

293

(TS)

A

Total No. of Questions - 24

Total No. of Printed Pages - 4

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No.

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Part - III

MATHEMATICS, Paper - II (B)
(Coordinate Geometry and Calculus)
(English Version)

Time : 3 hours

Max. Marks : 75

Note : This question paper consists of three sections A, B and C.

SECTION A

10 × 2 = 20

I. Very short answer type questions.

- Attempt all questions.
- Each question carries two marks.

- If the length of the tangent from (5, 4) to the circle $x^2 + y^2 + 2ky = 0$ is 1, then find k .
- Find the equation of the polar of (1, -2) with respect to circle $x^2 + y^2 - 10x - 10y + 25 = 0$.
- Find the angle between the circles $x^2 + y^2 - 12x - 6y + 41 = 0$ and $x^2 + y^2 + 4x + 6y - 59 = 0$.
- Find the equation of parabola whose focus is $S(1, -7)$ and vertex is $A(1, -2)$.
- If the eccentricity of a hyperbola is $\frac{5}{4}$, then find the eccentricity of its conjugate hyperbola.

6. Evaluate $\int \frac{\text{Log}(1+x)}{1+x} dx$ on $(-1, \infty)$.
7. Evaluate $\int \frac{1}{1+\cos x} dx$ on $I \subset \mathbb{R} - \{(2n+1)\pi : n \in \mathbb{Z}\}$.
8. Evaluate $\int_1^5 \frac{dx}{\sqrt{2x-1}}$.
9. Find the value of $\int_0^{2\pi} \sin^2 x \cdot \cos^4 x dx$.
10. Find the order and degree of the differential equation

$$x^{1/2} \left(\frac{d^2 y}{dx^2} \right)^{1/3} + x \cdot \frac{dy}{dx} + y = 0.$$

SECTION B

5 × 4 = 20

II. Short answer type questions.

- Attempt any five questions.
- Each question carries four marks.

11. Find the equation of the circle whose center lies on the X-axis and passing through the points $(-2, 3)$ and $(4, 5)$.
12. Show that the circles $S \equiv x^2 + y^2 - 2x - 4y - 20 = 0$,
 $S' \equiv x^2 + y^2 + 6x + 2y - 90 = 0$ touch each other internally.
 Find their point of contact.

13. Find the equation of the ellipse in the standard form whose distance between foci is 2 and the length of latus rectum is $\frac{15}{5}$.

14. Find the eccentricity and length of latus rectum of the ellipse

$$9x^2 + 16y^2 - 36x + 32y - 92 = 0.$$

15. Find the equation of the tangents to the hyperbola $x^2 - 4y^2 = 4$ which are :

i) Parallel

ii) Perpendicular to the line $x + 2y = 0$.

16. Obtain the reduction formula for $\int_0^{\pi/2} \sin^n x \, dx$ for an integer $n \geq 2$.

17. Solve the differential equation $(1 + x^2) \frac{dy}{dx} + y = e^{\tan^{-1} x}$.

SECTION C

5 × 7 = 35

III. Long answer type questions.

i) Attempt any five questions.

ii) Each question carries seven marks.

18. If (2, 0), (0, 1), (4, 5) and (0, c) are concyclic, then find c.

19. Find the direct common tangents of the circles

$$x^2 + y^2 + 22x - 4y - 100 = 0, \text{ and}$$

$$x^2 + y^2 - 22x + 4y + 100 = 0.$$

20. Prove that the area of the triangle formed by the tangents at

(x_1, y_1) , (x_2, y_2) and (x_3, y_3) to the parabola $y^2 = 4ax$ ($a > 0$)

is $\frac{1}{16a} |(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)|$ sq. units.

21. Evaluate $\int \frac{1}{1 + \sin x + \cos x} dx$.

22. Evaluate $\int \frac{2x + 5}{\sqrt{x^2 - 2x + 10}} dx$.

23. Evaluate $\int_0^{\pi} \frac{x \sin^3 x}{1 + \cos^2 x} dx$.

24. Solve the differential equation

$$(2x + y + 1)dx + (4x + 2y - 1)dy = 0.$$
