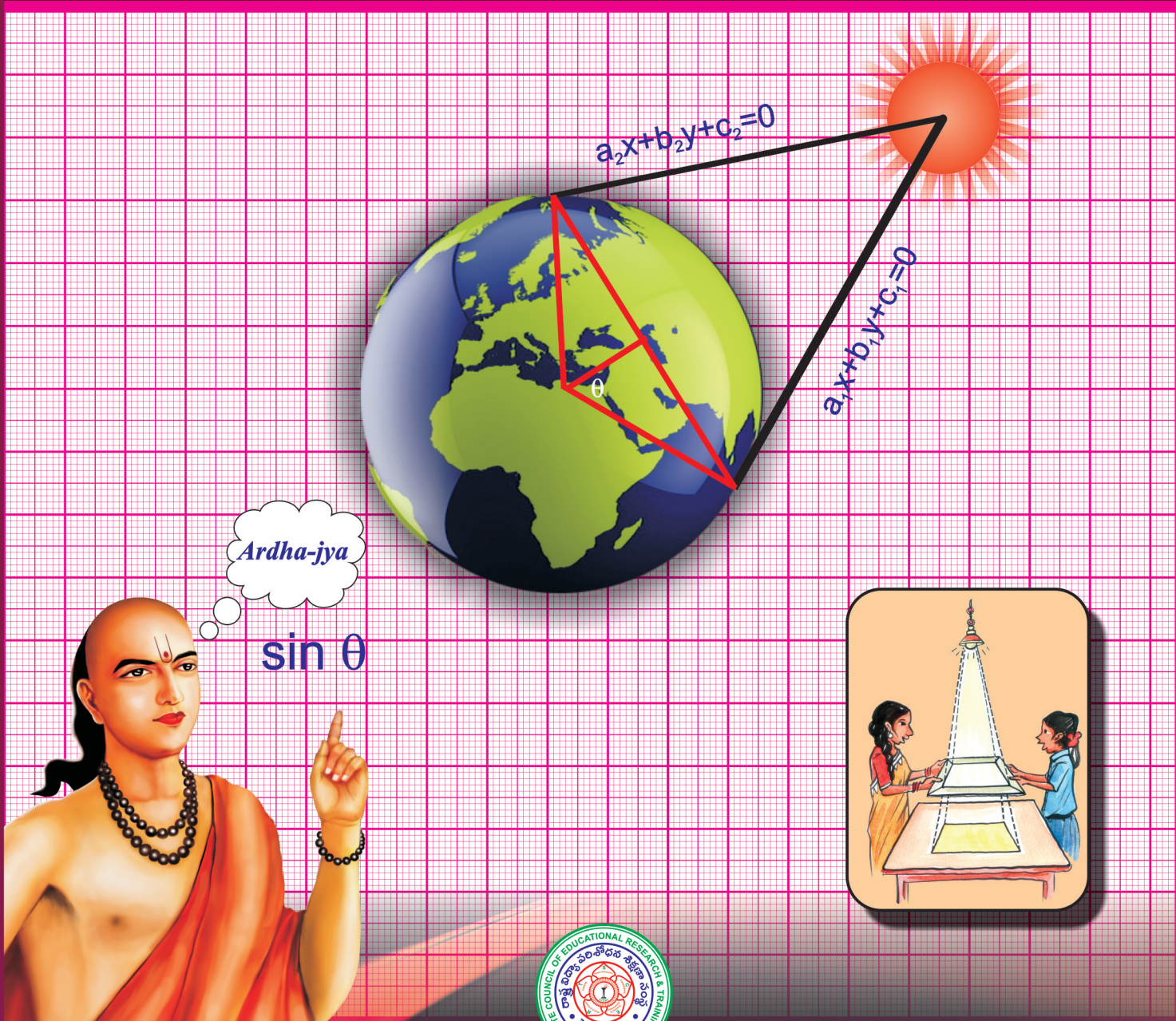


MATHEMATICS

ABHYASA DEEPIKA (Enrichment Material)

CLASS 10





MATHEMATICS

CLASS - 10

ABHYASA DEEPIKA



**State Council of Educational Research & Training
Telangana, Hyderabad.**



EDUCATION MINISTER
GOVERNMENT OF TELANGANA



MESSAGE

Keeping in view of the special conditions prevailing in this academic year, worksheets and digital classes are made available with the objective to facilitate the transaction of lessons in different subjects through alternate modes. Now that the SSC Board Examinations are round the corner, to facilitate easy self learning for the students, SCERT, TS has designed Enrichment Material for Class X, compiling all the major concepts of non-language subjects.

During all critical times and crises, teachers are taking initiative and doing their best to make the learning happen. In similar lines, they may guide the students to understand the aspects of this learning material. This learning material is quite useful to those who need help in different subjects to enhance their performance. I hope students will achieve good results by using this material.

March, 2021
Hyderabad.

Ms. Patlolla Sabitha Indra Reddy
Education Minister,
Government of Telangana.



SPECIAL CHIEF SECRETARY
GOVERNMENT OF TELANGANA



MESSAGE

Along with all other fields, the field of education has been severely affected by COVID 19 situation. The whole system, top-down, is struggling to save the academic year by reaching out to students and impart quality education. Teachers are playing a key role connecting to students through various online, social media and electronic media in addition to holding face to face classes for as many days as possible. SCERT, TS has designed an Enrichment Material for Class X to equip teachers and students to face the approaching examinations. Students can enhance their understanding of key concepts in every unit in different subjects using this material. Practice questions are given here to facilitate self assessment with the help of teachers where needed. I hope the students will make use of this material to achieve success.

March, 2021
Hyderabad.

Ms. Chitra Ramachandran, IAS
Special Chief Secretary,
Education Department, Telangana.



DIRECTOR OF SCHOOL EDUCATION
GOVERNMENT OF TELANGANA



MESSAGE

State Council of Educational Research and Training, Telangana, has prepared Enrichment Material to support the teachers and students in facilitating an effective transaction of key concepts in non-language subjects. Due to the special conditions prevailing due to COVID 19 situation, the syllabus for the examinations has been reduced up to 30% for the current academic year. The Enrichment Material covers the remaining 70% syllabus and helps the learners easily understand all the key concepts through self learning. I expect the students will make use of this material and perform well in the examinations.

March, 2021
Hyderabad.

Ms. A. Sridevasena, IAS
Director of School Education
Telangana

FOREWORD

The prevailing situations of COVID-19 have paved way for the development of a comprehensive learning material for class 10th students with an objective to cater the needs of students appearing for Public Examinations.

Department of School Education started online transmission through T-SAT and Doordarshan channels from 1st September, 2020. Apart from this, The District Educational Officers in some districts also started online classes on YouTube involving the subject experts. The ultimate objective is to help the students achieve prescribed Academic Standards. From 1st February, 2021 onwards face to face class room interactive classes started, in view of paucity of time it is not possible to cover all the concepts. Hence, this learning material helps to fill all those gaps.

This material gives an understanding and helps them achieve good results in the examination. The Mathematics syllabus of 10th class has 14 chapters out of which some concepts (30% of the syllabus) are meant for activity/project work and not considered for FA and SA. The key points of remaining concepts of 14 chapters (70% of the syllabus) are identified and made easy for the students to understand.

The self learning material is provided for further strengthening of the knowledge gained through classroom activities, worksheets and digital classes. The key concepts in each unit are dealt with definitions, formulae, practice problems, etc. under their specific headings. A variety of practice questions are given to facilitate self assessment.

Teachers are expected to go through the material thoroughly once to understand the purpose of the material and in turn guide the students in making effective use of the material. Students may be encouraged to approach their teachers to clarify doubts. I appreciate the efforts of the material developers. Further, I wish all the students to benefit from the material and come up with flying colours in examinations.

M. Radha Reddy
Director, SCERT,
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INSTRUCTIONS TO TEACHERS

- Focus on 14 units in Mathematics meant for evaluation in public examinations.
- This material is developed from the given 14 units meant for final examinations.
- Provide enough practice to students on key concepts and questions meant for practice.
- Provide practice on variety of questions given in the material.
- Focus on constructions (similar triangles, secants and tangents) and graphs (polynomials, linear equations, statistics).
- Explain the terms involved in the formulae and make the students to practice.
- Correlate with text books to clarify doubts in the Learning Material.
- Efforts should be made to make use of the learning material to the maximum extent for better result.

INSTRUCTIONS TO STUDENTS

- Read and practice the problems in the learning material.
- Understand the key concepts - explanation unit-wise to answer questions in the public examinations.
- Correlate with text books to clarify doubts in the reading material and approach your teachers.
- Good practice of the learning material helps you to achieve good grades.

Real Numbers

1. Division Algorithm:

For any two given positive integers ‘a’ and ‘b’, there exists a unique whole numbers “q” and “r” such that

$$a = bq + r \text{ where } 0 \leq r < b$$

Here “a” is dividend, “b” is divisor, “q” is quotient and “r” is remainder.

So that Dividend = (Divisor x Quotient) + Remainder.

2. Euclid’s Division Algorithm:

It is a technique to calculate the Highest Common Factor (HCF) of the two given positive integers.

To obtain the HCF of two positive integers c and d , with $c > d$, follow the steps below.

Step 1: Apply Division algorithm to c and d we find whole numbers q and r such that

$$c = dq + r, \quad 0 \leq r < d$$

Step 2 : If $r = 0$, d is the H.C.F. of c and d .

If $r \neq 0$, apply division algorithm to d and r .

Step 3: Continue this process till the remainder is zero. The divisor at this stage will be the required H.C.F

3. Fundamental Theory of Arithmetic / Unique Prime Factorization Theorem:

Every Composite number (positive) can be expressed as product of primes in a unique manner irrespective of their order.

Example: $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$

4. Rational number and their Decimal Expansion:

- i) If $a = p/q$ where p and q are co-primes and $q = 2^m \times 5^n$ (where m and n are whole numbers) then the rational number has terminating decimal expansion.

$$\text{Example : } \frac{7}{10} = \frac{7}{2 \times 5} = 0.7$$

- ii) If $a = p/q$, where p and q are Co-Primes and 'q' cannot be written in the form of $2^m \times 5^n$ (where m and n are whole numbers) then the rational number has a non-terminating repeating decimal expansion.

$$\text{Example : } \frac{7}{6} = \frac{7}{2 \times 3} = 1.166666 \dots\dots$$

5. Relationship between L.C.M and H.C.F of two numbers.

For any two positive integers 'a' and 'b'

$$\text{H.C.F of } a \text{ and } b \times \text{L.C.M of } a \text{ and } b = a \times b$$

Example : Let the numbers be 4 and 6 ,

$$\text{their H.C.F} = 2 \text{ and L.C.M} = 12$$

$$\text{H.C.F} \times \text{L.C.M} = 2 \times 12 = 24$$

$$\text{Product of numbers} = 4 \times 6 = 24.$$

6. Proving irrationality of numbers:

- i) \sqrt{a} is irrational if 'a' is not perfect square.
- ii) $a \pm \sqrt{b}$ is irrational if 'b' is not perfect square.
- iii) $\sqrt{a} \pm \sqrt{b}$ is irrational if 'a' and 'b' are not perfect squares.
- iv) 'p' is prime number, 'p' is a divisor of $a^2 \iff p$ is a divisor of 'a'.

7. Logarithms:

$$a^x = N \iff \log_a N = x \text{ Where } a \text{ and } N \text{ are positive real numbers, } a \neq 1$$

(Exponential form) (Logarithmic form)

8. Properties of Logarithms:

i) The Product Rule :

$$\log_a xy = \log_a x + \log_a y$$

ii) The Quotient Rule :

$$\log_a (x/y) = \log_a x - \log_a y$$

iii) The Power Rule :

$$\log_a x^m = m \log_a x.$$

iv) $\log_a 1 = 0$

v) $\log_a a = 1$

Practice problems

- Using Division Algorithm if 25 is expressed as, $25 = (4 \times q) + r$, then the value of 'r' is []
a) 0 b) 1 c) 2 d) 3
- The decimal expression of the number $\frac{441}{2^2 \times 5^3 \times 7}$ is []
a) a terminating decimal
b) non terminating but repeating decimal
c) non terminating, non repeating decimal
d) terminating after 2 decimal places
- Among the following, the number which is not irrational is []
a) $(2-\sqrt{3})^2$ b) $(\sqrt{2}+\sqrt{3})^2$ c) $(\sqrt{2}-\sqrt{3})(\sqrt{2}+\sqrt{3})$ d) $\frac{2\sqrt{7}}{7}$
- HCF of 26 and 91 is []
a) 15 b) 13 c) 19 d) 11
- The least number that is divisible by all the numbers from 1 to 5 is []
a) 15 b) 80 c) 70 d) 60
- Find the value of $\log_5 125$.
- Find the value of $\log_{\sqrt{2}} 64$.
- Is $\log_4 64$ rational or irrational? Justify.
- Find the HCF of 36 and 48 by using Euclid's Division Algorithm.
- Find HCF and LCM of 80, 120 by Prime Factorization Method.
- If HCF of 90 and 144 is 18, then find their LCM.
- Convert the following into logarithmic form.
i) $3^y = 25$ ii) $\frac{1}{49} = 7^z$
- Write the following in the exponential form.
i) $\log_3 27 = 3$ ii) $5 \log_2 32$

14. Expand the following:

i) $\log(200)$ ii) $\log\left(\frac{125}{64}\right)$

15. Express as a single logarithm.

i) $2 \log x + 3 \log y - 5 \log z$

ii) $5 \log 3 + 7 \log 2 - 3 \log 11 - 4 \log 5$

16. Show that $5+3\sqrt{2}$ is an irrational number.

17. If $x^2 + y^2 = 27xy$, then show that $2 \log(x-y) = 2 \log 5 + \log x + \log y$.

18. Show that the square of any positive integer is of the form $5m$ or $5m+1$ or $5m+4$ where 'm' is a whole number.

19. Express 225 as product of prime factors.

20. Show that $\sqrt{2} + \sqrt{3}$ is an irrational number.

A Set is a collection of well defined objects and the objects in a set are called elements.

Elements in a set are written in a curly bracket $\{ \}$ separated by commas and set can be represented by using capital letters of english alphabets such as A,B,C etc.

Example: $N = \{1,2,3,\dots\}$

“1 is in the set N” is represented symbolically as $1 \in N$ and “0 is not in the set N” is represented as $0 \notin N$. $x \in A$ is read as “x belongs to A” and $x \notin A$ is read as “x does not belong to A”

If a set is written by showing elements in it separated by commas, the form of writing the set is called “roster form” of the set and if a Set is written by ‘defining elements in it by a common property’, then that form of writing the set is called “set builder form” of the set.

Example: $A = \{3,6,9,12,15\}$ is in the roster form and the same set $A = \{x: x \text{ is a multiple of } 3 \text{ less than } 16\}$ is called set builder form of the set.

Sometimes a set with a common property could not have any element. Then, a set with no elements in it is called a null set.

Example: $A = \{x: x \text{ is an odd number which is divisible by } 2\}$ does not contain any element and this is a null set. A null set is represented as \emptyset . $\emptyset = \{ \}$

If a set contains an infinite number of elements, then it is called an ‘infinite set’ and if a set contains a finite number of elements, then it is called a ‘finite set’.

Example: $A = \{3,6,9,12\}$ is finite set and $B = \{3,6,9,12,\dots\}$ is infinite set

If 'all elements of set A are also present in another set B', then "A is a subset of B" and is represented symbolically as $A \subset B$.

Example: If $A = \{1, 3, 5\}$ and $B = \{1, 2, 3, 4, 5\}$, then we say that 'A is subset of B', and symbolically $A \subset B$.

When some sets are taken and analysed, the set from which these sets are taken is called a universal set. A universal set is generally represented as U or μ . Every set is a subset of the universal set.

When two sets have the same elements, the sets are called 'Equal sets'.

Example: If $P = \{x: x \text{ is multiple } 12\}$ and $Q = \{x: x \text{ is a multiple of both } 3 \text{ and } 4\}$, then $P = \{12, 24, 36, 48, \dots\}$ and also $Q = \{12, 24, 36, 48, \dots\}$. Hence P and Q are equal sets.

Operations on sets:

Union (\cup): $A \cup B$ is a set of elements which contains the which are in A or B or both A and B

Intersection (\cap): $A \cap B$ is a set of elements which are in both A and B.

Difference ($-$): $A - B$ is set of elements which are only in A but not in the set B.

Practice problems

- Which of the following is a set? []
 - A is a list of any five prime numbers
 - B is a list of first five prime numbers
 - C is a list of intelligent people in Indiaa) B b) C c) A d) None of these
- Which of the following is not a set builder form of $A = \{2, 4, 6, 8, 10\}$? []
 - $A = \{x: x \text{ is an even number}\}$
 - $A = \{x: x = 2n, n \leq 5, n \in \mathbb{N}\}$
 - $A = \{x: x \in \text{list of first five even numbers}\}$
 - $A = \{x: x \text{ is an even number, } x \leq 5\}$

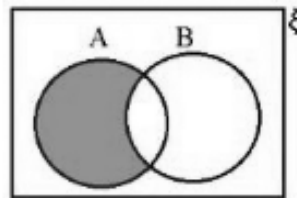
10. If $A = \{x: x \text{ is a multiple of } 2\}$, then which of the following is not a subset of A ? []

- a) $\{x: x \text{ is a multiple of } 4, x < 40\}$ b) $\{x: x \text{ is a multiple of } 6, x < 60\}$
c) $\{x: x \text{ is a multiple of } 8, x < 80\}$ d) $\{x: x \text{ is a multiple of } 9, x < 90\}$

11. Which of the following are not equal sets []

- a) $\{x: x \text{ is a letter from the word MADAM}\}$ and $\{x: x \text{ is a letter from the word DAM}\}$
b) $\{x: x \text{ is even number}\}$ and $\{x: x \text{ is a number divisible by } 2\}$
c) $\{x: x \text{ is a square number}\}$ and $\{x: x \text{ is sum of first 'n' odd numbers, } n \in \mathbb{N}\}$
d) $\{x: x \text{ is a prime number, } x < 8\}$ and $\{x: x \text{ is an odd number, } x < 8\}$

12. What does the shaded region represent in the Venn diagram? []



- a) $A - B$ b) $B - A$ c) $A \cup B$ d) $A \cap B$

13. If $A = \{2, 3, 5, 7, 11\}$ and $B = \{1, 3, 5, 7, 9\}$ then $A \cup B$ is []

- a) $\{3, 5, 7\}$ b) $\{1, 2, 3, 4, 5, 6, 7, 9, 11\}$
c) $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$ d) $\{1, 2, 3, 5, 7, 9, 11\}$

14. If $A = \{2, 3, 5, 7, 11\}$ and $B = \{1, 3, 5, 7, 9\}$ then $A \cap B$ is []

- a) $\{3, 5, 7\}$ b) $\{1, 2, 3, 4, 5, 6, 7, 9, 11\}$
c) $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$ d) $\{1, 2, 3, 5, 7, 9, 11\}$

15. If $A = \{2, 3, 5, 7, 11\}$ and $B = \{1, 3, 5, 7, 9\}$ then $A - B$ is []

- a) $\{3, 5, 7\}$ b) $\{2, 11\}$ c) $\{1, 7, 9\}$ d) $\{1, 2, 3, 5, 7, 9, 11\}$

Very Short Answer Questions (2 Marks)

- Write roster form of (i) $A = \{x: x \text{ is a multiple of } 5 \text{ and } x < 30\}$ (ii) $B = \{x: x = n^2 + 1, n \leq 5, n \in \mathbb{N}\}$.
- Write set builder form of (i) $P = \{1, 2, 3, 4, 6, 12\}$ (ii) $Q = \{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}\}$.
- Write any two set builder forms of null set.

4. A null set is represented by Rama as $\{\emptyset\}$ and Sridhar as $\{0\}$. Who is correct? Justify your answer.
5. Write all the subsets of $K = \{1, 3, 5, 7\}$
6. If $A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 3, 5, 7, 9\}$, then represent these sets in a Venn diagram.
7. If $A = \{2, 4, 6, 8, 10\}$ and $B = \{1, 3, 5, 7, 9\}$, then represent these sets in a Venn diagram.
8. If $A = \{1, 2, 3, 4, 5, 7, 9\}$ and $B = \{1, 3, 5, 7, 9\}$, then represent these sets in a Venn diagram.
9. Given that A and B are non-empty disjoint sets and they are not subsets to each other. Represent $A \cup B$ in a Venn diagram by shaded region.
10. Given that A and B are non-empty disjoint sets and they are not subsets to each other. Represent $A \cup B$ in a Venn diagram by shaded region.
11. Given that A and B are non-empty disjoint sets and they are not subsets to each other. Represent $A \cap B$ in a Venn diagram by shaded region.
12. If $D = \{x: x \text{ is odd prime number less than } 10\}$ and $S = \{x: x \text{ is prime number, } 1 < x < 9\}$, then can you say D and S are equal sets? Justify your answer.
13. If $A = \{2, 4, 8, 16, 32\}$ and $B = \{0, 2, 4, 6, 8\}$ then find $A \cup B$.
14. If $A = \{a, b, c, d, e, f\}$ and $B = \{a, e, i, o, u\}$ then find $A \cap B$.
15. If $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{1, 3, 5, 7, 9\}$ then find $A - B$.
16. A and B are non-empty sets and $A \subset B$, is it correct to say $A \cup B = B$? Why?
17. A and B are non-empty sets and $A \subset B$, is it correct to say $A \cap B = A$? Why?
18. A and B are non-empty sets and $A \subset B$, what would be $A - B$ and why?
19. Is it correct to say $\emptyset \cup A = A$? Justify your answer.
20. Is it correct to say $\emptyset \cap A = \emptyset$? Justify your answer.

Short Answer Questions (4 Marks)

1. If $A = \{x: x \text{ is a multiple of } 3 \text{ which is less than } 15\}$ and $B = \{x: x \text{ is a factor of } 12\}$. Represent A and B in a Venn diagram.
2. If $A = \{x: x \text{ is a factor of } 24\}$ and $B = \{x: x \text{ is a factor of } 120\}$. Represent A and B in a Venn diagram.
3. If $A = \{x: x \text{ is a prime number less than } 20\}$ and $B = \{x: x \text{ is a multiple of } 4\}$. Represent A and B in a Venn diagram.
4. $A = \{x: x \text{ is a name of the quadrilateral whose opposite sides are equal}\}$ and $B = \{x: x \text{ is a name of the quadrilateral whose diagonals are equal}\}$. Represent this information in a Venn diagram.

5. Given $P = \{x: x \text{ is a prime number less than } 7\}$ and $Q = \{x: x \text{ is a prime factor of } 30\}$. Check the equality of P and Q.
6. If $A = \{x: x \text{ is a multiple of } 4 \text{ and } x \leq 20\}$ and $B = \{x: x = 2^n, n < 5, n \in W\}$, then find $A \cup B$, $A \cap B$ and $A - B$.
7. If $A = \{x: x \text{ is a factor of } 36\}$ and $B = \{x: x = n^2, -5 \leq n \leq 5, n \in Z\}$, then find $A \cup B$, $A \cap B$ and $A - B$.
8. If $A = \{x: x = 6n, x \leq 30 \text{ and } n \in Z\}$ and $B = \{x: x = n(n+1), n < 6, n \in W\}$, then find $A \cup B$, $A \cap B$ and $A - B$.
9. Write any two non-empty sets A and B such as $A \subset B$ and verify the relationship $A \cup B = B$.
10. Write any two non-empty sets A and B such as $A \subset B$ and verify the relationship $A \cap B = A$.
11. Write any two non-empty sets A and B such as $A \subset B$ and verify the relationship $A - B = \emptyset$.
12. Write any two non-empty sets such that A and B are disjoint sets. Prove that $A - B = A$
13. Write any two non-empty sets such that A and B are disjoint sets. Prove that $B - A = B$
14. Write any two non-empty sets such that A and B are disjoint sets. Prove that $n(A \cap B) = 0$
15. Write any two non-empty sets such that A and B are disjoint sets. Prove that $n(A \cup B) = n(A) + n(B)$

Essay Type Questions (8 Marks)

1. If $A = \{x: x \text{ is a factor of } 12\}$ and $B = \{x: x \text{ is a factor of } 30\}$, then verify the relation between $n(A)$, $n(B)$, $n(A \cap B)$ and $n(A \cup B)$.
2. If $A = \{x: x \text{ is a multiple of } 4, x \leq 50\}$ and $B = \{x: x = 6n, n < 10, n \in Z\}$, then show that $n(A - B) = n(A) - n(A \cap B)$.
3. If $A = \{x: x = n^2, -3 < n < 5, n \in W\}$ and $B = \{x: x = 3n + 1, -3 \leq n \leq 2, n \leq 30 \text{ and } n \in Z\}$, then show that $n(B - A) = n(B) - n(A \cap B)$.
4. If $A = \{x: x = \log n, n = 10^m, m < 6, m \in N\}$ and $B = \{x: -5 < x < 5, n \in Z\}$ then represent sets A and B in a Venn diagram and write $A \cap B$, $A - B$ and $B - A$.
5. If $P = \{x: x \text{ is a composite number, } x \leq 15\}$ and $Q = \{x: x = 3n, n < 10, n \in Z\}$ then represent sets P and Q in a Venn diagram and write $P \cap Q$, $P - Q$ and $Q - P$.

Polynomials

1. **Polynomial:** An algebraic expression becomes a polynomial if the powers of variable(s) are whole numbers. A polynomial does not contain the terms like \sqrt{x} , x^{-2} , $\frac{1}{x}$, $x^{3/4}$ etc.

2. **Value of a polynomial at a specific value of the variable:**

For finding the value of $P(x) = x^2 + 2x + 3$ at $x=1$, we have to substitute $x = 1$ in the given polynomial.

$$P(x) = x^2 + 2x + 3$$

$$P(1) = (1)^2 + 2(1) + 3$$

$$= 1 + 2 + 3 = 6$$

The value of the polynomial $P(x)$ at $x=1$ is 6 i.e. $P(1) = 6$

3. **Degree of polynomial:**

The highest power of the variable of the all the terms of the given polynomial is the degree of that polynomial.

Ex:	$P(x) = x^2 + 2x + 4x^3 - 5$
	$\begin{array}{cccc} \downarrow & \downarrow & \downarrow & \downarrow \\ 2 & 1 & 3 & 0 \end{array}$
Degree of term	

Degree of $P(x) = 3$ (\because It is the highest power of all terms of the polynomial)

4. **Zero of a polynomial:**

For a polynomial $P(x)$, if $P(k) = 0$, then ' k ' is called zero of the polynomial $P(x)$.

Ex:	$P(x) = x^2 - 2x + 1$
	$P(1) = (1)^2 - 2(1) + 1$
	$= 1 - 2 + 1$
	$= 0$

So '1' is zero of a polynomial $P(x)$.

5. **Relation between zeroes and coefficients:**

i) For a quadratic polynomial $P(x) = ax^2 + bx + c$, let α, β are zeroes.

$$\text{Sum of zeroes } \alpha + \beta = \frac{-b}{a}$$

$$\text{Product of zeroes } \alpha \times \beta = \frac{c}{a}$$

ii) For a cubic polynomial $P(x) = ax^3 + bx^2 + cx + d$, let α, β, γ are zeroes.

$$\alpha + \beta + \gamma = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{-d}{a}$$

6. If α and β are zeroes of a quadratic polynomial $P(x)$, then

$$P(x) = x^2 - x(\alpha + \beta) + \alpha\beta$$

We write the same as

$$P(x) = x^2 - x(\text{sum of zeroes}) + (\text{product of zeroes})$$

7. The graph of a quadratic polynomial is a Parabola.

i) If this cuts X-axis at two points $(x_1, 0)$ and $(x_2, 0)$ then x_1, x_2 are zeroes of $P(x)$.

Further the two zeroes are real and distinct.

ii) If this touches X-axis at only one point $(x_1, 0)$, then the two zeroes are x_1, x_1 real and equal.

iii) If it does not cut X-axis, we understand that it has no real zeroes.

Practice problems

1. If the sum of the zeroes of the polynomial $P(x) = 2x^3 - 3kx^2 + 4x - 5$ is 6, then the value of 'k' is []
 a) 2 b) 4 c) -2 d) -4
2. Among the following, the expression that is not a polynomial is []
 a) $\sqrt{3x^2} - 2\sqrt{3x} + 3$ b) $\frac{3}{2}x^3 - 5x^2 - \frac{1}{\sqrt{2}}x - 1$
 c) $x + \frac{1}{x}$ d) $5x^2 - 3x + \sqrt{2}$
3. If $P(x) = 3x^4 - 5x^3 + x^2 + 8$, then the value of $P(-1)$ is []
 a) 2 b) 15 c) 17 d) -17
4. Degree of the polynomial $P(x) = x^3 - 2x^2 - \sqrt{3}x + \frac{1}{2}$ is []
 a) $\frac{1}{2}$ b) 2 c) 3 d) 4
5. If the sum and product of the zeroes of a quadratic polynomial are '2' and '-15' respectively, then the quadratic polynomial is []
 a) $x^2 - 2x + 15$ b) $x^2 - 2x - 15$ c) $x^2 + 2x - 15$ d) $x^2 + 2x + 15$
6. If one zero of the quadratic polynomial $2x^2 - 8x - k$ is $\frac{5}{2}$, then the value 'k' is []
 a) $\frac{15}{2}$ b) $\frac{5}{2}$ c) $\frac{45}{2}$ d) $\frac{-15}{2}$
7. The coefficient of x^2 term in the polynomial $P(x) = 2x^3 - 5x^2 - 3x + 7$ is []
 a) 2 b) -5 c) -3 d) 7
8. Find the zeroes of the quadratic polynomial $P(x) = 4x^2 - 4x + 1$.
9. Find the zeroes of the quadratic polynomial $P(x) = x^2 + 7x + 12$ and verify the relation between zeroes and its coefficients.
10. If $P(x) = x^2 - 5x + 6$, then find the values of $P(-1)$, $P(0)$, $P(1)$.
11. Write quadratic polynomials with the given numbers as sum and product of its zeroes respectively.
 i) $\frac{1}{4}, -1$ ii) $0, \sqrt{5}$

12. Write quadratic polynomials with given numbers as their zeroes.

i) 2, 3

ii) $\frac{1}{4}, 1$

13. Verify that 3, -1 and $\frac{-1}{3}$ are the zeroes of the cubic polynomial $P(x) = 3x^3 - 5x^2 - 11x - 3$.

14. Complete the following table for the polynomial $P(x) = 6x^2 - 13x + 6$

x	-2	-1	0	1	2
x^2					
$6x^2$					
$-13x$					
6					
y					

15. Draw the graphs of the following polynomial and find their zeroes from the graph.

i) $x^2 + 4x + 4$

ii) $2x^2 - 6x + 4$

Pair of Linear Equations in Two Variables

1. An equation of the form $ax + by + c = 0$ where a, b, c are real numbers ($a^2 + b^2 \neq 0$) is called a linear equation in two variables x and y .
2. In general, the system of pair of linear equations are represented by $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ where $a_1^2 + b_1^2 \neq 0$ and $a_2^2 + b_2^2 \neq 0$
3. **A pair of linear equations in two variables can be solved by**
 - i) Algebraic method
 - a) Substitution method
 - b) Elimination method
 - ii) Graphical method

a) Substitution method:

Step 1: Let the equation be $a_1x + b_1y + c_1 = 0$ (1) and

$a_2x + b_2y + c_2 = 0$ (2)

Step 2 : Select either of the two equations, say (1) and find the value of one variable , say “y” in terms of other variable i.e “x”.

Step 3 : Substitute the value of “y”, obtained in step (2) in the equation (2) to get a linear equation in “x”.

Step 4 : Solve the equation obtained in step (3) to get the value of “x”.

Step 5: Substitute the value of “x”, obtained in step (4) in the expression for ‘y’ in terms of ‘x’ obtained in step (2) to get the value of ‘y’.

b) Elimination method:

- Step 1: Obtain the two equations
- Step 2: Multiply the equations so as to make the coefficients of the variable to be eliminated equal.
- Step 3: Add or subtract the equations obtained in step (2) according as the terms having the same coefficients are of opposite or of the same sign.
- Step 4: Solve the equation in one variable obtained in step(3)
- Step 5: Substitute the value obtained in step(4) in any one of the given equations and find the value of other variable.

ii) Graphical method :

For graphical representation of a Linear equation in two variables, we require minimum three solutions of the equation. For this we express 'y' in terms of 'x' from the given equation. Then corresponding to any three convenient values of 'x', we find the corresponding value of 'y'. We then plot these three points obtained on a graph paper and join by a ruler. The line thus obtained represents the graph of the given linear equations.

Note: A pair of Linear Equations in two Variables is represented by two lines.

4. Nature of the Solution:

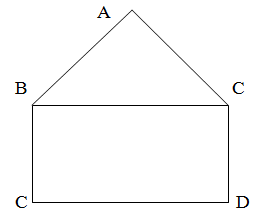
Let $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ form a pair of Linear Equations, then the following situations can be arrived.

Case 1: $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ Pair of linear equations is consistent.

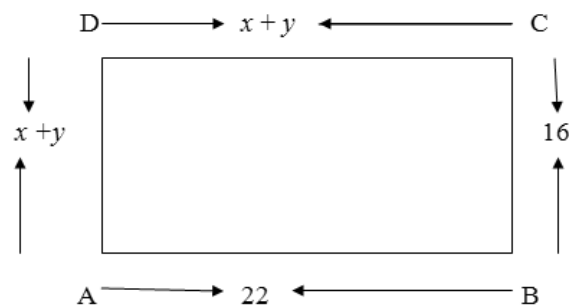
Case 2: $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ Pair of linear equations is inconsistent

Case 3: $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ Pair of linear equations is dependent and consistent

14. In the figure ABCDE is a pentagon with $BE \parallel CD$ and $BC \parallel DE$, BC is perpendicular to CD, $AB = 5\text{cm}$, $AE = 5\text{cm}$, $BE = 7\text{cm}$ $BC = x - y$ and $CD = x + y$. If the perimeter of the ABCDE is 27cm. Find the value of x and y given $x, y \neq 0$



15. Sum of the ages of a father and the son is 48 years. If the father's age is three times that of his son, then find their respective ages.
16. Seven times a two digit number is equal to four times the number obtained by reversing the order of its digits. If the difference of the digits is 3. Find the number.
17. In the given rectangle ABCD. Find the values of x and y .



18. If 2 is subtracted from the numerator and 1 is added to the denominator, a fraction becomes $\frac{1}{2}$, but when 4 is added to the numerator and 3 is subtracted from the denominator it becomes $\frac{3}{2}$. Find the fraction.
19. A fraction becomes $\frac{9}{11}$, if 2 is added to both numerator and denominator. If 3 is added to both numerator and denominator it becomes $\frac{5}{6}$. Find the fraction.
20. 4 Chairs and 3 tables cost Rs.2100/- where as the cost of 5 chairs and 2 tables of same kind is Rs1750. Find the cost of one chair and one table separately.
21. Two numbers are in the ratio 2 : 3. If 5 is added to each number, the ratio becomes 5:7. Find the numbers.
22. The larger of two supplementary angles exceeds the smaller by 18° . Find them.

Quadratic Equations

1. $ax^2 + bx + c = 0$ ($a \neq 0$) is called the general form of a quadratic equation.

Note: Here 'a' is called coefficient of x^2 , 'b' is called coefficient of x and 'c' is called the constant term.

2. If $a\alpha^2 + b\alpha + c = 0$ and $a\beta^2 + b\beta + c = 0$ ($a \neq 0$) then α and β are called the roots or solution of the quadratic equation the $ax^2 + bx + c = 0$.

Note: In general, the roots of quadratic equation are denoted by Greek letters α and β (α - Alpha and β - Beeta).

3. The quadratic equation formed by the roots α and β is $x^2 - (\alpha + \beta)x + \alpha\beta = 0$
4. If α and β are the roots of the equation $ax^2 + bx + c = 0$ ($a \neq 0$), then

i) Sum of the roots = $\alpha + \beta = \frac{-b}{a}$

ii) Product of the roots = $\alpha\beta = \frac{c}{a}$

5. Methods of solving a quadratic equation:

i) By factorization method

ii) By quadratic formula

i) By factorization method

In this method, we first express the quadratic polynomial into product of linear factor by using middle term splitting method or different identities and equate it to zero.

Note: Let us consider $(x - \alpha)(x - \beta) = 0$

$$\begin{array}{l|l} (x - \alpha) = 0 & (x - \beta) = 0 \\ x = \alpha & x = \beta \end{array}$$

$x = \alpha$ and $x = \beta$ are the solutions of quadratic equation.

ii) By quadratic formula

The roots of equation $ax^2+bx+c=0$ ($a \neq 0$) is given by $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Where $\alpha = \frac{-b + \sqrt{b^2-4ac}}{2a}$

$$\beta = \frac{-b - \sqrt{b^2-4ac}}{2a}$$

6. Nature of the roots

The value of b^2-4ac is called the discriminant of quadratic equation $ax^2+bx+c=0$. It is denoted by 'D'.

$$D = b^2-4ac$$

Note: The nature of the roots of a quadratic equation can be found, with the help of the value of its discriminant.

S.No.	D = b ² -4ac	Nature of the roots
1	D > 0	Distinct real roots
2	D = 0	Two equal roots
3	D < 0	No real root (imaginary)

Practice problems

- The sum of the roots of the equation $3x^2-7=0$ is []
a) 0 b) $\frac{7}{3}$ c) $-\frac{7}{3}$ d) $\frac{3}{7}$
- The product of the roots of the equation $x^2+x+1=0$ is []
a) -1 b) 1 c) 0 d) 3

3. If α and β are the roots of the equation $ax^2 + bx + c = 0$ ($a \neq 0$), then the value of $\frac{1}{\alpha} + \frac{1}{\beta} =$ []
- a) $\frac{-b}{a}$ b) $\frac{c}{a}$ c) $\frac{-a}{c}$ d) $\frac{-b}{c}$
4. If $\sin\alpha$ and $\cos\alpha$ are the zeroes of $px^2 + qx + r = 0$ ($p \neq 0$), then the relationship among p , q and r is []
- a) $q^2 - p^2 = 2pr$ b) $p^2 - q^2 = 2pr$ c) $p^2 - r^2 = 2qr$ d) $p^2 + q^2 = 2qr$
5. If 3 is a solution of $3x^2 + (k-1)x + 9 = 0$, then $k =$ []
- a) 11 b) -11 c) 13 d) -13
6. The roots of the equation $x^2 - 2x + 1 = 0$ are []
- a) 1, 1 b) 1, 7 c) -1, 7 d) -1, -7
7. The roots of the equation $3(x+3)^2 = 48$ are []
- a) 1, -7 b) 1, 7 c) -1, 7 d) -1, -7
8. The roots of the equation $x^2 + 7x = 0$ are []
- a) 0, -7 b) 0, 7 c) 7, -7 d) -7, -7
9. The quadratic equation having one of the roots is $3 + \sqrt{5}$ is []
- a) $x^2 - 6x + 4 = 0$ b) $x^2 - 6x - 4 = 0$
c) $x^2 + 6x + 4 = 0$ d) $x^2 + 6x + 5 = 0$
10. If the roots of $kx^2 + 2x + 3 = 0$ are equal, then $k =$ []
- a) $\frac{1}{3}$ b) $\frac{-1}{3}$ c) 3 d) -3
11. If the equation $x^2 - 4x + a = 0$ has no real roots, then []
- a) $a < 4$ b) $a \leq 4$ c) $a < 2$ d) $a > 4$

12. If the roots of $x^2-bx+c=0$ are two consecutive integers, then []

- a) $b^2-4c=0$ b) $b^2-4c=1$ c) $b^2-4c=2$ d) $b^2+4c=1$

13. Check whether the following are quadratic equations.

a) $x(2x+3) = x+2$

b) $y(8y+5) = y^2+3$

c) $(x-2)^2+1 = 2x-3$

d) $x(x+1) = 6$

14. The product of two consecutive positive odd integers is 63. Represent the data in the form of a quadratic equation.

15. Laxmi says “ $(2x-1)(x-3) = (2x+5)(x-1)$ is a quadratic equation”. Do you agree with her? Justify your answer.

16. The area of a rectangular plot is 528m^2 . The length of the plot (in meters) is one more than twice its breadth. Represent the data in the form of quadratic equation.

17. Check whether 1 and $\frac{2}{3}$ are the roots of $3x^2-5x+2=0$ or not.

18. If 2 is the root of $x^2-5x+k=0$, then find the value of ‘k’.

19. Write a quadratic equation whose roots are -2 and -3.

20. Find a quadratic equation whose roots are $4+\sqrt{15}$ and $4-\sqrt{15}$.

21. Find the roots of the following quadratic equations.

a) $x^2-3x-10=0$

b) $x^2-5x+6=0$

c) $x^2+5x+6=0$

d) $x^2-5x-6=0$

e) $x^2+5x+6=0$

f) $100x^2-20x+1=0$

g) $\sqrt{2}x^2+7x+5\sqrt{2}=0$

h) $2x^2-x+\frac{1}{8}=0$

22. Find the roots quadratic equations of the following equations by quadratic formula

a) $2x^2 - 6x + 3 = 0$

b) $2x^2 - 3x - 5 = 0$

c) $4x^2 + 4x + 1 = 0$

d) $3x^2 - 4\sqrt{3}x + 4 = 0$

23. Find the nature of the roots of the following quadratic equations.

a) $3x^2 - 5x + 2 = 0$

b) $x^2 + x + 1 = 0$

c) $x^2 + 4x + 5 = 0$

d) $x + \frac{1}{x} = 3 \ (x \neq 0)$

24. Dattu says “The equation $x^2+1=0$ has real roots”. Do you agree with him? Justify your answer.

25. If the roots of $(b-c)x^2+(c-a)x+(a-b) = 0$ ($b \neq c$) are equal then prove that $2b=a+c$.

26. The sum of two numbers is 18 and their product is 56. Find the numbers.

27. The sum of a number and its reciprocal is $\frac{10}{3}$. Find the numbers.

28. The hypotenuse of a right angled triangle is 6m more than the twice the shortest side. If the third side is 2m less than the hypotenuse, find the sides of the triangle.

29. Find two consecutive positive integers, sum of whose squares is 365.

Progressions

- An arithmetic progression is list of numbers in which each term, except the first term is obtained by adding a fixed number to the preceding term.
- The fixed number is called the common difference of the Arithmetic Progression (A.P).
- If $a_1, a_2, a_3, \dots, a_n$ is an Arithmetic Progression then $a_2 - a_1 = a_3 - a_2 = \dots = a_n - a_{n-1} = d$, where 'd' is the common difference.
- The n^{th} term of an A.P. with first term 'a' and common difference 'd' is given by $a_n = a + (n-1)d$, a_n is also called the general term of the A.P.
- If the first and last term of an A.P. are given and the common difference is not given then $S_n = \frac{n}{2}[a + a_n]$ or $S_n = \frac{n}{2}[a + l]$. Where a_n is the last term, l is the last term.
- Sum of 'n' terms of an A.P. is $S_n = \frac{n}{2}[2a + (n-1)d]$, where
a – first term, n – number of terms, d – common difference.

Practice problems

- The sum of Natural number from 1 to 100 is []
A) 4050 B) 10100 C) 55 D) 5050
- If $x-1, x+3, 3x-1$, are in A.P, then x is equal []
A) 5 B) 8 C) 6 D) 4
- Which term of the A.P 125, 120, 115, is the negative? []
A) 25th B) 26th C) 24th D) 27th

4. If $a_{26} - a_{25} = 15$, then the common difference of the A.P. is []
 A) 3 B) 5 C) 7 D) 15
5. How many numbers are divisible by 4 lying between 101 and 250? []
 A) 40 B) 62 C) 38 D) 37
6. The sum of 15 terms of the A.P. 4, 7, 10, is []
 A) 315 B) 475 D) 375 D) 325
7. In the A.P. 1, -1, -2, -5, then $d =$ _____ []
 A) -2 B) 1 C) 2 D) 10
8. In an A.P. $a_1 = -4$, $a_6 = 6$ then $a_2 =$ _____ []
 A) 3 B) 6 C) 1 D) -2
9. The 17th term of 1.1, 2.2, 3.3, 4.4 []
 A) 18.7 B) 19.8 C) 17.6 D) 17.17
10. Which term of the A.P. 100, 90, 80, is zero? []
 A) 10th B) 9th C) 11th D) 12th
11. How many terms are there in the A.P. 3, 6, 9, 12, 111?
12. Check whether -150 is a term of the A.P. 11, 8, 5, 2,?
13. Find the sum of two digit numbers which are multiple of 5 .
14. Which term of the sequence -1, 3, 7, 11, is 95?
15. What is the 18th term of A.P. defined by $a_n = n(n - 3) / n + 4$?
16. Find the 30th term of the A.P. 10, 7, 4,?
17. If in an A.P. 7 times of 7th term is equal to 11 times of 11th term. Then show that 18th term is equal to zero.

18. The sum of the 4th and 8th terms of the A.P. is 24, and the sum of the 6th and 10th terms is 44, find the first three terms of the A.P.?
19. The first and last terms of an A.P. are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?
20. The angles of a quadrilateral are in an A.P. The least angle is one – fourth of the greatest angle. Find the angles of the quadrilateral.

1. Distance Formula:

The distance between any two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Distance of a point from origin:

The distance of a point $P(x, y)$ from the origin 'O' is given by $OP = \sqrt{x^2 + y^2}$

2. Problems based on geometrical figure

To show that a given figure is a

- i) Parallelogram - Prove that the opposite sides are equal.
- ii) Rectangle - Prove that the opposite sides are equal and diagonals are equal.
- iii) Rhombus - Prove that 4 sides are equal.
- iv) Square - Prove that 4 sides are equal and diagonals are equal.
- v) Isosceles triangle - Prove that any 2 sides are equal.
- vi) Equilateral triangle - Prove that all 3 sides are equal.
- vii) Right angle triangle - Prove that sides of a triangle satisfies Pythagoras theorem

3. Section formula

The coordinates of the point $P(x, y)$ which divides the line segments joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m_1 : m_2$ are

$$\left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} \right)$$

4. Mid point formula

The coordinates of the point P(x, y) which is a mid point of the line segment joining the points A(x₁, y₁) and B (x₂, y₂) is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

5. Centroid

Centroid of the triangle whose vertices are A(x₁, y₁), B (x₂, y₂) and C (x₃, y₃) is

$$G(x, y) = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

6. Points of trisection

Points divide the line segment either in the ratio 1:2 and 2:1 are called **Points of trisection**.

7. **Slope of a line passing through** A(x₁, y₁), B (x₂, y₂) is $m = \tan \theta = \frac{y_2 - y_1}{x_2 - x_1}$ ('θ' lies between 0 and 90°).

Practice problems

Multiple Choice Questions

- The distance of a point P(4, -3) from the origin is []
a) 1 unit b) 7 units c) 5 units d) 3 units
- The distance between the points A(2, -3) and B(2, 2) is []
a) 2 units b) 4 units c) 5 units d) 3 units
- What is the midpoint of line segment with end points (-3, 4) and (10, -5) is []
a) (-13, -9) b) (-6.5, -4.5)
c) (3.5, 0.5) d) None of the these
- If the origin is midpoint of the line segments joined by the points (2, 3) and (x, y), then the value of (x, y) is []
a) (2, -3) b) (2, 3) c) (-2, 3) d) (-2, -3)
- The distance of the point P(2, 3) from the X-axis is []
a) 2 units b) 3 units c) 1 units d) 5 units

6. The point on the X-axis has coordinates []
 a) (a, 0) b) (0, a) c) (-a, a) d) (a, -a)
7. The point on the Y-axis has coordinates []
 a) (-a, 0) b) (a, 0) c) (0, b) d) (-a, -b)
8. Slope of the line joining the points (5, 3) and (6, 3) is []
 a) 2 b) 1 c) 0 d) not define
9. The ratio in which the centroid divides the median from the vertex of the triangle is []
 a) 1:3 b) 2:1 c) 3:1 d) 1:1
10. The coordinates of the centroid of triangle whose vertices are (0, 6), (8, 12), and (8, 0) is []
 a) (4, 6) b) (16, 6) c) (8, 6) d) $(\frac{16}{3}, 6)$

Very Short Answer Questions

- Find the distance between two points A(-3, 3) and B(3, -3)
- Find the slope of the line which makes the angle 60° with X-axis.
- What is the other end of the diameter of the circle whose center is (1, 2) and one end point is (3, 4)?
- Find the value of x if the distance between two points (8, x) and (x, 8) is $2\sqrt{2}$ units.
- Find the centroid of the triangle whose vertices are A(0, 0), B(1, 4) and C(2, -2).
- Find the coordinates of the point which divides line segments joining the points (4, -3), (8, 5) in the ratio 3:1 internally.
- Show that the points A(-3, 3), B(0, 0), C(3, -3) are collinear. (Use $\overline{AB} + \overline{BC} = \overline{AC}$).
- Slope of a line \overline{PQ} with points P(2, 5), Q(x, 3) is 2. Find 'x'?

Short Answer Questions

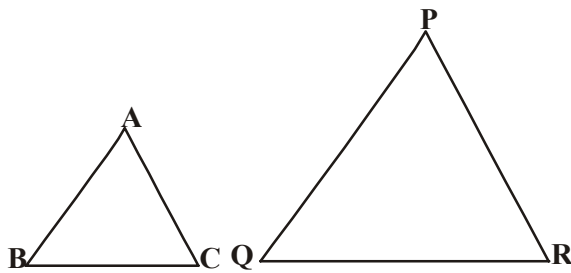
1. Find the ratio in which Y-axis divides the line segment joining the points A(3, 2), B(-1, 2).
2. Find the point on the X-axis which is equidistant from A(2, -5), B(-2, 9).
3. Find the value of 'k', if (7, -2), (5, 1), (3, k) are collinear (Use $\overline{AB} + \overline{BC} = \overline{AC}$).
4. A(3, 6), B(3, 2), C(8, 2) are the vertices of a rectangle \square ABCD, then find the fourth vertex D.
5. Name the shape of the quadrilateral formed by joining the points A(-1, 2), B(1, 0), C(-1, 2) and D(-3, 0). Justify your answer.
6. Show that A(a, 0), B(-a, 0), C(0, $a\sqrt{3}$) are vertices of equilateral triangle.
7. Show that (-4, -7), (-1, 2), (8, 5), (5, -4) are vertices of rhombus.
8. In what ratio does the point (-4, 6) divide the line segment joining the points (-6, 6) and (2, 6).
9. Find a point on the Y-axis which is equidistant from the points A(6, 5) and B(-4, 3).
10. Find the coordinates of the point which divides the line segment joining the points (4, -3), (8, 5) in the ratio 3:1 internally.

Essay Type Questions

1. Show that the points A(1, 2), B(5, 4), C(3, 8) and D(-1, 6) are vertices of a square.
2. Prove that the points A(-2, -1), B(1, 0), C(4, 3) and D(1, 2) are vertices of parallelogram.
3. Find the coordinates of the points of trisection of the line segment joining (4, -1) and (-2, -3).
4. If (a, b) is the midpoint of the line segment joining the points A(10, -6) and (k, 4) and $a - 2b = 18$. Find the value of 'a' and 'b'.
5. The centroid of a triangle whose vertices are (-8, 4), (P, 6) and (-3, 9) is $\left(\frac{-17}{3}, \frac{19}{3}\right)$, then find the value of 'P'.
6. Find the value of 'm' if the points (5, 1), (-2, -3) and (8, 2m) are collinear. (Use $\overline{AB} + \overline{BC} = \overline{AC}$)

Similar Triangles

- Similar Figures :** Two geometrical figures having the same shape but not necessarily the same size are called similar figures.
- Similar Triangles :** Two triangles are said to be similar if
 - Their corresponding angles are equal.
 - Their corresponding sides are in proportion (same ratio)



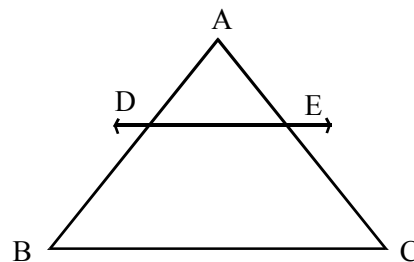
$$\triangle ABC \sim \triangle PQR \Leftrightarrow \text{i) } \angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$$

$$\text{ii) } \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

3. Basic Proportionality Theorem (Thales Theorem)

If a straight line is drawn parallel to one side of a triangle intersecting the other two sides, then it divides the two sides in the same ratio.

$$\text{In } \triangle ABC, DE \parallel BC \Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$$



Corollary

i) $\frac{AD}{DB} = \frac{AE}{EC}$

ii) $\frac{DB}{AD} = \frac{EC}{AE}$

iii) $\frac{AD}{AB} = \frac{AE}{AC}$

iv) $\frac{AB}{AD} = \frac{AC}{AE}$

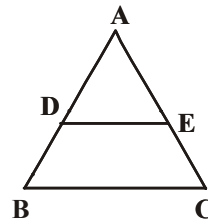
v) $\frac{DB}{AB} = \frac{EC}{AC}$

vi) $\frac{AB}{DB} = \frac{AC}{EC}$

4. Converse of Basic Proportionality Theorem :

If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

In $\triangle ABC$, $\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow DE \parallel BC$



5. Criterion for similarity:

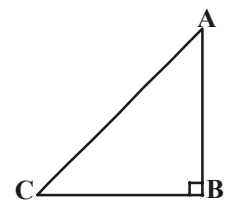
i) AAA Criterion for similarity (Corollary AA Similarity)

ii) SAS Criterion for similarity

iii) SSS Criterion for similarity

6. Pythagoras Theorem

In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.



$$AC^2 = AB^2 + BC^2$$

Practice problems

1. If 'a' is a side of an equilateral triangle then what is its altitude? []

A) $\frac{\sqrt{3}}{4}a^2$

B) $\frac{\sqrt{3}}{2}a$

C) $\sqrt{3}a$

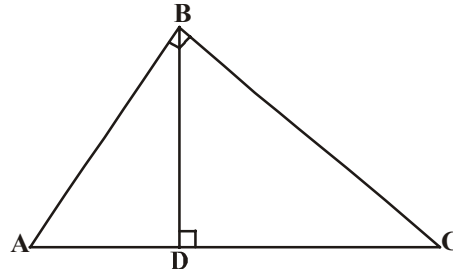
D) $\sqrt{2}a$

2. In $\triangle ABC$, $AC^2 = AB^2 + BC^2$, then $\angle B$? []

- A) 30° B) 60° C) 48° D) 90°

3. Which of the following is true from the given figure? []

- A) $\triangle ABC \sim \triangle ADB$
 B) $\triangle ABC \sim \triangle BDC$
 C) $\triangle ADB \sim \triangle BDC$
 D) All the above



4. $\triangle ABC \sim \triangle PQR$; if $\angle A + \angle B = 100^\circ$ then, $\angle R = ?$ []

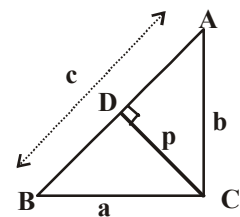
- A) 100° B) 80° C) 90° D) 50°

5. If $\angle A = 50^\circ$, $\angle B = 60^\circ$, $\angle C = 70^\circ$ and $\angle P = 60^\circ$, $\angle Q = 50^\circ$, $\angle R = 70^\circ$ then, which of the following is true? []

- A) $\triangle ABC \sim \triangle PQR$ B) $\triangle ABC \sim \triangle QPR$
 C) $\triangle ABC \sim \triangle PRQ$ D) all the above

6. Which of the following is true from the given figure. []

- A) $pc = ab$ B) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2}$ C) $p^2 = \frac{a^2 b^2}{a^2 + b^2}$

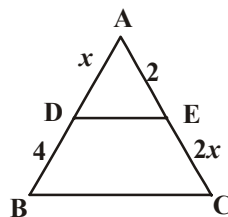


D) all the above

7. In a right angle $\triangle ABC$, $\angle B = 90^\circ$, $AB = 20$, $BC = 15$, then $AC = ?$ []

- A) 20 B) 10 C) 25 D) 225

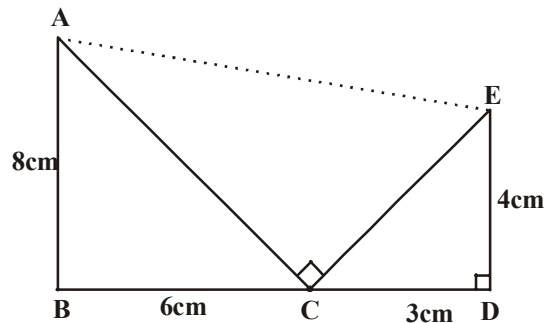
8. Find the value of 'x' from the figure. $DE \parallel BC$ []



- A) 2 B) 4 C) 6 D) 8

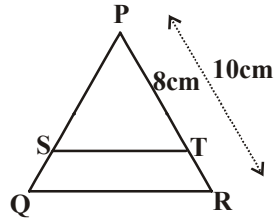
Very Short Answer Questions

1. Koushik walks 12m due East and turns left and walks another 5m, how far is he from the place started?
2. ABC is an isosceles triangle and $\angle B=90^\circ$, then show that $AC^2=2AB^2$.
3. In a triangle DEW, $AB \parallel EW$, if $AD=4\text{cm}$, $DE=12\text{cm}$ and $DW=24\text{cm}$ then find the value of 'DB'?
4. If in triangle ABC, $AB=6\text{cm}$, $BC=12\text{cm}$ and $CA=6\sqrt{3}\text{cm}$, then the measure of $\angle A$ is Justify your answer.
5. $\triangle ABC \sim \triangle DEF$, if $DE=2AB$ and $BC=3\text{cm}$, then EF is equal to 3cm. Are you agree this? Justify your answer with appropriate reasons.
6. From the figure, find the length of AE?

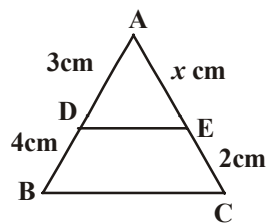


Short Answer Questions

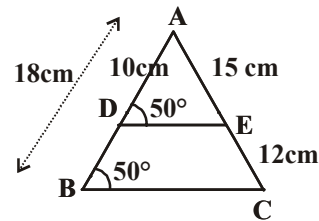
1. In figure if $ST \parallel QR$, $PT=8\text{cm}$ and $PR=10\text{cm}$, then what is the value of $\frac{PS}{SQ}$?



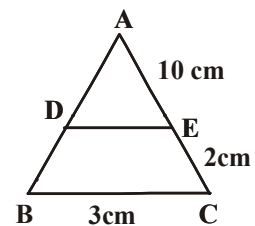
2. In the figure, find AE if $DE \parallel BC$



3. In the figure name the similar triangles



4. An isosceles triangle ABC similar to triangle PQR. $AC=AB=4\text{cm}$, $PQ=10\text{cm}$ and $BC=6\text{cm}$. What is the length of the PR? Which type of triangle is $\triangle PQR$?
5. In the adjoining figure, $DE \parallel BC$. What is the value of DE?



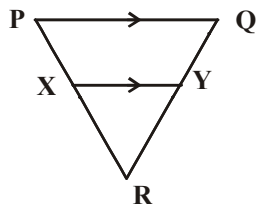
6. In triangles ABC and PQR if $\angle B = \angle Q$ and $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{1}{2}$,

then what is the value of $\frac{PR}{QR}$?

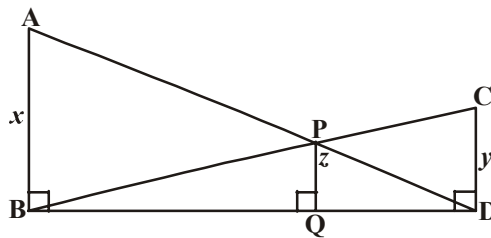
7. Is the line $DE \parallel BC$ in given $\triangle ABC$? Where $AD=8\text{cm}$, $DB=6\text{cm}$, $AE=12\text{cm}$, $EC=9\text{cm}$.
8. Is the line $AB \parallel PQ$ in given $\triangle PQR$? Where $RA=4\text{cm}$, $RB=4\text{cm}$, $AP=2\text{cm}$ and $BQ=8\text{cm}$.

Essay Type Questions

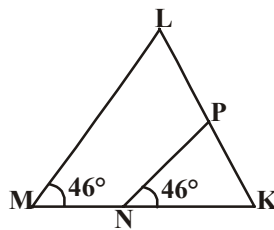
- Construct a triangle of sides 5cm, 6cm and 7cm. Then construct a triangle similar to it, whose sides are $\frac{2}{3}$ times of the corresponding sides of the first triangle.
- In $\triangle PQR$, if $XY \parallel PQ$ and $\frac{PX}{XR} = \frac{5}{3}$ and $QR=7.2\text{cm}$, then find the length of RY .



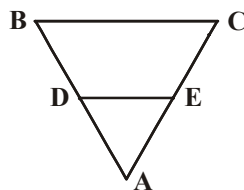
- In the figure AB, CD, PQ are perpendicular to BD , $AB=x$, $CD=y$ and $PQ=z$. Prove that $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$.



- In figure, $\angle M = \angle N = 46^\circ$, express x in terms of a, b and c , where a, b and c are lengths of LM, MN and NK respectively.

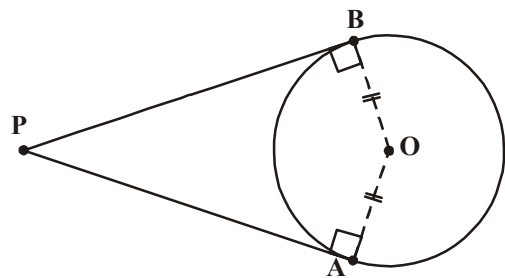


- In figure, $DE \parallel BC$ in $\triangle ABC$ such that $BC=8\text{cm}$, $AB=6\text{cm}$ and $DA=1.5\text{cm}$. Find DE .



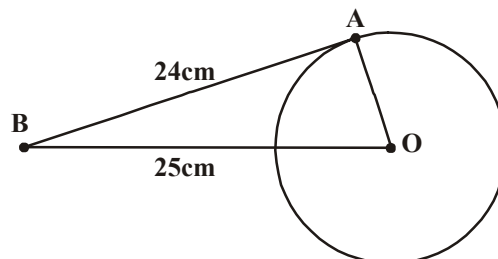
Tangents and Secants to a Circle

- The Radius drawn from the points of contact is perpendicular to the tangents.
- The number of tangent drawn from the external point is two (2).
- The lengths of the tangents drawn from the external points are equal.

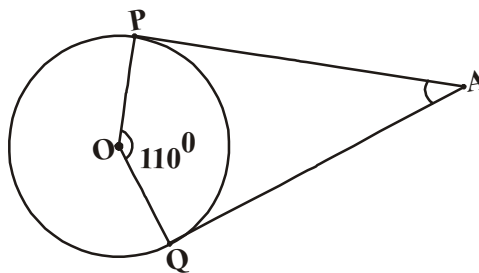


Practice problems

1. From a point B, the length of the tangent to a circle is 24cm and the distance of B from the centre is 25cm. The radius of the circle is []

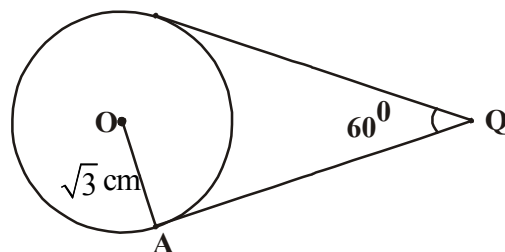


- A) 7 cm B) 12 cm C) 15 cm D) 24.5 cm
2. If AP and AQ are the two tangents to a circle with centre 'O' so that $\angle QOP = 110^\circ$ then, then $\angle PAQ = ?$ []

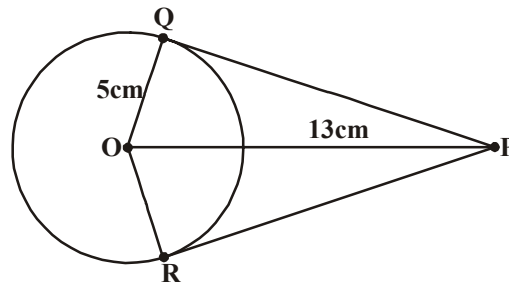


- A) 60° B) 70° C) 80° D) 90°

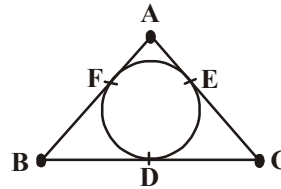
3. The angle between a tangent to a circle and the radius drawn at point of contact is []
- A) 60° B) 30° C) 45° D) 90°
4. The number of tangents drawn from the external point is []
- A) 2 B) 3 C) 1 D) infinite
5. A tangent to a circle touches it in points []
- A) 2 B) 1 C) 3 D) infinite
6. We can draw tangents to a given circle []
- A) 2 B) 1 C) 3 D) infinite
7. The length of tangent from 'C' point 15 cm away from the centre of a circle of radius 9 cm is []
- A) 10 cm B) 13 cm C) 12 cm D) 14 cm
8. If a quadrilateral ABCD is drawn to circumscribe a circle, then show that $AB+CD=AD+BC$.
9. Prove that the parallelogram circumscribing a circle is rhombus.
10. The tangents making an angle of 60° between them are drawn to a circle of radius $\sqrt{3}$ cm, then find the length of each tangent.



11. From 'O' point 'P', which is at a distance of 13cm from the centre of circle of radius 5cm, the pair of tangents PQ and PR are drawn to the circle, then find the area of the quadrilateral PQOR (in cm^2)



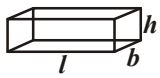
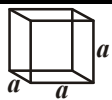
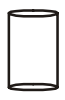

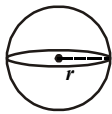
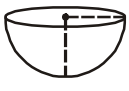
12. A triangle ABC is drawn to circumscribe a circle. If $AB=13\text{cm}$, $BC=14\text{cm}$, $AE=7\text{cm}$ then find AC.





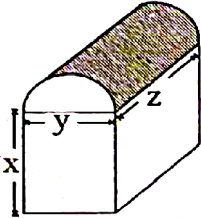




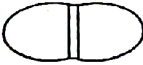

13. Draw a pair of tangents to a circle of radius 5cm which are inclined to each other at an angle 60° .
14. Draw a circle of radius 6 cm. From a point 10cm away from its centre, construct the pair of tangents to the circle.
15. Draw a circle of radius 5cm. Make a point 'A' which is 8cm away from its centre 'O', construct the tangents AB and AC.

Mensuration

Surface areas and volumes of different solid shapes.

S. No.	Name of the solid	Figure	Lateral / Curved surface area	Total surface area	Volume	Nomenclature
1.	Cuboid		$2h(l+b)$	$2(lb+bh+hl)$	lbh	l :length b :breadth h :height
2.	Cube		$4a^2$	$6a^2$	a^3	a :side of the cube
3.	Regular circular Cylinder		$2\pi rh$	$2\pi r(r+h)$	$\pi r^2 h$	r :radius of the base h :height
4.	Right circular cone		πrl	$\pi r(l+r)$	$\frac{1}{3} \pi r^2 h$	r :radius of the base h :height l :slant height
5.	Sphere		$4\pi r^2$	$4\pi r^2$	$\frac{4}{3} \pi r^3$	r :radius
6.	Hemisphere		$2\pi r^2$	$3\pi r^2$	$\frac{2}{3} \pi r^3$	r :radius

Some solid figures and their combination shapes

Name of the solid	Figure	Shape
Tanker		Its shape is a combination of a cylinder with two hemispherical ends.
Toy (Lattu)/ Water drop		Its shape is a combination of cone and hemisphere
Box		Its shape is a combination of a cuboid and a half cylinder.
Sharpened pencil		Its shape is a cylinder with a cone at one end and a hemisphere at the other end.
Ice-cream cone		Its shape is a combination of cone and a hemisphere.
Funnel		Its shape is a combination of cone and a cylinder.
Test tube		The shape is a combination of a cylinder and a hemisphere.
Capsule		Its shape is cylindrical with two hemispherical ends; used as medicine.
Rocket		Its shape is a combination of a cone and a cylinder.

Practice problems

Multiple Choice Questions (1 Mark)

1. The surface areas of two spheres are in the ratio 4:9 then, ratio of their volumes []
A) 27 : 8 B) 8:27 C) 1:9 D) 2:3
2. If radius, height and slant height of a cone are r , h , and l respectively then which of the following is true []
A) $l^2 > r^2 + h^2$ B) $l^2 < r^2 + h^2$ C) $l^2 = r^2 + h^2$ D) $l^2 = h^2 - r^2$
3. The ratio of volume of a cylinder and cone of equal diameter and height is []
A) 1:3 B) 3:1 C) 2:3 D) 3:2
4. Total surface area of a hemisphere whose radius ' r ' []
A) $2\pi r^2$ B) $3\pi r^2$ C) $4\pi r^2$ D) $\frac{4}{3}\pi r^3$
5. The total surface area of cube is 864cm^2 then, its side is []
A) 10cm B) 12cm C) 14cm D) 16cm
6. The base diameter and height are 8cm and 3cm of a cone, then the slant height
..... cm []
A) 4 B) 5 C) 6 D) 7
7. The diameter of a sphere which is inscribed in a cube of side ' a ' units is []
A) $\sqrt{2a}$ B) $\sqrt{3a}$ C) $2a$ D) a
8. Volume of a cylindrical shaped bowl is $125\pi \text{ cm}^3$ and its height is 5cm then its []
A) 25 B) 625 C) 5 D) 125
9. The shape formed by rotating a right angled triangle about its hypotenuse is []
A) sphere B) cone C) hemisphere D) pyramid
10. If ' a ' is length, ' b ' is breadth and ' c ' is height of a cuboid, then curved surface area is units. []
A) $2a(b+c)$ B) $2b(a+c)$ C) $2c(a+b)$ D) $2a+2b+2c$

Very Short Answer Questions (2 Marks)

1. Write the formula to find the curved surface area of a right circular cone and explain each term.
2. Write the formula to find the total surface area of a right circular cylinder and explain each term.
3. Write the formula to find the total surface area of a hemisphere and explain each term.
4. The base diameter and height of a cone is 12cm and 8cm, then find the curved surface area of the cone.
5. If the radius of a hemispherical bowl is 21 cm, then find the volume of the bowl.

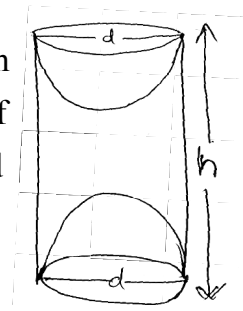
Short Answer Questions (4 Marks)

1. A cylinder, a cone and a hemisphere have same base and same height, then show that the ratio of their volumes is 3 : 1 : 2.
2. Two cubes each of volume 27cm^3 are joined end to end together. Find the total surface area of the resulting cuboid.
3. The curved surface area of a cylinder is 264m^2 and its volume is 924m^3 . Find the ratio of its base diameter to its height.
4. “The Total Surface Area (TSA) of a sphere is 16π sq.units and the TSA of a hemisphere with same radius is 8π .” Do you agree with this statement? Justify your answer with a reason.
5. Find the corresponding difference of CSA and volume of a cylinder when its radius is doubled and height is halved.
6. If the volume of an inverted cone is ‘V’ and it is filled with water upto half of it then find volume of the water.
7. “A sphere is exactly inscribed in a cylinder. Is the CSA of cylinder equal to TSA of sphere.” Explain with reason.

Essay Type Questions (8 Marks)

1. A vessel is in the shape of hemisphere and a cylinder surmounted on it. If the diameter of the vessel is 14cm and complete height of vessel is 13cm, then find volume of vessel.
2. From a face of cube whose length is 21cm, a hemisphere is scooped out from it. Find TSA of remaining part.
3. A solid wooden toy is in the shape of a right circular cone mounted on a hemisphere. If the radius of the hemisphere is 4.2cm and the total height of the toy is 10.2cm. Find the volume of the wooden toy.
4. A solid is in the form of a cylinder with hemispherical ends. The total height of the solid is 19cm and the diameter of the cylinder is 7cm. find the volume and total surface area of the solid (Use $\Pi = \frac{22}{7}$).
5. A solid toy is in the form of a right circular cylinder with an hemispherical shape at one end and a cone at other end. Their common diameter is 4.2cm and the height of the cylindrical and conical portions are 12cm and 7cm respectively. Find the volume of solid toy.

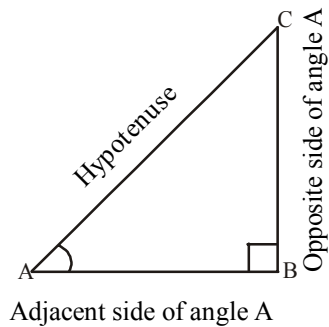
6. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in the adjacent figure. If height of the cylinder is 10cm, and its base radius is 3.5cm, find the total surface area of the article.



7. A solid right-circular cone of height 60cm and radius 30cm is dropped in a right-circular cylinder full of water of height 180cm and radius 60cm. Find the volume of water left in the cylinder in cubic meter (Use $\Pi = \frac{22}{7}$).
8. From a right circular cylinder of height 2.4cm and base diameter 1.4cm, a right circular cone of same radius and same height is cut out. Find the total surface area of remaining solid.

Trigonometry

1. Naming the sides in a right triangle:



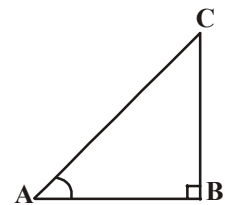
2. Trigonometric Ratios:

In a right angled triangle $\triangle ABC$, with right angle at B

$$\sin A = \frac{\text{side opposite to angle A}}{\text{Hypotenuse}}$$

$$\cos A = \frac{\text{side adjacent to angle A}}{\text{Hypotenuse}}$$

$$\tan A = \frac{\text{side opposite to angle A}}{\text{Side adjacent to angle A}}$$



3.
$$\operatorname{Cosec} A = \frac{1}{\sin A}$$

$$\operatorname{Sec} A = \frac{1}{\cos A}$$

$$\tan A = \frac{\sin A}{\cos A}$$

$$\tan A = \frac{1}{\cot A}$$

4. If one of the Trigonometric ratios of acute angle is known, the remaining trigonometric ratios of the angle can be determined.
5. The values of the trigonometric ratios for angle $0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ$.

$\angle A$	0°	30°	45°	60°	90°
$\sin A$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos A$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan A$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined
$\cot A$	not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
$\sec A$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	not defined
$\operatorname{cosec} A$	not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

6. Trigonometric identities

i) $\sin^2 A + \cos^2 A = 1$

ii) $\sec^2 A - \tan^2 A = 1$ (for $0^\circ < A < 90^\circ$)

iii) $\operatorname{cosec}^2 A - \cot^2 A = 1$ (for $0^\circ < A < 90^\circ$)

Practice problems

Multiple Choice Questions (MCQs)

1. The value of $(\sin 30^\circ + \cos 30^\circ) - (\sin 60^\circ + \cos 60^\circ)$ is []

- A) -1 B) 0 C) 1 D) 2

2. If $\sin\theta = \frac{a}{b}$, then $\cos\theta$ is equal to []
- A) $\frac{a}{\sqrt{b^2-a^2}}$ B) $\frac{b}{a}$ C) $\frac{\sqrt{b^2-a^2}}{b}$ D) $\frac{b}{\sqrt{b^2-a^2}}$
3. If A is an acute angle of triangle ABC, right angled at B, then the value of $(\sin A + \cos A)$ is []
- A) Equal to one B) Greater than one
C) Less than one D) Equal to two
4. If $\tan\theta = \cot\theta$, then value of $\sec\theta =$ []
- A) 2 B) 1 C) $\frac{1}{\sqrt{3}}$ D) $\sqrt{2}$
5. The value of $\frac{2\tan 30^\circ}{1+\tan^2 30^\circ}$ is []
- A) $\sin 60^\circ$ B) $\cos 60^\circ$ C) $\tan 60^\circ$ D) $\sin 30^\circ$
6. $\sin\theta - \cos\theta = 0$, then the value of $\sin^4\theta + \cos^4\theta$ is []
- A) 1 B) $\frac{3}{4}$ C) $\frac{1}{2}$ D) $\frac{4}{3}$
7. If $\sec\theta + \tan\theta = p$, then $\tan\theta =$ []
- A) $\frac{p^2+1}{p}$ B) $\frac{p^2-1}{p}$ C) $\frac{p^2+1}{2p}$ D) $\frac{p^2-1}{2p}$
8. $5 \operatorname{cosec}^2\theta - 5 \cot^2\theta =$ []
- A) 1 B) 5 C) 0 D) 10
9. If $\cos\theta + \cos^2\theta = 1$, $\sin^2\theta + \sin^4\theta =$ []
- A) -1 B) 0 C) 1 D) 4
10. If $\tan\theta$ is expressed in $\sin\theta$ []
- A) $\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$ B) $\frac{\sin\theta}{1-\sin^2\theta}$ C) $\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$ D) $\frac{\sqrt{\sin\theta}}{\sqrt{1-\sin^2\theta}}$

11. $(1 + \tan^2 \theta)(1 + \sin \theta) = \underline{\hspace{2cm}}$ []
 A) $(1 - \sin \theta)$ B) 1 C) $\frac{1}{1 - \sin \theta}$ D) None of these
12. If $\sin 45^\circ + \tan 45^\circ - \cos x = 1$, then $\tan x = \underline{\hspace{2cm}}$ ($x < 90^\circ$) []
 A) 30° B) 60° C) 90° D) 45°
13. The value of $\sqrt{\frac{1 + \cos \theta}{1 - \cos \theta}}$ is $\underline{\hspace{2cm}}$ []
 A) $\cot \theta - \operatorname{cosec} \theta$ B) $\operatorname{cosec} \theta + \cot \theta$
 C) $\operatorname{cosec}^2 \theta + \cot^2 \theta$ D) $(\cot \theta + \operatorname{cosec} \theta)^2$
14. If $\operatorname{cosec} \theta + \cot \theta = p$, then $\cos \theta = \underline{\hspace{2cm}}$ []
 A) $\frac{p^2 - 1}{p^2 + 1}$ B) $\frac{p^2 - 1}{2p^2}$ C) $\frac{p^2 + 1}{p^2 - 1}$ D) $\frac{p^2 - 1}{2p}$
15. If $\tan \theta = \frac{3}{4}$, then $\cos^2 \theta - \sin^2 \theta = \underline{\hspace{2cm}}$ []
 A) $\frac{7}{25}$ B) 1 C) $\frac{-7}{25}$ D) $\frac{4}{25}$

Very Short Answer Type Questions (2 Marks)

1. If $\tan\left(\frac{5\theta}{2}\right) = \sqrt{3}$, and θ is acute angle, then find the value of 2θ .

2. If in a right angled triangle ABC, $\tan A = \frac{12}{5}$, then find $\sin A \cdot \cos A$.

3. Find the value of the expression.

$$2 \left(\frac{\sin 45^\circ + \cos 45^\circ}{\sec 45^\circ + \operatorname{cosec} 45^\circ} \right) + \left(\frac{\sin 30^\circ \cdot \cos 60^\circ + \cos 30^\circ \cdot \sin 60^\circ}{\sin^2 55^\circ + \cos^2 55^\circ} \right)$$

4. If in a right angled triangle PQR, angle Q is 90° , $\operatorname{cosec} P = \frac{17}{15}$, then find the ratio of $\tan P$, $\cos P$.

5. $2 \times \left(\frac{\sin 30^\circ + \cos 60^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ} \right) + 3 \times \left(\frac{\sec^2 75^\circ - \tan^2 25^\circ}{\sin 90^\circ + \cos 90^\circ} \right)$

6. Express $\cot \theta$ in terms of 'cos θ ' and sin θ (separately).
7. If $\tan(A-B) = \frac{1}{\sqrt{3}}$ and $\sin A = \frac{\sqrt{3}}{2}$ then find the values of $\angle B$ and $\cos B$. ($A, B < 90^\circ$)
8. If $\sin \theta + \cos \theta = a$, then show that $\sin \theta \times \cos \theta = \frac{a^2 - 1}{2}$.

Short Answer Questions (4 Marks)

1. Show that $(\sin \theta + \operatorname{cosec} \theta)^2 + (\cos \theta - \sec \theta)^2 - (\tan \theta + \cot \theta)^2 = 1$
2. Evaluate $\frac{2 \operatorname{cosec}^2 30^\circ + 3 \sin^2 60^\circ - \frac{3}{4} \tan^2 30^\circ}{\sin^2 30^\circ + \cos^2 45^\circ}$
3. Evaluate $\frac{\tan^2 60^\circ + 4 \cos^2 45^\circ + 3 \sec^2 30^\circ + 5 \cos^2 90^\circ}{\operatorname{cosec}^2 30^\circ + \sec 60^\circ - \cot^2 30^\circ}$
4. If $7 \sin^2 \theta + 3 \cos^2 \theta = 4$. Show that $\tan \theta = \frac{1}{\sqrt{3}}$.
5. If $\tan \theta = \frac{3}{4}$, find the value of $\frac{1 - \cos \theta}{1 + \cos \theta}$.
6. $\sqrt{3} \tan \theta = 3 \sin \theta$, find the value of $\sin^2 \theta - \cos^2 \theta$.
7. If $x = a \sec \theta + b \tan \theta$ and $y = a \tan \theta + b \sec \theta$. Prove that $x^2 - y^2 = a^2 - b^2$.
8. Prove that $\frac{\sin \theta - \cos \theta + 1}{\sin \theta + \cos \theta - 1} = \frac{1}{\sec \theta - \tan \theta}$.
9. In a right triangle ABC, right angled at B, the ratio of AB to AC is $1 : \sqrt{2}$. Find the value of $\frac{2 \tan A}{1 + \tan^2 A}$.
10. If $\sin \theta = \frac{4}{5}$, find the value of $\frac{4 \tan \theta - 5 \cos \theta}{\sec \theta + 4 \cot \theta}$.

Essay Type Questions

1. If $\tan \theta = \frac{1}{\sqrt{7}}$, find the value of $\frac{\operatorname{cosec}^2 \theta - \sec^2 \theta}{\operatorname{cosec}^2 \theta + \sec^2 \theta}$.

2. Prove that $\frac{\sin^3 \theta - \cos^3 \theta}{\sin \theta - \cos \theta} + \frac{\sin^3 \theta + \cos^3 \theta}{\sin \theta + \cos \theta} = 2$.

3. If $\sec \theta + \tan \theta = p$, show that $\sin \theta = \frac{p^2 - 1}{p^2 + 1}$.

4. Prove the following.

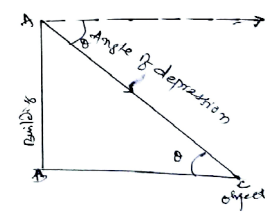
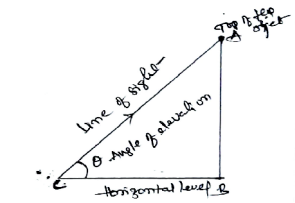
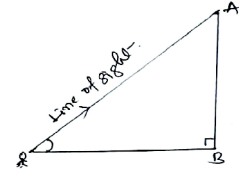
$$\frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} = \cos A + \sin A$$

5. Prove that $\frac{\sec \theta + \tan \theta - 1}{\tan \theta - \sec \theta + 1} = \frac{\cos \theta}{1 - \sin \theta}$.

6. Prove that $\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A} = 1 + \tan A + \cot A$.

Applications of Trigonometry

- Line of sight:** The line of sight is the line drawn from the eye of an observer to the object viewed.
- Angle of elevation:** The angle of elevation of the object viewed, is the angle formed by the line of sight with the horizontal when it is above the horizontal level i.e. the case when we raise our head to look at the object.
- Angle of depression:** The angle of depression of an object viewed, is the angle formed by the line of sight with the horizontal when it is below the horizontal level i.e. the case when we lower our head to look at the object.
- The height or length of an object or the distance between two distinct objects can be determined with the help of trigonometric ratios.



Practice problems

Multiple Choice Questions

- If the ratio of the height of a tower and the length of its shadow is $1 : \sqrt{3}$, what is the angle of elevation of sun is []
 a) 30° b) 60° c) 45° d) 90°
- If the angle of elevation of a tower from a distance 200m from its foot is 60° , then the height of the tower is []
 a) $100\sqrt{3}$ m b) $200\sqrt{3}$ c) $\frac{200}{\sqrt{3}}$ m d) $\frac{100}{\sqrt{3}}$ m

3. A kite is flying at a height of 60m above the ground. The inclination of the string with ground is 60° . Find the length of string (assuming that there is no slack in the string) []
- a) $40\sqrt{3}$ b) $30\sqrt{3}$ c) $20\sqrt{3}$ d) $60\sqrt{3}$
4. If a person observes the top of a tree at an angle of elevation 45° from 20m distance from the foot of the tree, then height of the tree is []
- a) 40m b) 10m c) 20m d) 60m
5. If the height of a tower and its shadow's length are equal at a particular time, then the angle of elevation of the sun []
- a) 45° b) 60° c) 30° d) 90°
6. It is found that on walking 'd' meters towards a light house in a horizontal line through its base, the elevation of top changes from 30° to 60° . The height of light house is []
- a) $3\sqrt{2}d$ b) $2\sqrt{3}d$ c) $\frac{\sqrt{3}}{2}d$ d) $\frac{2}{\sqrt{3}}d$
7. The angle of depression of a car, standing on the ground from the top of a 75m tower is 30° . The distance of the car from the base of the tower is []
- a) $2\sqrt{3}m$ b) $50\sqrt{3}m$ c) $75\sqrt{3}m$ d) 150m
8. The angle of elevation of the top of a tower at a point on the ground 50m away from the foot of the tower is 45° . Then the height of tower is (in meters)..... []
- a) $50\sqrt{3}m$ b) 50m c) 100m d) $\frac{50}{\sqrt{3}}m$
9. A ladder made an angle of 60° with the ground when placed against a wall. If the foot of ladder is 2m away from the wall, then the length of the ladder is (in meters) []
- a) $\frac{4}{\sqrt{3}}$ b) $4\sqrt{3}$ c) $2\sqrt{2}$ d) 4

10. If the angles of elevation of the top of the tower from two points distance 'x' and 'y' from the base and in the straight line with it are complementary, then the height of the tower is
- a) xy b) \sqrt{xy} c) $\frac{x}{y}$ d) $\sqrt{\frac{x}{y}}$ []

Very Short Answer Questions

1. A person observed the top of a tower at angle of elevation of 45° , when the observation point is 15 meters away from the foot of the tower. Find the height of the tower.
2. Ramu says that "If the length of the shadow of a tower is increasing then the angle of elevation of the sun is also increasing" Is it true or false? Justify.
3. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag containing the foot of the tower such that the angle of elevation of top of the tower and top of the flag are α and β respectively. Represent the above through a figure.
4. A tree is $20\sqrt{3}$ meters length. Find the angle of elevation, if its top from a point 20 meters away from its foot.
5. The angle of elevation of a ladder leaning against a wall is 60° and the foot of the ladder is 10m away from the wall. Find the length of the wall.
6. A pole 6m high casts a shadow $2\sqrt{3}$ meters on the ground, then find the sun's elevation.
7. If the length the shadow of a tower on the ground is equal to its height, then find the angle of elevation of the sun at that particular time.
8. A ship is sighted at sea from the top on a light house of 75m height. If the angle of depression is found to be 30° , find the distance of ship from the light house (in meters).

Short Answer Type Questions

1. The shadow of a tower standing on a level ground is found to be 40m longer when sun's altitude is 30° than when it was 60° . Find the height of the tower.

2. From the top of a hill, the angles of depressions of two consecutive kilometre stones due east are found to 30° and 45° . Find the height of the hill?
3. The angle of elevation of the top of a tower from a certain point is 30° ; If the observer moves 20 metres towards the tower, the angle of elevation of the top increases by 15° . Find the height of the tower.
4. As observed from the top of a 100m high light house from the sea level, the angles of depression of two ships are 30° and 45° . If one ship is exactly behind the other on the same side of the light house, find the distance between the two slips (use $\sqrt{3}=1.732$).
5. The angles of depression of the top and bottom of a building 50 metres high as observed from the top of a tower are 30° and 60° respectively. Find the height the tower.

Essay Type Answer Questions (8m)

1. An aeroplane at an altitude of 250m, observes the angle of depression of two boats on the opposite banks of a river to be 45° and 60° respectively. Find the width of the river (use $\sqrt{3}=1.732$).
2. A 20m high vertical pole and a vertical tower are on the same level ground in such a way that the angle of elevation of the top of the tower, as seen from the foot of the pole is 60° and the angle of elevation of the top of the pole as seen from the foot of the tower is 30° . Find: (i) the height of the tower; (ii) the horizontal distance between the pole and the tower.
3. A man observes the angle of elevation of top of the building to be 30° . He walks towards it in a horizontal line through its base. On covering 60m, the angle of elevation changes to 60° . Find the height of the buildings.
4. An aeroplane when flying at a height of 4000m from the ground passes through vertically above another aeroplane at an instant, when the angle of the elevation of two planes from the same point on the ground are 60° and 45° respectively. Find the vertical distance between two aeroplanes at that time.

Probability

- Probability is used to find the chance that something will happen. How likely it is that some event will occur.
- **Random experiment** : For random experiments, the results are known well in advance, but the result of the specific performance cannot be predicted.
- The theoretical (classical) probability of an event E, written as P(E), is defined as P(E) as the ratio of 'Number of outcomes favourable to E' to the 'Total number of all possible outcomes' of a random experiment where we assume that the outcomes of the experiment are equally likely.

$$P(E) = \frac{\text{Number of outcomes favourable to E}}{\text{Total number of possible outcomes of the experiment}}$$

- An event which will definitely occur is called a sure event. The collection of all outcomes of a sure event is the entire sample space. The probability of a sure event (or certain event) is 1.
- An event which cannot occur on any account is called an impossible event. The probability of an impossible event is 0.
- The probability of an event E is a number P(E) such that $0 \leq P(E) \leq 1$
- An event having only one outcome is called an elementary event. The sum of the probabilities of all the elementary events of an experiment is 1.
- For any event E, $P(E) + P(\bar{E}) = 1$, where \bar{E} stands for 'not E'. E and \bar{E} are called complementary events.
- **Equally likely events** : Two or more events are said to be equally likely if each one of them has an equal chance of occurrence.

- **Mutually Exclusive events** : Two or more events are mutually exclusive if the occurrence of each event prevents the every other event.
- **Exhaustive events** : Two or more events are said to be exhaustive, if the union of their outcomes is the entire sample space.
- **Complementary events** : Two events are said to be complementary, if they are mutually exclusive and also exhaustive. (OR) Two events are said to be complementary if occurrence of an event prevents the occurrence of the other and the union of their outcomes is the entire sample space.

Practice problems

Multiple Choice Questions (MCQs)

- Which of the following is not possible value of the probability of an event? []
 a) 0 b) 1 c) $\overline{0.75125}$ d) 1.1
- Which of the following is a certain(sure) event? []
 a) The event of getting a head when a dice is rolled
 b) The event of getting a head when a coin is tossed
 c) The event of getting a tail when a coin is tossed
 d) The event of getting a head or tail when a coin is tossed []
- Which of the following is an impossible event?
 a) The event of getting a head when a dice is rolled
 b) The event of getting a head when a coin is tossed
 c) The event of getting a tail when a coin is tossed
 d) The event of getting a head or tail when a coin is tossed
- What is the probability of a sure event? []
 a) 0 b) 0.5 c) 1 d) 100
- What is the probability of an impossible event? []
 a) 0 b) 0.5 c) 1 d) 100

6. If there are 8 elementary events which are equally likely, then what is the probability of each elementary event? []
 a) 8 b) $\frac{1}{8}$ c) 0.8 d) 1.8
7. What is the probability getting a prime number when a dice is rolled? []
 a) 0.5 b) 2.6 c) 0.6 d) $0.\bar{3}$
8. There are 12 boys and 18 girls in a class. If a student is chosen at random, what is the probability of selecting a girl? []
 a) 0.4 b) 0.45 c) 0.9 d) 0.6
9. If a card is chosen at random from a bag containing cards with numbers 1 to 10, then the probability of getting a multiple of 3 is []
 a) 0.3 b) 0.5 c) $0.\bar{3}$ d) 0.4
10. If a ball is chosen at random from a box containing 6 white balls, then the event of getting a white ball is []
 a) Impossible event b) Sure event
 c) Ambiguous event d) White event
11. If a ball is chosen at random from a box containing 10 Red balls, then the event of getting a black ball is []
 a) Impossible event b) Sure event
 c) Ambiguous event d) Red event
12. The range of probability of an event []
 a) $0 < P(E) < 1$ b) $-1 < P(E) < 1$ c) $0 \leq P(E) \leq 1$ d) $-1 \leq P(E) \leq 1$
13. There are three balls with colours white, black and red in a bag. What is the probability of not getting a red ball? []
 a) $\frac{1}{3}$ b) $\frac{2}{3}$ c) $\frac{3}{2}$ d) $\frac{11}{3}$
14. The probability of raining on a certain day in July month in Hyderabad is 0.65. What is the probability of not raining on a particular day? []
 a) 1.65 b) 0.45 c) 0.35 d) 0.25

15. Which of the following pair events is not a pair of mutually exclusive events?
- a) Getting head and getting tail when coin is tossed
 - b) Coming up 3 and coming up 6 when a dice is rolled
 - c) Selecting a black card and a king card when a card is drawn from a pack of cards
 - d) Selecting a prime number and selecting a composite number from numbers 1 to 100

Very Short Answer Questions

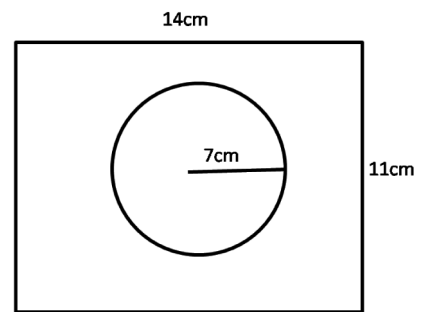
1. Write any two examples of equally likely events.
2. Write any two examples of sure events.
3. Write any two examples of impossible events.
4. Write any two examples of complementary events.
5. When we say that an event is not a sure event? Why?
6. What are mutually exclusive events?
7. What is the probability of getting a composite number when a dice is rolled?
8. There are cards with numbers 1 to 15 in a box. What is the probability of getting a prime number when a card is selected at random?
9. There are 2 white and 3 blue balls in a bag. What is the probability of getting a blue ball when a ball is drawn at random from the bag.
10. There cards with all alphabets on them in a bag. What is the probability of getting a vowel when a card is drawn at random?
11. There are 15 boys and 25 girls in class. If a student is selected at random, what is the probability of selecting a girl?
12. If the probability that Tanish will take an examination is 0.83, then what is the probability that she will fail the examination?
13. The probability that a person is affected by corona in an area is 0.23. If Anjali lives in that area, what is the probability that she is not affected by corona?
14. Sakshi went to a mobile shop to select a mobile. There are 89 features which she likes in a mobile out of 100 features. What is the probability that she will not buy that mobile?
15. If there are 10 primary events in a random experiment which are equally likely, then what is the sum of the probabilities of all the events?

Short Answer Questions (4 Marks)

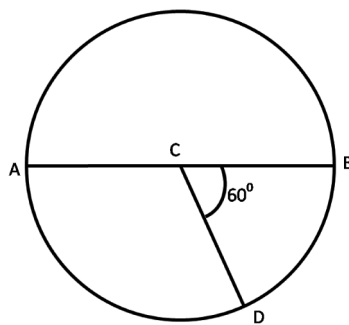
1. Two dice are rolled at random. What is the probability of getting a sum of the numbers on them as 7.
2. Two coins are tossed simultaneously. What is the probability of getting a head and a tail on those coins?
3. There are 4 blue, 5 orange and 6 red balls in a bag. When a ball is drawn at random, what is the probability of getting (i) an orange ball (ii) not an orange ball?
4. There are cards with numbers 1 to 100 in a bag. When a ball is drawn out from the bag, what is the probability that the number on the card will contain 9 in either units or tens place?
5. There are mobile numbers of 20 teachers, 16 lawyers and 14 doctors on a page. If a number is selected at random, then what is the probability of selecting a mobile number of (i) a teacher (ii) a person who is not a teacher.?
6. Two dice are rolled at random. What is the probability of getting sum of the numbers on them (i) more than 20 (ii) less than 20
7. A manufacturer supplied 75 good LED bulbs along with 15 defective bulbs. When a bulb is taken out at random total bulbs, what is the probability of selecting (i) a good bulb (ii) a defective bulb?
8. In a village, 75 people were affected by coronavirus and 2925 were not affected. When the police searched for the affected persons, what is the probability to select a person affected by coronavirus from the village?
9. A box contains 144 ball pens of which 20 are defective. Saniya buys a pen if it is only a good pen. The shopkeeper draws a pen, gives it to her. What is the probability that (i) she would buy the pen (ii) she would not buy the pen?
10. A box contains cards with numbers 11 to 99 on them. If a card is drawn at random, What is the probability of getting a (i) square number (ii) a composite number?

Essay Type Questions (8 Marks)

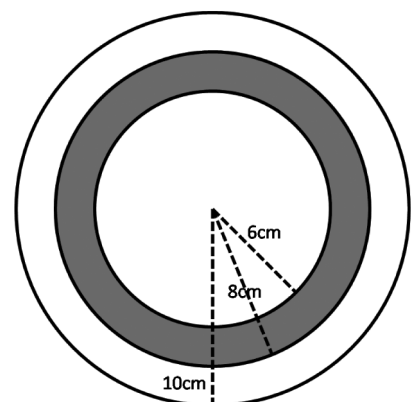
1. Suppose a coin is dropped in a rectangular area with the dimensions $14\text{cm} \times 11\text{cm}$. There is a circle with radius 7cm drawn in it as shown in the figure. What is the probability of dropping the coin in the circle?



2. Two customers wished to visit a shop on any day in a week. What is the probability that they would visit the shop (i) on the same day together (ii) different days?
3. A circle with diameter AB and center C is drawn and another radius CD is also drawn as shown in the figure. A dice is thrown into the circle. What is the probability that the dice will fall in the sector region of ACD ?



4. An equilateral triangle is circumscribed by a circle with radius $7\sqrt{3}\text{ cm}$. If a coin is thrown into the circle, what is the probability that it will fall outside the triangle?
5. A target consists of three concentric circles of radii $6, 8$ and 10cm respectively shown in the figure. A dart is shot on the target. What is the probability that the dart will land on the shaded region?

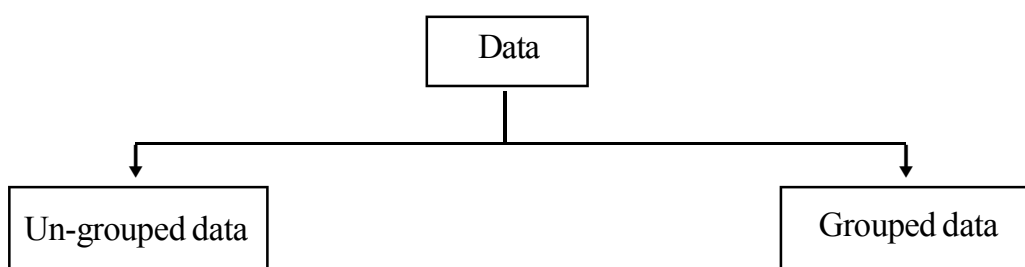


Data

Information which is in the form of numbers and words and helps in taking decisions or drawing conclusions is called 'data'.

The numerical entries in the data are called 'observations'.

Data is in two types.

**Measures of central tendencies**

Usually we collect data and draw certain conclusions based on the nature of data. Understanding its nature, we do certain computations like 'mean', 'mode' and 'median' which are referred as measures of central tendencies.

In this chapter we can discuss central tendencies on both (a) ungrouped data and (b) grouped data.

1. Arithmetic mean (or) Mean or Average

a) Mean of ungrouped data: Sum of the observations by number of observations is called 'mean of ungrouped data'.

If $x_1, x_2, x_3, \dots, x_n$ are the observation of a data, then

$$\text{Mean } (\bar{x}) = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n} \text{ or}$$

$$\text{Mean } (\bar{x}) = \frac{\sum x_i}{n}$$

Here, $\sum x_i = x_1 + x_2 + x_3 + \dots + x_n$

$n =$ number of observations.

b) Mean of grouped data:

i) Direct method

$$\text{Mean } (\bar{x}) = \frac{\sum f_i x_i}{\sum f_i}$$

ii) Assumed mean method or Deviation method

$$\text{Mean } (\bar{x}) = a + \frac{\sum f_i d_i}{\sum f_i}$$

$a =$ Assumed mean

$f_i = i^{\text{th}}$ class frequency

$d_i = x_i - a$ ($d_i =$ deviation)

$\sum f_i =$ sum of frequencies

2. Mode

a) Mode of ungrouped data: The most frequently occurring value for a set of observations is called the 'mode'.

- A data may or may not contains mode.
- A data can have more than one mode.

b) Mode of grouped data:

$$\text{Mode} = l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right] \times h$$

Here, l = lower boundary of modal class

h = height of modal class

f_1 = frequency of modal class

f_0 = frequency of preceding class of modal class

f_2 = frequency of succeeding class of modal class

3. Median

a) Median of ungrouped data:

The middle most observation of the given data that is written in ascending or descending order is called 'median'.

Note:

- If the number observation in the given data is an odd number, then first write the data in ascending or descending order and the middle most observation is called 'median'.
- If the number observations in the given data is an even number, then first write the data in ascending or descending order and the take the average of the middle most two observations as 'median'.

b) Median of grouped data:

$$\text{Median} = l + \left[\frac{\frac{n}{2} - cf}{f} \right] \times h$$

Here, l = lower boundary of median class

n = sum of all frequencies

cf = cumulative frequency of the preceding class of median class

f = frequency of median class

h = height of median class

Less than cumulative frequency curve

To draw the less than type cumulative frequency curve, we can take upper boundaries on X-axis and less than cumulative frequency on Y-axis.

Practice problems

- Median of the data $\frac{1}{7}, \frac{1}{10}, \frac{1}{2}, \frac{1}{5}, \frac{1}{15}$ []
A) $\frac{1}{10}$ B) $\frac{1}{2}$ C) $\frac{1}{7}$ D) $\frac{1}{15}$
- $\bar{x} = a + \frac{\sum f_i d_i}{\sum f}$ formula $d_i =$ []
A) $a + x_i$ B) $x_i - a$ C) $x_i + \bar{x}$ D) $a - x_i$
- For what value of 'k', the median of the data 4, 6, k, 9, 10, 19 is 7.5 which are given in ascending order []
A) 7 B) 6 C) 9 D) 8
- Mode of first 10 natural numbers []
A) 1 B) 10 C) 5.5 D) Not define
- The mean of data is 12. If each observation is multiplied by 4 and then 2 added to each result, find the mean of the new observations so obtained []
A) 28 B) 50 C) 18 D) 12
- The most stable measures of central tendencies is []
A) range B) median C) mean D) mode
- Mean of first four multiples of 5 is []
A) 15.5 B) 12.5 C) 10.5 D) 20.5
- For drawing the less than type cumulative frequency, we can use following values []
A) More than cumulative frequency on X-axis and upper boundaries on Y-axis.
B) More than cumulative frequency on X-axis and lower boundaries on Y-axis.
C) Lower boundaries on X-axis and more than cumulative frequency on Y-axis.
D) Upper boundaries on X-axis and less than cumulative frequency on Y-axis.

9. Mean of $a - 2d, a - d, a, a + d$ and $a + 2d$ is []

- A) $5a$ B) $3a$ C) a D) $5d$

10. Mid values are used to find which of the following central tendencies []

- A) mean B) mode C) median D) range

Very Short Answer Questions

1. Write the formula to find the mean of the grouped data using assumed mean method and explain each term in it.
2. Write the formula to find the mode of the grouped data and explain each term in it.
3. Write the formula to find the median of the grouped data and explain each term in it.
4. Find the mean of the prime numbers below 20.
5. “Median of first 10 composite numbers is 13.” Do you agree with this statement? Give the reason.
6. Find the mode of 3, 9, 4, 5, 3, 7, 2, 8, 3.
7. If mean of 9, 11, 13, k , 18, 19, is ‘ k ’, then find the ‘ k ’.
8. If median of $\frac{x}{3}, \frac{x}{2}, \frac{x}{5}, \frac{x}{9}, \frac{x}{4}$ ($x > 0$) is 5, then find the value of ‘ x ’.

Short Answer Questions

1. In the following table marks 20 students is given. Find the mean of following data.

Marks	1	2	3	4	5
Number of students	3	5	7	4	1

2. Find the median of $\frac{1}{4}, \frac{2}{5}, \frac{4}{5}, \frac{1}{2}, \frac{3}{4}$.

3. Find the mean of following grouped data using direct method.

C.I	0-10	10-20	20-30	30-40	40-50	50-60
f	1	2	4	6	3	1

4. The following distribution shows the marks scored by 140 students in an examination. Calculate the mode of the distribution.

Marks	0-10	10-20	20-30	30-40	40-50
No. of students	20	24	40	36	20

5. Find the median of the following grouped data.

Height (cm)	less than 120	less than 140	less than 160	less than 180	less than 200
No. of students	12	26	34	40	50

6. The following frequency distribution shows the number of runs scored by some batsmen of India in one day cricket matches. Find the mode for the given data.

Runs scored	2000-4000	4000-6000	6000-8000	8000-10,000	10000-12000
No. of batsmen	9	8	10	2	1

Essay Type Questions

1. If mean of the following grouped data is 25, then find the value of 'P'.

Class Interval	0-10	10-20	20-30	30-40	40-50
Frequency	4	6	10	6	P

2. The following table shows the age distribution of case of COVID patients joined in a hospital. Find mode for the above grouped data.

C.I.	5-14	15-24	25-34	35-44	45-54	55-64
f	6	11	21	23	14	5

3. If median of the following data is 240, then find the value of 'f'.

C.I	0-100	100-200	200-300	300-400	400-500	500-600	600-700
f	15	17	f	12	9	5	2

4. Draw the less than type cumulative frequency curve for the following grouped data.

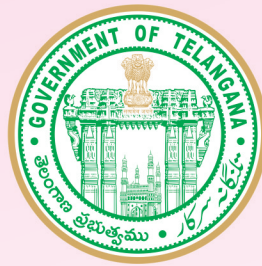
C.I	0-10	10-20	20-30	30-40	40-50	50-60
f	1	3	5	12	6	3

5. On the Annual Day of a school, age-wise participation of students is given in the following frequency distribution table. Draw the less than type cumulative frequency curve and find the median of the data from the graph.

Height (cm)	less than 6	less than 8	less than 10	less than 12	less than 14	less than 16	less than 18
No. of students	2	6	12	22	42	67	76

6. The following distribution gives the daily income of 50 workers of a factory. Draw the distribution to a less than type cumulative frequency curve. Hence, obtain the median of daily income.

Daily income	200-250	250-300	300-350	350-400	400-450	450-500
No. of workers	10	5	11	8	6	10



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