II

293

Total No. of Questions - **24**

Total No. of Printed Pages - **3**

Regd. No.

Part - III

MATHEMATICS, Paper - II (B)

(Co-ordinate Geometry and Calculus)

(English Version)

Time : 3 hours ***Max. Marks : 75***

Note : This question paper consists of **three** sections *A, B* and *C*.

SECTION A

I. **Very short answer** type questions. **10 x 2 = 20**

i) Answer **all** questions.

ii) Each question carries **two** marks.

1. If $x^2 + y^2 - 6x + 4y - 12 = 0$ represents a circle, then find the parametric equations of the circle.

2. Find the centre and radius of the sphere $x^2 + y^2 + z^2 - 2x - 4y - 6z = 11$.

3. Find the value of 'k' if the lines $2x + 3y + 4 = 0$ and $x + y + k = 0$ are conjugate with respect to $y^2 = 8x$.

4. If the length of latus rectum is equal to half of its minor axis of an ellipse in the standard form, then find the eccentricity of the ellipse.

5. Find the n^{th} derivative of $Log\left(4 - x^2\right)$, $|x| < 2$.

6. Evaluate $\int \frac{e^x (1+x)}{Cos^2\left(xe^x\right)} dx$ on $I \subset R \setminus \left\{x \in R : Cos\left(xe^x\right) = 0\right\}$.

7. Evaluate : $\int e^x \left(\frac{1+x\ Log\ x}{x} \right) dx$ on $(0, \infty)$.

8. Evaluate : $\int_0^{\pi/2} Sin^4 x\ Cos^5 x\ dx$.

9. Find the area of the region enclosed by the given curves $x = 4 - y^2$, $x = 0$.

10. Find the order and degree of the differential equation $\frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{5/3}$.

SECTION B

II. Short answer type questions. **5 x 4 = 20**

i) Attempt **any five** questions.

ii) Each question carries **four** marks.

11. Find the equation of the circle whose centre lies on the X-axis and passing through (–2, 3) and (4, 5).

12. Show that the equation of common tangents to the circle $x^2 + y^2 = 2a^2$ and the parabola $y^2 = 8ax$ are $y = \pm (x + 2a)$.

13. Find the eccentricity, foci and directrices of the ellipse $4x^2 + y^2 - 8x + 2y + 1 = 0$.

14. Prove that the polar equation of conic in the standard form is $\frac{l}{r} = 1 + e\ Cos\ \theta$.

15. Evaluate : $\int \frac{dx}{5 + 4\ Cos\ x}$.

16. Solve the differential equation $\left(x^2 + y^2 \right) dx = 2xy\ dy$.

17. Solve the differential equation $\left(1 + x^2 \right) \frac{dy}{dx} + y = e^{Tan^{-1}x}$.

SECTION C

III. Long answer type questions. **5 x 7 = 35**

i) Attempt **any five** questions.

ii) Each question carries **seven** marks.

18. Show that the circles $x^2+y^2-6x-2y+1=0$ and $x^2+y^2+2x-8y+13=0$ touch each other. Find the point of contact and the equation of the common tangent at their point of contact.

19. Find the coordinates of the limiting points of the coaxial system to which the circles $x^2+y^2+10x-4y-1=0$ and $x^2+y^2+5x+y+4=0$ are two members.

20. Show that the poles with respect to the parabola $y^2=4ax$ of the tangents to the rectangular hyperbola $x^2-y^2=a^2$ lies on the ellipse $4x^2+y^2=4a^2$.

21. If $y=Cos\,(m\,Log\,x), x>0$ then show that $x^2y_2+xy_1+m^2y=0$ and hence deduce that $x^2y_{n+2}+(2n+1)xy_{n+1}+\left(m^2+n^2\right)y_n=0$.

22. Evaluate : $\int\frac{x+1}{x^2+3x+12}dx$.

23. Evaluate : $\int_0^1\frac{Log\,(1+x)}{1+x^2}dx$.

24. Find the approximate value of π from $\int_0^1\frac{1}{1+x^2}dx$ using Simpson's rule by dividing [0, 1] into four equal parts.