = \log(x)$$

80. The solution of the differential equation $y' = \frac{1}{e^{-y} - x}$, **is**

1) $x = e^{-y}(y + c)$ 2) $y + e^{-y} = x + c$ 3) $x = e^{y}(y + c)$ d) $x + y = e^{-y} + c$

Key: 1

Sol: $\frac{dy}{dx} = \frac{1}{e^{-y} - x} = \frac{e^{y}}{1 - xe^{y}}$

$$\frac{dx}{dy} + x = e^{-y}$$

If= $e^{\int 1.dy} = e^{y}$

Solution $x.e^{y} = \int e^{-y} e^{y} dy = y + c$

$$x = e^{-y}(y + c)$$

PHYSICS

81. Electron microscope is based on the principle

1) Photoelectric effect   2) Wave nature of electron

3) Superconductivity   4) Laws of electromagnetic induction

Key: 1

Sol: Conceptual.

82. Force is given by the expression, $F = A\cos(Bx) + C\cos(Dt)$ **where x is displacement and t is time. The dimension of** $\left(\frac{D}{B}\right)$ **is same as that of**

1) Velocity   2) Velocity gradient   3) Angular velocity   4) Angular momentum

Key: 1

Sol: Bx & Dt have no dimension

$\therefore Bx = m^0 L^0 T^0$ and $Dt = M^0 L^0 T^0$

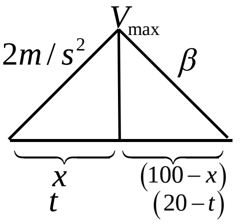
$$B = \frac{1}{x} = \frac{1}{L^1} = L^{-1} \text{ and } D = \frac{1}{t} = \frac{1}{T^1} = T^{-1}$$

$$\frac{D}{B} = \frac{T^{-1}}{L^{-1}} = L^1 T^{-1} \rightarrow velocity$$

83. A car accelerates from rest with $2 m/s^2$ **on a straight line path and then comes to rest after applying brakes. Total distance travelled by the car is 100 m in 20 seconds. Then the maximum velocity attained by teh car is**

1) 10 m/s   2) 20 m/s   3) 15 m/s   4) 5 m/s

Key: 1



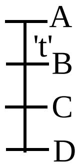
Sol: $V_{max} = \left(\frac{\alpha\beta}{\alpha+\beta}\right)t \rightarrow 1$

$S = \frac{1}{2}\left(\frac{\alpha\beta}{\alpha+\beta}\right)t^2 \rightarrow 2$

$$\frac{V_{max}}{s} = \frac{\left(\frac{\alpha\beta}{\alpha+\beta}\right)t}{\frac{1}{2}\left(\frac{\alpha\beta}{\alpha+\beta}\right)t^2}$$

$$\frac{V_{max}}{100} = \frac{2}{t} = \frac{200}{20} = 10m/s$$

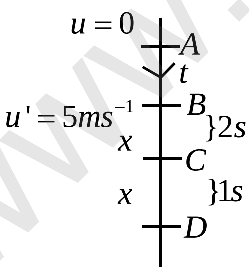
84. A body is falling freely from a point A at a certain height from the ground and passes through points B,C and D (vertically as shown below) so that BC = CD. The time taken by the particle to move from B to C is 2 seconds and from C to D 1 second. Time taken to move from A to B in seconds is



1) 0.6 2) 0.5 3) 0.2 4) 0.4

Key:2

Sol:



$BC = CD = x$

$x = (u' \times 2) + \frac{1}{2}g \times 2^2$

$x = 2u' + 2g \rightarrow 1$

$$2x = (u' \times 3) + \frac{1}{2} g \times 3^2$$

$$2x = 3u' + \frac{9g}{2} \rightarrow 2$$

$$2(2u' + 2g) = \frac{6u' + 9g}{2}$$

$$2(4u' + 4g) = 6u' + 9g$$

$$8u' + 8g = 6u' + 9g$$

$$8u' - 6u' = 9g - 8g$$

$$2u' = g$$

$$u' = \frac{10}{2} = 5m/s$$

$$u' - 0 = gt$$

$$u' = gt$$

$$t = 0.5\sec$$

85. A particle moves from (1,0,3) to the point (-3,4,5), when a force $\overline{F} = (\hat{i} + 5\hat{k})$ acts on it. amount of work done in Joules is

1) 14 2) 10 3) 6 4) 15

Key: 3

Sol: $w = \hat{F}.\hat{S}$ $\overline{S} = \overline{r_2} - \overline{r_1}$

$W = (\hat{i} + 5\hat{k}).(-4\hat{i} + 4\hat{j} + 2\hat{k})$ $= (-3\hat{i} + 4\hat{j} + 5\hat{k}) - (1\hat{i} + 3\hat{k})$

$= -4 + 10$ $= -4\hat{i} + 4\hat{j} + 2\hat{k}$

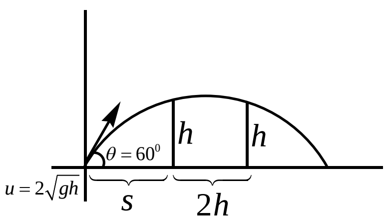
$= 6J$

86. A particle is projected with velocity $2\sqrt{gh}$ **and at an angle** 60^0 **to the horizontal so that it just clears two walls of equal height 'h'** which are at a distance 2h from each other. The time taken by the particle to travel between these two walls is

1) $2\sqrt{\frac{2h}{g}}$ 2) $\sqrt{\frac{h}{2g}}$ 3) $2\sqrt{\frac{h}{g}}$ 4) $\sqrt{\frac{h}{g}}$

Key: 3

Sol:



isothermal

$$S = u\cos\theta . t_1$$

$= 2\sqrt{gh}\cos 60 \times t_1$

$= \sqrt{gh}\ t_1 \rightarrow 1$

$S + 2h = 2\sqrt{gh}\cos 60 \times t_2$

$S + 2h = \sqrt{gh}\, t_2 \rightarrow 2$

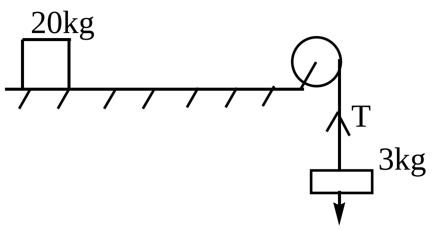
$\sqrt{gh}t_1 + 2h = \sqrt{gh}\, t_2$

$2h = \sqrt{gh}\left[t_2 - t_1\right]$

$$\left(t_2 - t_1\right) = \frac{2h}{\sqrt{gh}} = 2\sqrt{\frac{h^2}{gh}}$$

$$\Delta T = \left(t_2 - t_1\right) = 2\sqrt{\frac{h}{g}}$$

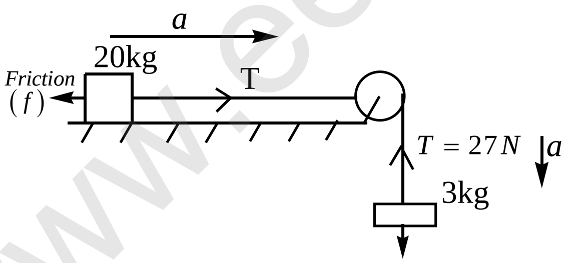
87. A body of mass 20 kg is moving on a rough horizontal plane. A block of mass 3 kg is connected to the 20 kg mass by a string of negligible mass through a smooth pulley as shown in the figure. The tension in the string is 27 N. The coefficient of kinetic friction between the heavier mass and the surface is $\left(g = 10 m/s^2\right)$.



1) 0.025 2) 0.035 3) 0.35 4) 0.25

Key: 2

Sol:



For 3*kg block*

$3a = 3g - T$

$3a = 30 - 27$

$a = \frac{3}{3} = 1 m/s^2$

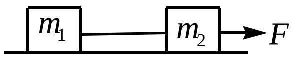
for 20*kg block*

$T - f = 20a \Rightarrow 27 - \mu \times 20 \times 10 = 20 \times 1$

$$27-20=200\mu$$

$$\mu=\frac{7}{200}=0.035$$

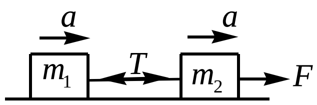
88. Two masses m_1 and m_2 are placed on a smooth horizontal surface and are connected by a string of negligible mass. A horizontal force F is applied on the mass m_2 as shown in the figure. The tension in the string is



1) $\left(\frac{m_1}{m_1+m_2}\right)F$ 2) $\frac{m_2F}{m_1+m_2}$ 3) $\left(\frac{m_1}{m_2}\right)F$ 4) $\frac{m_2F}{m_1}$

Key: 1

Sol:



$$a=\frac{F}{m_1+m_2}$$

Tension is the string $T=m_1a$

$$=\frac{m_1F}{m_1+m_2}$$

89. A body of mass 3 kg moving with a velocity $\left(2\hat{i}+3\hat{j}+3\hat{k}\right)m/s$ collides with another body of mass 4kg moving with a velocity $\left(3\hat{i}+2\hat{j}-3\hat{k}\right)m/s.$ The two bodies stick together after collision. The velocity of the composite body is

1) $\frac{1}{7}\left(4\hat{i}+6\hat{j}-3\hat{k}\right)$ 2) $\frac{1}{7}\left(18\hat{i}+17\hat{j}-3\hat{k}\right)$ 3) $\frac{1}{7}\left(6\hat{i}+4\hat{j}-6\hat{k}\right)$ 4) $\frac{1}{7}\left(9\hat{i}+8\hat{j}-6\hat{k}\right)$

Key: 2

Sol: $m_1\overline{u_1}+m_2\overline{u_2}=\left(m_1+m_2\right)\bar{v}$

$$6\hat{i}+9\hat{j}+9\hat{k}+12\hat{i}+8\hat{j}-12\hat{k}=7\bar{v}$$

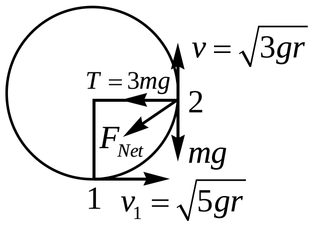
$$\bar{v}=\frac{18\hat{i}+17\hat{j}-3\hat{k}}{7}$$

90. A simple pendulum of length L carries a bob of mass m. When the bob is at its lowest position, it is given the minimum horizontal speed necessary for it to move in a vertical circle about the point of suspension. When the string is horizontal the net force on the bob is

1) $\sqrt{10}mg$ 2) $\sqrt{5}mg$ 3) 4 mg 4) 1 mg

Key: 1

Sol:



$F_{Net} = \sqrt{(3mg)^2 + (mg)^2}$

$= \sqrt{10(mg)^2}$

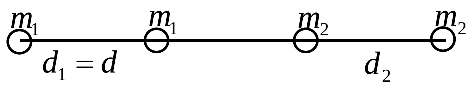
$= \sqrt{10}mg$

91. **A system of two particles is having masses m_1 and m_2. If the particle of mass m_1 is pushed towards the center of particles through a distance d, by what distance the particle of mass m_2 should be moved so as to keep the centre of mass of particles at the orignal position?**

1) $\frac{m_1}{m_1+m_2}d$ 2) d 3) $\frac{m_1}{m_2}d$ 4) $\frac{m_2}{m_1}d$

Key: 3

Sol:



$m_1 d_1 = m_2 d_2$

$d_2 = \frac{m_1}{m_2}.d_1 = \frac{m_1}{m_2}d$

92. **A thin uniform circular disc of mass M and radius R is rotating in a horizontal plane about an axis passing through its centre and perpendicular to its plane with an angular velocity ω. Another disc of same thickness and radius but of mass $\frac{1}{8}M$ is placed gently on the first disc co-axially. The angular velocity of the system is now**

1) $\frac{8}{9}\omega$ 2) $\frac{5}{9}\omega$ 3) $\frac{1}{3}\omega$ 4) $\frac{2}{9}\omega$

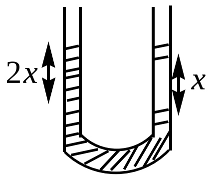
Key: 1

Sol: $I_1\omega_1 = I_2\omega_2$

$\frac{MR^2}{2} \times \omega_1 = \left(M + \frac{M}{8}\right)\frac{R^2}{2} \times \omega_2$

$\omega_2 = \frac{8\omega}{9}$

93. 9 kg solution is poured into a glass U-tube as shown in the figure below. The tube's inner diameter is $2\sqrt{\frac{\pi}{5}}m$ and the solution oscillates freely up and down about its position of equilibrium $(x=0)$. The period of oscillation in seconds is ($1m^3$ of solution has a mass $\mu = 900kg, g = 10m/s^2$, ignore frictional and surface tension effects)



1) 0.1 2) 10 3) $\sqrt{\pi}$ 4) 1

Key: 1

Sol: Net pressure force on liquid $= \rho Ag(2x)$

Equation of SHM

$$ma = -2\rho Agx$$

$$a = -\frac{2\rho Ag}{m}x = -\omega^2 x$$

$$\omega = \sqrt{\frac{2\rho Ag}{m}} = \sqrt{\frac{2\rho \times \pi \times D^2 \times g}{4m}} \qquad D = diameter\ of\ tube$$

$$or\ \omega = \sqrt{\frac{2\times 900 \times \frac{\pi}{4} \times \frac{\pi}{5} \times 4 \times 10}{4\times 9}}$$

$$\omega = 20\pi\ rad/s$$

$$T = \frac{2\pi}{\omega} = 0.1s$$

94. The bodies of masses 100kg and 8100 kg are held at a distance of 1m. The gravitational field at a point on the line joining them is zero. The gravitational potential at that point in J/kg is $\left(G = 6.67\times 10^{-11} N.m^2/kg^2\right)$

1) -6.67×10^{-7} 2) -6.67×10^{-10} 3) -13.34×10^{-7} 4) -6.67×10^{-9}

Key: 1

Sol: $x = \frac{1}{\sqrt{81}+1} = 0.1m$

$$\therefore V = -\frac{GM_1}{r_1} - \frac{GM_2}{r_2}$$

$$= -6.67\times 10^{-11}\left[\frac{100}{0.1} + \frac{8100}{0.9}\right]$$

$$= -6.67\times 10^{-8}[1+9] \qquad = -6.67\times 10^{-7}.$$

95. An elastic spring of unstretched length L and force constant K is stretched by a small length x. It is further stretched by another small length y. Work done during the second stretching is

1) $\frac{ky}{2}(x+2y)$ 2) $\frac{k}{2}(2x+y)$ 3) $ky(x+2y)$ 4) $\frac{ky}{2}(2x+y)$

Key: 4

Sol: Work done during second stretching

= Change in elastic PE

$$=\frac{1}{2}k(x+y)^2-\frac{1}{2}kx^2$$

$$=\frac{1}{2}k(2x+y)y$$

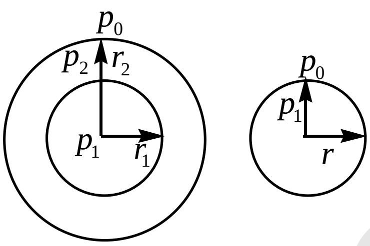
$$=\frac{k}{2}y(y+2x)$$

96. A soap bubble of radius 1.0 cm is formed inside another soap bubble of radius 2.0 cm. The radius of an another soap bubble which has the same pressure difference as that between the inside of the smaller and outside of large soap bubble, in meters is

1) 6.67×10^{-3} 2) 3.34×10^{-3} 3) 2.23×10^{-3} 4) 4.5×10^{-3}

Key: 1

Sol:



$$P_1-P_2=\frac{4T}{r_1}----(1)$$

$$P_2-P_0=\frac{4T}{r_2}----(2)$$

$$P_1-P_0=\frac{4T}{r}----(3)$$

$$(1)+(2)\Rightarrow P_1-P_0=\frac{4T}{r_1}+\frac{4T}{r_2}----(4)$$

From (3) and (4) $\frac{1}{r_1}+\frac{1}{r_2}=\frac{1}{r}$

$$\Rightarrow r=\frac{r_1r_2}{r_1+r_2}=\frac{2}{3}=0.67cm$$

or $r=6.67\times10^{-3}m$

97. A slab of stone area 3600 cm^2 and thickness 10cm is exposed on the lower surface to steam at $100^0 C$. A block of ice at $0^0 C$ rests on upper surface of the slab. In one hour 4.8kgof ice is melted. The thermal conductivity of stone in $Js^{-1}m^{-1}k^{-1}$ is (Latent heat of ice $= 3.36 \times 10^5\, J/kg$)

1) 12.0 2) 10.5 3) 1.02 4) 1.24

Key: 4

Sol: $\frac{d\theta}{dt} = \frac{KA(\theta_1 - \theta_2)}{L} = \left(\frac{dm}{dt}\right)L$

$$\frac{k \times 3600 \times 10^{-4} \times (100-0)}{0.1} = \left(\frac{4.8}{3600}\right) \times 3.36 \times 10^5$$

$\Rightarrow k = 1.24\, JS^{-1}m^{-1}k^{-1}$

98. The surface of a black body is at a temperature $727^0 C$ and its cross section is $1m^2$. Heat radiated from this surface in one minute in Joules is (Stefan's constant $= 5.7 \times 10^{-8} W/m^2/k^4$)

1) 34.2×10^5 2) 2.5×10^5 3) 3.42×10^5 4) 2.5×10^6

Key: 1

Sol: $\rho = e\sigma AT^4$

$e = 1$ for black body

$\rho = 5.7 \times 10^{-8} \times 1 \times (727 \times 273)^4\, w$

$\therefore$ Heat radiated in one minute

$= 60 \times \rho = 5.7 \times 10^4 \times 60$

$= 342 \times 10^4\, Joules$

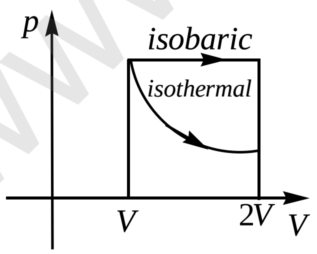
or $heat = 34.2 \times 10^5\, J$

99. Two moles of a gas is expanded to double its volume by two different processes. One is isobaric and the other is isothermal. If w_1 and w_2 are the works done respectively, then

1) $w_2 = \frac{w_1}{\ln 2}$ 2) $w_2 = w_1$ 3) $w_2 = w_1 \ln 2$ 4) $w_1^2 = w_2 \ln 2$

Key: 3

Sol:



$W_1 = \rho \Delta V$

$= \rho V$

$$W_2 = nRT \ln\left(\frac{2v}{v}\right)$$

$$W_2 = \rho V \ln 2$$

$$\therefore W_2 = W_1 \ln 2$$

100. Uranium has two isotopes of masses 235 and 238 units. If both of them are present in Uranium hexafluoride gas, find the percentage ratio of difference in rms velocities of two isotopes to the rms velocity of heavier isotope.

1) 1.64 2) 0.064 3) 0.64 4) 6.4

Key: 3

Sol: $V_{rms} = \sqrt{\frac{3RT}{m}}$

$$\Rightarrow V_{rms} \alpha \frac{1}{\sqrt{m}}$$

$$\% = \frac{V_{rms1} - V_{rms2}}{V_{rms}} \times 100$$

$$\% = \frac{\frac{k}{\sqrt{m_2}} - \frac{k}{\sqrt{m_2}}}{\frac{k}{\sqrt{m_1}}} \times 100$$

$$= \frac{\sqrt{m_2} - \sqrt{m_1}}{\sqrt{m_1}} \times 100$$

$$= \frac{\sqrt{238} - \sqrt{235}}{\sqrt{235}} \times 100$$

$$= \left(\sqrt{\frac{235+3}{235}} - 1\right) \times 100$$

$$= \left(\left[1 + \frac{1}{2} \times \frac{3}{235}\right] - 1\right) \times 100$$

$$= \frac{3}{470} \times 100 = 0.64$$

101. A source of frequency 340 Hz is kept above a vertical cylindrical tube closed at lower end. The length of the tube is 120 cm. Water is sowly poured in just enough to produce resonance. Then the minimum height (velocity of sound = 340 m/s) of the water level in the tube for that resonce is

1) 0.75 m 2) 0.25 m 3) 0.95 m 4) 0.45

Key: 4

Sol: n= 340 Hz

First resonance = $\frac{V}{4n} = 25cm$

Second resonance = $\frac{3V}{4n} = 75cm$

Third resonance = $\frac{5V}{4n} = 125cm$

$\therefore$ **The minimum height = (120-75) cm = 45 cm**

= 0.45 m

102. A thin convex lens of focal length 'f' made of crown glass is immersed in a liquid of refractive index μ_l . ($\mu_l > \mu_c$) where μ_c is the refractive index of the crown glass. Then convex lens now is

1) A convex lens of longer focal length
2) A convex lens of shorter focal length
3) A divergent lens
4) A convex lens of focal lenght $\left(\mu_c - \mu_l\right)$f

Key: 3

Sol: $\frac{1}{f} = \left(\mu_c - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Let $\frac{1}{R_1} - \frac{1}{R_2} = k$

$\Rightarrow \frac{1}{f} = \left(\mu_c - 1\right)k$

In liquid, $\frac{1}{f^1} = \left(\frac{\mu_c}{\mu_l} - 1\right)k$

$\mu_l < \mu_c$ given

$\Rightarrow f^1 < 0 \Rightarrow$ divergent lens

103. Two convex lenses of focal lengths f_1 and f_2 form images with magnification m_1 and m_2, when used individually for an object kept at the same distance from the lenses. Then f_1 / f_2 is

1) $\frac{m_1\left(1+m_1\right)}{m_2\left(1+m_2\right)}$
2) $\frac{m_1\left(1+m_1\right)}{m_2\left(1+m_1\right)}$
3) $\frac{m_2\left(1+m_1\right)}{m_1\left(1+m_2\right)}$
4) $\frac{m_2\left(1+m_2\right)}{m_1\left(1+m_1\right)}$

Key: 2

Sol: $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} = \frac{1}{f} + \frac{1}{u}$

$v = \frac{fu}{u+f}$

Taking -ve sign for u

$-m = \frac{f}{u + f}$

$-m_1 = \frac{v}{u} = \frac{f}{u+f_1}$ -----(1)

$-m_2 = \frac{f_2}{u+f_2}$ ------(2)

From (1) $fu + f_1 = -\frac{f_1}{m_1}$

From (2) $fu + f_2 = -\frac{f_2}{m_2}$

$f_1 - f_2 = -\frac{f_1}{m_1} + \frac{f_2}{m_2}$

Dividing by f_2

$\frac{f_1}{f_2} - 1 = \left(\frac{f_1}{f_2}\right)\left(\frac{-1}{m_1}\right) + \frac{1}{m_2}$

$\left(\frac{f_1}{f_2}\right)\left(1+\frac{1}{m_1}\right) = \left(1+\frac{1}{m_2}\right)$

$\frac{f_1}{f_2} = \frac{m_1}{m_2}\frac{(m_2+1)}{(m_1+1)}$

104. With the help of a telescope that has an objective of diameter 200 cm, it is proved that light of wavelengths of the order of 6400 $\overset{o}{A}$ coming from a star can be easily resolved. Then the limit of resolution is

1) 39×10^{-8} deg 2) $39\times10^{-8} rad$ 3) $19.5\times10^{-8} rad$ 4) 19.5×10^{-8} deg

Key: 2

Sol: $\Delta\theta = \frac{1.22\lambda}{a}$

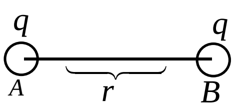
$\Delta\theta = \frac{1.22\times6400\times10^{-10}}{200\times10^{-2}}$

$\Delta\theta = 39\times10^{-8}\, rad$

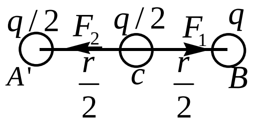
105. Two charged identical metal spheres A and B repel each other with a force of $3\times10^{-5} N$. Another identical uncharged sphere C is touched with sphere A and then it is placed midway between A and B. Then the magnitude of Net force on C is

1) $1\times10^{-5} N$ 2) $3\times10^{-5} N$ 3) $2\times10^{-5} N$ 4) $5\times10^{-5} N$

Key: 2

Sol: 

$F = 9\times10^9 \times \frac{q^2}{r^2} = 3\times10^{-5}$ -----(1)



$$F_1 = \frac{1}{4\pi \in_0} \frac{q^2}{4 \times \frac{r^2}{4}}$$

$$= 9 \times 10^9 \frac{q^2}{r^2} = 3 \times 10^{-5} ------(2)$$

$$F_2 = 9 \times 10^9 \times \frac{q \times \frac{q}{r}}{\left(\frac{r^2}{4}\right)}$$

$$= q \times \frac{9 \times 10^9 q^2}{r^2}$$

$$= 2 \times 3 \times 10^{-5} N$$

$$(F_{Net})_c = F_2 - F_1 = 2 \times 3 \times 10^{-5} - 1 \times 3 \times 10^{-5}$$

$$= 3 \times 10^{-5} N$$

106. The electrostatic potential inside a charged sphere is given as $V = Ar^2 + B$, where r is the distance from the center of the sphere; A and B are constants. Then the charge density in the sphere is

1) $16 A \in_0$ 2) $-6 A \in_0$ 3) $20 A \in_0$ 4) $-15 A \in_0$

Key: 2

Sol: $V = Ar^2 + B$

$$E = \frac{-dv}{dr} = -2Ar$$

$$\int \overline{E}.\overline{dA} = \frac{q_{enc}}{\in_0}$$

$$\Rightarrow -2Ar 4\pi r^2 = \frac{\int \rho 4\pi r^2 dr}{\in_0}$$

as ρ is constant

$$-8A\pi r^3 = \frac{4\pi\rho}{\in_0} \frac{r^3}{3}$$

$$\Rightarrow \rho = -6A \in_0$$

107. Three unequal resistances are connected in parallel. Two of these resistance are in the ratio 1 : 2. The equivalent resistance value among these three connected in parallel is $1\,\Omega$. What is the highest resistance value among these resistances if no resistance is fractional?

1) $10\,\Omega$ 2) $8\,\Omega$ 3) $15\,\Omega$ 4) $6\,\Omega$

Key: 4

Sol: Let $R_1 = R \Rightarrow R_2 = 2R$ **& Let** R_3

$\therefore t = \frac{1}{R} + \frac{1}{2R} + \frac{1}{R_3}$

$\Rightarrow 1 = \frac{2R_3 + R_3 + 2R}{2RR_3}$

$2RR_3 = 3R_3 + 2R$

$\Rightarrow R_3(2R-3) = 2R$

$R_3 = \frac{2R}{2R-3}$

Now $R_1\, R_2 \,\&\, R_3$ **are +ve integers**

$\therefore$ **By trail & error**

Let R = 3 then $R_1 = 3\Omega$

$R_2 = 6\Omega \,\&\, R_3 = \frac{6}{3} = 2\Omega$

$\therefore$ highest = 6

108. Two electric resistors have eqwual values of resistance R. Each can be operated with a power of 320 watts (w) at 220 volts. If the two resistors are connected in series to a 110 volts electric supply, then the power generated in each resistor is

1) 90 watts 2) 80 watts 3) 60 watts 4) 20 watts

Key: 4

Sol: $R = \frac{V^2}{\rho} = \frac{220 \times 22}{320}$

In series $R = \frac{220 \times 11}{16}$

$\therefore \rho = \frac{V^2}{R} = \frac{110 \times 110}{220 \times 11} \times 16$

$= 40\,w$ **together**

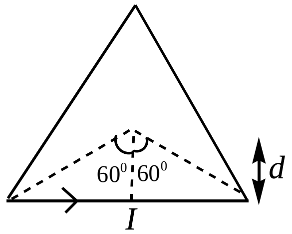
in each = 20 w

109. A current of 1 A is flowing along the sides of an equilateral triangle of side $4.5 \times 10^{-2}\, m$. **The magnetic field at the centroid of the triangle is** $\left(\mu_0 = 4\pi \times 10^{-7}\, H/m\right)$

1) $4 \times 10^{-5} T$ 2) $2 \times 10^{-5} T$ 3) $4 \times 10^{-4} T$ 4) $2 \times 10^{-4} T$

Key: 1

Sol:



$d = l\sin 60^0 \times \frac{1}{3} = \frac{l}{2\sqrt{3}}$

$B = 3\times \frac{\mu_0 I}{4\pi d}\left(\sin 60 + \sin 60\right)$

$= 3\times 10^{-7} \times \frac{1\times 2\sqrt{3}}{4.5\times 10^{-2}}\left(\sqrt{3}\right) = 4\times 10^{-5} T$

110. A charged particle (charge = q; mass = m) is rotating in a circle of radius 'R' with uniform speed 'V'. Ratio of its magnetic moment (μ) to the angular momentum (L) is

1) $\frac{q}{2m}$ 2) $\frac{q}{m}$ 3) $\frac{q}{4m}$ 4) $\frac{2q}{m}$

Key: 1

Sol: $\frac{\mu}{L} = \frac{q}{2m}$

111. Two small magnets have their masses and lengths in the ratio 1 : 2. The maximum torques experienced by them in a uniform magnetic field are the same. For small oscillations, the ratio of their time period is

1) $\frac{1}{2\sqrt{2}}$ 2) $\frac{1}{\sqrt{2}}$ 3) $\left(\frac{1}{2}\right)$ 4) $2\sqrt{2}$

Key: 1

Sol: $T = 2\pi\sqrt{\frac{I}{\mu B}}$

$= 2\pi\sqrt{\frac{\frac{m\left(l^2 + b^2\right)}{12}}{\mu B}}$

$T \,\alpha\, \sqrt{m}\, l$ [$\therefore$ b is neglected]

$\frac{T_1}{T_2} = \sqrt{\frac{m_1}{m_2}} \times \frac{l_1}{l_2}$

$= \sqrt{\frac{1}{2}} \times \frac{1}{2}$

$\frac{T_1}{T_2} = \frac{1}{2\sqrt{2}}$

112. Two coils have mutual inductance 0.005 H. THe current changes in the first coil according to equation $I = I_0 \sin \omega t$, where $I_0 = 10\,A$ and $\omega = 100\pi\, rad\, s^{-1}$. The maximum value of emf in the second coil is

1) 5 2) 5π 3) 0.5π 4) π

Key: 2

Sol: $\varepsilon = M\dfrac{di}{dt}$ & $I = I_0 \sin \omega t$

$= -\omega M I_0 \cos \omega t$

$\therefore \varepsilon_{max} = \omega M I_0$

$= 100\pi \times 0.005 \times 10$

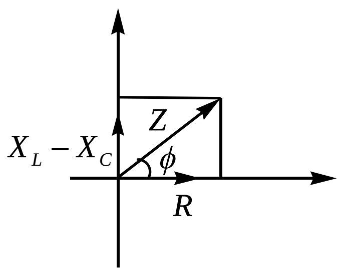
$= 5\pi$

113. A capacitance of $\left(\dfrac{10^{-3}}{2\pi}\right)F$ and an inductance of $\left(\dfrac{100}{\pi}\right)mH$ and a resistance of 10Ω are connected in series with an AC voltages source of 220 V, 50 Hz. The phase angle of the circuit is

1) 60^0 2) 30^0 3) 45^0 4) 90^0

Key: 3

Sol:



$\tan\phi = \dfrac{X_L - X_C}{R}$

$= \dfrac{\omega L - \dfrac{1}{\omega c}}{R}$

$= \dfrac{10^{-3} \times 2\pi \times 50 \times \dfrac{100}{\pi} - \dfrac{1}{2\pi \times 50 \times \dfrac{10^{-3}}{2\pi}}}{10}$

$= \dfrac{10-20}{10} = -1$

$\therefore$ **Current leads voltage by** 45^0**.**

114. Two equations are given below :

(A) $\oint \overline{E}.d\overline{A} = \dfrac{Q}{\epsilon_0}$ **;** **(B)** $\oint \overline{B}.d\overline{A} = 0$

They are

1) (A) - Ampere's law
(B) - Gauss law for electricity

2) (A) - Gauss law for electric fields
(B) - Gauss law for magnetic fields

3) (A) - Faraday law
(B) - Gauss law for electric fields
4) Both (A) and (B) represent Faraday law
Key: 2

Sol: $\oint \overline{E}.\overline{dA} = \frac{Q}{t_0} \rightarrow$ **Gauss law for electric field**

$\int \overline{B}.\overline{dA} = 0 \rightarrow$ **Gauss law for magnetic field**

115. A charged particle is accelerated from rest through a certain potential difference. The de Broglie wavelength is λ_1 when it is accelerated through V_1 and is λ_2 when accelerated through V_2. The ratio λ_1/λ_2 is

1) $V_1^{3/2} : V_2^{3/2}$ 2) $V_2^{1/2} : V_1^{1/2}$ 3) $V_1^{\frac{1}{2}} : V_2^{\frac{1}{2}}$ 4) $V_1^{2} : V_2^{2}$

Key: 2

Sol: $\lambda = \frac{h}{\sqrt{2mqv}} \Rightarrow \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{V_2}{V_1}}$

116. If the first line of Lymann series has a wavelength 1215.4 $\overset{o}{A}$, the first line of Balmer series is approximately

1) 4864 $\overset{o}{A}$ 2) 1025.5 $\overset{o}{A}$ 3) 6563 $\overset{o}{A}$ 4) 6400 $\overset{o}{A}$

Key: 3

Sol: $\frac{1}{\lambda_1} = Z^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$

$\frac{1}{\lambda_2} = Z^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$

$\therefore \frac{\lambda_2}{\lambda_1} = \frac{3}{4} \times \frac{36}{5} = \frac{27}{5}$

$\therefore \lambda_2 = \frac{27}{5} \times 1215.4 \overset{0}{A}$

$= 6563.16 \overset{0}{A}$

117. A certain radioactive element disintegrates with a decay constant of 7.9×10^{-10} / sec. At a given instant of time, if the activity of the sample is equal to 55.3×10^{11} disintegration/sec, then number of nuclei at that instant of time

1) 7.0×10^{21} 2) 4.27×10^{13} 3) 4.27×10^{3} 4) 6×10^{23}

Key: 1

Sol: $\frac{dN}{dt} = \lambda N$

$\therefore N = \frac{dN/dt}{\lambda} = \frac{55.3 \times 10^{11}}{7.9 \times 10^{-10}}$

$= 7 \times 10^{21}$

118. The change in current through a junction diode is 1.2 mA when the forward bias voltage is changed by 0.6 V. The dynamic resistance is

1) $500\,\Omega$ 2) $300\,\Omega$ 3) $150\,\Omega$ 4) $250\,\Omega$

Key: 1

Sol: $r = \frac{\Delta V}{\Delta I} = \frac{0.6}{1.2 \times 10^{-3}}$

$= \frac{6000}{12} = 500\,\Omega$

119. A semiconductor has equal electron and hole concentration of $2 \times 10^{8} m^{-3}$. On doping with a certain impurity, the electron concentration increases to $4 \times 10^{10} m^{-3}$, then the new hole concentration of the semiconductor is

1) $10^{6} m^{-3}$ 2) $10^{8} m^{-3}$ 3) $10^{10} m^{-3}$ 4) $10^{12} m^{-3}$

Key: 1

Sol: $np = n_i^2$

$\Rightarrow P = \frac{n_i^2}{n} = \frac{\left(2 \times 10^{8}\right)^2}{4 \times 10^{10}}$

$= 10^{6} m^{-3}$

120. A message signal of 12 kHz and peak voltage 20 V is used to modulate a carrier wave of frequency 12 MHz and peak voltage 30 V. Then the modulation index is

1) 0.32 2) 6.7 3) 0.67 4) 67

Key: 3

Sol: $\mu = \frac{A_m}{A_c} = \frac{20}{30} = \frac{2}{3} = 0.67$

CHEMISTRY

121. Assertion(A) : Atom with completely filled and half filled subshells are stable.
Reason (R) : Completely filled and half filled cubshells have symmatrical distribution of electrons and have maximum exchange energy

1) (A) and (R) are correct, (R) is the correct explanation of (A)
2) (A) and (R) are correct, (R) is not correct explanation of (A)
3) (A) is correct, (R) is not correct 4) (A) is not correct, but (R) is correct

Key: 1

Sol: Half filled and completely filled subshells are more stable due to symmetrical distribution of electrons and have maximum exchange energy.

122. The element with the electronic configuration $1s^2\, 2s^2\, 2p^6\, 3s^2\, 3p^2\, 3d^{10}\, 4s^1$ **is**

1) Cu 2) Ca 3) Cr 4) Co

Key: 1

Sol: Cu : $1s^2\, 2s^2\, 2p^6\, 3s^2\, 3p^6\, 3d^{10}\, 4s^1$

This configuration more stable due to completely filled electronic configuration of 'd'-orbital.

123. Among the following, the isoelectronic species is/are

(i) $O^{2-}, F^-, Na^+, Mg^{2+}$ **(ii)** $Na^+, Mg^+.Al^{3+}, F^-$ **(iii)** N^{3-}, O^{2-}, F^-, Ne

1) (i) & (ii) 2) (i), (ii) & (iii) 3) (ii) & (iii) 4) (i) & (iii)

Key: 4

Sol: $\underset{(10e^-)}{O^{-2}}\ \underset{(10e^-)}{F^-}\ \underset{(10e^-)}{Na^+}\ \underset{(10e^-)}{Mg^{+2}}$

$\underset{(10e^-)}{Na^+}\ \underset{(11e^-)}{Mg^+}\ \underset{(10e^-)}{Al^{+3}}\ \underset{(10e^-)}{F^-}$

$\underset{(10e^-)}{N^{-3}}\ \underset{(10e^-)}{O^{-2}}\ \underset{(10e^-)}{F^-}\ \underset{(10e^-)}{Ne}$.

124. What is the atomic number of the element with symbol Uus?

1) 117 2) 116 3) 115 4) 114

Key: 1

Sol: Uus - Un un septium (117) .

125. Match the following

List-I	List-II
(A) PCl_3	**(I) Square planar**
(B) BF_3	**(II) T-shape**
(C) ClF_3	**(III) Trigonal-pyramidal**
(D) XeF_4	**(IV) See-saw**
	(V) Trigonal planar

	(A)	**(B)**	**(C)**	**(D)**
1)	(IV)	(II)	(I)	(III)
2)	(III)	(V)	(II)	(IV)
3)	(III)	(V)	(II)	(I)
4)	(II)	(IV)	(III)	(V)

Key: 3

Sol: $PCl_3(SP^3, 1l.p) - Trigonal - Pyramidal$

$BF_3(SP^2, 0l.p) - Trigonal - Planar$

$ClF_3(SP^3d, 2l.p) - T - Shape$

$XeF_4(SP^3d^2, 2l.p) - Square\ Planar$.

126. The order of covalent character of KF, KI, KCI is

1) $KCl < KF < KI$ 2) $KI < KCl < KF$ 3) $KF < KI < KCl$ 4) $KF < KCl < KI$

Key: 4

Sol: According to FaJan's Rule

Covalent Character α Size of Anion

Size of Anions $I^- > Br^- > Cl^- > F^-$.

127. If the kinetic energy in J. of CH_4 (molar mass=16g mol^{-1}) at T(K) is X, the kinetic energy in J, of O_2 (molar mass=32g mol^{-1}) at the same temperature is

1) X 2) 2X 3) X^2 4) $\frac{X}{2}$

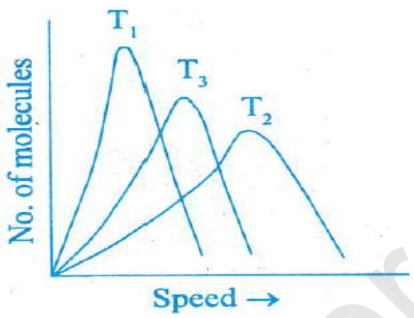
Key: 1

Sol: Average Kinetic Energy **=x**

$$KE = \frac{3}{2}RT$$

Average Kinetic Energy αT .

128. The given figure shows the Maxwell distribution of molecular speeds of a gas at three different temperatures T_1, T_2 and T_3.The correct order of temperatures is



1) $T_1 > T_2 > T_3$ 2) $T_1 > T_3 > T_2$ 3) $T_3 > T_2 > T_1$ 4) $T_2 > T_3 > T_1$

Key: 4

Sol: Conceptual .

129. In Haber's process 50.0 g of $N_2(g)$ and 10.0 g of $H_2(g)$ are mixed to produce $NH_3(g)$. What is the number of moles of $NH_3(g)$ formed ?

1) 3.33 2) 2.36 3) 2.01 4) 5.36

Key: 1

Sol: $N_2 + 3H_2 \rightleftharpoons 2NH_3$

$\frac{50}{28} moles \frac{10}{2} moles$

1.785moles 5moles

Limiting reagent is H_2

3moles of $H_2 \rightarrow 2moles\ NH_3$

5moles of $H_2 \rightarrow$?.

$\frac{5\times2}{3} = 10/3 = 3.33$

130. The following reaction occurs in acidic medium

$KMnO_4 + 8H^+ + 5e^- \rightarrow K^+ + Mn^{2+} + 4H_2O$

1) 79.0 2) 31.6 3) 158.0 4) 39.5

Key: 2

Sol: $\underset{(+7)}{KMnO_4} \rightarrow \underset{(+2)}{Mn^{+2}}$

n-factor (or) valancy factor=7-2=5

Equivalent weight = $\frac{Molecular\ weight}{n-factor}$ =158/5=31.6.

131. Given that $N_2(g)+3H(g) \rightarrow 2NH_3(g); \Delta_r H^{\Theta} = -92kJ$,the standard molar enthalpy of formation in kJ mol^{-1} of $NH_3(g)$ is

1) -92 2) +46 3) +92 4) -46

Key: 4

Sol: Standard molar enthalpy (for 1mole)=-92/2=-46KJ/mole.

132. Which one of the following is correct ?

1) The equilibrium constant ((K_c) is independent of temperature

2) The value of K_c is independent of initial concemtrations of reactants and products

3) At equilibrium, the rate of the forward reaction is twice the rate of the backward reaction

4) The equilibrium constant (K_c) for the reaction

$$Ni(S)+4CO(g) \rightleftharpoons Ni(CO)_4(g) is, \frac{[Ni(CO)_4]}{[CO]}$$

Key: 2

Sol: Equilibrium Constant (K_c) is only dependent on temperature but not on initial and final concentrations of reactants and products respectively.

133. pH of an aqueous solution of NH_4Cl is

1) 7 2) >7 3) <7 4) 1

Key: 3

Sol: This is formed from strong acid and weak base

$NH_4Cl + H_2O \rightleftharpoons NH_4OH + HCl$

Cationic hydrolysis so solution is acidic

$P^H < 7$

134. What is the change in the oxidation state of Mn, in the reaction of $MnO^-{}_4$ with H_2O_2 in acidic medium ?

1) $7 \rightarrow 4$ 2) $6 \rightarrow 4$ 3) $7 \rightarrow 2$ 4) $6 \rightarrow 2$

Key: 3

Sol: In Acidic medium

$\underset{(+7)}{MnO^-{}_4} \rightarrow \underset{(+2)}{Mn^{+2}}$

135. Which one of the following will not give flame test ?

1) Ca 2) Ba 3) Sr 4) Be

Key: 4

Sol: Berilium does not give flame test in II(A) group elements

136. Which one of the following forms a basic oxide ?

1) B 2) Tl 3) Al 4) Ga

Key: 2

Sol: $B_2O_3 \rightarrow Acidic\,oxide$

$Al_2O_3 \rightarrow Amphoteric\,oxide$

$Ga_2O_3 \rightarrow Amphoteric\,oxide$

$Tl_2O_3 - Basic\,oxide$

137. The gas produced by the passage of air over hot coke is

1) Carbon monoxide 2) Carbon dixode 3) Producer gas 4) Water gas

Key: 3

Sol: Producer gas $(CO + N_2)$

138. In environmental chemistry the medium which is affected by a pollutant is called as the ____

1) Sink 2) Slag 3) Solvent 4) Receptor

Key: 4

Sol: Conceptual

139. The hybridisation of each carbon in the following compound is

$$CH_3 - \overset{\overset{\displaystyle O}{\|}}{C} - CH_2 - CN$$

	i	ii	iii	iv		i	ii	iii	iv
1)	sp^3	sp^2	sp^3	sp	2)	sp^3	sp^3	sp^2	sp
3)	sp^3	sp	sp^3	sp^2	4)	sp^3	sp^2	sp	sp^3

Key: 1

Sol: $$CH_3 - \overset{\overset{\displaystyle O}{\|}}{C} - CH_2 - C \equiv N$$

$$(sp^3)\ (sp^2)\ (sp^2)\ (sp)$$

140. The product Z of the following reaction is:

$$H_3CC \equiv CH \xrightarrow{2HBr} Z$$

1) $H_3CCH_2CHBr_2$ 2) $H_3CCBr_2CH_3$ 3) $H_3CCHBrCH_2Br$ 4) $BrCH_2CH_2CH_2Br$

Key: 2

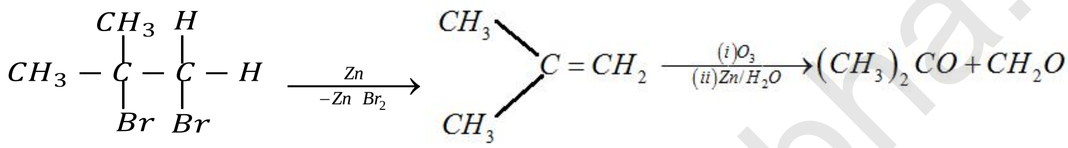
Sol: $$CH_3 - C \equiv C - H \xrightarrow{HBr} CH_3 - \underset{\underset{\displaystyle Br}{|}}{C} = CH_2 \xrightarrow{HBr} CH_3 - \overset{\overset{\displaystyle Br}{|}}{\underset{\underset{\displaystyle Br}{|}}{C}} - CH_3$$

141. Identify X and Y in the following reaction sequence

$$X \xrightarrow{Zn} Y \xrightarrow[Zn-H_0O]{O_3} (CH_3)_2 CO + CH_2O$$

	X	Y
1)	$(CH_3)_2 CHCH_3$ with Br on the CH carbon	$CH_3CH = CHCH_3$
2)	$(CH_3)_2 CHCH_2Br$	$CH_3CH = CHCH_3$
3)	$(CH_3)_2 CBrCH_2Br$	$(CH_3)_2 C = CH_2$
4)	$(CH_3)_2 CHCHBr_2$	$(CH_3)_2 C = CH_2$

Key: 3

Sol: 

142. The packing efficiency of simple cubic (sc), body centred cubic (bcc) and cubic close packing (ccp) lattices follow the order

1) bcc < ccp < sc 2) ccp < bcc < sc 3) sc < ccp < bcc 4) sc < bcc < ccp

Key: 4

Sol: Packing efficiency of

(i) Simple cubic lattice (sc) 52.4%

(ii) Body centred cubic lattice (bcc) 68%

Cubic close packing (ccp).......74%

143. The experimental depression in freezing point of a dilute solution is 0.025 K. If the van't Hoff factor (i) is 2.0, the calculated depression in freezing point (in K) is

1) 0.00125 2) 0.025 3) 0.0125 4) 0.05

Key: 3

Sol: $i = \dfrac{Experimental\ \ colligative\ \ \Pr operty}{Calculated\ colligative \Pr operty}$

$$2 = \frac{0.025}{(\Delta T_f)cal}$$

$$(\Delta T_f)cal = \frac{0.025}{2} = 0.0125$$

144. The molality of an aqueous dilute solution containing non- volatile solute is 0.01 M. What is the boiling temperature (in ^0C) of solution? (Boiling point elevation constant, $K_b = 0.52$ Kg mol^{-1}K; boiling temperature of water $= 100^0C$).

1) 100.0052 2) 100.052 3) 100.0 4) 100.52

Key: 2

Sol: $\Delta T_b = K_b$ **molality**

$(T_s - T_0) = K_b$ **molality**

$T_s - 373 = 0.52 \times 0.1$

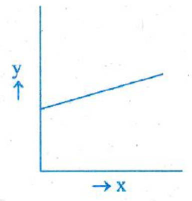
$T_s = 0.052 + 373$

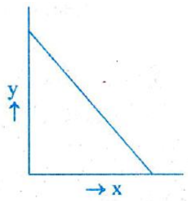
$T_s = 373.052\ k$

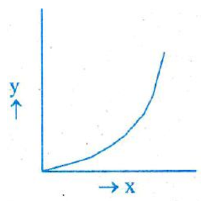
$T_s = 373.052 - 273$

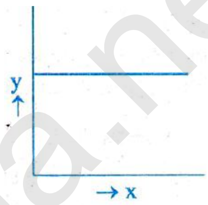
$T_s = 100.052^0 C$

145. Which one of the following is the correct plot of $\wedge_m$ (in $S\ cm^2\ \ mol^{-1}$) and $\sqrt{c}$ $(in\ mol/L)^{1/2}$ for KCl solution? $\left(y = \wedge_m ; x = \sqrt{c}\right)$.

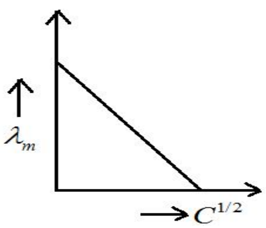
1) 

2) 

3) 

4) 

Key: 2

Sol: 

146. For the reaction

$5Br^-(aq) + 6H^+(aq) + BrO_3^-(aq) \rightarrow 3Br_2(aq) + 3H_2O(l)$

if, $-\dfrac{\Delta\left[BrO_3^-\right]}{\Delta t} = 0.01\, mol\ \ L^{-1}\ \min^{-1}$**,**

$-\dfrac{\Delta\left[Br_2\right]}{\Delta t} in\ \ mol\ \ L^{-1}\ \min^{-1}$ **is**

1) 0.01 2) 0.3 3) 0.03 4) 0.005

Key: 3

Sol: Rate of reaction

$$\frac{-1}{5}\frac{\Delta\left(Br^-\right)}{\Delta t} = \frac{-1}{6}\frac{\Delta\left(H^+\right)}{\Delta t} = -1\frac{\Delta\left(BrO_3^-\right)}{\Delta t} = \frac{1}{3}\frac{\Delta\left(Br_2\right)}{\Delta t}$$

$$\frac{\Delta\left(Br_2\right)}{\Delta t} = -3\frac{\Delta\left(BrO_3\right)}{\Delta t} = 3 \times 0.01 = 0.03$$

147. Which one of the following is an emulsion?

1) Milk 2) Soap lather 3) Butter 4) Vanishing Cream

Key: 1, 4

Sol: Conceptual

148. Copper matte contains__________

1) Cu_2O, Cu_2S 2) Cu_2O, FeO 3) Cu_2S, FeS 4) Cu_2S, FeO

Key: 3

Sol: $Cu_2S + FeS$ is called copper matte.

149. $\underline{X}$ reacts with dilute nitric acid to form 'laughing gas'. What is $\underline{X}$?

1) Cu 2) P_4 3) S_8 4) Zn

Key: 4

Sol: $4Zn + 10HNO_3(dilute) \rightarrow 4Zn(NO_3)_2 + 5H_2O + N_2O(Laughing\ gas)$

150. Xenon reacts with fluorine at 873 K and 7 bar to form XeF_4. In this reaction the ratio of Xenon and fluorine required is:

1) 1:5 2) 10:1 3) 1:3 4) 5:1

Key: 1

Sol: $Xe(g) + 2F_2(g) \rightarrow XeF_4(g)$

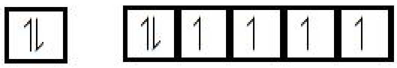
(1:5 Ratio)

151. Which of the following metal ions has a calculated magnetic moment value of $\sqrt{24}$ B.M.?

1) Mn^{2+} 2) Fe^{+2} 3) Fe^{3+} 4) Co^{2+}

Key: 2

Sol: $Fe^{+2}[Ar]$ $4s^0$ $3d^6$



4- unpaired electrons n=4

magnetic momentum $(\mu) = \sqrt{n(n+2)}$

$= \sqrt{4(4+2)} = \sqrt{24}$ B. M.

152. Which one of the following does not exhibit geometrical isomerism?

1) Octahedral complex with formula $[MX_2L_4]$

2) Square planar complex with formula $[MX_2L_2]$

3) Tetrahedral complex with formula $[MABXL]$

4) Octahedral complex with formula $[MX_2(L-L)_2]$

Key: 3

Sol: Conceptual

153. The P Dispersity Index (PDI) of a polymer is ($\overline{M}_w =$ weight average molecular mass and $\overline{M}_n =$ number average molecular mass)

1) The product of $\overline{M}_n$ and $\overline{M}_w$ 2) The sum of $\overline{M}_n$ and $\overline{M}_w$

3) The difference between $\overline{M}_w$ and $\overline{M}_n$ 4) The ratio between $\overline{M}_w$ and $\overline{M}_n$

Key: 4

Sol: Poly Despersity index (PDI) $= \dfrac{\overline{M}_w}{\overline{M}_n}$

154. Hormone that maintains the blood glucose level within the limit is:

1) Thyroxine 2) Insulin 3) Testosterone 4) Epinephrine

Key: 2

Sol: Insulin maintains glucose level in blood.

155. Chloroxylenol is an example of _______________

1) Antiseptic 2) Antipyretic 3) Analgesic 4) Tranquilizer

Key: 1

Sol: Chloroxylenol is a constituent in dettol which is Anti septic

156. Which one of the following has highest boiling point?

1) $H_3CCH_2CH_2CH_2Cl$ 2) $(H_3C)_2\,CHCH_2Cl$

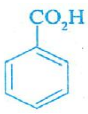
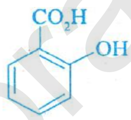
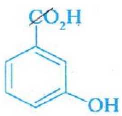
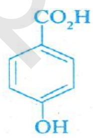
3) $(H_3C)_3\,CCl$ 4) $H_3CCH_2\underset{|\atop Cl}{C}HCH_3$

Key: 1

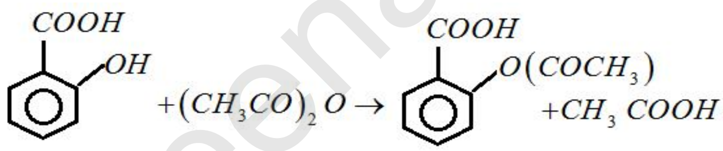
Sol: Boiling point α length of chain.

157. $X + Y \xrightarrow{H^+}$ **Aspirin** $+ H_3CCOOH$

Identify X and Y from the following:

	X	Y		X	Y
1)		H_3CCOCl	2)		$(H_3CCO)_2\,O$
3)		H_3CCO_2H	4)		H_3CCOCH_3

Key: 2

Sol: 

Ortho - Acetyl Salicylic acid (aspirin)

158. $R-CN \xrightarrow[(ii)H_3O^+]{(i)SnCl_2+HCl} R-CHO$

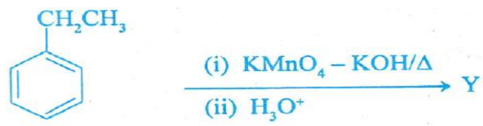
What is the name of the above reaction?

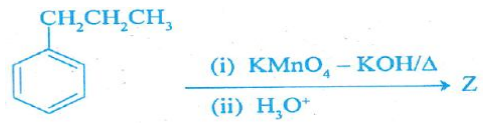
1) Resenmund 2) Williamson

3) Stephen 4) Kolbe

Key: 3

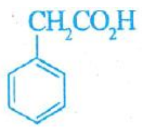
Sol: Conceptual

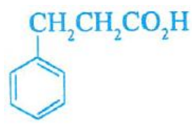
159. 

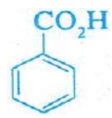


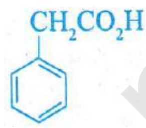
What are the structures of Y and Z?

Y **Z** **Y** **Z**

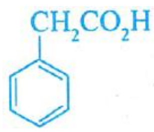
1) 

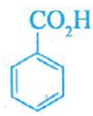


2) 

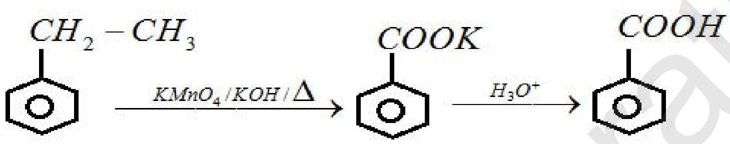


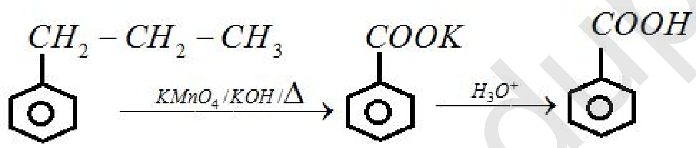
3) 



4) 

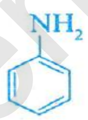
Key: 4

Sol: 

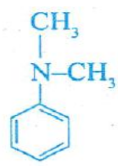


160. Which is the strongest base among the following?

1) H_3CNH_2

2) 

3) 

4) 

Key: 1

Sol: Aliphatic amines are more basic than aromatic amines.

*****

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