293(N)

(NEW SYLLABUS)

III

Total No. of Questions : 24
Total No. of Printed Pages : 4

Regd. No. | | | | | | | | | | |

Part-III

MATHEMATICS, Paper - II (B)

(English version)

Time : 3 Hours] ***[Max. Marks : 75***

***Note :** This question paper consists of **three** sections **A**, **B** and **C**.*

SECTION - A **10×2=20**

I . *Very short answer type questions.*

(i) Answer **all** questions.

(ii) Each question carries **TWO** marks.

1. Find the equation of the Circle, which is concentric with $x^2 + y^2 - 6x - 4y - 12 = 0$ and passing through $(-2, 14)$.

2. Obtain the parametric equation of the circle represented by $x^2 + y^2 = 4$.

3. Show that the angle between the circles

$$x^2 + y^2 = a^2, \quad x^2 + y^2 = ax + ay \text{ is } \frac{3\pi}{4}.$$

4. Find the co-ordinates of the points on the parabola $y^2 = 8x$, whose focal distance is 10.

5. Define Rectangular Hyperbola and find its eccentricity.

6. Evaluate :

$$\int \frac{1}{(x+3)\sqrt{x+2}}\,dx\ , \quad \text{on} \quad x \in \mathrm{I} \subset (-2,\ \infty)$$

7. Evaluate :

$$\int \frac{dx}{(x+1)(x+2)}\ .$$

8. Find the value of the integral

$$\int_0^{2\pi} \sin^2 x \cos^4 x \cdot dx$$

9. Evaluate :

$$\int_0^{\pi/2} \frac{\sin^5 x}{\sin^5 x + \cos^5 x} \cdot dx$$

10. Find the order and degree of

$$\left(\frac{d^3y}{dx^3}\right)^2 - 3\left(\frac{dy}{dx}\right)^2 - e^x = 4\ .$$

SECTION-B

5×4=20

II. *Short answer type questions.*

(i) Attempt **ANY FIVE** questions.

(ii) Each question carries **FOUR** marks.

11. If the abscissae of points A, B are the roots of the equation $x^2 + 2\,ax - b^2 = 0$ and ordinates of A, B are roots of $y^2 + 2py - q^2 = 0$, then find the equation of a circle for which $\overline{\mathrm{AB}}$ is a diameter.

12. Show that the circles $x^2 + y^2 - 8x - 2y + 8 = 0$ and $x^2 + y^2 - 2x + 6y + 6 = 0$ touch each other and find the point of contact.

13. Find the length of Major axis, Minor axis, Latus rectum and eccentricity of the ellipse $9x^2 + 16y^2 = 144$.

14. Find the condition for the line $x \cos\alpha + y \sin\alpha = p$ to be a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

15. Find the equation of the tangents to the hyperbola $x^2 - 4y^2 = 4$; which are *(i)* parallel, *(ii)* perpendicular to the line $x + 2y = 0$.

16. Evaluate :

$$\int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx$$

17. Solve : $\cos x \cdot \frac{dy}{dx} + y \sin x = \sec^2 x$

SECTION-C 5×7=35

III. *Long answer type questions.*

(i) Attempt **ANY FIVE** questions.

(ii) Each question carries **SEVEN** marks.

18. Find the equation of a Circle which passes through (4, 1), (6, 5) and having the centre on $4x + 3y - 24 = 0$.

19. Find the Transverse common tangents of the circles $x^2 + y^2 - 4x - 10y + 28 = 0$ and $x^2 + y^2 + 4x - 6y + 4 = 0$.

20. Evaluate $\int \frac{2\sin x + 3\cos x + 4}{3\sin x + 4\cos x + 5}\, dx$.

21. Obtain reduction formula for $I_n = \int \sin^n x \; dx$,

n being a positive integer, $n \geq 2$ and hence deduce the value of $\int \sin^4 x \, dx$.

22. Prove that the two parabolas $y^2 = 4ax$ and $x^2 = 4by$ intersect (other than

the origin) at an angle of $\tan^{-1}\left[\frac{3a^{1/3}\, b^{1/3}}{2\left(a^{2/3} + b^{2/3}\right)}\right]$.

23. Show that the area of the region bounded by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (ellipse)

is πab. Also deduce the area of the circle $x^2 + y^2 = a^2$.

24. Give the solution of $x \sin^2 \frac{y}{x} \, dx = y\, dx - x\, dy$ which passes through

the point $\left(1, \frac{\pi}{4}\right)$.
