

Total No. of Questions - 24

Total No. of Printed Pages - 4

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Part - III

MATHEMATICS, Paper - II (A)

(Algebra and Probability)

(English Version)

Time : 3 Hours

Max. Marks : 75

Note: This question paper consists of three Sections A, B and C.

SECTION A

10 × 2 = 20

I. Very Short Answer Type Questions.

- i) Answer all questions.
- ii) Each question carries two marks.

1. Write the conjugate of the complex number, $\frac{5i}{7+i}$.
2. If $z_1 = -1$, $z_2 = -i$, then find $\text{Arg}(z_1, z_2)$.
3. If $1, w, w^2$ are the cube roots of unity, then prove that $(a+b)(aw+bw^2)(aw^2+bw) = a^3 + b^3$.
4. If α, β are the roots of $ax^2 + bx + c = 0$, then find the value of $\alpha^2 + \beta^2$ in terms of a, b, c .
5. If the product of roots of $4x^3 + 16x^2 - 9x - a = 0$ is 9, then find a .

6. Find the number of ways of arranging the letters of the word SINGING, so that they begin and end with I.

7. If ${}^{12}C_{r+1} = {}^{12}C_{3r-5}$, then find r .

8. Find the set of values of x for which the binomial expansion of $(3 - 4x)^{3/4}$ is valid.

9. Find the mean deviation from the mean for the following discrete data :

6, 7, 10, 12, 13, 4, 12, 16.

10. 8 coins are tossed simultaneously. Find the probability of getting 6 heads.

SECTION B

5 × 4 = 20

II. Short Answer Type Questions.

- i) Attempt **any five** questions.
- ii) Each question carries **four** marks.

11. If $x + iy = \frac{3}{2 + \cos \theta + i \sin \theta}$, then show that $x^2 + y^2 = 4x - 3$.

12. If $x \in R$, then determine the range of the expression $\frac{x+2}{2x^2 + 3x + 6}$.

13. If the letters of the word MASTER are permuted in all possible ways and the words thus formed are arranged in the dictionary order, then find the rank of the word REMAST.

14. Find the number of ways of forming a committee of 5 members out of 6 Indians and 5 Americans, so that always the Indians will be in majority in the committee.

15. Resolve $\frac{2x^2 + 3x + 4}{(x - 1)(x^2 + 2)}$ into partial fraction.

16. Find the probability of drawing an ace or a spade from a well shuffled pack of 52 playing cards.

17. If A and B are two events with $P(A \cup B) = 0.65$, $P(A \cap B) = 0.15$, then find the value of $P(A^c) + P(B^c)$.

SECTION C

5 × 7 = 35

III. Long Answer Type Questions.

- i) Attempt any five questions.
- ii) Each question carries seven marks.

18. If n is a positive integer, then show that

$$(1+i)^n + (1-i)^n = 2^{\frac{n+2}{2}} \cos\left(\frac{n\pi}{4}\right).$$

19. Find the algebraic equation whose roots are the translates of the roots of the equation $x^4 - 5x^3 + 7x^2 - 17x + 11 = 0$ by -2 .

20. For $r = 0, 1, 2, \dots, n$, prove that

$$C_0 \cdot C_r + C_1 \cdot C_{r+1} + C_2 \cdot C_{r+2} + \dots + C_{n-r} \cdot C_n = {}^{2n}C_{n+r} \text{ and hence}$$

deduce that $C_0^2 + C_1^2 + C_2^2 + \dots + C_n^2 = {}^{2n}C_n$.

21. Find the sum of infinite series $\frac{3}{4 \cdot 8} - \frac{3 \cdot 5}{4 \cdot 8 \cdot 12} + \frac{3 \cdot 5 \cdot 7}{4 \cdot 8 \cdot 12 \cdot 16} - \dots$

22. Calculate the variance and standard deviation for the discrete frequency distribution given below :

x_i	4	8	11	17	20	24	32
f_i	3	5	9	5	4	3	1

23. A fair die is rolled. Consider the events $A = \{1, 3, 5\}$ $B = \{2, 3\}$ and $C = \{2, 3, 4, 5\}$ find :

- i) $P = (A \cup B)$ ii) $P = (A/B)$
 iii) $P = (A/C)$ iv) $P = (B/C)$

24. The probability distribution of a random variable X is given below.

$X = x_i$	1	2	3	4	5
$P(X = x_i)$	k	$2k$	$3k$	$4k$	$5k$

Find the value of k and the mean and variance of X .