MATHS

1. Match the differential equation in List 1 to thier Integrating factors in List-II

List-I Differential Equation	**List-II** Integrating factor
(i) $\left(x^3+1\right)\frac{dy}{dx}+x^2y=3x^2$	**(a)** x^3
(ii) $x^2\frac{dy}{dx}+3xy=x^6$	**(b)** $\left(x^3+1\right)^2$
(iii) $\left(x^3+1\right)^2\frac{dy}{dx}+6x^2\left(x^3+1\right)y=x^2$	**(c)** $\left(x^2+1\right)^2$
(iv) $\left(x^2+1\right)\frac{dy}{dx}+4xy=\ln x$	**(d)** x^2+1
	(e) $\left(x^3+1\right)^{1/3}$
	(f) $\left(x^3+1\right)^{1/2}$

The correct match is

	(i)	(ii)	(iii)	(iv)
(1)	(d)	(a)	(b)	(c)
(2)	(e)	(a)	(b)	(c)
(3)	(e)	(b)	(c)	(f)
(4)	(e)	(a)	(c)	(d)

Key: 2

Sol: (i) $\frac{dy}{dx}+\frac{x^2}{1+x^3}y=\frac{3x^2}{1+x^3}$

$\int Pdx=\frac{1}{3}\int\frac{3x^2}{1+x^3}dx=\log\left(1+x^3\right)^{\frac{1}{3}}$

$\text{I.F}=\left(1+x^3\right)^{\frac{1}{3}} \longrightarrow key(e)$

(ii) $\frac{dy}{dx}+\frac{3x}{x^2}y=x^4$

$\int Pdx=3\int\frac{1}{x}dx=\log x^3$

$\text{I.F}=x^3 \longrightarrow key(a)$

(iii) $\frac{dy}{dx}+\left(\frac{6x^2}{x^3+1}\right)y=\frac{x^2}{\left(x^3+1\right)^2}$

$\int Pdx=2\int\frac{3x^2}{1+x^3}dx=\log\left(1+x^3\right)^2$

$\text{I.F}=\left(1+x^3\right)^2 \longrightarrow key(b)$

(iv) $\frac{dy}{dx}+\frac{4x}{x^2+1}y=\frac{\log x}{x^2+1}$

$\int Pdx = 2.\int\frac{2x}{x^2+1}dx=\log\left(x^2+1\right)^2$

I.F $=\left(x^2+1\right)^2$ $\longrightarrow key(c)$

2. The solution of the differential equation $xy' = 2x\,e^{-y/x}+y$ **is**

1) $e^{y/x}+1n|cx|=0$ 2) $e^{-y/x}=x+c$ 3) $e^{y/x}=\ln|cx|$ 4) $e^{y/x}=2\ln|cx|$

Key: 4

Sol: $x.\frac{dy}{dx}=2xe^{\frac{-y}{x}}+y$

$\frac{dy}{dx}=2e^{-y/x}+\frac{y}{x}$

$v+x.\frac{dv}{dx}=2e^{-v}+v$

$\int e^v dv = 2.\int\frac{1}{x}dx$

$e^v=2\log x+2\log c$

$e^{\frac{y}{x}}=2\log cx$

3. The differential equation of the family of curves $y=ax+\frac{1}{a}$, **where** $a\neq 0$ **is an arbitary constant, has the degree**

1) 4 2) 3 3) 1 4) 2

Key: 4

Sol: $y=ax+\frac{1}{a}$

$\therefore \frac{dy}{dx}=a$

$y=xy_1+\frac{1}{y_1}$

$yy_1=xy_1^2+1$

degree $=2$

4. **The area of the region bounded by the curves** $y=9x^2$ **and** $y=5x^2+4$ **(in square units) is**

1) 64 2) $\frac{64}{3}$ 3) $\frac{32}{3}$ 4) $\frac{16}{3}$

Key: 4

Sol: $y = 9x^2$

$y = 5x^2 + 4$

$4x^2 = 4$

$x = \pm 1$

$$R_1 = \int_0^1 \left(5x^2 + 4 - 9x^2\right) dx$$

$$= \int_0^1 \left(4 - 4x^2\right) dx$$

$$= 4\left(x\right)_0^1 - \frac{4}{3}\left(x^3\right)_0^1$$

$$= \frac{8}{3}$$

Total area $= R_1 + R_2 = 2R_1 = \frac{16}{3}$

5. $\int_0^{\pi/2} \frac{16x \sin x \cos x dx}{\sin^4 x + \cos^4 x} =$

1) $\frac{\pi^2}{4}$ 2) $\frac{\pi^2}{2}$ 3) π^2 4) $2\pi^2$

Key: 3

Sol: $\int_0^{\frac{\pi}{2}} \frac{16 \sin x \cos x}{\sin^4 + \cos^4 x} dx$

$$= 16\int_0^{\frac{\pi}{2}} \frac{\left(\frac{\pi}{2} - x\right)\cos x \sin x}{\sin^4 x + \cos^4 x} dx$$

$$2I = 8\pi \int_0^{\frac{\pi}{2}} \frac{\frac{\sin x \cos x}{\cos^4 x}}{\frac{\sin^4 x}{\cos^4 x} + \frac{\cos^4 x}{\cos^4 x}} dx$$

$$= \frac{4\pi}{2}\int_0^{\frac{\pi}{2}} \frac{2\tan x . \sec^2 x}{\tan^4 x + 1} dx$$

$$= 2\pi . \int_0^{\infty} \frac{1}{1+t^2} dt$$

$$2\pi\left(\frac{\pi}{2} - 0\right) = \pi^2$$

6. $\int_0^{I} \sqrt{\frac{1-x}{1+x}} dx =$

1) $\frac{\pi}{2} - 1$ 2) $\frac{\pi}{2} + 1$ 3) $\pi - 1$ 4) $\frac{3\pi}{2}$

Key: 1

Sol: $\int_0^1 \frac{1-x}{\sqrt{1-x^2}} dx$ $= \int_0^1 \frac{1}{\sqrt{1-x^2}} dx + \frac{1}{2}\int_0^1 \frac{-2x}{1-x^2} dx$

$=\left(\sin^{-1}x\right)_0^1+\frac{1}{2}.2\left(\sqrt{1-x^2}\right)_0^1 \quad =\frac{\pi}{2}-1$

7. **If** $\int\frac{x+5}{x^2+4x+5}dx=a\log\left(x^2+4x+5\right)+b\tan^{-1}\left(x+k\right)+$ **constant then** $\left(a,b,k\right)=$

1) $\left(\frac{1}{2},3,2\right)$ 2) $\left(\frac{1}{2},1,2\right)$ 3) $\left(\frac{1}{2},3,1\right)$ 4) $\left(1,3,2\right)$

Key: 1

Sol: $\int\frac{x+5}{x^2+4x+5}dx$

$=\frac{1}{2}\int\frac{2x+10}{x^2+4x+5}dx$

$=\frac{1}{2}\left[\int\frac{2x+4}{x^2+4x+5}dx+\int\frac{6}{x^2+4x+5}dx\right]$

$=\frac{1}{2}\left[\log\left(x^2+4x+5\right)+6\tan^{-1}\left(x+2\right)\right]$

$=\frac{1}{2}\log\left(x^2+4x+5\right)+3\tan^{-1}\left(x+2\right)+c$

$a=\frac{1}{2},b=3,k=2$

8. $\int\sqrt{e^x-4}\ dx=$

1) $\tan^{-1}\left(\frac{\sqrt{e^x-4}}{2}\right)+\sqrt{e^x-4}+c$

2) $2\sqrt{e^x-4}-4\tan^{-1}\left(\frac{\sqrt{e^x-4}}{2}\right)+c$

3) $2\sqrt{e^x-4}-4\cot^{-1}\left(\frac{\sqrt{e^x-4}}{2}\right)+c$

4) $\sqrt{e^x-4}-4\tan^{-1}\left(\sqrt{e^x-4}\right)+c$

Key: 2

Sol: $\int\sqrt{e^x-4}\,dx$

$=\int\frac{e^x-4}{\sqrt{e^x-4}}dx$

$=\int\frac{e^x}{\sqrt{e^x-4}}-4\int\frac{e^x}{e^x\sqrt{e^x-4}}dx$

$=e^x-4=t^2$

$e^xdx=2t\,dt$

$=2\sqrt{e^x-4}-8.\int\frac{1}{4+t^2}dt$

$=2\sqrt{e^x-4}-8\frac{1}{2}.\tan^{-1}\left(\frac{t}{2}\right)+c$

$=2\sqrt{e^x-4}-4\tan^{-1}\left(\frac{\sqrt{e^{x-4}}}{2}\right)+c$

9. $\int e^{-x}\tan^{-1}\left(e^x\right)dx=f\left(x\right)-\frac{1}{2}\log\left(1+e^{2x}\right)+c\Rightarrow f\left(x\right)=$

1) $e^x-e^{-x}\tan^{-1}\left(e^x\right)$ 2) $x^2+e^{-x}\tan^{-1}\left(e^x\right)$ 3) $-e^x\tan^{-1}\left(e^x\right)$ 4) $x-e^{-x}\tan^{-1}\left(e^x\right)$

Key: 4

Sol: Put $e^x = t, e^x dx = dt, dx = \frac{1}{t}dt$

$= \int \frac{\tan^{-1} t}{t^2} dt$

$= \tan^{-1} t\left(\frac{-1}{t}\right) - \int \left(\frac{-1}{t}\right)\frac{1}{1+t^2} dt$

$= \frac{-\tan^{-1} t}{t} + \frac{1}{2}\int \frac{2t}{t^2\left(1+t^2\right)}dt$

$t^2 = z, \quad 2tdt = dz$

$= \frac{-\tan^{-1} t}{t} + \frac{1}{2}\int \frac{(z+1)-z}{z(1+z)} dz$

$= \frac{-\tan^{-1} t}{t} + \frac{1}{2}\left[\int \frac{1}{z} dz - \int \frac{1}{z+1} dz\right]$

$= \frac{-\tan^{-1} t}{t} + \frac{1}{2}\log\left|\frac{t^2}{1+t^2}\right| + c$

$= \frac{-\tan^{-1} t}{t} + \frac{1}{2}\log\left|\frac{e^{2x}}{1+e^{2x}}\right| + c$

$= -e^{-x}\tan^{-1}\left(e^x\right) - \frac{1}{2}\log\left|1+e^{2x}\right| + \frac{1}{2}.2x + c$

$= x - e^{-x}\tan^{-1}\left(e^x\right) - \frac{1}{2}\log\left|1+e^{2x}\right| + c$

10. $\int \sqrt{\frac{2+x}{2-x}} dx =$

1) $2\sin^{-1}\left(\frac{x}{2}\right) + \sqrt{4-x^2} + c$

2) $\cos^{-1}\left(\frac{x}{2}\right) - \sqrt{4-x^2} + c$

3) $\sin^{-1}\left(\frac{x}{2}\right) - \sqrt{4-x^2} + c$

4) $2\sin{-1}\left(\frac{x}{2}\right) - \sqrt{4-x^2} + c$

Key: 4

Sol: $= 2.\int \frac{1}{\sqrt{4-x^2}} dx - \frac{1}{2}\int \frac{-2^x}{\sqrt{4-x^2}} dx$

$= 2.\sin^{-1}\left(\frac{x}{2}\right) - \frac{1}{2}.2\sqrt{4-x^2} + c$

11. **Two particles P and Q located at the point with coordinates** $P\left(t, t^3-16t-3\right)$, $Q\left(t1, t^3-6t-6\right)$ **are moving in a plane . The minimum distance between them in their motion is**

1) 1 2) 5 3) 169 4) 49

Key: 1

Sol: $P\left(t, t^3-16t-3\right)$

$Q\left(t+1, t^3-6t-6\right)$

$PQ^2 = 1 + (10t-3)^2 \qquad \left(PQ^2\right)^1 = 0$

$2(10t-3) = 0 \qquad t = \frac{3}{10} \qquad \therefore PQ = 1$

12. Define

$$f(x)=\begin{cases}x & (0\le x\le 1)\\ 2-x & (1\le x\le 2)\end{cases}$$

Then Rolle's theorem is not applicable to $f(x)$ because

1) $f(x)$ is not defined everywhere on $[0,2]$ 2) $f(x)$ is not continuous on $[0,2]$

3) $f(x)$ is not differential on $(1,2)$ 4) $f(x)$ is not differentiable on $(0,2)$

Key: 4

Sol: $f'(x)=\begin{cases}1, & 0\le x\le 1\\ -1, & 1\le x\le 2\end{cases}$

$f'(1-)\ne f'(1+)$

$f(x)$ is not differentiable is $(0,2)$

13. An Equilateral triangle is of side 10 units. In measuring the side, an error of 0.05 units i made. Then th percentage error in the area of the triangle is

1) 5 2) 4 3) 1 4) 0.5

Key: 3

Sol: $A=\frac{\sqrt{3}}{4}x^2$

$\log A=\log\frac{\sqrt{3}}{4}+2\log x$

$\frac{\delta A}{A}=2.\frac{\delta x}{x}$

$\frac{\delta A}{A}\times 100=\frac{2\times 5}{10}=1$

14. If the lines $y=-4x+b$ are tangent to the curve $y=\frac{1}{x}$, then b =

1) ± 4 2) ± 2 3) ± 1 4) ± 8

Key: 1

Sol: $-4x+b=\frac{1}{x}$

$4x^2-bx+1=0$

$\Delta=0$

$b^2-4(4)(1)=0$

$b^2=16$

$b^2=\pm 4$

15. **If** $x=\frac{1-\sqrt{y}}{1+\sqrt{y}}$, **then** $(x+1)\frac{d^2y}{dx^2}+\left(\frac{3\sqrt{y}+1}{\sqrt{y}}\right)\frac{dy}{dx}=$

1) $-2y$ 2) 0 3) $-y$ 4) y

Key: 2

Sol: Applying compenando and dividando

16. **If** $x^2+y^2=t+\frac{2}{t}$ **and** $x^4+y^4=t^2+\frac{4}{t^2}$, **then** $x^3y\frac{dy}{dx}=$

1) -1 2) -2 3) $\frac{y}{x}$ 4) xy

Key: 2

Sol: $x^4+y^4=t^2+\frac{4}{t^2}$

$=x^4+y^4+2x^2y^2-4$

$\Rightarrow 2y.\frac{dy}{dx}=2\left(\frac{-2}{x^3}\right)$

$\Rightarrow x^3y\frac{dy}{dx}=-2$

17. **If** $y=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)+\tan^{-1}\left(\frac{4x-4x^3}{1-6x^2+x^4}\right)$ **then** $\frac{dy}{dx}=$

1) $\frac{2}{1+x^2}$ 2) $\frac{4}{1+x^2}$ 3) $\frac{6}{1+x^2}$ 4) $\frac{7}{1+x^2}$

Key: 4

Sol: $y=7\tan^{-1}x$

$\frac{dy}{dx}=\frac{7}{1+x^2}$

18. **Two teams A and B have the same mean and their coefficient of variation are 4,2 respectively. If $\sigma_A.\sigma_B$ are the standard deviation of teams A,B respectively then the relation between them is**

1) $\sigma_A=\sigma_B$ 2) $\sigma_B=2\sigma_A$ 3) $\sigma_A=2\sigma_B$ 4) $\sigma_B=4\sigma_A$

Key: 3

Sol: Cofficients variancie $=\frac{\sigma A}{\overline{x}_A}=4$, $=\frac{\sigma B}{\overline{x}_B}=2$

$\Rightarrow \frac{\sigma A}{\sigma B}=\frac{2}{1}$

$\Rightarrow \sigma A=2(\sigma B)$

19. **If A and B are events such that** $P(A\cup B)=\frac{5}{6}, P(\overline{A})=\frac{1}{4}, P(B)=\frac{1}{3}$, **Then A and B are**

1) mutually exclusive 2) independent
3) exhaustive events 4) exhaustive and independent

Key: 2

Sol: $P(A\cup B)=\frac{5}{6}, P(\bar{A})=\frac{1}{4}, P(B)=\frac{1}{3}$

$\Rightarrow \frac{3}{4}+\frac{1}{3}-P(A\cap B)=\frac{5}{6}$

$\Rightarrow P(A\cap B)=\frac{1}{4}$

$P(A\cap B)=P(A).P(B)$

$=\frac{3}{4}.\frac{1}{3} \ =\frac{1}{4}$

20. If A and B are two events such that $P(\bar{A}|B)=0.6$ $P(B|A)=0.3$ $P(A)=0.1$ then $P(\bar{A}\cap\bar{B})=$

(Here $\bar{E}$ is the complement of the event E)

1) 0.88 2) 0.12 3) 0.6 4) 0.4

Key: 1

Sol: $\frac{P(A\cap B)}{P(B)}=0.6, \frac{P(A\cap B)}{P(A)}=0.3$

$P(A)=2P(B)$

$P(\bar{A}\cap\bar{B})=1-P(A\cap B)$

$=1-0.1-0.05+0.03$

$=0.88$

21. In a certain college 4% of the men and 1% of the women are taller than 1.8 meters.Also 60% of the students are women. If the student selected at random is found to be taller than 1.8 meters, then the probability that the students being a woman is

1) $\frac{3}{11}$ 2) $\frac{5}{11}$ 3) $\frac{6}{11}$ 4) $\frac{8}{11}$

Key: 1

Sol: $P=\dfrac{\frac{60}{100}\times\frac{1}{100}}{\frac{60}{100}\times\frac{1}{100}+\frac{40}{100}\times\frac{4}{100}}=\frac{60}{60+160}=\frac{6}{22}$

$=\frac{3}{11}$

22. X is a binomial variate with parameters $n=6$ and p. If $4P(X=4)=P(X=2)$, then p is

1) $\frac{1}{2}$ 2) $\frac{1}{3}$ 3) $\frac{1}{4}$ 4) $\frac{1}{6}$

Key: 2

Sol: $4P^2=q^2$

$\Rightarrow q=2P$, $P+Q=1$

$P=1/3, Q=2/3$

23. The Probability of a coin showing head is p. 100 such coins are tossed. If the probability of 50 coins showing heads is same as the probability of 51 coins showing heads, then p=

1) $\frac{1}{2}$ 2) $\frac{49}{100}$ 3) $\frac{51}{101}$ 4) $\frac{50}{101}$

Key: 3

Sol: $P(X=50)=P(X=51)$

$\Rightarrow {}^{100}C_{50}\, P^{50}Q^{50} = {}^{100}C_{51}.P^{50}Q^{50}$

$\Rightarrow 51(1-P)=50P$

$\Rightarrow 51-51P=50P$

$\Rightarrow P=\frac{51}{101}$

24. The locus of the point P which is equidistant from $3x+4y+5=0$ **and** $9x+12y+7=0$ **is**

1) a hyperbola 2) an ellipse 3) a parabola 4) a straight line

Key: 4

Sol: $\frac{3x+4y+5}{5}=\frac{9x+12y+7}{25}$

$15x+20y+25-9x-12y-7=0$

$6x+8y+18=0$

$3x+4y+9=0$

Straight line

25. If the origin of a coordinate system is shifted to $(-\sqrt{2},\sqrt{2})$ **and then th coordinate system is rotated anticlockwise through an angle** 45^0, **the point** $P(1,-1)$ **in the original system has new coordinates**

1) $(\sqrt{2},-2\sqrt{2})$ 2) $(0,-2\sqrt{2})$ 3) $(0,-2-\sqrt{2})$ 4) $(0,-2+\sqrt{2})$

Key: 3

Sol:

	X	Y
$x+\sqrt{2}$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$
$y-\sqrt{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$

$X=(x+\sqrt{2})\frac{1}{\sqrt{2}}+(y-\sqrt{2})\frac{1}{\sqrt{2}}$

$=\frac{1+\sqrt{2}}{\sqrt{2}}+\frac{-1-\sqrt{2}}{\sqrt{2}}$

$=\frac{1+\sqrt{2}-1-\sqrt{2}}{\sqrt{2}}=0$

$$Y=\frac{-\left(x+\sqrt{2}\right)}{\sqrt{2}}+\frac{y-\sqrt{2}}{\sqrt{2}}$$

$$=\frac{-1-\sqrt{2}-1-\sqrt{2}}{\sqrt{2}}$$

$$=\frac{-2-2\sqrt{2}}{\sqrt{2}}$$

$$=-\sqrt{2}-2$$

$$\left(0,-2-\sqrt{2}\right)$$

26. The combined equation of the straight lines passing through th point $(4,3)$ and each line making intercepts on the coordinate axes whose sum is -1 is

1) $(3x-2y-6)(x-2y+2)=0$ 2) $(3x-2y+6)(x-2y+2)=0$

3) $(3x-2y-6)(x-2y-2)=0$ 4) $(3x-2y+6)(x-2y-2)=0$

Key: 1

Sol: $\frac{x}{a}+\frac{y}{-1-a}=1\in(4,3)$

$\frac{4}{a}+\frac{3}{-1-a}=1$ $\frac{-4-4a+3a}{a(-1-a)}=1$

$a^2=4$ $a=\pm 2$

$\frac{x}{2}+\frac{y}{-3}=1$ $\frac{x}{-2}+\frac{y}{1}=1$

$\frac{-3x+2y}{-6}=1$ $x-2y=-2$

$-3x+2y=-6$ $x-2y+2=0 \longrightarrow 2$

$3x-2y+6=0 \longrightarrow 1$

$(3x-2y+6)(x-2y+2)=0$

27. The value of $k>0$ such that the angle between the lines $4x-y+7=0$ and $kx-5y-9=0$ is 45^0 is

1) $\frac{25}{3}$ 2) $\frac{5}{3}$ 3) 3 4) 5

Key: 3

Sol: $m_1=4, m_2=\frac{k}{5}$

$\tan 45^0=\left|\frac{4-\frac{k}{5}}{1+\frac{4k}{5}}\right|$ $\Rightarrow 5k=15, k=3$

28. An equation of a line whose segement between the coordinate axes is divided by the point $\left(\frac{1}{2},\frac{1}{3}\right)$ in the ratio 2 :3 is

1) $6x+9y=5$ 2) $9x+6y=5$ 3) $4x+9y=5$ 4) $9x+4y=5$

Key: 3

Sol: $\frac{29}{5}=\frac{1}{2}$ $\frac{3b}{5}=\frac{1}{3}$

$a=\frac{5}{4}$ $b=\frac{5}{9}$

$\frac{x}{\left(\frac{5}{4}\right)}+\frac{y}{\left(\frac{5}{9}\right)}=1$

$\frac{4x}{f}+\frac{9y}{5}=1$

$4x+9y=5$

29. Two pairs of straight lines with combined equations $xy+4x-3y-12=0$ and $xy-3x+4y-12=0$ form a square. then the combined equation of its diagonal is

1) $x^2-3x+4y-12=0$ 2) $x^2+2xy+y^2+x+y=0$

3) $x^2-y^2+x-y=0$ 4) $x^2-y^2+x+y=0$

Key: 3

Sol: $xy+4x-3y-12=0$

$x=3, y=-4$

$xy-3x+4y-12=0$

$x=-4, y=3$

$A(-4,-4)$

$B(3,-4)$

$C(3,3)$

$D(-4,3)$

Equation of AC $m=\frac{7}{7}=1$; $x-y=0$

Equation of BD $m=\frac{7}{-7}=-1$ $x+y+1=0$

$(x-y)(x+y+1)=0$

$x^2-y^2+x-y=0$

30. The line $x+y=k$ meets the pair of straight lines $x^2+y^2-2x-4y+2=0$ in two points A and B. If O is the origin and $\angle AOB=90^0$ then the value of $k>1$ is

1) 5 2) 4 3) 3 4) 2

Key: 4

Sol: $x^2+y^2-2x\left(\frac{x+y}{k}\right)-4y\left(\frac{x+y}{k}\right)+2\left(\frac{x+y}{k}\right)^2=0$

$\left(1-\frac{2}{k}+\frac{2}{k^2}\right)+\left(1-\frac{4}{k}+\frac{2}{k^2}\right)=0$ $\quad$ $k^2-3k+2=0$

$(k-1)(k-2)=0$

$k=2$

31. The value of a such that the power of the point $(1,6)$ with respect to the circle $x^2+y^2+4x-6y-a=0$ is

1) 7 $\quad$ 2) 11 $\quad$ 3) 13 $\quad$ 4) 21

Key: 4

Sol: $S_{11}=-16$

$1+36+4-36-a=-16$

$a=5+16=21$

32. The area (in square units) of the triangle formed by the tangent, normal at $(1,\sqrt{3})$ to the circle $x^2+y^2=4$ and the X-axis is

1) $4\sqrt{3}$ $\quad$ 2) $\frac{7}{2}\sqrt{3}$ $\quad$ 3) $2\sqrt{3}$ $\quad$ 4) $\frac{1}{2}\sqrt{3}$

Key: 3

Sol: $m=\frac{-1}{\sqrt{3}}$ $\quad$ Area $=\frac{3}{2}\left|\frac{-1}{\sqrt{3}}-\sqrt{3}\right|$

$=\frac{6}{\sqrt{3}}=2\sqrt{3}$

33. If $(4,2)$ and $(k,-3)$ conjugate points with respect to $x^2+y^2-5x+8y+6=0$, then k =

1) $\frac{28}{3}$ $\quad$ 2) $-\frac{28}{3}$ $\quad$ 3) $\frac{3}{28}$ $\quad$ 4) $-\frac{3}{28}$

Key: 1

Sol: $S_{12}=0$

$(4)(k)+(2)(-3)-\frac{5}{2}(4+k)+4(2-3)+6=0$

$3k=28$ $\quad$ $k=\frac{28}{3}$

34. The length of the common chord of the two circles $(x-a)^2+y^2=a^2$ and $x^2+(y-b)^2=b^2$ is

1) $\frac{ab}{\sqrt{a^2+b^2}}$ $\quad$ 2) $\frac{2ab}{\sqrt{a^2+b^2}}$ $\quad$ 3) $\frac{a+b}{\sqrt{a^2+b^2}}$ $\quad$ 4) $\sqrt{a^2+b^2}$

Key: 2

Sol: $\frac{2r_1r_2}{\sqrt{r_1^2+r_2^2}} = \frac{2ab}{\sqrt{a^2+b^2}}$

35. **The equation of the circles passing through $(1,2)$ and the points of intersection of the circles $x^2+y^2-8x-6y+21=0$ and $x^2+y^2-2x-15=0$ is**

1) $x^2+y^2+6x-2y+9=0$
2) $x^2+y^2-6x-2y+9=0$
3) $x^2+y^2-6x-4y+9=0$
4) $x^2+y^2-6x+4y+9=0$

Key: 3

Sol: $x+y-6=0$

$x^2+y^2-8x-6y+21+\lambda(x+y-6)=0$ at $(1,2)$

$\Rightarrow \lambda=2$

$x^2+y^2-6x-4y+9=0$

36. **The equation of the parabola with focus $(1,-1)$ and directrix $x+y+3=0$ is**

1) $x^2+y^2-10x-2y-2xy-5=0$
2) $x^2+y^2-10x-2y-2xy-5=0$
3) $x^2+y^2+10x+2y-2xy-5=0$
4) $x^2+y^2+10x+2y+2xy-5=0$

Key: 1

Sol: $sp^2 = pm^2$

$(x-1)^2+(y+1)^2=\frac{(x+y+3)^2}{2}$

$x^2+y^2-2xy-10x-2y-5=0$

37. **If P is a point on the parabola $y^2=8x$ and A is the point $(1,0)$, then the locus of the midpoint of the line segment AP is**

1) $y^2=4\left(x-\frac{1}{2}\right)$
2) $y^2=2(2x+1)$
3) $y^2=x-\frac{1}{2}$
4) $y^2=2x+1$

Key: 1

Sol: $\alpha=2x_1-1$, $\beta=2y_1$

$\beta^2=8\alpha$

$4y^2=8(2x-1)$

$y^2=4x-2$

$y^2=4\left(x-\frac{1}{2}\right)$

38. **For the ellipse** $\frac{x^2}{25}+\frac{y^2}{16}=1,$ **a list of lines given in List-I are to be matched with their equations given in List-II**

List-I	**List-II**
(i) Directrix corresponding to the focus $(-3,0)$	**(a)** $y=4$
(ii) Tangent at the vertex $(0,4)$	**(b)** $3x=25$
(iii) latus rectum through $(3,0)$	**(c)** $x=3$
(iv) $(x^2+1)\frac{dy}{dx}+4xy=\ln x$	**(d)** $y+4=0$
	(e) $x+3=0$
	(f) $3x+25=0$

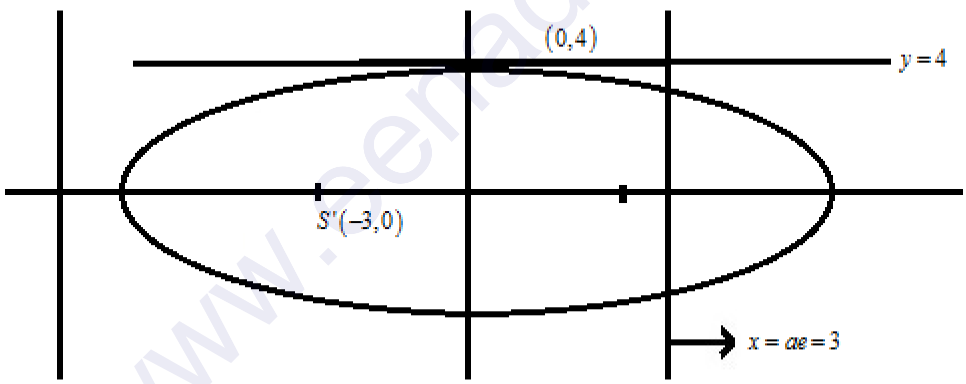
The correct matching is

	(i)	(ii)	(iii)
(1)	(b)	(a)	(e)
(2)	(f)	(a)	(c)
(3)	(b)	(d)	(c)
(4)	(f)	(a)	(e)

Key: 2

Sol: $e=\sqrt{1\frac{-16}{25}}=\frac{3}{5}$

$S=(ae,0)=(3,0)$



$x=\frac{-a}{e}$ $\quad=\frac{-5}{\frac{3}{5}}$

$=\frac{-25}{3}$ $\quad 3x+25=0$

(i) $\longrightarrow f$

(ii) $y=4\longrightarrow a$

(iii) $x=3\longrightarrow c$

39. The centre of the ellipse $\frac{(x+y-3)^2}{9}+\frac{(x-y+1)^2}{16}=1$ **is**

1) $(-1,2)$ 2) $(1,-2)$ 3) $(-1,-2)$ 4) $(1,2)$

Key: 4

Sol: $\left.\begin{array}{l} x+y-3=0 \\ x-y+1=0 \end{array}\right\}$

$\frac{x}{1-3}=\frac{y}{-3-1}=\frac{1}{-1-1}$

Centre $=(1,2)$

40. The product of length of perpndicular from any point on the hyperbola $x^2-y^2=16$ **to is symptoes is**

1) 2 2) 4 3) 8 4) 16

Key: 3

Sol: $P=(4\sec\theta, 4\tan\theta)$

$x+y=0$

$d_1=\frac{4(\sec\theta+\tan\theta)}{\sqrt{2}}$

$x-y=0$

$d_2=\frac{4(\sec\theta-\tan\theta)}{\sqrt{2}}$

$d_1d_2=\frac{16(1)}{2}=8$

41. $A(4,3,5), B(0,-2,2)$ **and** $C(3,2,1)$ **are three points. The coodinates of the point in which the bisector of** $\angle BAC$ **meets the side** $\overline{BC}$ **is**

1) $\left(\frac{15}{8},\frac{4}{8},\frac{11}{8}\right)$ 2) $\left(\frac{12}{7},\frac{2}{7},\frac{10}{7}\right)$ 3) $\left(\frac{9}{5},\frac{2}{5},\frac{7}{5}\right)$ 4) $\left(\frac{3}{2},0,\frac{3}{2}\right)$

Key: 1

Sol: 'D' divide BC is ratio AB:AC

$AB=\sqrt{16+25+9}=\sqrt{50}=5\sqrt{2}$

$AC=\sqrt{1+1+16}=\sqrt{18}=3\sqrt{2}$

$AB:AC=5:3$

$D=\left[\frac{15+0}{8},\frac{10-6}{8},\frac{5+6}{8}\right]$

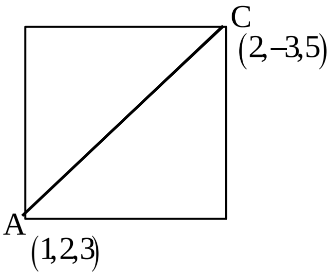
$=\left(\frac{15}{8},\frac{4}{8},\frac{11}{8}\right)$

42. If the extremities of diagonal of a square are (1,2,3) and (2,-3,5), then its side of length

1) $\sqrt{6}$ 2) 15 3) $\sqrt{15}$ 4) 3

Key: 3

Sol:



$\sqrt{2}\times side=\sqrt{1+25+4}$

$=\sqrt{30}$ $side=\sqrt{15}$

43. A plane meets the coordinate axes in P,Q,R respectively. If the centroid of ΔPQR is $\left(1,\frac{1}{2},\frac{1}{3}\right)$, then teh equation of the plane is

1) $2x+4y+3z=5$ 2) $x+2y+3z=3$ 3) $x+4y+6z=5$ 4) $2x-2y+6z=3$

Key: 2

Sol: $(a,b,c)=\left(3,\frac{3}{2},1\right)$

$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

$\frac{x}{3}+\frac{2y}{3}+\frac{z}{1}=1$

$\Rightarrow x+2y+3z=3$

44. $\lim_{x\to 0}\left[\tan\left(x+\frac{\pi}{4}\right)\right]^{1/x}=$

1) e^2 2) e 3) $e^{3/2}$ 4) e^{-1}

Key: 1

Sol: $\underset{x\to 0}{lt}\left[Tan\left(x+\frac{\pi}{4}\right)\right]^{\frac{1}{x}}$

$=e^{\underset{x\to 0}{lt}\frac{1}{x}\left[Tan\left(x+\frac{\pi}{4}\right)-1\right]}$

$=e^{\underset{x\to 0}{lt}\frac{\sec^2\left(x+\frac{\pi}{4}\right)}{1}}$

$=e^2$

45. The value that should be assigned to $f(0)$ so that the function $f(x)=(x+1)^{\cot x}$ is continuous at $x=0$, is

1) e 2) 1 3) 2 4) e^{-1}

Key: 1

Sol: $\underset{x\to 0}{lt} f(x)=f(x)=f(0)$

$\Rightarrow f(0)\underset{x\to 0}{lt}(1+x)^{\cot x}$

$=e^{\underset{x\to 0}{lt}\cot x\times x}$

$=e^{\underset{x\to 0}{lt}\frac{x}{Tanx}}=e^1=e$

46. If $f(x)$ is a real function defined on $[-1,1]$, then the real function $g(x)=f(5x+4)$ is defined on the interval

1) $[-4,9]$ 2) $[-1,9]$ 3) $[-2,9]$ 4) $[-3,9]$

Key: 2

Sol: $-1\le 5x+4\le 1$

$-1\le x\le \frac{-3}{5}$

$-1\le x\le 1$

$-5+4\le 5x+4\le 9$

$[-1,9]$

47. If $f:N\to R$ si defined by $f(1)=-1$ and $f(n+1)=3f(n)+2$ for $n>1$ then f is

1) one-one 2) onto 3) a constant function 4) $f(n)>0$ for $n>1$

Key: 3

Sol: Constant function

48. The remainder of $n^4-2n^3-n^2+2n-26$ when divided by 24 is

1) 20 2) 21 3) 22 4) 23

Key: 3

Sol: $n=1\Rightarrow -26+22-22$

$\Rightarrow -48+22\Rightarrow 24(-2)+22$

Remainder = 22

49. $A(x)=\begin{vmatrix}1 & 2 & 3\\ x+1 & 2x+1 & 3x+1\\ x^2+1 & 2x^2+1 & 3x^2+1\end{vmatrix}$ then $\Rightarrow \int_0^1 A(x)dx=$

1) 0 2) 1 3) 2 4) 4

Key: 1

Sol: $A(x)=\begin{vmatrix}1 & 2 & 3\\ x+1 & 2x+1 & 3x+1\\ x^2+1 & 2x+1 & 3x+1\end{vmatrix}$

$c_2 \to c_2 - c_1, c_3 \to c_3 - c_2$

$$A(x)=\begin{vmatrix}1 & 1 & 1\\ x+1 & x & x\\ x+1 & x^2 & x^2\end{vmatrix}=0$$

50. Let $\begin{vmatrix}x^2+x+1 & x+1 & 2x-3\\ 3x^2 & x+2 & x-1\\ x^2+5x+1 & 2x+3 & x+4\end{vmatrix}=ax^4+bx^3+cx^2+dx+e$ **be an identify in x. If a,b,c, are known, then the value of e is**

1) 29 2) 24 3) 16 4) 9

Key: 1

Sol: $put x = 0$

$$\begin{vmatrix}1 & 1 & -3\\ -1 & 2 & -1\\ 1 & 3 & 4\end{vmatrix}=e$$

$e = 29$

51. The system of equations

$4x + y + 2z = 5$

$x - 5y + 3z = 10$

$9x - 3y + 7z = 20$

has

1) no solution 2) unique solution

3) two solutions 4) Infinite number of solution

Key: 4

Sol: $\begin{vmatrix}4 & 1 & 2\\ 1 & -5 & 3\\ 9 & -3 & 7\end{vmatrix}=4(-35+9)-1(7-27)+2(-3+45)=0$

$$\begin{bmatrix}1 & -5 & 3 & 10\\ 4 & 1 & 2 & 5\\ 9 & -3 & 7 & 20\end{bmatrix}$$

$R_2 \to R_2 - 4R_1$

$R_3 \to R_3 - 9R_1$

$$\begin{bmatrix} 1 & -5 & 3 & 10 \\ 0 & 21 & -10 & -35 \\ 0 & 42 & -20 & -70 \end{bmatrix}$$

Infinite meny solutions

52. If $1, w, w^2$ **are the cube roots of unity and if** $\alpha = w + 2w^2 - 3$ **then** $\alpha^3 + 12\alpha^2 + 48\alpha + 3 =$

1) -63 2) -62 3) -61 4) -60

Key: 4

Sol: $\alpha = \omega + \omega^2 + \omega^2 - 3$

$= -1 + \omega^2 - 3$

$= \omega^2 - 4$

$\alpha + 4 = \omega^2$ cubing on both sides

$\alpha^3 + 12\alpha^2 + 48\alpha = -63$

$\alpha^3 + 12\alpha^2 + 48\alpha + 3 = -63 + 3 = -60$

53. If α, β **are the roots of** $1 + x + x^2 = 0$ **then the value of** $\alpha^4 + \beta^4 + \alpha^{-4}\beta^{-4} =$

1) 0 2) 1 3) -1 4) 2

Key: 1

Sol: ω, ω^2 are roots

$= \alpha^4 + \omega^8 + \dfrac{1}{\alpha^4\beta^4}$

$= \omega^4 + \omega^8 + \dfrac{1}{\omega^{12}}$

$= \omega + \omega^2 + 1$

$= 0$

54. If α, β **are roots of the equation** $x^2 + 4x + 8 = 0$ **then for any** $n \in N, \alpha^{2n} + \beta^{2n} =$

1) $2^{2n+1}\cos\dfrac{n\pi}{2}$ 2) $2^{3n}\cos\dfrac{n\pi}{2}$ 3) $2^{3n+1}\cos\dfrac{n\pi}{2}$ 4) $2^{3n}\cos\dfrac{n\pi}{4}$

Key: 3

Sol: $x^2 - 4x + 8 = 0$

$x = \dfrac{4 \pm \sqrt{16 - 4.8}}{2}$

$= \dfrac{4 \pm 4x}{2}$

$= 2 \pm 2i$

$= 2(1 \pm i)$

$\alpha = 2(1+i) = 2\sqrt{2}\left[cis\dfrac{\pi}{4}\right]$

$\beta = 2(1-i) = 2\sqrt{2}\left[cis\left(\frac{-\pi}{4}\right)\right]$

$\alpha^{2n}+\beta^{2n} = \left(2\sqrt{2}\right)\left[cis\left(\frac{2n\pi}{4}\right)\right]+cis\left(\frac{-2n\pi}{4}\right)$

$= 2^{3n+1}\cos\left(\frac{n\pi}{2}\right)$

55. If α,β are the non-real cube roots of 2 then $\alpha^6+\beta^6 =$

1) 8 2) 4 3) 2 4) 1

Key: 1

Sol: $x = 2^{\frac{1}{3}}$

$x = 2^{\frac{1}{3}}, 2^{\frac{1}{3}}\omega, 2^{\frac{1}{2}}\omega^2$

$\alpha = 2^{\frac{1}{3}}\omega \quad \beta = 2^{\frac{1}{3}}\omega^2$

$\alpha^6 = 4 \quad \beta^6 = 4$

$\alpha^6+\beta^6 = 8$

56. Let $\alpha \ne \beta$ satisfy $\alpha^2+1=6\alpha, \beta^2+1=6\beta$. then, the qudratic equation whose roots are $\frac{\alpha}{\alpha+1}, \frac{\beta}{\beta+1}$ is

1) $8x^2+8x+1=0$ 2) $8x^2-8x-1=0$ 3) $8x^2-8x+1=0$ 4) $8x^2+8x-1=0$

Key: 3

Sol: $\alpha+\beta=6, \alpha\beta=1$

$S.R \quad \frac{\alpha}{\alpha+1}+\frac{\beta}{\beta+1} = \frac{2(\alpha\beta)+(\alpha+\beta)}{\alpha\beta+(\alpha+\beta)+1}$

$= \frac{2+6}{1+6+1} = \frac{1}{8} = 1$

$P.R \quad \frac{\alpha\beta}{\alpha\beta+(\alpha+\beta)+1} = \frac{1}{1+6+1} = \frac{1}{8}$

$8x^2-8x+1=0$

57. The set of solutions of $|x|^2-5|x|+4<0$ is

1) $(-4,-1)$ 2) $(1,4)$ 3) $(-4,-1)\cup(1,4)$ 4) $(-4,4)$

Key: 3

Sol: $|x|^2-5|x|+4<0$

$\Rightarrow (|x|-1)(|x|-4)<0$

$If\ x>0 \Rightarrow 1<+x<4 \quad x\in(1,4)$

$x < 0 \quad \Rightarrow -4 < x < -1 \quad x \in (-4, -1)$

58. Let α, β, γ **be the roots of** $x^3 + x + 10 = 0$ **,write** $\alpha_1 = \frac{\alpha+\beta}{\gamma^2}, \beta_1 = \frac{\beta+\gamma}{\alpha^2}, \gamma 1 = \frac{\gamma+\alpha}{\beta^2}$ **, then the value of** $\left(\alpha_1^3 + \beta_1^3 + \gamma_1^3\right) - \frac{1}{10}\left(\alpha_1^2 + \beta_1^2 + \gamma_1^2\right)$ **is**

1) $\frac{1}{10}$ 2) $\frac{1}{5}$ 3) $\frac{3}{10}$ 4) $\frac{1}{2}$

Key : 3

Sol : $\alpha + \beta + \gamma = 0$

$\alpha_1 = -\frac{1}{\gamma}, \beta_1 = -\frac{1}{\alpha}, r_1 = -\frac{1}{\beta}$

$\left(\alpha_1^3 + \beta_1^3 + r_1^3\right) - \frac{1}{10}\left(\alpha_1^2 + \beta_1^2 + \gamma_1^2\right)$

$= -\left(\frac{1}{\alpha^3} + \frac{1}{\beta^3} + \frac{1}{\gamma^3}\right) - \frac{1}{10}\left(\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}\right)$

$= -\left(\frac{\alpha^3\beta^3 + \beta^3\gamma^3 + \gamma^3\alpha^3}{(\alpha\beta\gamma)^3}\right) - \frac{1}{10}\left(\frac{\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2}{(\alpha\beta\gamma)^2}\right)$

$= \frac{3}{10}$

59. Suppose α, β, γ **are the roots of** $x^3 + x^2 + x + 2 = 0$ **then the value of** $\left(\frac{\alpha+\beta-2\gamma}{\gamma}\right)\left(\frac{\beta+\gamma-2a}{\alpha}\right)\left(\frac{\gamma+\alpha-2\beta}{\beta}\right)$ **is**

1) $-\frac{47}{2}$ 2) $\frac{47}{2}$ 3) -47 4) 47

Key: 1

Sol: $\left(\frac{-1-3\gamma}{\gamma}\right)\left(\frac{-1-3\alpha}{\alpha}\right)\left(\frac{-1-3\beta}{\beta}\right)$

$= -\left[\frac{1-3+9-54}{-2}\right]$

$= -\frac{47}{2}$

60. $\sum_{y=0}^{10}(40-r)_{C_5}=$

1) $41_{C_5}-30_{C_5}$ 2) $41_{C_6}-30_{C_6}$ 3) $41_{C_5}+30_{C_5}$ 4) 41_{C_6}

Key: 2

Sol: $40_{C_5}+39_{C_5}+38_{C_5}+38_{C_5}+...+30_{C_5}$

$=\left[\left(30_{C_6}+30_{C_5}\right)+31_{C_5}+32_{C_5}+....\right]-30_{C_6}$

$=41_{C_6}-30_{C_6}$

61. The number of diagonal of a regular polyon is 35. Then the number of sides of the polygon is

1) 12 2) 54 3) 27 4) 11

Key: 3

Sol: $\frac{n(n-3)}{2}=35$

$n=10$

62. $x=1+\frac{3}{11}\times\frac{1}{6}+\frac{3\times7}{2!}\left(\frac{1}{6}\right)^2+\frac{3\times7\times11}{3!}\left(\frac{1}{6}\right)^+......\Rightarrow x^4=$

1) 81 2) 54 3) 27 4) 8

Key: 3

Sol: $=1+\frac{p}{\lfloor 1}\left(\frac{x}{q}\right)+\frac{p(p+q)}{\lfloor 2}\left(\frac{x}{q}\right)^2$

$p=3,\frac{x}{q}=\frac{1}{6},q=4$

$p=3,\frac{x}{q}=\frac{1}{6},q=4$

$x^4=3^3=27$

63. If $|x|$ is so small so that x^2 and higher powers of x may be neglected, then an approximate value of $\frac{\left(1+\frac{2}{3}x\right)^{-3}(1-15x)^{-1/5}}{(2-3x)^4}$ is

1) $\frac{1}{8}(1+7x)$ 2) $\frac{1}{16}(1-7x)$ 3) $1-7x$ 4) $\frac{1}{16}(1+7x)$

Key: 4

Sol: $\frac{(1-2x)(1+3x)}{(1+x)}(2)^{-4}\left(1-\frac{3x}{2}\right)^{-4}$

$=\frac{1}{16}[1+6x+x]=\left(\frac{1+7x}{16}\right)$

64. The coefficient of x^n **in the expression of** $\frac{1}{x^2-5x+6}$ **for** $|x|<1$ **is**

1) $\frac{1}{2^{n-1}}-\frac{1}{3^{n-1}}$ 2) $\frac{1}{2^{n+2}}-\frac{1}{3^{n+2}}$ 3) $\frac{1}{2^{n+1}}-\frac{1}{3^{n+1}}$ 4) $\frac{1}{2^n}-\frac{1}{3^n}$

Key: 3

Sol: $\frac{1}{(x-2)(x-3)}=\frac{1}{(x-3)}-\frac{1}{(x-2)}$

$=(2-x)^{-1}-(3-x)^{-1}$

$=\frac{1}{2}\left[1-\frac{x}{2}\right]^{-1}-\frac{1}{3}\left[1-\frac{x}{3}\right]^{2}$

Coeff of $x^2=\frac{1}{2}\left[\left(\frac{1}{2}\right)^2\right]-\left(\frac{1}{3}\right)^n$

$=\frac{1}{2^{n+1}}-\frac{1}{3^{n+1}}$

65. In a ΔABC, **the value of** $\angle A$ **is obtained from the equation** $3\cos A+2=0$. **The quadratic equation. whose roots are** $\sin A$ **and** $\tan A$ **is**

1) $3x^2+\sqrt{5x}-5=0$ 2) $6x^2-\sqrt{5x}-5=0$ 3) $6x^2+\sqrt{5x}-5=0$ 4) $6x^2+\sqrt{5x}+5=0$

Key: 3

Sol: $CosA=\frac{2}{3}, TanA=-\frac{\sqrt{5}}{2}, SinA=\frac{\sqrt{5}}{3}, \quad A is \quad II\ Q$

$x^2-\left(\frac{\sqrt{5}}{3}-\frac{\sqrt{5}}{2}\right)x+\frac{-5}{6}=0$

$\Rightarrow 6x^2+\sqrt{5}x-5=0$

If A is IIIQ

$x^2-\left(\frac{\sqrt{5}}{2}-\frac{\sqrt{5}}{3}\right)x-\frac{5}{6}=0$

$\Rightarrow 6x^2-\sqrt{5}x-5=0$

66. If $A=\sin^2\theta+\cos^4\theta$, **thn for all values of** θ, A **lies in the interval**

1) $[1,2]$ 2) $\left[\frac{3}{4},1\right]$ 3) $\left[\frac{1}{2},\frac{3}{4}\right]$ 4) $\left[\frac{3}{4},\frac{19}{16}\right]$

Key: 2

Sol: $A=\sin\theta+\cos\theta(1-\sin\theta)$

$=1-\frac{1}{4}\sin^2(2\theta)$

$=1-\frac{1}{4}[0,1]$

$=1-\left[0,\frac{1}{4}\right]$

$=\left(\frac{3}{4},1\right)$

67. In a $\Delta ABC, \angle C=\frac{\pi}{3}$, **then** $\frac{3}{a+b+c}-\frac{1}{a+c}=$

1) $\frac{1}{a+b}$ 2) $\frac{1}{b+c}$ 3) $\frac{1}{2a+b}$ 4) $\frac{1}{b+2c}$

Key: 2

Sol: $\frac{1}{c+a}+\frac{1}{b+c}=\frac{3}{a+b+c}\Rightarrow \underline{|c}=\frac{\pi}{3}$

68. The number of solution of $\sec x\cos 5x+1=0$ **in the interval** $[0,2\pi]$ **is**

1) 5 2) 8 3) 10 4) 12

Key: 2

Sol: $\frac{cos5x+cosx}{\cos x}=\frac{2\cos 3x\cos 2x}{\cos x}=0$, $2\frac{\left(4\cos^3 x-3\cos x\right)}{\cos x}\left(2\cos^2 x-1\right)=0$

$4\cos^2 x-3=0$

$\cos x=\frac{\sqrt{3}}{2}$ $\cos x=\frac{-\sqrt{3}}{2}$

$x=\frac{\pi}{6},2\pi-\frac{\pi}{6}$ $x=\frac{5\pi}{6}$

$\frac{\pi}{6},\frac{11\pi}{6}$ $=\frac{5\pi}{6}$

$2\cos^2 x-1=0$

$\cos x=\frac{1}{\sqrt{2}}$ $\cos x=\frac{-1}{\sqrt{2}}$

$x=\frac{\pi}{4}$ $x=\frac{-\pi}{4}$

Total number of solution $=8$

69. If $\cos\left(\cot^{-1}\left(\frac{1}{2}\right)\right)=\cot\left(\cos^{-1}x\right)$, **hen a value of x is**

1) $\frac{1}{\sqrt{6}}$ 2) $\frac{-1}{\sqrt{12}}$ 3) $\frac{2}{\sqrt{6}}$ 4) $\frac{-2}{\sqrt{6}}$

Key: 1

Sol: $\cos\left(\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)=\cot\left[\cot^{-1}\frac{(x)}{\sqrt{1-x^2}}\right]$

$\Rightarrow \frac{1}{\sqrt{5}}=\frac{x}{\sqrt{1-x^2}}$ squaring on both sides

$\Rightarrow x=\pm\frac{1}{\sqrt{6}}$

70. If $\cosh 2x = 199$**, then** $\coth x =$

1) $\frac{5}{3\sqrt{11}}$ 2) $\frac{5}{6\sqrt{11}}$ 3) $\frac{7}{3\sqrt{11}}$ 4) $\frac{10}{3\sqrt{11}}$

Key: 4

Sol: $\cosh^2 x-\sinh^2 x=1$

$\cosh^2 x+\sinh^2 x=199$

on solving above to equations

$\Rightarrow \cosh x=10$

$\Rightarrow \sinh x=3\sqrt{11}$

$\Rightarrow \coth x=\frac{10}{3\sqrt{11}}$

71. The angles of a triangle ABC are in an arithmetic progression. The larger sides a,b satisfy the relation $\frac{\sqrt{3}}{2}<\frac{b}{a}<1$**, then the possible values of the smallest side are**

1) $\frac{a\pm\sqrt{4b^2-3a^2}}{2a}$ 2) $\frac{a\pm\sqrt{4b^2-3a^2}}{2b}$ 3) $\frac{a\pm\sqrt{4b^2-3a^2}}{2c}$ 4) $\frac{a\pm\sqrt{4b^2-3a^2}}{2}$

Key: 4

Sol: $a^2+c^2+b^2=ac$

$c^2-ac+\left(a^2-b^2\right)=0$

$c=\frac{a\pm\sqrt{a^2-4\left(a^2-b^2\right)}}{2}$

$=\frac{1\pm\sqrt{4b^2-3a^2}}{2}$

72. $\frac{1}{r^2}+\frac{1}{r_1^2}+\frac{1}{r_2^2}+\frac{1}{r_3^2}=$

1) $\frac{a^2+b^2+c^2}{\Delta}$ 2) $\frac{a^2+b^2+c^2}{\Delta}$ 3) $\frac{\Delta^2}{a^2+b^2+c^2}$ 4) $\frac{\Delta}{a^2+b^2+c^2}$

Key: 2

Sol: $\frac{1}{r^2}+\frac{1}{r_1^2}+\frac{1}{r_2^2}+\frac{1}{r_3^2}=\frac{a^2+b^2+c^2}{\Delta^2}$

Conceptual

73. If in a $\Delta ABC, r_1 = 2r_2 = 3r_3$, then $b:c=$

1) $4:3$ 2) $5:4$ 3) $2:1$ 4) $3:2$

Key: 1

Sol: $lr_1 = mr_2 = nr_3$

$\therefore a:b:c := (m+n):(n+l):(l+m)$

$=5:4:3$

$\therefore b:c = 4:3$

74. P is the point of intersection of the diagonals of the parallelogram ABCD. If is any point in the space and $\overline{SA}+\overline{SB}+\overline{SC}+\overline{SD}=\lambda\overline{SP}$, then $\lambda=$

1) 2 2) 4 3) 6 4) 8

Key: 2

Sol: $\therefore P$ is

$\overline{OP}=\frac{\overline{OA}+\overline{OC}}{2}$

$\overline{OP}=\frac{\overline{OB}+\overline{OD}}{2}$

$\overline{OA}+\overline{OC}=2(\overline{OP})$

$\overline{OB}+\overline{OD}=2(\overline{OP})$

$\overline{OA}+\overline{OB}+\overline{OC}+\overline{OD}=4(\overline{OP})$

75. If M and N are the mid points of the side BC and CD respectively of a parallelogram ABCD, the n $\overline{AM}+\overline{AN}=$

1) $\frac{4}{3}\overline{AC}$ 2) $\frac{5}{3}\overline{AC}$ 3) $\frac{3}{2}\overline{AC}$ 4) $\frac{6}{5}\overline{AC}$

Key: 3

Sol: $\overline{AM}=\overline{AB}+\frac{1}{2}\overline{BC}$

$\overline{AN}=\overline{AD}+\frac{1}{2}\overline{DC}$

$\overline{AM}+\overline{AN}=\overline{AB}+\frac{1}{2}\overline{BC}+\overline{BC}+\frac{1}{2}\overline{AB}$

$=\frac{3}{2}(\overline{AB}+\overline{BC})$

$=\frac{3}{2}\overline{AC}$

76. ABCD is a parallelogram and P is a point on the segment $\overline{AD}$ dividing it internally in the ratio $3:1$. The line $\overline{BP}$ meets the diagonal AC in Q. Then $AQ:QC=$

1) 3 : 4 2) 4 : 3 3) 3 : 2 4) 2 : 3

Key: 1

Sol: $\overline{AB}=\overline{a}$

$\overline{BC}=\overline{b}$

$\overline{AC}=\overline{a}+\overline{b}$

$\overline{AD}=\overline{b}$

$\overline{AP}=\frac{3}{4}\left(\overline{b}\right)$

77. The position vectors of the vertices of ΔABC are $3\overline{i}+4\overline{j}+\overline{k},\overline{3j}+\overline{k},5\left(\overline{i}+\overline{j}+\overline{k}\right)$ respectively. The magnitude of the altitude from A onto the side BC is

1) $\frac{4}{3}\sqrt{5}$ 2) $\frac{5}{3}\sqrt{5}$ 3) $\frac{7}{3}\sqrt{5}$ 4) $\frac{8}{3}\sqrt{5}$

Key: 1

Sol: $\overline{AB}=\left(2,1,-2\right)$

$\overline{BC}=\left(4,2,4\right)$

$BD=\frac{\overline{AB}.\overline{BC}}{\left|\overline{BC}\right|}$

$\frac{\left|8+2-8\right|}{\sqrt{16+4+16}}$

$=\frac{2}{6}=1/3$

$AD^2=9-\frac{1}{9}$

$=\frac{81-1}{9}$

$=\frac{80}{9}$

$AD=\frac{\sqrt{80}}{\sqrt{9}}=\frac{4\sqrt{5}}{3}$

78. The shortest distance between the skew lines $\frac{x-3}{-1}=\frac{y-4}{2}=\frac{z+2}{1},\frac{x-1}{1}=\frac{y+7}{3}=\frac{z+2}{2}$

1) 6 2) 7 3) $3\sqrt{5}$ 4) $\sqrt{35}$

Key: 4

Sol: $(x, y_1, z_1) = (3, 4, -2)$

$(x_2, y_2, z_2) = (1, -7, -2)$

$a, b, c_1 = -1, 2, 1$

$a_2, b_2 c_2 = 1, 3, 2$

$$= \frac{\begin{vmatrix} -2 & -11 & 0 \\ -1 & 2 & 1 \\ 1 & 3 & 2 \end{vmatrix}}{\sqrt{(4-3)^2 + (1+2)^2 + (-3-2)^2}} = \frac{35}{\sqrt{35}} = \sqrt{35}$$

79. If $\bar{a} = 2\bar{i} - 3\bar{j} + 5\bar{k}, \bar{b} = 3\bar{i} - 4\bar{j} + 5\bar{k}$ and $\bar{c} = 5\bar{i} - 3\bar{j} - 2\bar{k}$, then the volume of the parallelopiped with co-terminus edges $\bar{a} + \bar{b}, \bar{b} + \bar{c}, \bar{c} + \bar{a}$ is

1) 1 2) 5 3) 8 4) 16

Key: 4

Sol: Volume of paralleopiped

$= 2\left[\bar{a}\bar{b}\bar{c}\right]$

$$= 2\begin{bmatrix} 2 & -3 & 5 \\ 3 & -4 & 5 \\ 5 & -3 & -2 \end{bmatrix}$$

$= 2\left|46 - 93 + 55\right| = 16$

80. in a data the number is is repeated i times for $i = 1, 2, \ldots n$. Then the mean of the data is

1) $\frac{2n+1}{6}$ 2) $\frac{2n+1}{4}$ 3) $\frac{2n+1}{3}$ 4) $\frac{2n+1}{2}$

Key: 3

Sol: Put $n = 2$

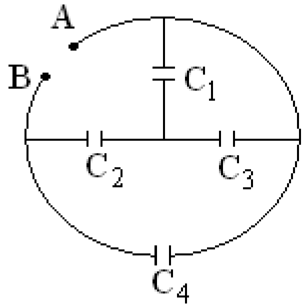
Mean $= \frac{1 + 2(2)}{3} = \frac{5}{3}$

put $n = 3$

Mean $= \frac{1 + 2(2) + 3(3)}{6} = \frac{7}{3}$

PHYSICS

81. In the arrangement of capacitors shown in the figure, if each capacitor is 9 pF, then the effective capacitance between the points A and B is



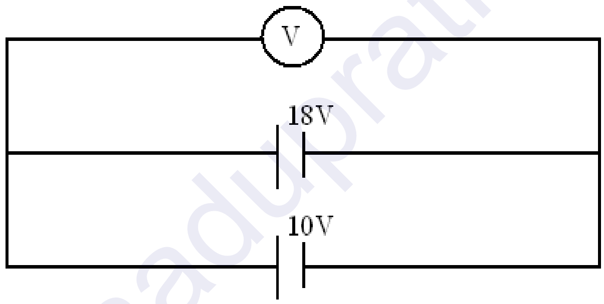
1) 10 pF 2) 15 pF 3) 20 pF 4) 5 pF

Key : 2

Sol : $\left[(C \parallel C) \square C\right] \parallel C$

$= \frac{5C}{3} = \frac{5\times 9}{3} = 15\,pF$

82. A battery of the emf 18 V and internal resistance of 3Ω and another battery of emf 10 V and internal resistance of 1Ω are connected as shown in figure. THen the voltmeter reading is



1) 10 V 2) 12 V 3) 16 V 4) 8 V

Key : 2

Sol : $\frac{\frac{\varepsilon_1}{r_1}+\frac{\varepsilon_2}{r_2}}{\frac{1}{r_1}+\frac{1}{r_2}} = \frac{\frac{18}{3}+\frac{10}{1}}{\frac{1}{3}+\frac{1}{1}}$

$= 12V$

83. A wire of Aluminium and a wire of Germanium are cooled to a temperature of $77^{o}K$. Then

1) resistance of each of them decreases.
2) resistance of each of them increases
3) resistance of Aluminium wire increases and that of Germanium wire decreases
4) Resistance of Aluminium wire decreases and that of Germanium wire increases

Key : 4

Sol : Al has +ve α

& Ge -ve $\propto$

$\therefore$ as T is decreased

R of Al <<

& Ge >>

84. A voltmeter of 250 mV range having a resistance of 10Ω is converted into an ammeter of 250 mA range. The value of necessary shunt is (nearly)

1) 2Ω 2) 0.1Ω 3) 1Ω 4) 10Ω

Key : 3

Sol : $S = \frac{G}{n-1}$

$n = \frac{i}{i_g} \quad ;\left[i_g = \frac{V_g}{G} = \frac{250mV}{10} = 25mA\right]$

$= \frac{250mA}{25mA} = 10$

$\therefore S = \frac{10}{9} = 1.1\Omega$

85. A circular loop and a square loop are formed from two wires of same length and cross section. Same current is passed through them. Then the ratio of their dipole moments is

1) 4 2) $\frac{2}{\pi}$ 3) 2 4) $\frac{4}{\pi}$

Key : 4

Sol : $2\pi r = 4l$

$\Rightarrow \pi r = 2l$

$\therefore \frac{M_{circle}}{M_{square}} = \frac{i\pi r^2}{il^2} = \pi\left(\frac{r}{l}\right)^2$

$= \pi \times \left(\frac{2}{\pi}\right)^2 = \frac{4}{\pi}$

86. At a certain place a magnet makes 30 oscillations per minute. At another place where the magnetic field is doubled, its time period will be

1) $\sqrt{2}\,\text{sec}$ 2) $2\,\text{sec}$ 3) $4\,\text{sec}$ 4) $\frac{1}{2}\,\text{sec}$

Key : 1

Sol : $T = 2\pi\sqrt{\frac{I}{MB}}$

$\therefore T \propto \frac{1}{\sqrt{B}}$

$T_1 = \frac{60}{30} = 2s$

$\therefore T_2 = \sqrt{2}s$

87. A small square loop of wire of side 'ℓ' is placed inside a large square loop of side $L(L>\ell)$. If the loops are coplanar and their centres coincide, the mutual induction of the system is directly proportional to

1) $\frac{\ell}{L}$ $\frac{\ell^2}{L}$ 3) $\frac{\ell}{L^2}$ 4) $\frac{\ell^2}{L^2}$

Key : 2

Sol : $M=\frac{\phi}{i}=\frac{BA}{i}$

$=\frac{4\times B_{one\,side}A}{i}\propto\frac{l^2}{L}$ (as $A\propto l^2\ \&\ B\propto\frac{1}{L}$)

88. In a circuit L, C and R are connected in series with an alternating voltage source of frequency f. When current in the circuit leads the voltage by 45^0, the value of C

1) $\frac{1}{2\pi f(2\pi fL+R)}$ 2) $\frac{1}{2\pi f(2\pi fL+L)}$ 3) $\frac{1}{2\pi f(R+L)}$ 4) $\frac{1}{2\pi f\left(R+\frac{1}{L}\right)}$

Key : 1

Sol : $\tan\phi=\frac{X_L-X_C}{R}=-1$

(as current leads voltage by 45^0)

$\Rightarrow\frac{1}{\omega C}-\omega L=R$

$\Rightarrow\frac{1}{\omega C}=\omega L+R$

$\Rightarrow C=\frac{1}{\omega(\omega L+R)}$

89. Suppose that the electric flux inside a parallel plate capacitor changes at a rate of 7×10^{14} units/ sec, then the magnetic induction field density at any point inside the capacitor is

[Area of the plate of the capacitor = 1 m^2

permittivity of free space $=8.8\times10^{-12}Nm^2c^{-2}$

Permeability of free space $=4\pi\times10^{-7}$ Tesla m/Amp]

1) 7.79×10^{-3} T 2) 0.779×10^{-5} T 3) 8.85×10^{-4} T 4) 88.5×10^{-12} T

Key :

Sol : $\frac{d\phi_E}{dt}=A\frac{dE}{dt}=7\times10^4$

B varies with distance from axis

Hence Question is wrong

90. If an electron has an energy such that its De Broglie wavelength is 5500 $\overset{o}{A}$, then the energy value of that electron is $\left(h = 6.6\times10^{34} Js, m_c = 9.1\times10^{31} kg\right)$.

1) $8\times10^{-20} J$ 2) $8\times10^{-10} J$ 3) $8J$ 4) $8\times10^{-25} J$

Key : 4

Sol : $\lambda = \frac{h}{p}$ and $\frac{p^2}{2m} = E$

$p = \frac{h}{\lambda} = \sqrt{2mE} \Rightarrow E = \frac{h^2}{2m\lambda^2}$

Substitutions the given values

$E = 8\times10^{-25} J$

91. The following statements are given about Hydrogen atom
A) The wavelengths of the spectral lines of Lyman series are greater than the wavelength of the second spectral line of Balmer series
B) The orbits correspond to circular standing waves in which the circumference of the orbit equals a whole number of wavelengths

1) A is false, B is true 2) A is true, B is false 3) A is false, B is false 4) A is true, B is true

Key : 1

Sol : $\lambda_{Lyman} < \lambda_{Balmer}$

$\therefore A$ is false

& $2\pi r_n = n\lambda$

hence B is true

92. A radioactive nucleus can decay by two different processes. The half lives of the first and second decay processes are 5×10^3 and 10^5 years respectively. Then, the effective half-life of the nucleus is

1) 105×10^5 yrs 2) 4762 yrs 3) 10^4 yrs 4) 47.6 yrs

Key : 2

Sol : $\frac{1}{T} = \frac{1}{T_1} + \frac{1}{T_2}$

$= \frac{1}{5\times10^3} + \frac{1}{10^5}$

$\Rightarrow T = \frac{10^5}{21} = 4761.9$ yrs

93. In a half wave rectifier the AC input source of frequency 50 Hz is used. The fundamental frequency of the output is

1) 50 Hz 2) 150 Hz 3) 200 Hz 4) 75 Hz

Key : 1

Sol : For half wave rectifier we get one pulse per cycle in the output

94. If n_e and n_h are electron and hole concentrations in an extrinsic semiconductor and n_i is electron concentration in an intrinsic semiconductor, then

1) $\left(\frac{n_e}{n_h}\right) = n_i$ 2) $(n_e + n_h) = n_i$ 3) $(n_e - n_h) = n_i^2$ 4) $n_e n_h = n_i^2$

Key : 4

Sol : By mass action law

$n_e n_h = n_i^2$

95. A carrier wave of peak voltage 12 volts is used to transmit a signal. If the modulation index is 75%, the peak voltage of the modulating signal is

1) 18 V 2) 22 V 3) 16 V 4) 28 V

Key : No key in the options

Sol : Modulation index

$\mu = \frac{V_m}{V_c}$

$\frac{75}{100} = \frac{V_m}{12} \Rightarrow V_m = 9V$

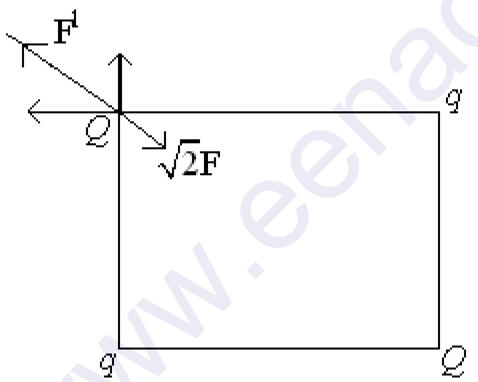
96. Charges 'Q' are placed at the ends of a diagonal of a square and charges 'q' are placed at the other two corners. The condition for the net electric force on 'Q' to be zero is

1) $Q = -2\sqrt{2}q$, q being −ve 2) $Q = -\frac{q}{2}$, q being −ve

3) $Q = 2\sqrt{2}q$, q being −ve 4) $Q = 2q$, q being −ve

Key : 1

Sol :



$F^1 + \sqrt{2}F = 0$

$F^1 = -\sqrt{2}F$

$\frac{1}{4\pi\varepsilon_0}.\frac{Q^2}{\left(\sqrt{2}a\right)^2} = -\sqrt{2}\frac{1}{4\pi\varepsilon_0}.\frac{qQ}{a^2}$

$\frac{Q}{2} = -\sqrt{2}q$

$Q = -2\sqrt{2}q$

97. Through a narrow slit of width 2mm, diffraction pattern is formed on a screen kept at a distance 2 m from the slit. The wavelength of the light used is 6330 Å and falls normal to the slit and screen. Then, the distance between the two minima on either side of the central maximum is

1) 12.7 mm 2) 1.27 mm 3) 2.532 mm 4) 25.3 mm

Key : 2

Sol : $\frac{2\lambda D}{d} = \frac{2 \times 6330 \times 10^{-10} \times 2}{2 \times 10^{-3}}$

$= 1266 \times 10^{-6} = 1.266mm$

98. A convex lens of glass $\left(\mu_g = 1-45\right)$ has a focal length f_a in air. The lens is immersed in a liquid of refractive index $\left(\mu_1\right)$ 1.3. The ratio of the f_{liquid} / f_a is

1) 3.9 2) 0.23 3) 0.43 4) 0.39

Key : 1

Sol : $\frac{1}{f} \propto \left(\frac{\mu_2}{\mu_1} - 1\right)$

$\Rightarrow \frac{f_l}{f_a} = \frac{(1.45-1)}{\left(\frac{1.45}{1.3} - 1\right)} = 3.9$

99. Three thin lenses are combined by placing them in contact with each other to get more magnification in an optical instrument. Each lens has a focal length of 3 cm. If the least distance of distinct vision is taken as 25 cm, the total magnification of the lens-combination in normal adjustment is

1) 9 2) 26 3) 300 4) 3

Key : 2

Sol : $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} = \frac{3}{f} = \frac{3}{3}$

$\Rightarrow f = 1cm$

$M = \frac{D}{f}$ (for normal adjustment)

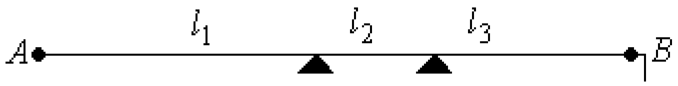
$= \frac{25}{1} = 25$

$M = 1 + \frac{D}{f} = 26$

(when final image is at D)

$\therefore$ nearest

100. A thin wire of length of 99 cm is fixed at both ends as shown in the figure. The wire is kept under a tension and is divided into three segments of lengths l_1, l_2 and l_3 as shown in figure. When the wire is made to vibrate, the segments vibrate respectively with their fundamental frquencies in the ratio $1:2:3$ Then, the lengths l_1, l_2, l_3 of the segments respectively are (in cm)



1) 27, 54, 18   2) 18, 27, 54   3) 54, 27, 18   4) 27, 9, 14

Key : 3

Sol: $l_1 : l_2 : l_3 : \frac{1}{n_1} : \frac{1}{n_2} : \frac{1}{n_3}$

$= \frac{1}{1} : \frac{1}{2} : \frac{1}{3}$

$= 6:3:2$

$\therefore \ l_1 = \frac{6}{11} \times 99 = 54cm$

$l_2 = \frac{3}{11} \times 99 = 27cm$

$l_3 = \frac{2}{11} \times 99 = 18cm$

101. R.M.S velocity of oxygen molecules at N.T.P is 0.5 km/s. The R.M.S veloxity for the hydrogen molecule at N.T.P is

1) 4 km/s   2) 2 km/s   3) 3 km/s   4) 1 km/s

Key: 2

Sol: . $V_{rms} = \sqrt{\frac{3RT}{M}}$

$\therefore V_{rms} \propto \sqrt{\frac{1}{M}}$

$\therefore \frac{V_{O_2}}{V_{H_2}} = \sqrt{\frac{2}{32}} = \frac{1}{4}$

$\therefore V_{H_2} = 4V_{O_2} = 2km/s$

102. 1g of water of at $100^o C$ is completely converted into sream at $100^o C$. 1g steam occupis a volume of 1650 cc. (Neglect the volume of 1g of water at $100^o C$). At the pressure of $10^5 N/m^2$, latent heat of steam is 540 cals/g (1 Caloric =e=4.2 Jolues) . The increase in the internal energy in Joules is

1) 2310   2) 2103   3) 1650   4) 2150

Key:2

Sol: . $Q = 540cal$

$W = P_o (V_2 - V_1)$ $V_1 = 0$

$\therefore W = 10^5 (1650x10^{-6}) = 165J$

$\therefore \Delta U = 540x4.2 - 165 = 2103J$

103. A copper rod of length 75 cm and an iron rod of length 125 cm are joining together end to end. Both are of circular cross section with diameter 2cm. The free ends of the copper and iron are maintained at 100^oC and 0^oC respetively. The surfaces of the bars are insulated thermally the temperature of the copper -iron junction is
[Thermal conductivity of copper is 386.4W/m -K and that of iron is 48.46K]

1) 100^oC 2) 0^oC 3) 93^oC 4) 50^oC

Key: 3

Sol: . $\frac{100-\theta}{R_1}=\frac{\theta-0}{R_2}$

$$\frac{100-\theta}{\frac{L_1}{K_1A_1}}=\frac{\theta-0}{\frac{L_2}{K_2A_2}} \quad (A_1=A_2)$$

$$(100-\theta)\frac{K_1}{L_1}=\theta\frac{K_2}{L_2}$$

$$(100-\theta)\frac{386.4}{0.75}=\theta x\frac{48.46}{1.25}$$

$$\Rightarrow \theta=93^oC$$

104. A thermos flask contains 250g of coffee 90^oC. To this 20g of milk at 5^oC is added. After equilibrium is established , the temperature of the liquid is (Assume no heat loss to the theroms bottle. Take specific heat of coffee and milk as 1.00 cal/g^o C]

1) 3.23^oC 2) 3.17^oC 3) 83.7^oC 4) 37.8^oC

Key: 3

Sol: .$12.5x1x(90-\theta)=20x1(\theta-5)$

$$12.5x90-12.5\theta=2\theta-100$$
$$25x90-25\theta=2\theta-10$$
$$2260=27\theta$$
$$\theta=\frac{2260}{27}=83.7^oC$$

105. 1000 spherical drops of water each 10^{-8} m in diametere coalesce to form one large speherical drop. The amount of energy liberated in this process in Joules is
(surface tension of the water is 0.075N/m)

1) $10.75\pi x10^{-15}$ 2) $6.75\pi x10^{-15}$ 3) $8.65\pi x10^{-15}$ 4) $3.88\pi x10^{-15}$

Key: 2

Sol: . $W=4\pi R^2T\left[n^{\frac{1}{3}}-1\right]$

$$=4x\pi x\left(0.5x10^{-7}\right)^2 x75x10^{-3}[10-1]$$

$$=4\pi x0.25x10^{-14}x75x10^{-3}x9$$

$$=675x\pi x10^{-17}$$

$$=6.75x\pi x10^{-15}J$$

106. When a force F_1 is applied on a metallic wire, the legth of the wire is L_1. If a force F_2 is applied on the same wire, the length of the wire is L_2. The original length of the wire L is

1) $\frac{L_1F_1+L_2F_2}{F_1+F_2}$ 2) $\frac{L_2-L_1}{F_1+F_2}$ 3) $\frac{F_1L_2-F_2L_1}{F_1-F_2}$ 4) $\frac{F_2L_2-F_2L_2}{F_1-F_2}$

Key: 3

Sol: . $F \propto l$

$F_1 \propto (l_1 - l) \Rightarrow (1)$

$F_2 \propto (l_2 - l) \Rightarrow (2)$

$\frac{F_1}{F_2} = \frac{l_1 - l}{l_2 - l}$

$F_1l_2 - F_1l = F_2l_1 - F_2l$

$(F_1 - F_2)l = F_1l_2 - F_2l_1$

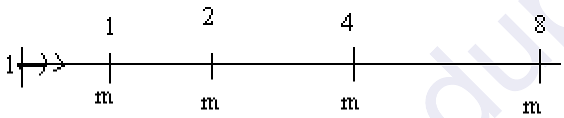
$l = \frac{F_1l_2 - F_2l_1}{F_1 - F_2}$

107. Infinite number of spheres, each of mass m are placed on the X-axis at distance 1,2,4,8,16,..... meters from origin. The magnitude of the gravity field at the origin is

1) $\frac{2}{3}$Gm 2) $\frac{4}{3}$Gm 3) Gm 4) 6Gm

Key: 2

Sol: .



$I = Gm\left[\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{8^2} + ...\right]$

$= Gm\left[\frac{1}{2^o} + \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} + ...\right]$

$= Gm\left[\frac{1}{1-\frac{1}{4}}\right] = \frac{Gm}{\left(\frac{3}{4}\right)} = \frac{4Gm}{3}$

108. A particle of mass 4kg is executing S.H.M. Its displacement is given by the equation Y=8 cos$[100t + \pi/4]$ cm. Its maxium kinetic energy is

1) 128J 2) 64J 3) 16j 4) 32J

Key: 3

Sol: . $K.E_{max} = \frac{1}{2}mV_{max}^2$

$= \frac{1}{2}m\omega^2A^2 \qquad \omega = 100$

$= \frac{1}{2}x4x10^4x64x10^{-4}$

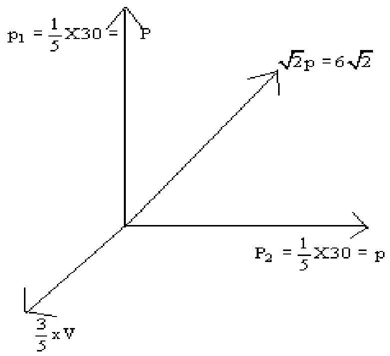
$= 128J$

109. A body of mass 1 kg, initially at rest explodes and breaks into three parts. The masses of the parts are in the ratio 1:1:3. The two pieces of equal mas fly off perpendicular to each other with a speed of 30m/s each. The velocity of the heavier part in m/s is

1) $10\sqrt{2}$ 2) 6 3) 3 4) $6\sqrt{2}$

Key: 1

Sol:



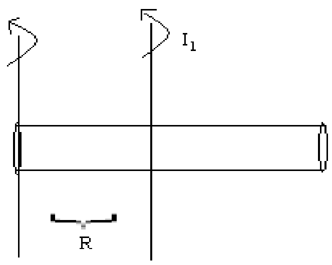
$$\frac{3}{5}v = 6\sqrt{2}$$

$$v = 10\sqrt{2} m/s$$

110. The moment of inertia of a soild cylinder of mass M, length 2R and radius R about an axis passing through the center of mass and perperdicular the to the axis of the cylinder is I_1 and about an axis pasing through one end of the cylinder and perpendicular to the axis of cylinder is I_2, then

1) $I_2 < I_1$ 2) $I_2 - I_1 = MR^2$ 3) $\frac{I_2}{I_1} = \frac{19}{12}$ 4) $\frac{I_2}{I_1} = \frac{7}{6}$

Key:2



Sol: . $I_1 = \frac{M}{12}\left[4R + R^2\right] = \frac{5MR^2}{12} \rightarrow (1)$

$$I_2 = \frac{17MR^2}{12}$$

$$I_2 - I_1 = \frac{17}{12}MR^2 - \frac{5}{12}MR^2$$

$$= \frac{12MR^2}{12} = MR^2$$

$$I_2 = I_1 + MR^2$$

$$= \frac{5MR^2}{12} + MR^2$$

$$= \frac{17MR^2}{12}$$

111. Match the following:.

A	B
a) Rocket propulsion	**e) Bernoulli's principle in fluid dynamics**
b) Aeroplane	**f) Total internal reflection of light**
c) Optical fibers	**g) Newton's laws of motion**
d) Fusion test reactor	**h) Magnetic confinement of plasma**
	i) Photoelectric effect s

	a	b	c	d
1)	g	f	e	h
2)	g	e	f	i
3)	i	e	f	g
4)	g	e	f	h

Key: 4

Sol: .Conceptual

112. Force F is given by the equation $F = \frac{X}{\text{Linear density}}$ **. Then dimension of X are**

1) $M^2L^0T^{-2}$ 2) $M^0L^0T^{-1}$ 3)- L^2T^{-2} 4) $M^0L^2T^{-2}$

Key: 1

Sol: .X=F x linear density

$= MLT^{-2} x \frac{M}{L}$

$= M^2T^{-2}$

113. The displacment of a particle moving in a stright line is given by the expression $x = At^2 + Bt^2 + Ct + D$ **in meters, where t is in seconds and A, B, C and D are constants. The ratio beteween the initial acceleration and initial velocity is**

1) $\frac{2C}{B}$ 2) $\frac{2B}{C}$ 3) 2C 4) $\frac{C}{2B}$

Key: 2

Sol: . $x = At^3 + Bt^2 + Ct + D$

$v = 3At^2 + 2Bt + C$

at $t = 0$ $v = c$

at $t = 0$ $a = 2B$

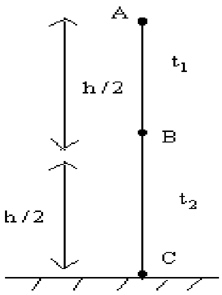
$\therefore \frac{a}{v} = \frac{2B}{C}$

114. A,B,C are pointed in a verticle line AB=BC. If a body falls freely from rest at A, and t_1 **and** t_2 **are time to travel distance AB and BC, then ratio** (t_2 / t_1) **is**

1) $\sqrt{2}+1$ 2) $\sqrt{2}-1$ 3) $2\sqrt{2}$ 4) $\frac{1}{\sqrt{2}+1}$

Key:2,4

Sol: .



$$\frac{t_2}{t_1}=\frac{\sqrt{\frac{2h}{g}}-\sqrt{\frac{2}{g}\left(\frac{h}{2}\right)}}{\sqrt{\frac{2h}{g}\left(\frac{h}{2}\right)}}$$

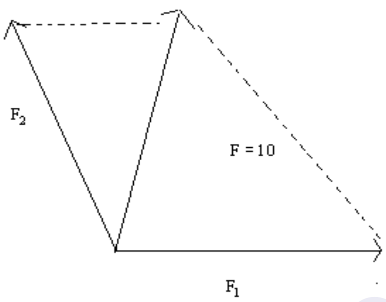
$$=\sqrt{2}-1$$

115. Sum of magnitude of two forces is 25N. The resultant of these forces is normal to the smaller force and has a magnitude of 10N. Then the force are

1) 14.5N,10.5N 2) 16N,9N 3) 13N, 12N 4) 20N,5N

Key: 1

Sol: .



$$F=F_1+F_2=25N\left(F_1<F_2\right)$$

$$F^2+F_1^2=F_2^2$$

$$\Rightarrow F_2^2-F_1^2=100$$

$$\Rightarrow\left(F_2-F_1\right)\left(F_2+F_1\right)=100$$

$$\Rightarrow F_2-F_1=4$$

$$\therefore F_2=14.5N \;\&\; F_1=10.5N$$

116. A body of mass 'm' thrown up vertically with velocity v_1 reaches a maximum height h_1 in t_1 seconds. Another body of mass 2 m is projected with a velocity v_2 at an angle θ. The second body reaches a maximum height h_2 in time t_2 seconds. If t_1 seconds . If $t_1=2t_2$, ratio $\left(\frac{h_1}{h_2}\right)$ is

1) 1:2 2) 4:1 3) 1:1 4) 3:2

Key:2

Sol: . $h_1 = \frac{V_1^2}{2g}$ & $t_1 = \frac{v_1}{g}$

$h_2 = \frac{V_2^2 \sin^2\theta}{2g}; \& t_2 = \frac{v_2 \sin\theta}{g}$

Now $t_1 = 2t_2$

$\frac{v_1}{g} = \frac{2v_2 \sin\theta}{g}$

$\therefore \frac{h_1}{h_2} = \frac{v_1^2}{v_2^2 \sin^2\theta} = 4$

117. Hammer of mass M stricks a nail of mass 'm' with a velocity 20m/s into a fixed wall. The nail penetraction into the wall to a depth of 1cm. The average resistance of the wall to the penetration of the nail is

1) $\left(\frac{M^2}{M+m}\right) x10^3$ 2) $\frac{2M^2}{M+m} x10^4$ 3) $\frac{M+m}{M^2} x10^2$ 4) $\frac{M^2}{M+m} x10^2$

Key:2

Sol: .By conservation of momentum

Mx20=(M+m) V

By work energy therom, $\frac{1}{2}(M+m)V^2 = fx1cm$

$f = \frac{M^2}{M+m} x2x10^4 N$

118. A body of mass 10 kg is acted upon by a given by equation F=$\left(3t^2 - 30\right)$ Newtons. The initial velocity of the body is 10m/s. The velocity of the body after 5sec. is

1) 4.5m/s 2) 6m/s 3) 7.5m/s 4) 5m/s

Key:3

Sol: .By Impulse equation

$\int Fdt = mv - mu$

$\Rightarrow \int_0^5 \left(3t^2 - 30\right)dt = 10(v - 10)$

$\Rightarrow \left[\frac{3t^3}{3} - 30t\right]_0^5 = 10v - 100$

$\Rightarrow V = 7.5ms^{-1}$

119. A ball (initially at rest) is released from the top of a tower. the ratio of work done by the force of gravity in the first, second and thitd seconds is

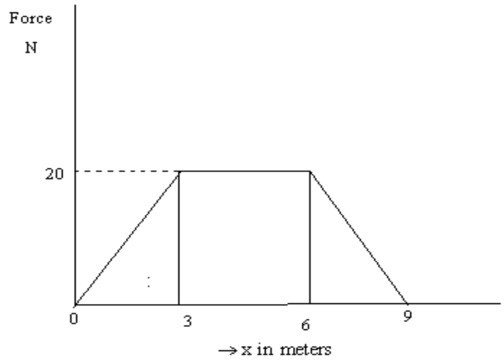
1) 1:3:5 2) 1:4:16 3) 1:9:25 4) 1:2:3

Key:1

Sol: . $W_1 : W_2 : W_3 = mg.s_1 : mg.s_2 : mgs_3$

$\Rightarrow W_1 : W_2 : W_3 = S_1 : S_2 : S_3 = 1:3:5$

120. A body of mass 2.4kg is subjecteed to a force which varies with distance as shown in the figure. The body starts from rest at x=0. Its velcoity at x=9m is



1) $5\sqrt{3}m/sec$ 2) $20\sqrt{3}m/sec$ 3) $10m/sec$ 4) $40m/sec$

Key: 3

Sol: . Work done =Area under the curve

By work -energy therom

$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$\Rightarrow 120 = \frac{1}{2} x 2.4 x v^2$

$v = 10m/s$

CHEMISTRY

121. What is the weight (in g) of Na_2CO_3 (molar mass=106) present in 250ml. of its 0.2M solution?

1) 0.53 2) 5.3 3) 1.06 4) 10.6

Key: 2

Sol: . $0.2 = \frac{x}{106} \times \frac{\cancel{1000}^{4}}{\cancel{250}}$

$x = \frac{0.2 \times 106}{4} = 0.1 \times 53 = 5.3g$

122. An aqueous dilute solution containing non-volatile solute boils at 100.052 °C. What is the molality of solution? ($K_b = 0.52\,kg.mol^{-1}.K$; boiling temperature of water= $100°C$)

1) 0.1 m 2) 0.01 m 3) 0.001 m 4) 1.0 m

Key: 1

Sol: $0.052 = 0.52 \times m$

$m = \frac{0.052}{0.52} = 0.1$

123. A lead storage battery is discharged. During the charging of this battery, the reaction that occurs at node is

1) $PbSO_4(s)+2e^- \rightarrow Pb(s)+SO_4^{2-}(aq)$

2) $PbSO_4(s)+2H_2O(l) \rightarrow PbO_2(s)+SO_4^{2-}(aq)+4H^+(aq)+2e^-$

3) $PbSO_4(s) \rightarrow Pb^{2+}(aq)+SO_4^{2-}(aq)$

4) $PbSO_4(s)+2H_2O(l)+2e^- \rightarrow PbO_2(s)+SO_4^{2-}(aq)+2H^+(aq)$

Key: 2

Sol: .Conceptual

124. For the reaction $5Br^-(aq)+BrO_3^-(aq)+6H^+(aq) \rightarrow 3Br_2(aq)+3H_2O(l)$

if, $-\frac{\Delta[Br^-]}{\Delta t}=0.05l\,L^{-1}min^{-1}$

1) 0.005 2) 0.05 3) 0.5 4) 0.01

Key: 4

Sol: . $-\frac{1}{5}.\frac{d[Br^-]}{dt}=\frac{d[BrO_3^-]}{dt}$

$\Rightarrow \frac{1}{5}\times 0.05=\frac{d[BrO_3^-]}{dt}$

$\Rightarrow 0.01=\frac{d[BrO_3^-]}{dt}$

125. Which one of the following is used in the hardening of leather?

1) Light sensitive silver bromide in gelatin 2) Sodium lauryl sulphate

3) Alum 4) Tannin

Key: 4

Sol: Conceptual.

126. German silver contains which of the following metals?

1) Cu, Zn 2) Fe, Zn 3) Zn, Fe, Ni 4) Cu, Zn, Ni

Key: 4

Sol: Conceptual

127. The key step in the manufacturing of H_2SO_4 by contact process is

1) Absorption of SO_3 in H_2SO_4 to give oleum

2) Dilution of oleum with water

3) Burning of sulphur in air to generate SO_2

4) Catalytic oxidation of SO_2 with O_2 to give SO_3

Key: 4

Sol: .Conceptual

128. Ammonia on reaction with chlorine forms an explosive NCl_3. What is the mole ratio of NH_3 and Cl_2 required for this reaction?

1) 8:3 2) 1:1 3) 1:3 4) 10:1

Key: 3

Sol: . $NH_3 + 3Cl_2 \rightarrow NCl_3 + 3HCl$

$\Rightarrow 1:3$

129. Which one of the following lanthanide ions does not exhibit paramagnetism?

1) Lu^{3+} 2) Ce^{3+} 3) Eu^{3+} 4) Yb^{3+}

Key: 1

Sol: .Conceptual

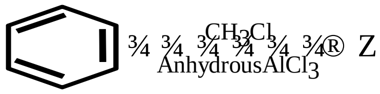
130. An example of covalent solid is

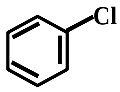
1) MgO 2) Mg 3) SiC 4) CaF_2

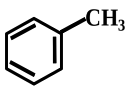
Key: 3

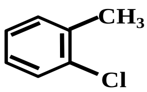
Sol: .Conceptual

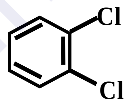
131. The product (Z) of the following reaction is



1) 

2) 

3) 

4) 

Key: 2

Sol: .Friedal craft's alkylation

132. Assertion (A): Reaction of 1-butene with HBr gives 1-bromobutane as major product.
Reason (R): Addition of hydrogen halides to alkenes proceeds according to Markovnikov's rule
The correct answer is

1) (A) and (R) are correct (R) is the correct explanation of (A)
2) (A) and (R) are correct (R) is not the correct explanation of (A)
3) (A) is correct but (R) is not correct
4) (A) is not correct but (R) is correct

Key: 4

Sol: Anti mar conikoff rule.

133. The two bonds N=O and N-O in H_3CNO_2 are of same bond length due to

1) Inductive effect 2) Hyperconjugation 3) Electrometric effect 4) Resonance effect

Key: 4

Sol: Conceptual

134. Municipal sewage BOD values (ppm) are ________

1) 1 - 5 2) 100 - 4000 3) 50 - 90 4) 20 - 40

Key: 2

Sol: Conceptual.

135. The buffer system which helps to maintain the pH of blood between 7.26 to 7.42 is

1) H_2CO_3 / HCO_3^- 2) NH_4OH / NH_4Cl 3) CH_3COOH / CH_3COO^- 4) CH_3COONH_4

Key: 1

Sol: Conceptual

136. Which one of the following elements does not form triiodide on reacting with iodine?

1) B 2) Tl 3) Al 4) Ga

Key: 2

Sol: Conceptual

137. White metal s an alloy of

1) Na, Mg 2) Na, Pb 3) Li, Mg 4) Li, Pb

Key: 4

Sol: Conceptual

138. Which one of the following is not a method to remove permanent hardness of water?

1) Clark's method 2) Calgon method
3) Ion-exchange method 4) Synthetic resins method

Key: 1

Sol: Conceptual

139. If the solubility of $Ca_3\left(PO_4\right)_2$ in wate is 'X' $mol\,L^{-1}$, its solubility product in $mol^5\,L^{-5}$ is

1) $6X^5$ 2) $36X^5$ 3) $64X^5$ 4) $108X^5$

Key: 4

Sol: $Ca_3\left(PO_4\right)_2 \rightarrow \underset{3s}{3Ca^{2+}} + \underset{2s}{2PO_4^{3-}}$

$K_{sp} = (3s)^3(2s)^2$

$= 27\times 4\times s^5$

$= 108s^5$

140. When one mole of A and one mole of B were heated in a one litre flask at T(K), 0.5 molesw of C was formed in the equilibrium

$A + B \rightleftharpoons C + D$

The equilibrium constant K_C is

1) 0.25 2) 0.5 3) 1 4) 2

Key: 3

Sol: $\underset{1}{A} + \underset{1}{B} \rightleftharpoons \underset{0}{C} + \underset{0}{D}$

1-x 1-x 0.5 0.5
0.5 0.5 0.5 0.5

$K_C = \dfrac{0.5\times 0.5}{0.5\times 0.5} = 1$

141. Which one of the following is not a state function?

1) Internal energy 2) Work 3) Entropy 4) Free energy

Key: 2

Sol: Conceptual

142. An organic compound having C, H and O has 13.13% H, 52.14% C, its molar mass is 46.068 g. What are its empirical and molecular formula?

1) $C_2H_6O, C_4H_{12}O_2$ 2) $CH_3O, C_2H_6O_2$ 3) C_2H_6O, C_2H_6O 4) $C_2H_6O_2, C_3H_9O_4$

Key: 3

Sol: . $C = \dfrac{52.14}{12} = 4.34$ $\quad \dfrac{4.34}{2.16} = 2$

H = $\dfrac{13.13}{1} = 13.13$ $\quad \dfrac{13.13}{2.16} = 6$

O = $\dfrac{34.63}{16} = 2.16$ $\quad \dfrac{2.16}{2.16} = 1$

$\therefore E.F = C_2H_6O$

$\therefore$ Mol.wt given =46.068

E.F wt = 46

$$\therefore n = \frac{Mol.wt}{E.F\ wt} = \frac{46.068}{46} = 1$$

$$\therefore M.F = E.F \times n = C_2H_6O \times 1 = C_2H_6O$$

143. According to significant figure convention the result obtained by adding 12.11, 18.0 and 1.012 is

1) 31.12 2) 31.1 3) 31 4) 31.122

Key: 2

Sol: .Conceptual

144. The most probable speed of O_2 molecules at T(K) is

1) $\sqrt{\frac{RT}{4\pi}}$ 2) $\sqrt{\frac{RT}{16\pi}}$ 3) $\sqrt{\frac{RT}{16}}$ 4) $\sqrt{\frac{3RT}{32}}$

Key: 3

Sol: .Most probable velocity of oxygen $= \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2RT}{32}} = \sqrt{\frac{RT}{16}}$

145. Match the following

List - I	List-II
A) Viscosity	**I) Critical temperature**
B) Ideal gas behaviour	**II) Isobar**
C) Liequefaction of gases	**III) Compressibility factor**
D) Charle's law	**IV)** $kg\ s^{-2}$
	V) $kg\ m^{-1}s^{-1}$

The correct answer is

	(A)	(B)	(C)	(D)
1)	**(IV)**	**(III)**	**(I)**	**(II)**
2)	**(V)**	**(III)**	**(I)**	**(II)**
1)	**(V)**	**(III)**	**(II)**	**(I)**
1)	**(IV)**	**(III)**	**(II)**	**(I)**

Key: 2

Sol: .Conceptual

146. What is the bond order of N_2 ?

1) 3 2) 4 3) 2 4) 1

Key: 1

Sol: Sol: N_2 molecule, $N \equiv N$

$\therefore$ B.O = 3.

147. Number of bonding electron pairs and number of lone pairs of electrons in CIF_3, SF_4, BrF_5 respectively are.

1) 3,2; 4,2; 5,2 2) 3,1; 4,1; 5,2 3) 3,1; 4,2; 5,1 4) 3,2; 4,1; 5,1

Key: 4

Sol: No.of e^- pairs $=\dfrac{G+M-C+A}{2}$

G= Group No. of central atom

M= No. of Movalent atom attached to central atom

C= Cationic charge

A= Anionic charge

For $ClF_3=\dfrac{7+3}{2}=5$ in which 3B.P+ 2LP

$SF_4=\dfrac{6+4}{2}=5$ in which 4 B.P + 1LP

$BrF_5=\dfrac{7+5}{2}=6$ in which 5 B.P+ 1 LP .

148. An element in +2 oxidation state has 24 electrons. The atomic number of the element and the number of unpaired electrons present in it respectively are

1) 24, 4 2) 26, 4 3) 24, 2 4) 26, 5

Key: 2

Sol: .Element with +2 oxdation state has 24 electrons

So for Neutral atom, it has 26 electrons

So it is - Fe

$Fe \rightarrow 1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,4s^2\,3d^6$

$Fe^{2+} \rightarrow$ $4s^2\,3d^6$ ↑↓|↑|↑|↑|↑

149. The equaion used to represent the electron gain enthalpy is

1) $X(g)+e^- ® X^-(g)$ 2) $X(s)+e^- ® X^-(g)$

3) $X(g) ® X^+(g)+e^-$ 4) $X(s) ® X^+(g)+e^-$

Key: 1

Sol: . $X(g)+e^- ® X^-(g)$

150. The radiation with maximum frequency is

1) X-rays 2) Radio waves 3) UV rays 4) IR rays

Key: 1

Sol: Increasing orde of wavelength is

cosmic rays $<\gamma$ rays < X rays < u.v rays < visible < I.R rays

< Micro waves < Television waves < Radio waves

$\therefore \upsilon \,\alpha\, \dfrac{1}{\lambda}$

$\therefore$ X-Rays have maximum frequency

151. The number of radial nodes present in 3p orbital is

1) 0 2) 1 3) 2 4) 3

Key: 2

Sol: No. of Radial Nodes = n-l-1

For 3P- orbital = 3-1-1=1

152. $H_3CCONH_2 + Br_2 + 4NaOH ® Y + Na_2CO_3 + 2NaBr + 2H_2O$ **What is Y in the reaction?**

1) H_3CCONH_2 2) H_3CNH_2 3) H_3CCOBr 4) $HCONH_2$

Key: 2

Sol: $CH_3-\overset{O}{\overset{\|}{C}}-NH_2+Br_2+4NaoH\rightarrow CH_3NH_2+Na_2CO_3+2NaBr+2H_2O$

(Hoffmann Bromamide Degradation)

153. $H_3CCH_2CO_2H \xrightarrow[\Delta]{P_2O_5} X \xrightarrow{H_2O} Y \xrightarrow{SOCl_2} Z$ **Identify X,Y and Z**

1) $H_2C=CHCO_2H$ $HOH_2CCHOHCO_2H$ $HOH_2CCHOHCOCl$

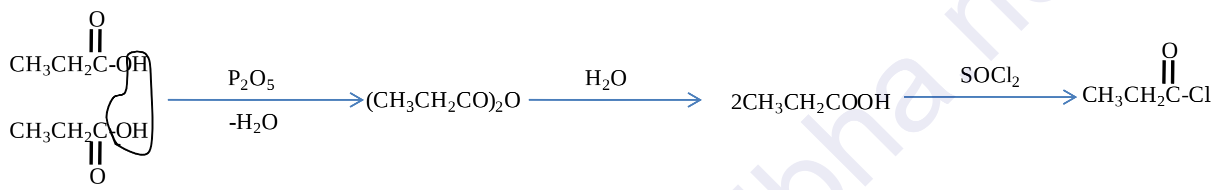
2) $(H_3CCH_2CO)_2O$ $H_3CCH_2CO_2H$ H_3CCH_2COCl

3) $(H_3CCO)_2O$ H_3CCO_2H $ClCH_2COCl$

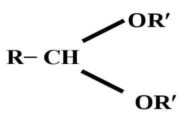
4) $(H_3CCH_2CO)_2O$ H_3CCO_2H H_3CCOCl

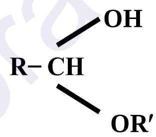
Key: 2

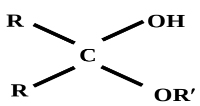
Sol:

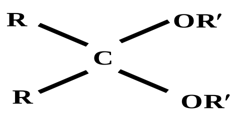


154. Which one of the followuing is an acetal?

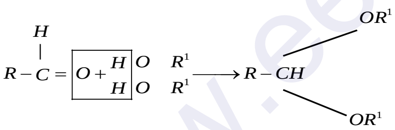
1) 

2) 

3) 

4) 

Key: 1

Sol: 

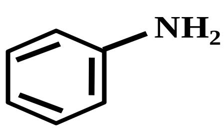
155. Which intermediate is formed in the Reimer-Tiemann rection?

1) Adehyde
2) Carbocation
3) Carbanion
4) Substituted bezal chloride

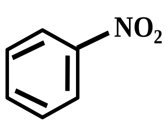
Key: 3

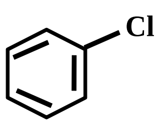
Sol: Carbanion

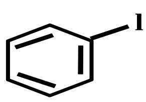
156. Identify Z in the following reaction

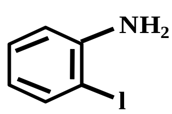


$\xrightarrow[2.KI]{1.NaNO_2,HCl,273-278K} Z$

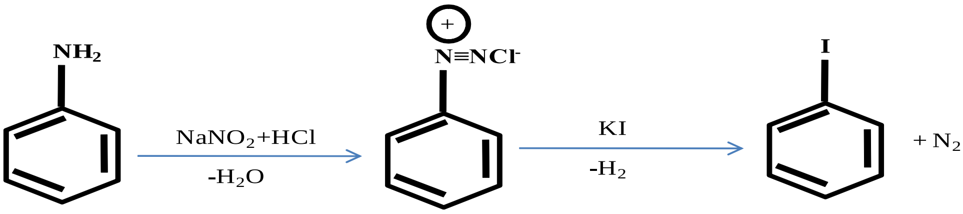
1) 

2) 

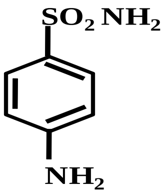
3) 

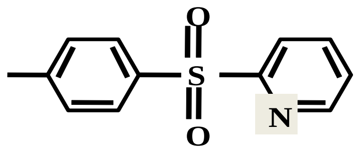
4) 

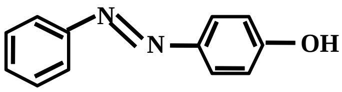
Key: 3

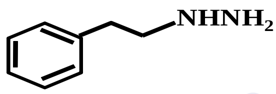
Sol: 

157. Which one of the following is the correct structure of sulphapyridine?

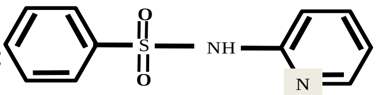
1) 

2) 

3) 

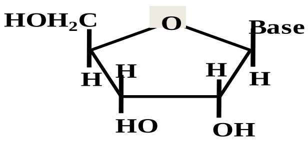
4) 

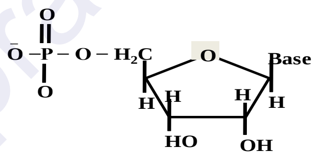
Key:2

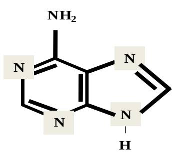
Sol: 

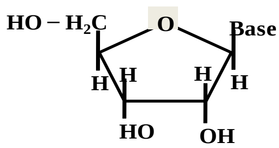
Sulphapyridine

158. Identify the nucleoside from the following?

1) 

2) 

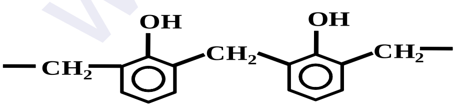
3) 

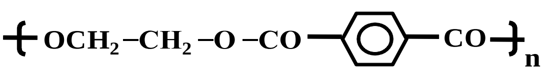
4) 

Key: 1

Sol: $\left[\overset{O}{\overset{\|}{C}} - (CH_2)_5 - NH \right]_n$ **Nylon – 6 (or) Perlon –L Caprolactum**

159. Identify condensation homopolymer from the following?

1) 

2) 

3) $\left[CO - (CH_2)_5 - NH \right]_n$

4) $\left[NH - (CH_2)_6 - NH - CO - (CH_2)_4 - CO \right]_n$

Key:3

Sol: Attachment of base to position "i" of sugar is called Nucleoside

160. The increasing order of field of ligands is?

1) $NH_3 < H_2O < Cl^- < CO < CN^-$
2) $Cl^- < H_2O < NH_3 < CN^- < CO$
3) $Cl^- < CO < CN^- < H_2O < NH_3$
4) $CN^- < CO < NH_3 < Cl^- < H_2O$

Key: 2

Sol: Increasing order of field strength of ligands is

$Cl^- < H_2O < NH_3 < CN^- < CO$

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